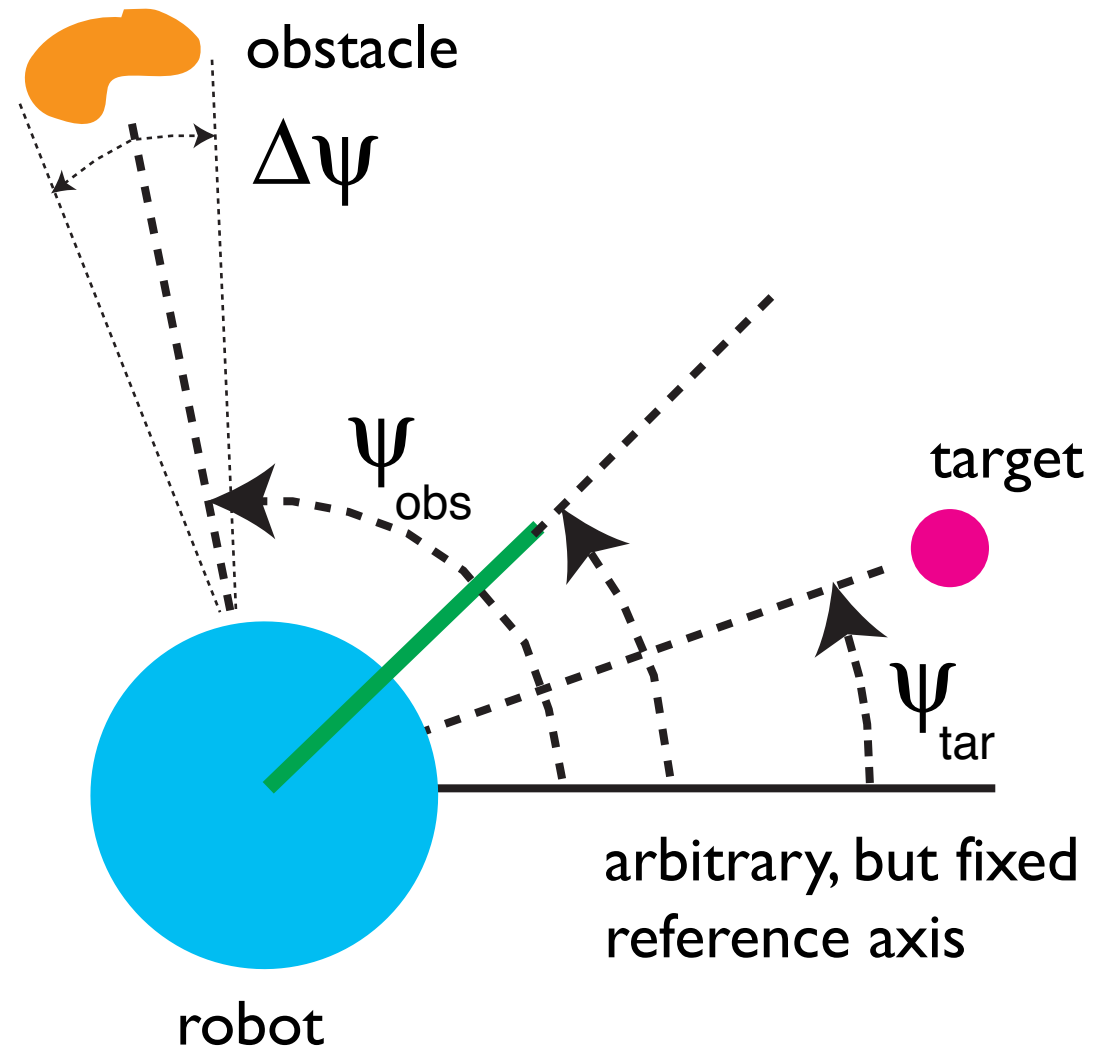


# Attractor dynamics approach to vehicle movement generation: Part 2: “sub-symbolic” approach

Gregor Schöner  
Institute for Neural Computation, RUB

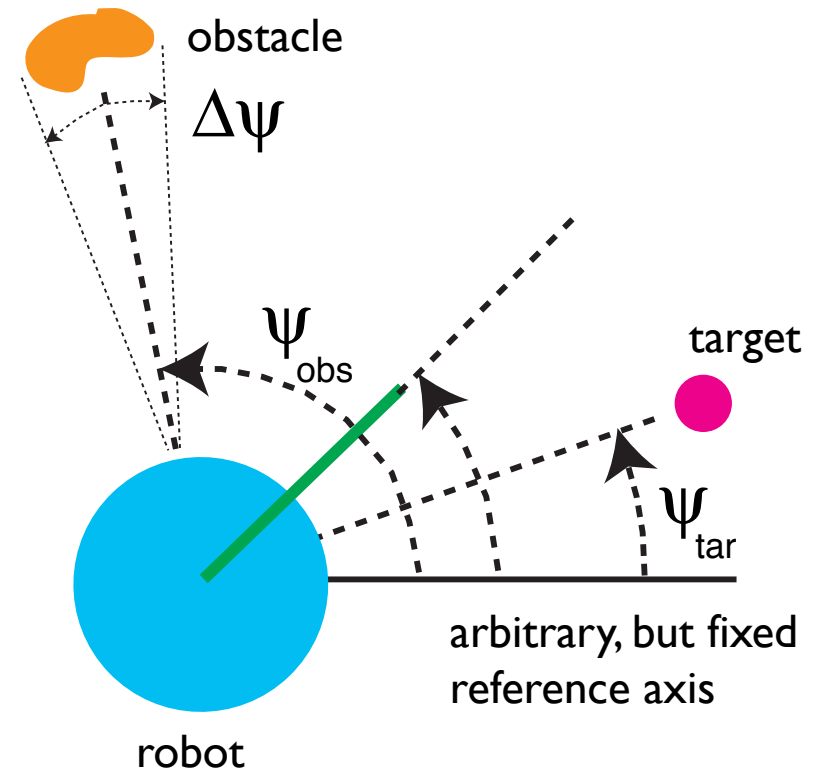
# Behavioral dynamics

■ constraints:  
obstacle avoidance  
and target  
acquisition



# Behavioral dynamics

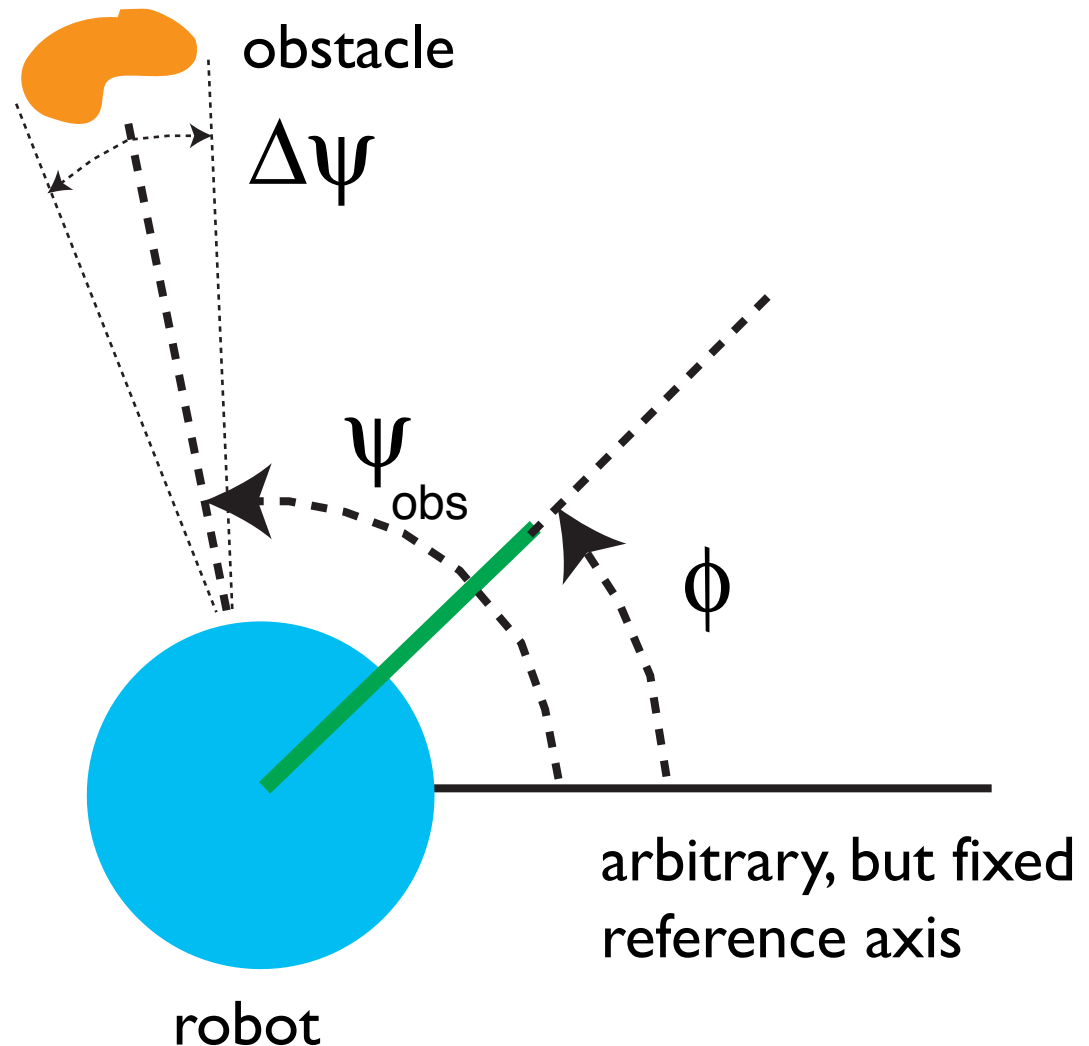
- so far, we had a “symbolic” approach to behavioral dynamics: the “obstacles” and “targets” were objects, that have identity, are preserved over time...and are represented by contributions to the behavioral dynamics



