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Autonomous robotics: Action, Perception, and Cognition

June 5, 2025

Exercise 7 Timing: Neural oscilator

Please upload solutions on the web page before midnight on **June 16, 2025** (Monday).

Read the review article "Timing, Clocks, and Dynamical Systems" by Schöner (*Brain and Cognition* **48**:31-51(2002)), the paper is available as a pdf download on the course web page). You can safely drop sections covering "stochastic timer models" (2.1, 3.1, and parts of the Discussion section). Not all details are comprehensible to you, so navigate around that, by deciding where to dig in and where to tolerate limited understanding. The goal is to understand the dynamic timer models.

1. The "Amari oscillator" of Equations (6) and (7) can be understood by identifying the fixed points to which the system moves within each quadrant. To understand that, approximate the sigmoid function as a step function. For each quadrant (1) $u > 0$, $v > 0$, (2) $u > 0$, $v < 0$, (3) $u < 0$, $v < 0$, (4) $u < 0$, $v > 0$, the equation is thus linear with different constant offsets. Compute the fixed point (solution of $\dot{u} = \dot{v} = 0$) in each quadrant. For the right choices of the coupling parameters, the fixed point for each quadrant lies in the neighboring quadrant.The vector-field in each quadrant points toward the fixed point, which drives all initial values in that quadrant in the direction of the neighboring quadrant. Make a sketch of that vector-field and argue intuitively why a limit cycle may emerge. [An additional resource is the 1977 Amari paper available on the course web page, Section 8.1 there.]

2. Simulation exercise: An interactive web-based simulator is available at
   `https://dynamicfieldtheory.org`
   specially here:
   `https://dynamicfieldtheory.org/examples/two_neurons.html`

   The sliders on the right give you control over the resting levels and inputs to two neurons, u_1, and u_2. With these you can build the equations (6) and (7) of the Schöner 2002 paper. One neuron plays the role of the exitatory, the other of the inhibitory neuron. You task is to find the oscillatory regime of the model. You can use the information in the appendix of the paper Schoner (2002) to find the right parameter values. Try varying the duration and the amplitude of the oscillations by changing parameters. Document your simulation results for one of those variations

   (a) stating the goal of the simulations

(b) specifying the model tha was simulated

(c) describing what you observed/looked at to decide whether the goal was reached

(d) describing the parameter values you adjusted and how your observations depended on the change of parameter values