# “symbolic” approach

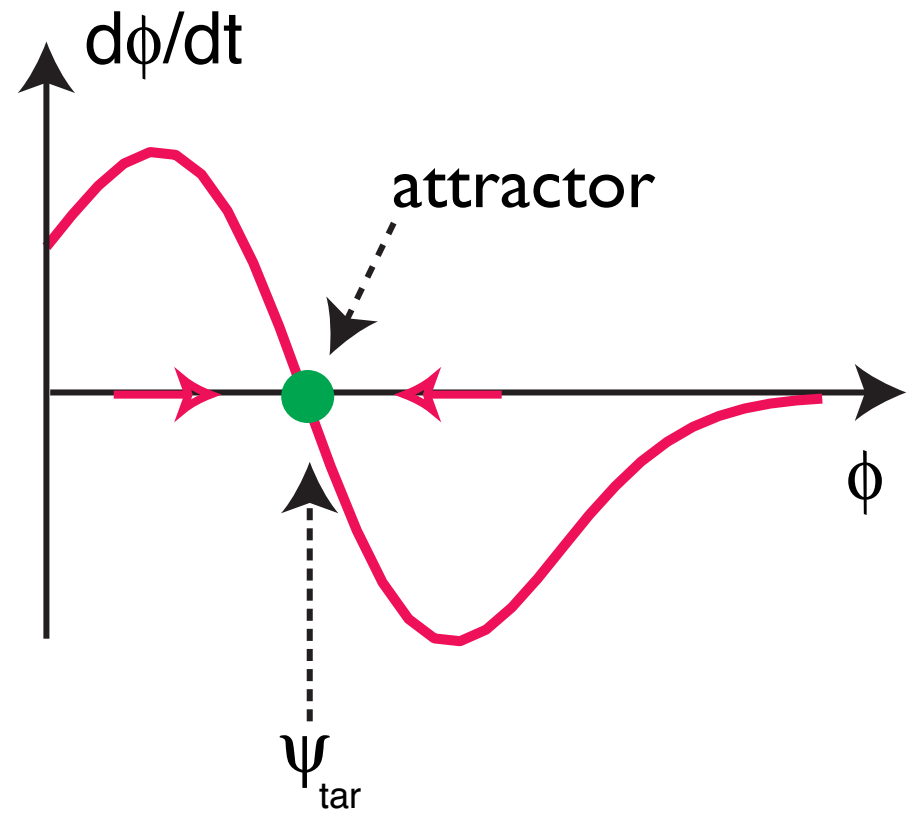
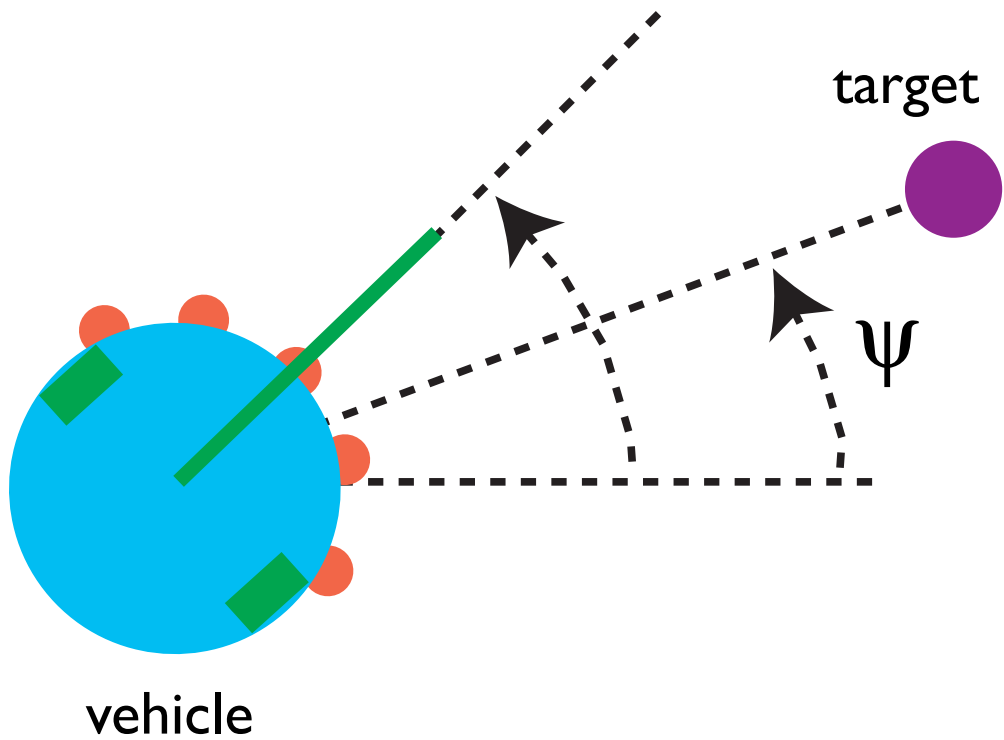
- requires high-level knowledge about objects in the world (“obstacles”, “targets”, etc) and perceptual systems that extract parameters about these...

- is that necessary?



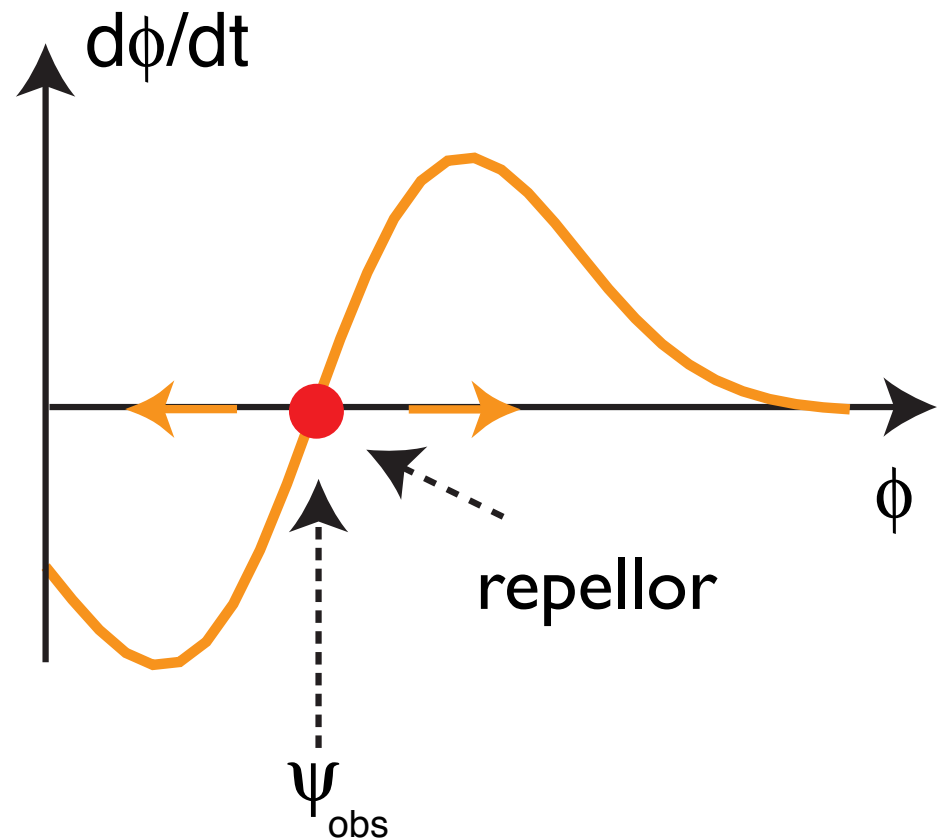
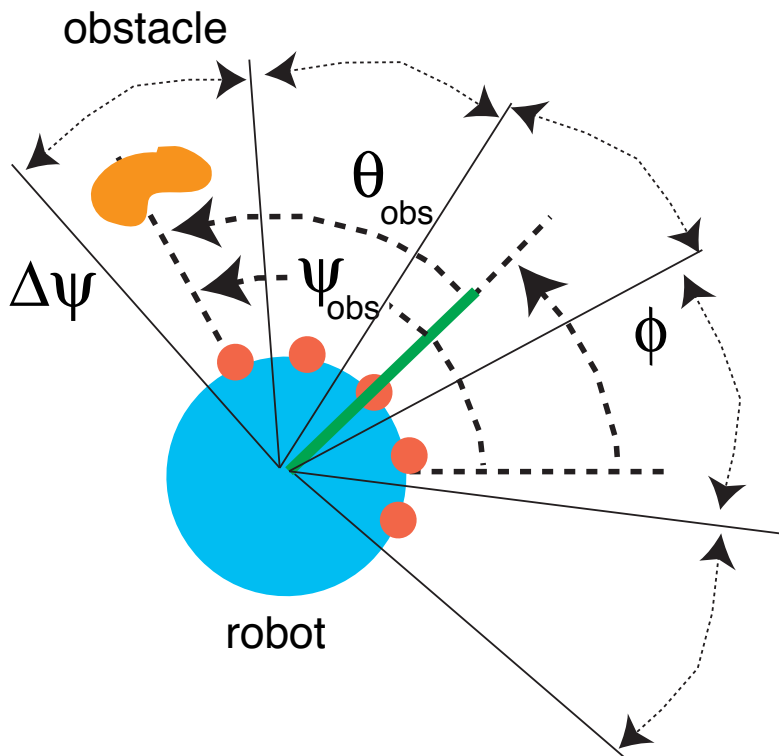
# Targets....

- are segmented... in the foreground
- => neural fields to perform this segmentation from low-level sensory information: Dynamic Field Theory ...



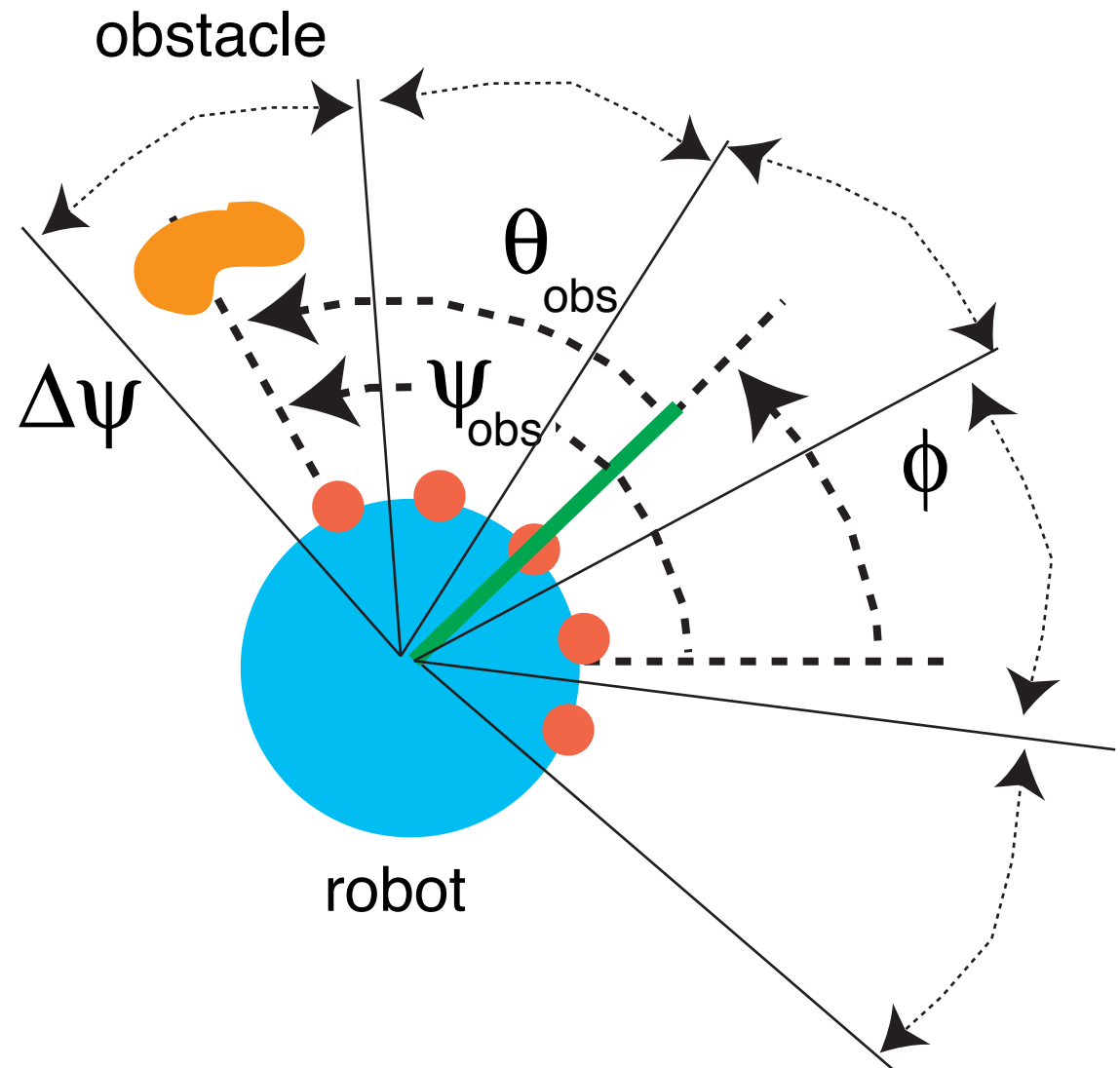
# Obstacles ...

- obstacles need not be segmented ... does not matter if obstacles are one or multiple objects...
- avoidance is about free space...



# “sub-symbolic” approach

- use low-level sensory information directly, without first detecting, segmenting, and estimating objects



# Obstacle avoidance: sub-symbolic

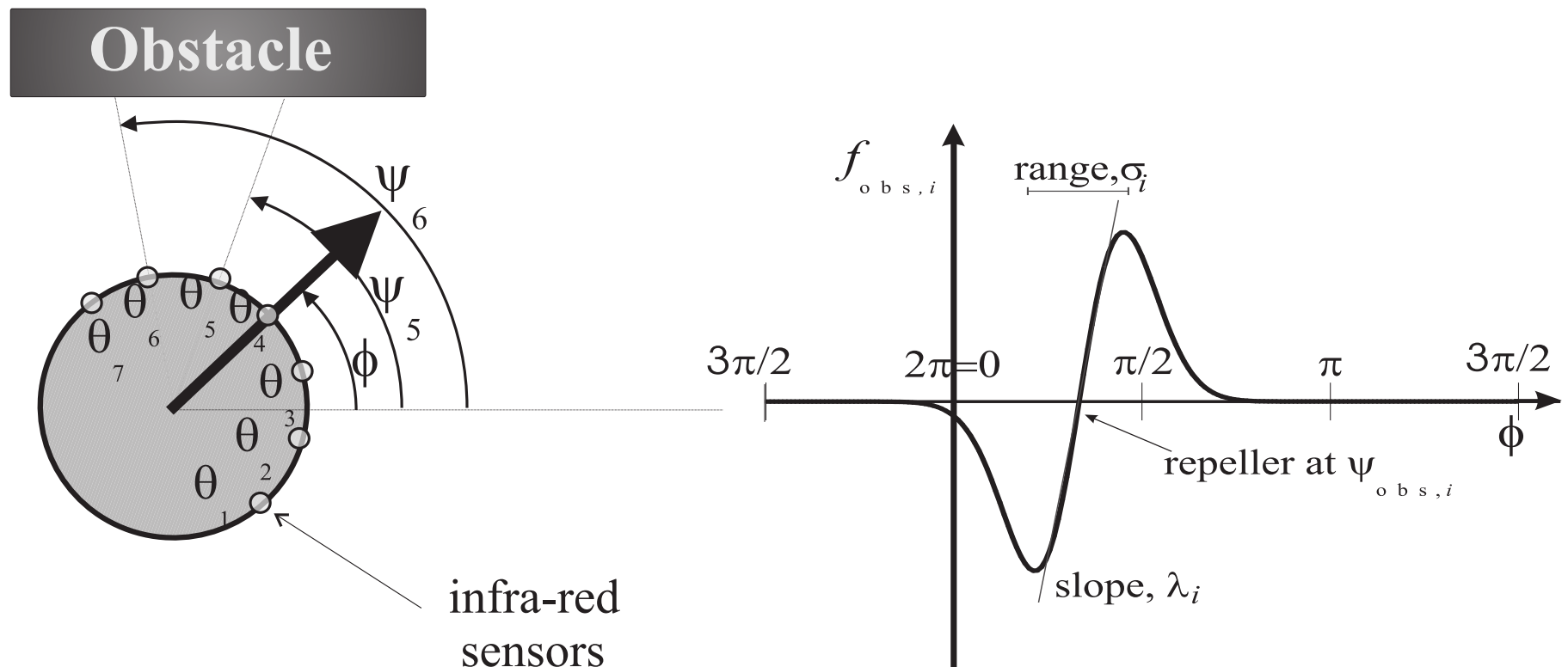
Figures and results from:

Estela Bicho: Dynamic Approach to Behavior-Based Robotics  
Design, Specification, Analysis, Simulation and Implementation  
Doctoral disseration, Univ. Minho, Guimarães, Portugal, 1999]

<https://core.ac.uk/download/pdf/55601836.pdf>

# Obstacle avoidance: sub-symbolic

- each sensor mounted at fixed angle  $\theta$
- that points in direction  $\psi = \phi + \theta$  in the world
- erect a repeller at that angle



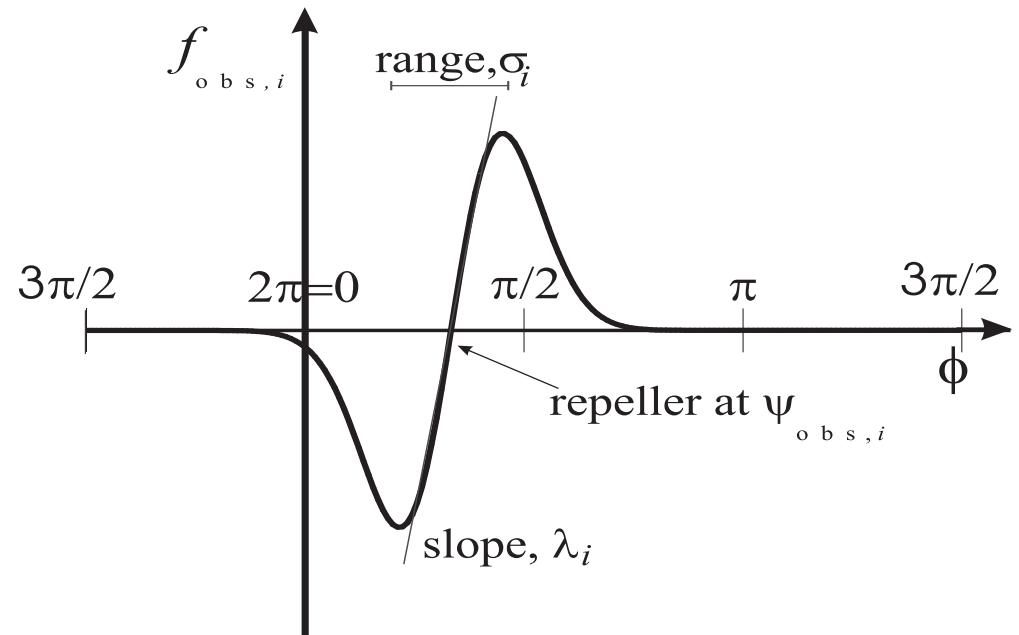
[Bicho, 1999]

# Obstacle avoidance: sub-symbolic

$$f_{\text{obs},i}(\phi) = \lambda_i(\phi - \psi_i) \exp \left[ -\frac{(\phi - \psi_i)^2}{2\sigma_i^2} \right] \quad i = 1, 2, \dots, 7$$

■ Note: only  $\Phi - \psi = -\theta$  shows up, which is constant!

■  $\Rightarrow$  force-let does not depend on  $\Phi$  !

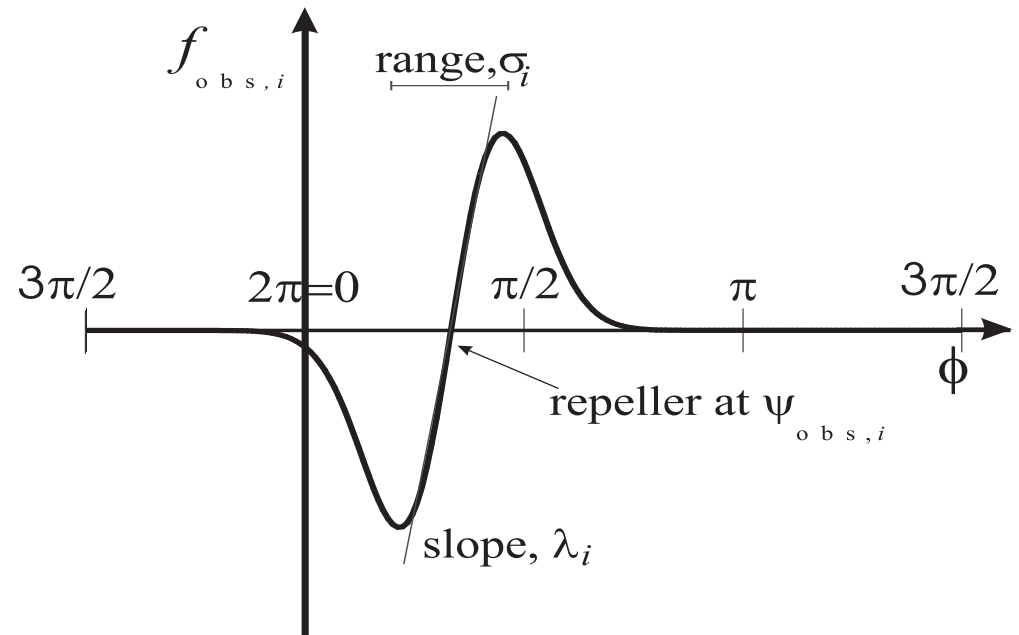


# Obstacle avoidance: sub-symbolic

$$f_{\text{obs},i}(\phi) = \lambda_i(\phi - \psi_i) \exp \left[ -\frac{(\phi - \psi_i)^2}{2\sigma_i^2} \right] \quad i = 1, 2, \dots, 7$$

$$\lambda_i = \beta_1 \cdot \exp \left[ -\frac{d_i}{\beta_2} \right]$$

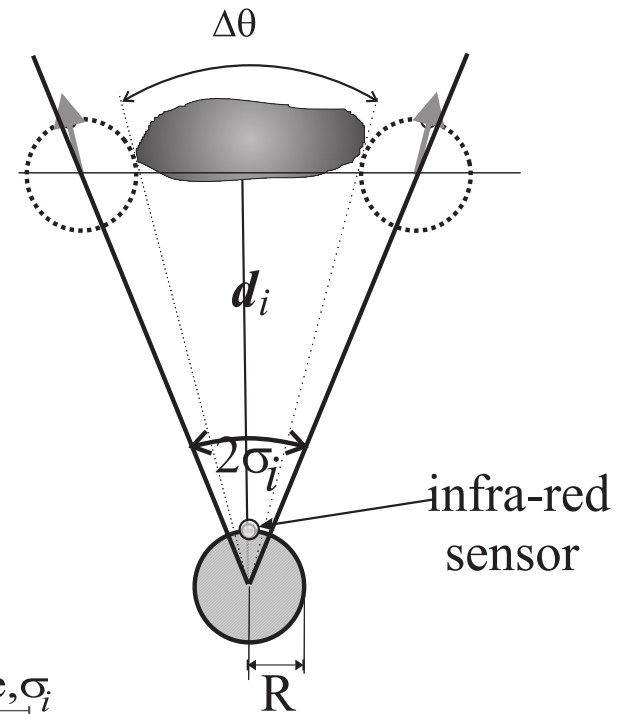
- Repulsion strength decreases with distance,  $d_i$
- $\Rightarrow$  only close obstacles matter



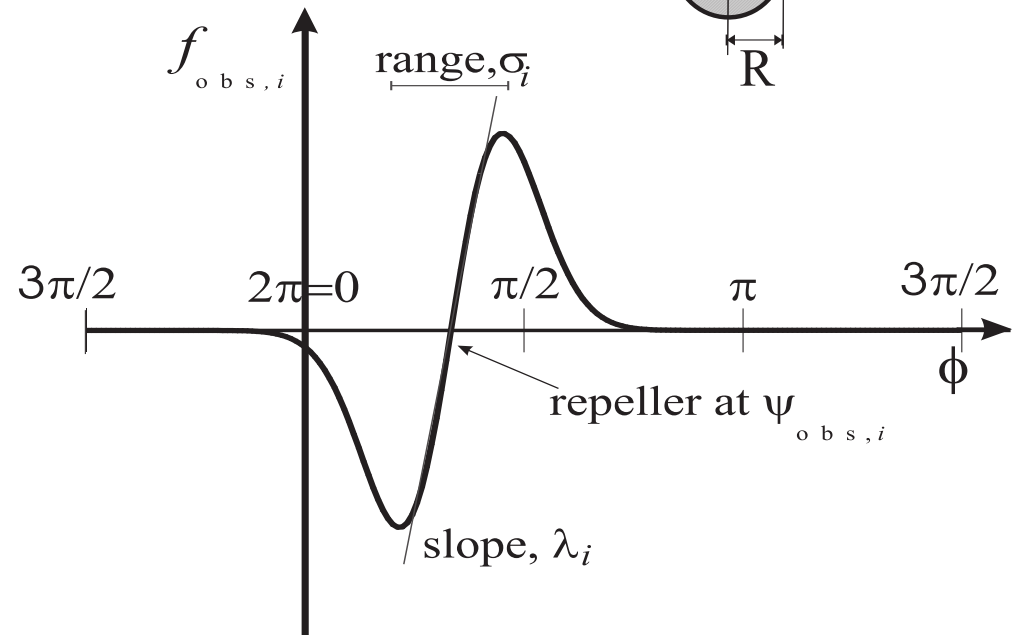
# Obstacle avoidance: sub-symbolic

$$f_{\text{obs},i}(\phi) = \lambda_i(\phi - \psi_i) \exp \left[ -\frac{(\phi - \psi_i)^2}{2\sigma_i^2} \right]$$

$$\sigma_i = \arctan \left[ \tan \left( \frac{\Delta\theta}{2} \right) + \frac{R_{\text{robot}}}{R_{\text{robot}} + d_i} \right].$$

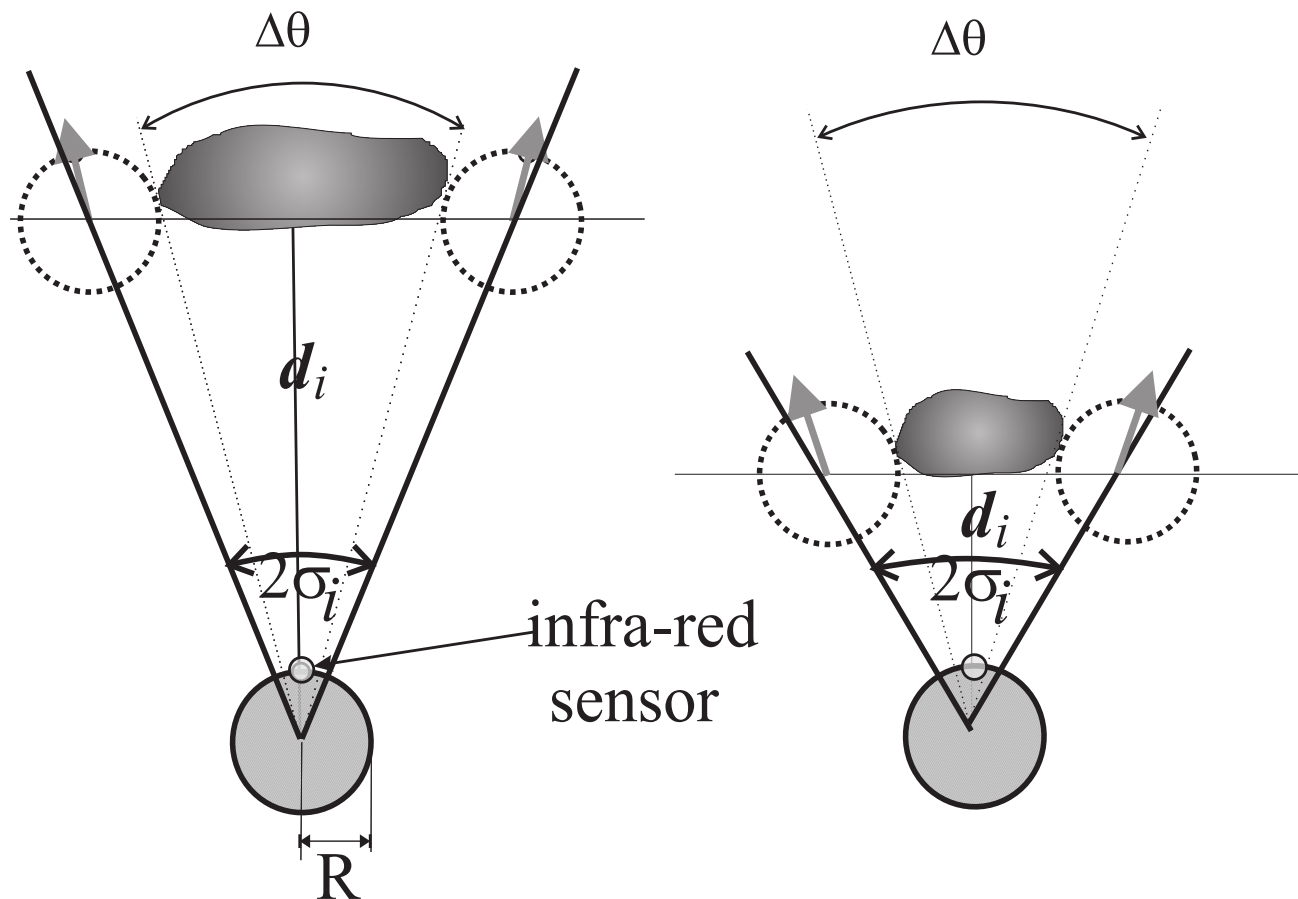


- angular range depends on sensor cone  $\Delta\theta$  and size over distance



# Obstacle avoidance: sub-symbolic

■  $\Rightarrow$  as a result, range becomes wider as obstacle moves closer

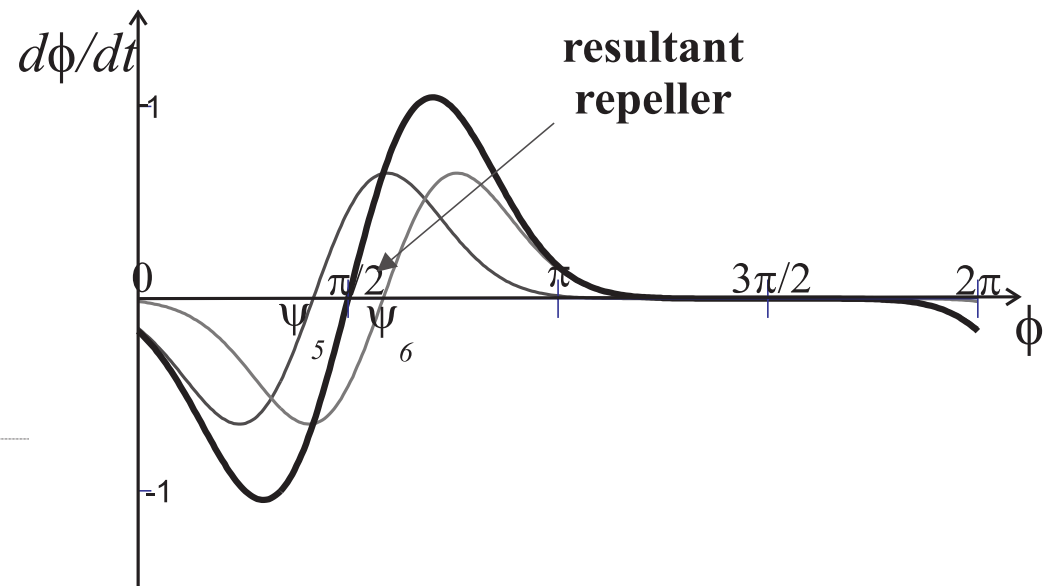
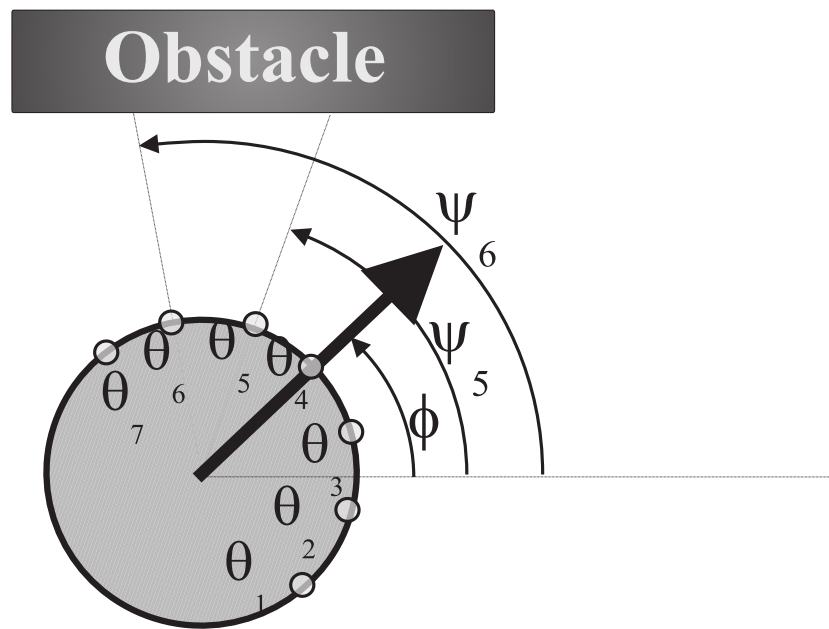


[Bicho, 1999]

# Obstacle avoidance: sub-symbolic

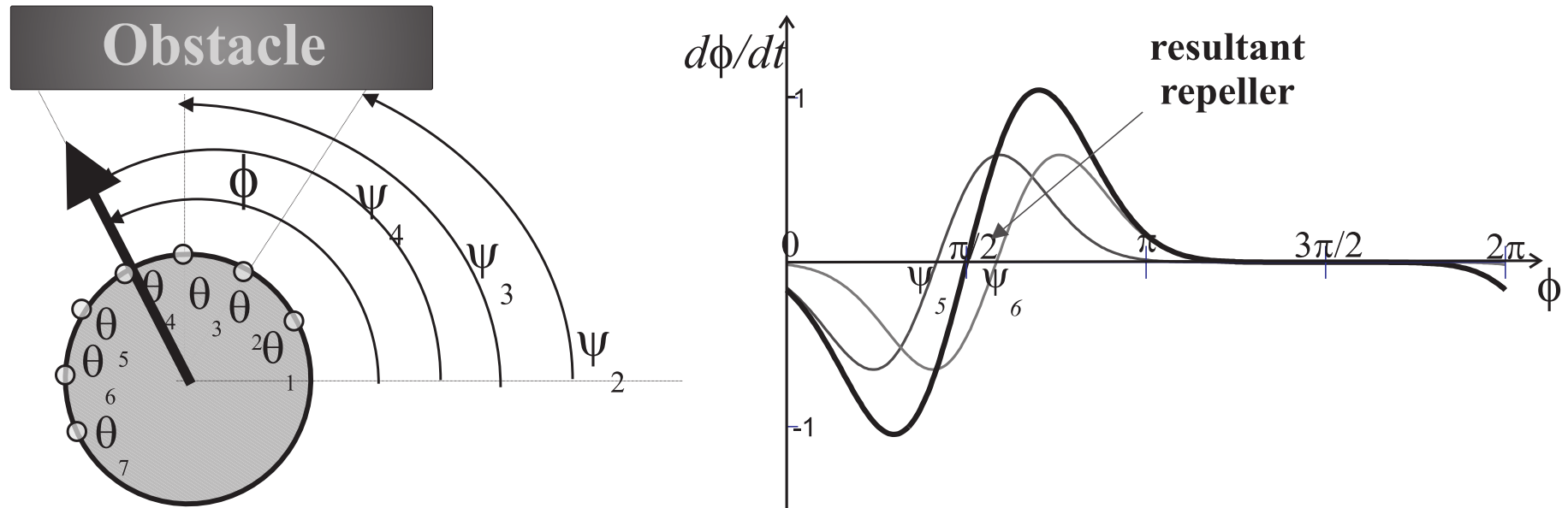
- summing contributions from all sensors

$$\frac{d\phi}{dt} = f_{\text{obs}}(\phi) = \sum_{i=1}^7 f_{\text{obs},i}(\phi)$$



# Obstacle avoidance: sub-symbolic

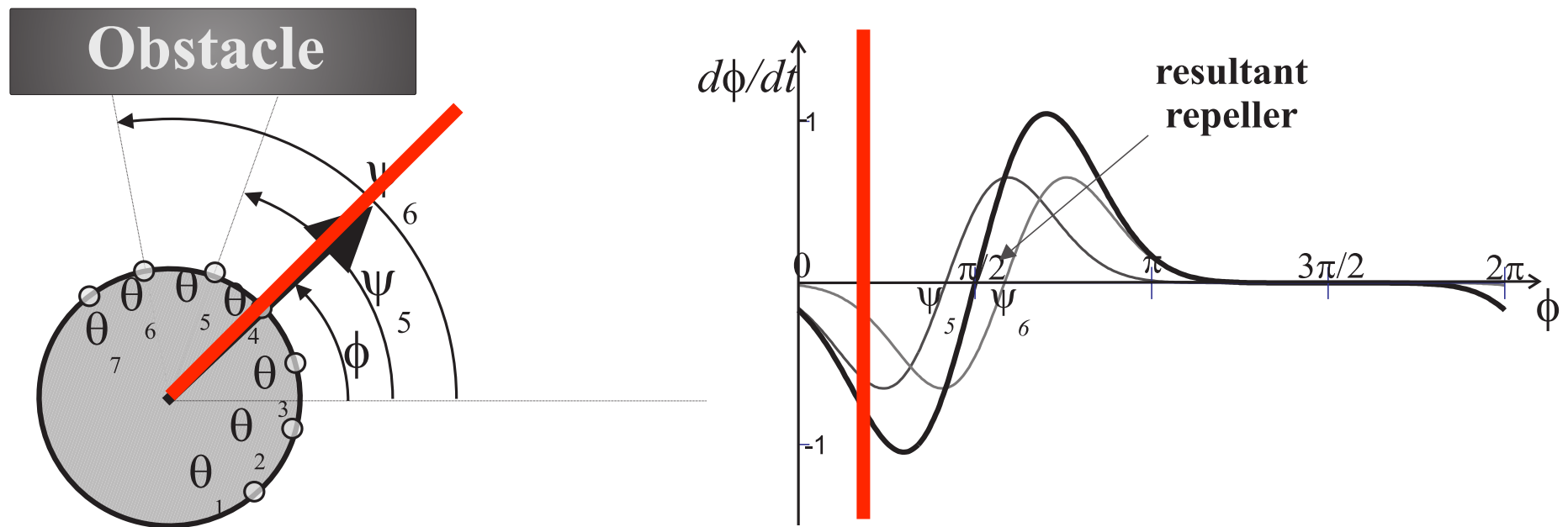
- but why does it work?
- shouldn't there be a problem when heading changes (e.g. from the dynamics itself)?



[Bicho, 1999]

# Obstacle avoidance: sub-symbolic

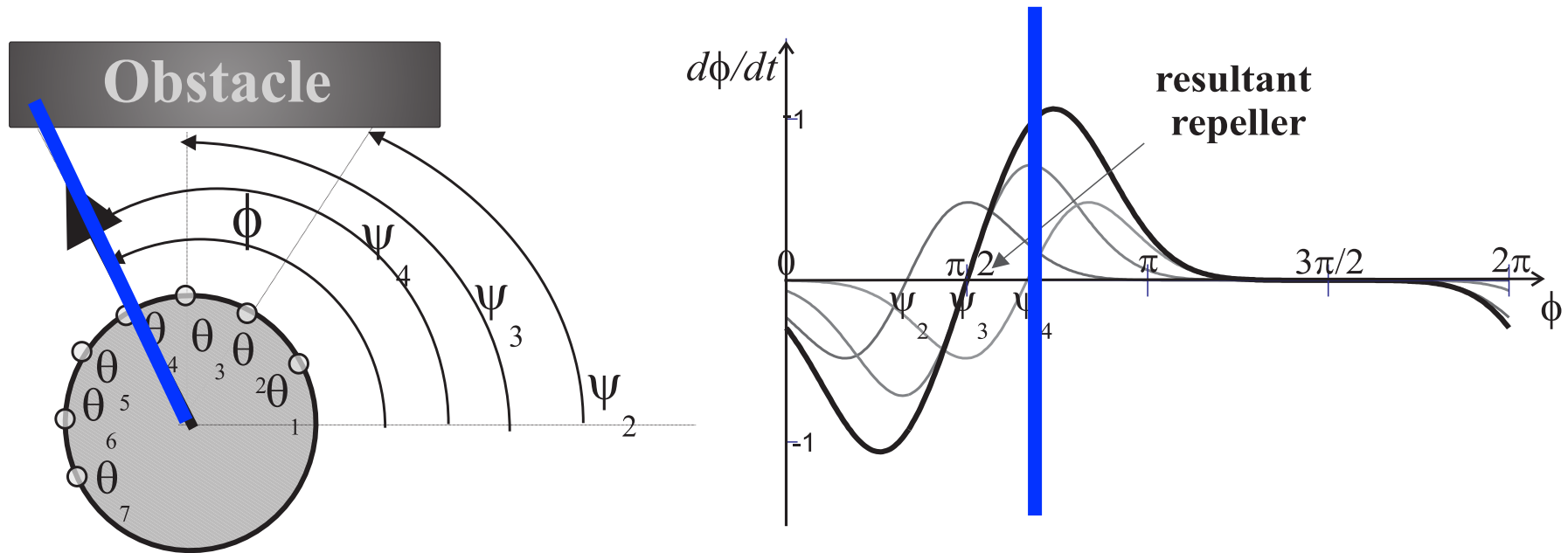
- but why does it work?
- shouldn't there be a problem when heading changes (e.g. from the dynamics itself)?



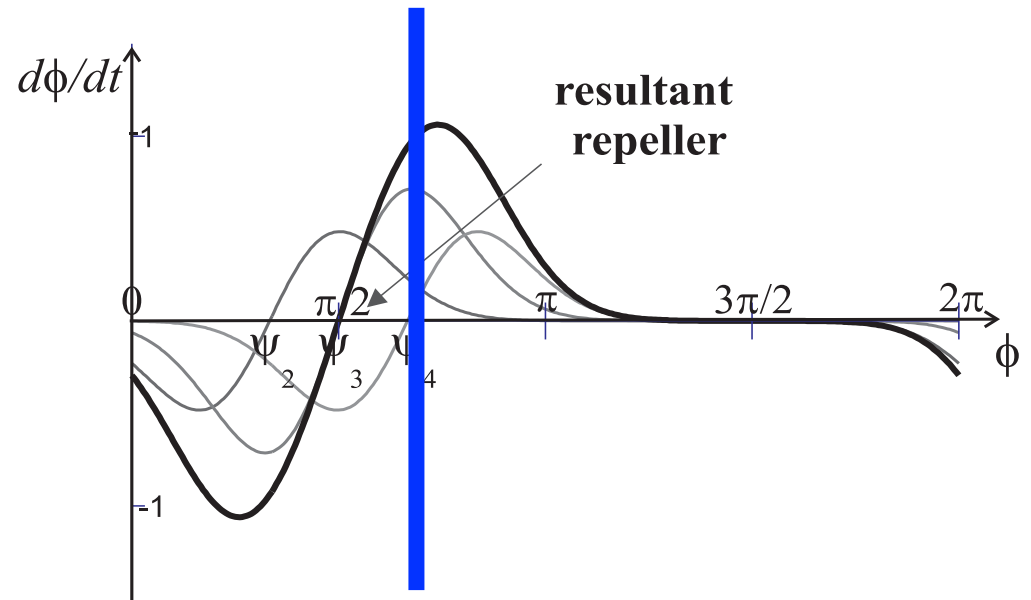
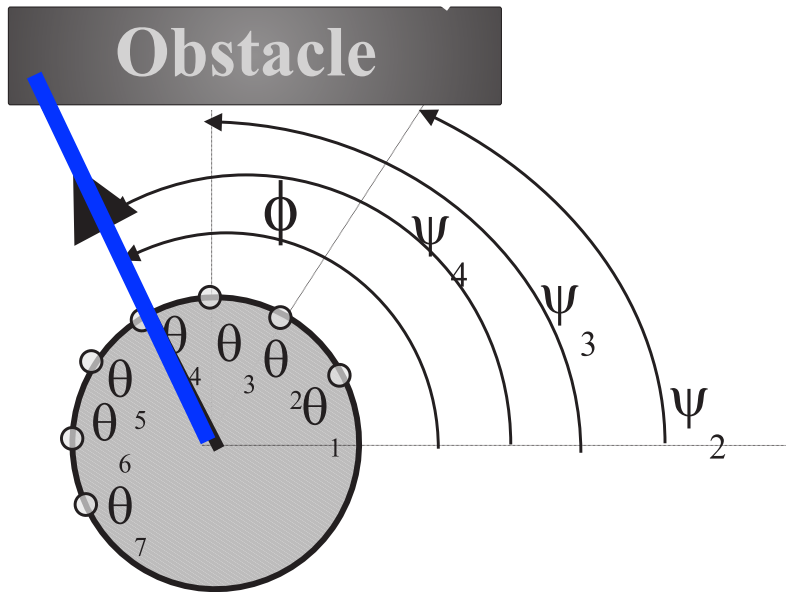
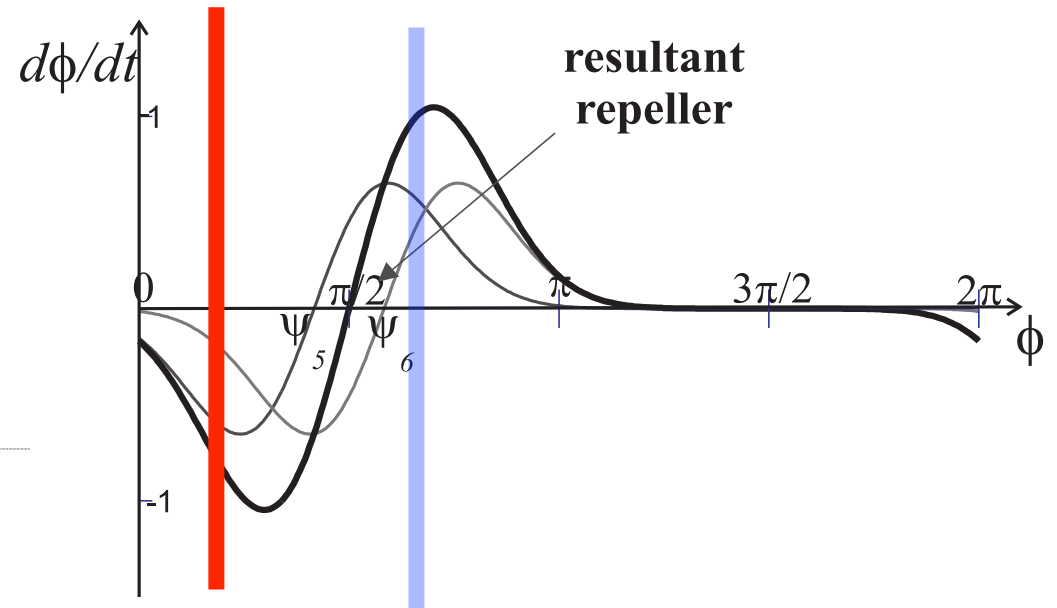
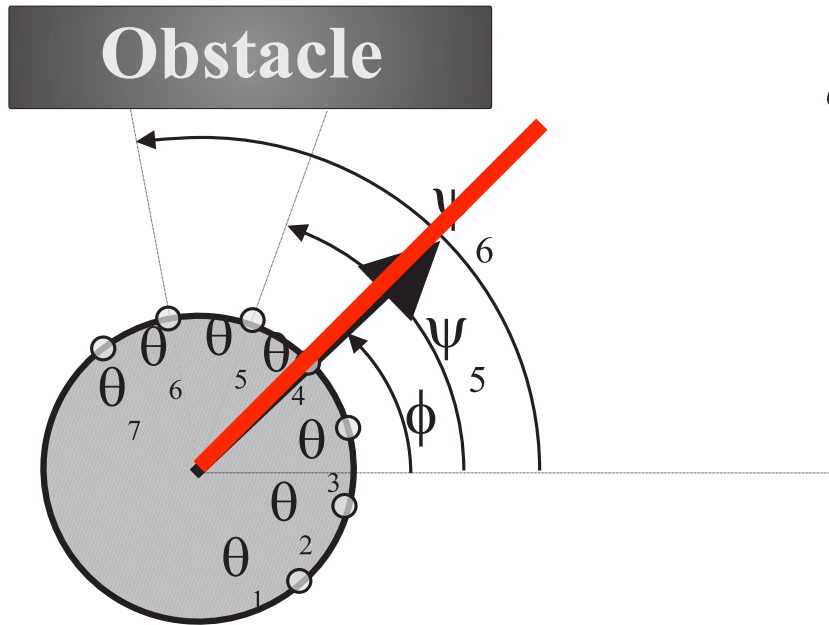
[Bicho, 1999]

# Obstacle avoidance: sub-symbolic

- but why does it work?
- shouldn't there be a problem when heading changes (e.g. from the dynamics itself)?



[Bicho, 1999]



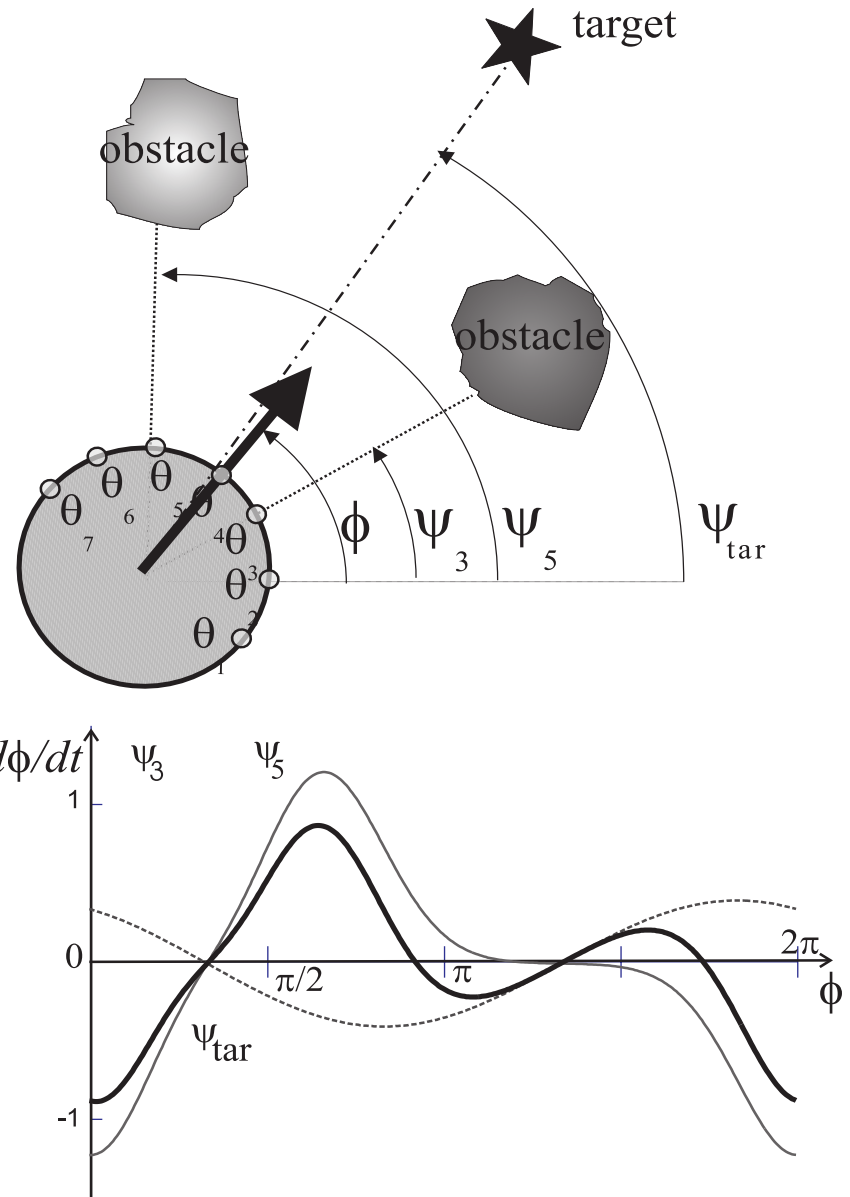
[Bicho, 1999]

■ => dynamics invariant!

# Behavioral Dynamics

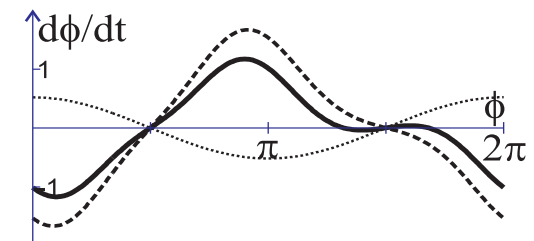
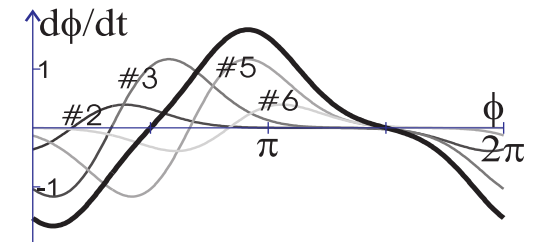
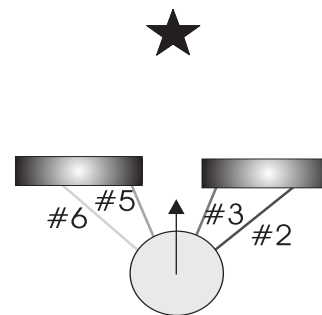
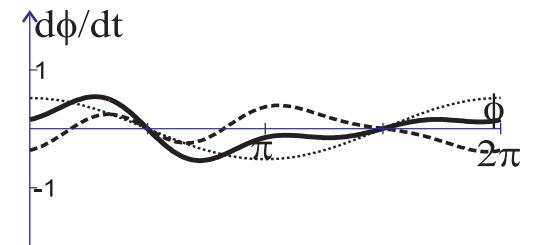
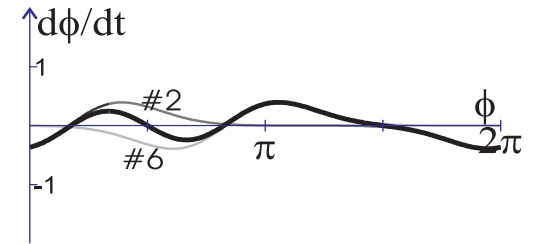
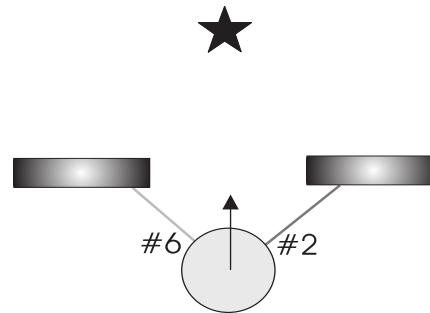
- integrating the two behaviors

$$\frac{d\phi}{dt} = f_{\text{obs}}(\phi) + f_{\text{tar}}(\phi)$$



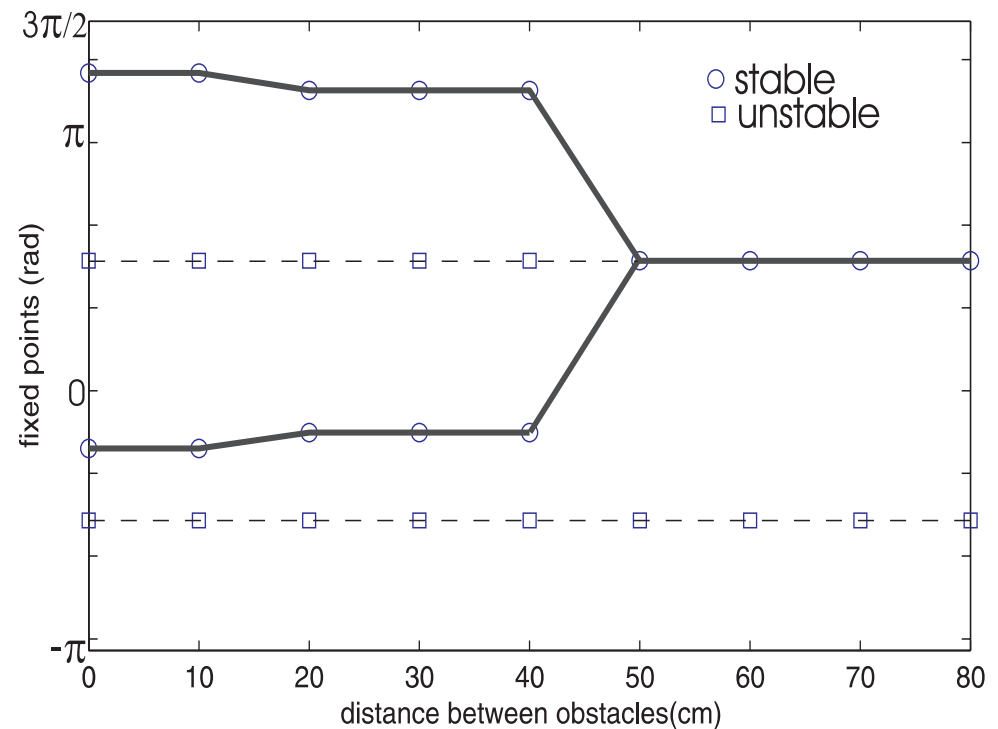
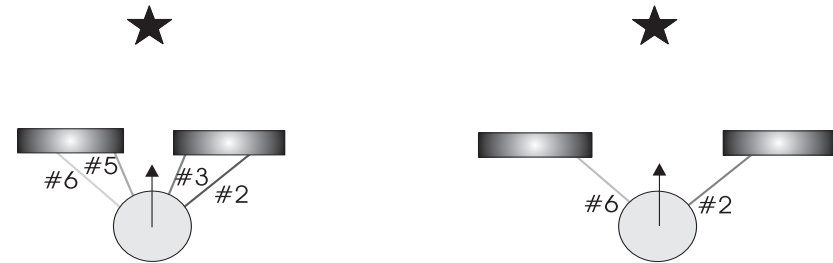
# Bifurcations

■ bifurcation as a function of the size of the opening between obstacles

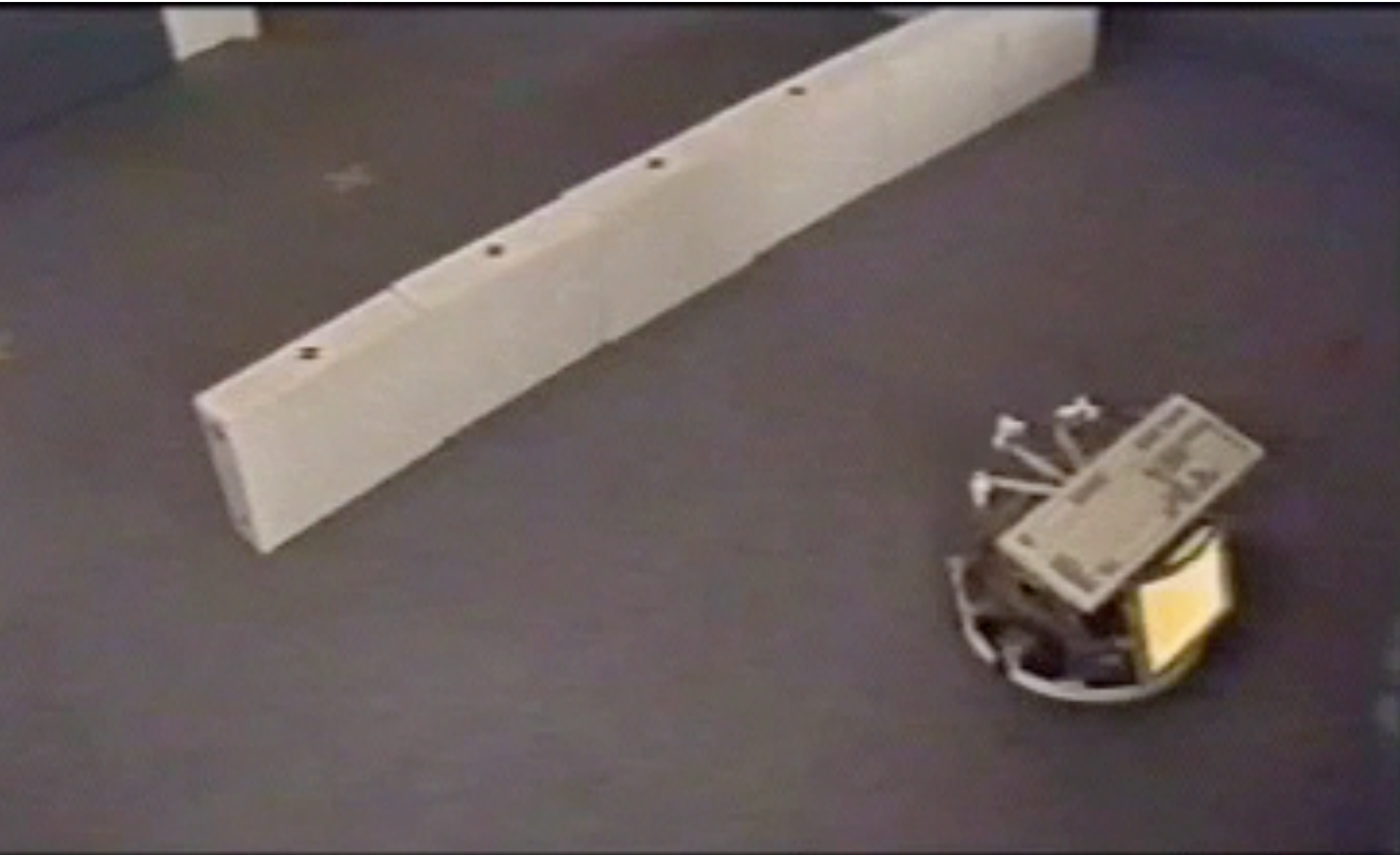


# Bifurcations

- bifurcation as a function of the size of the opening between obstacles
- => tune distance dependence of repulsion so that bifurcation occurs at the right opening

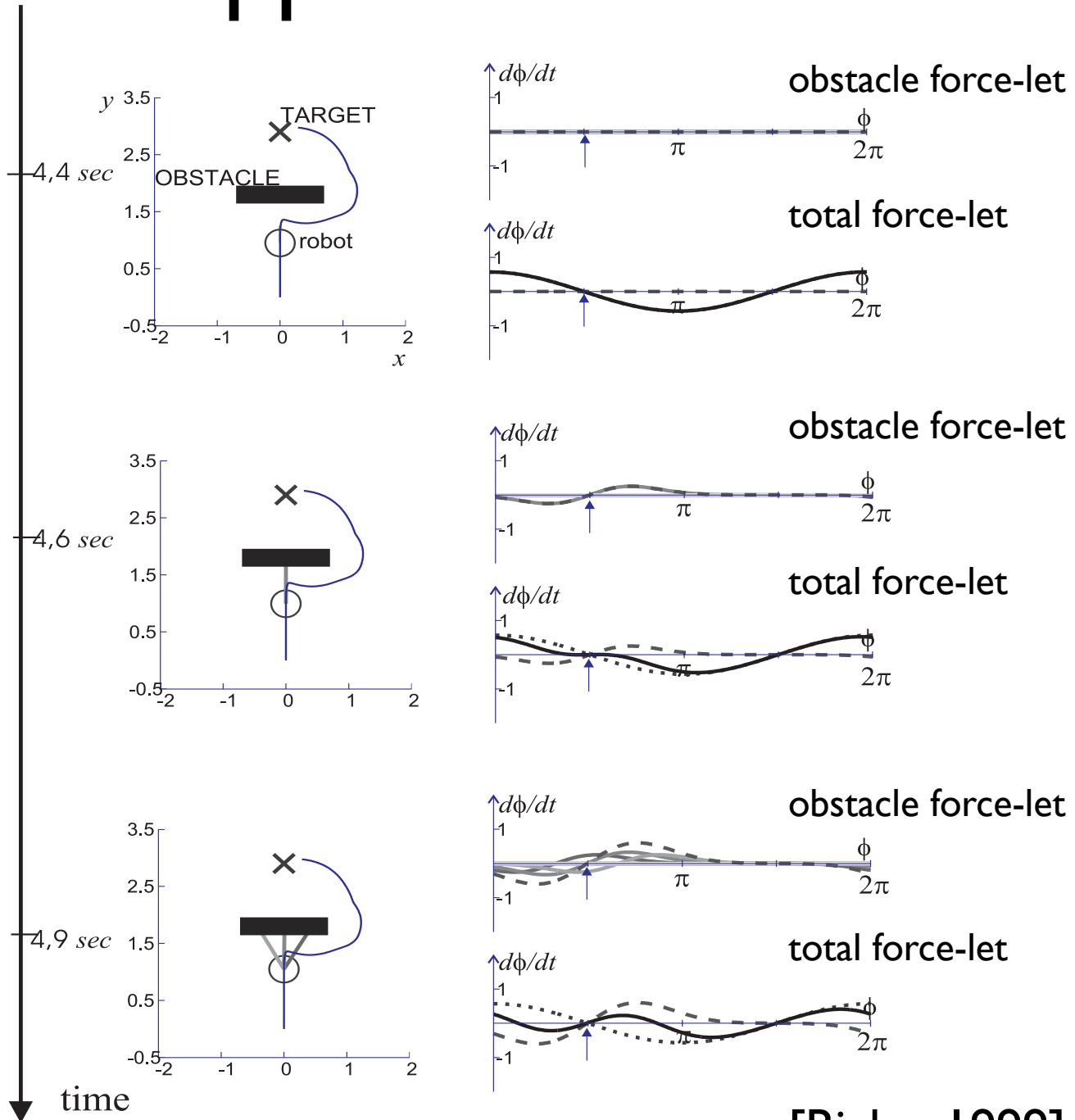


# Bifurcations



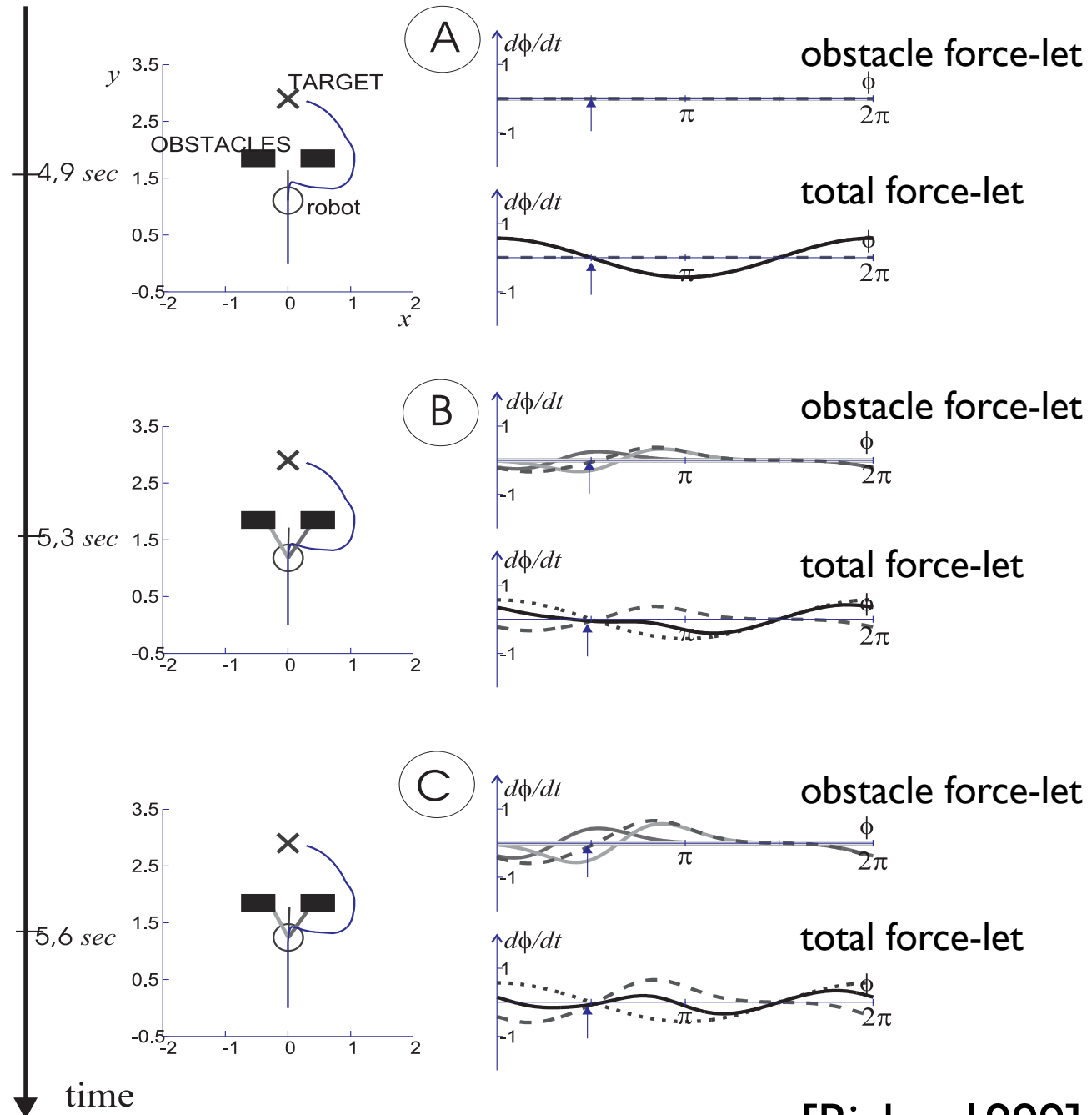
# Bifurcation on approach to wall

- initially attractor dominates: weak repulsion
- bifurcation
- then obstacles dominate: strong repulsion and total repulsion



# Bifurcation on approach to wall

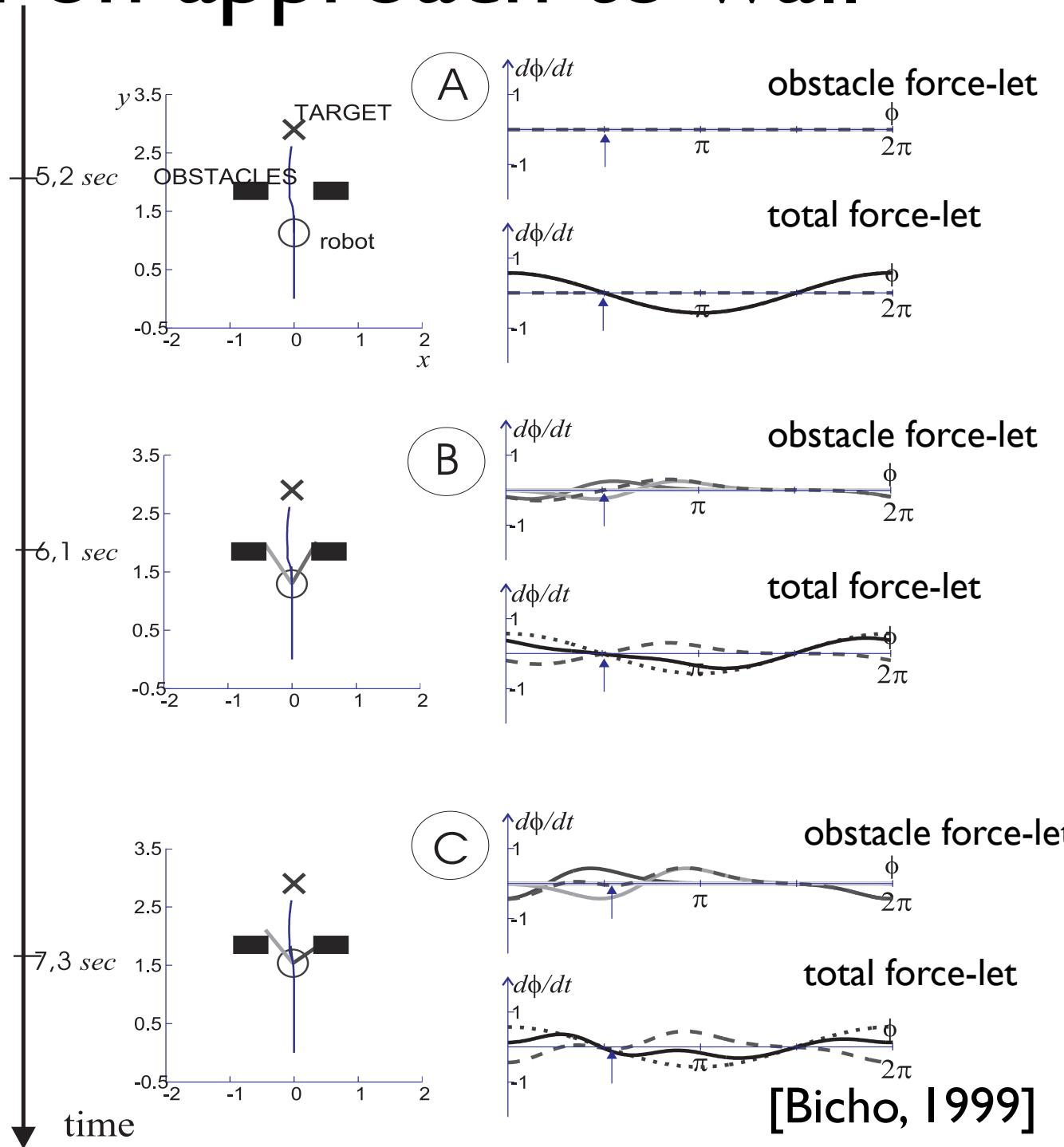
■ same with small opening



[Bicho, 1999]

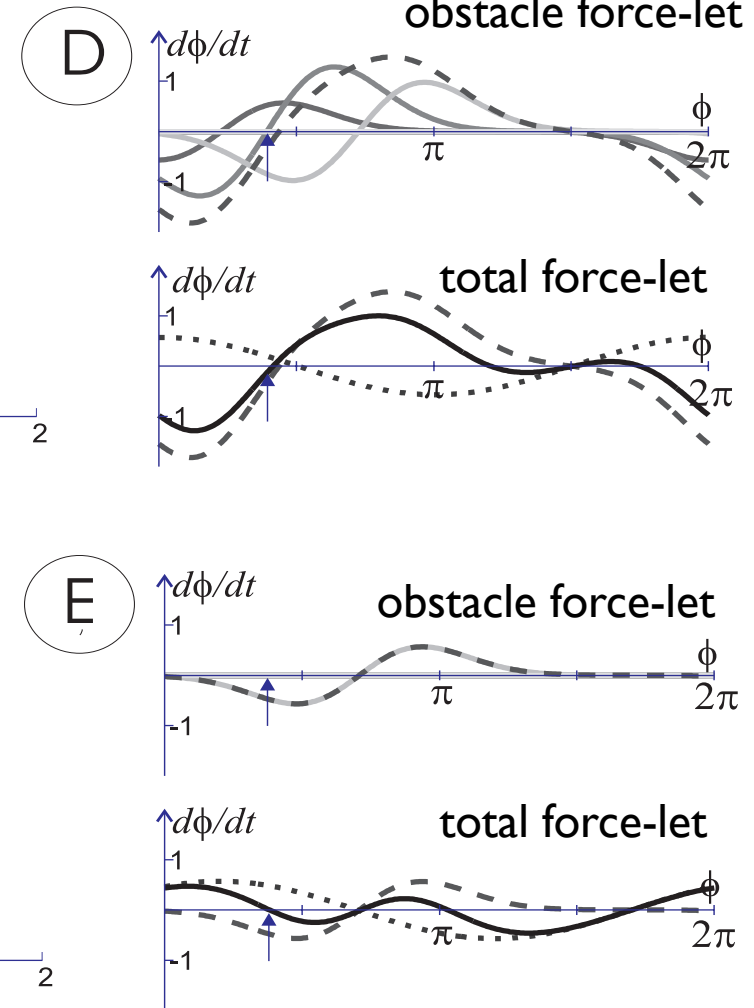
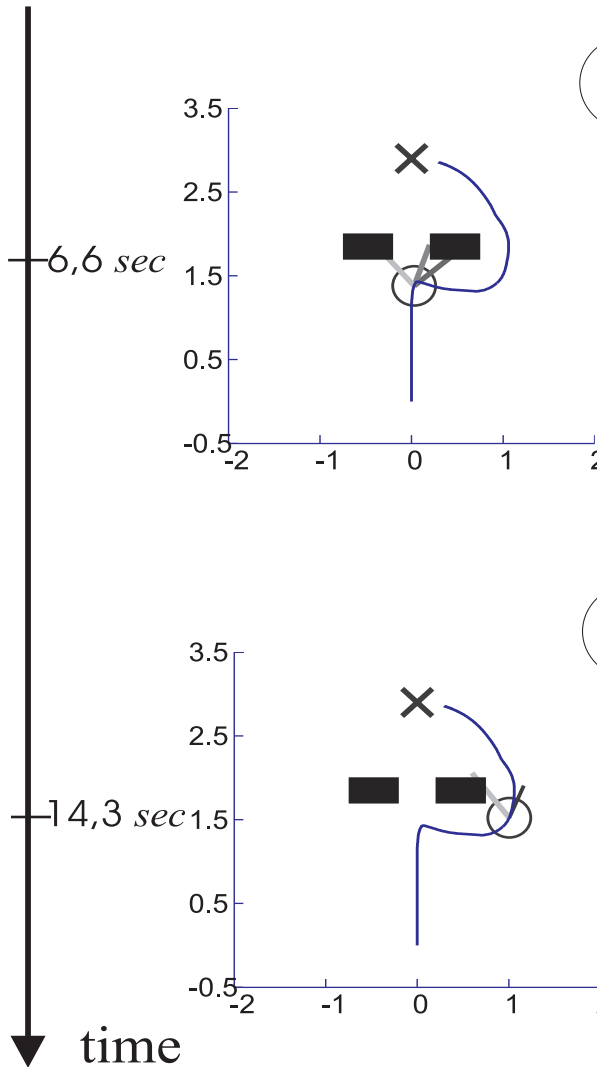
# Bifurcation on approach to wall

■ at larger opening:  
repulsion  
weak all the  
way through:  
attractor  
remains stable



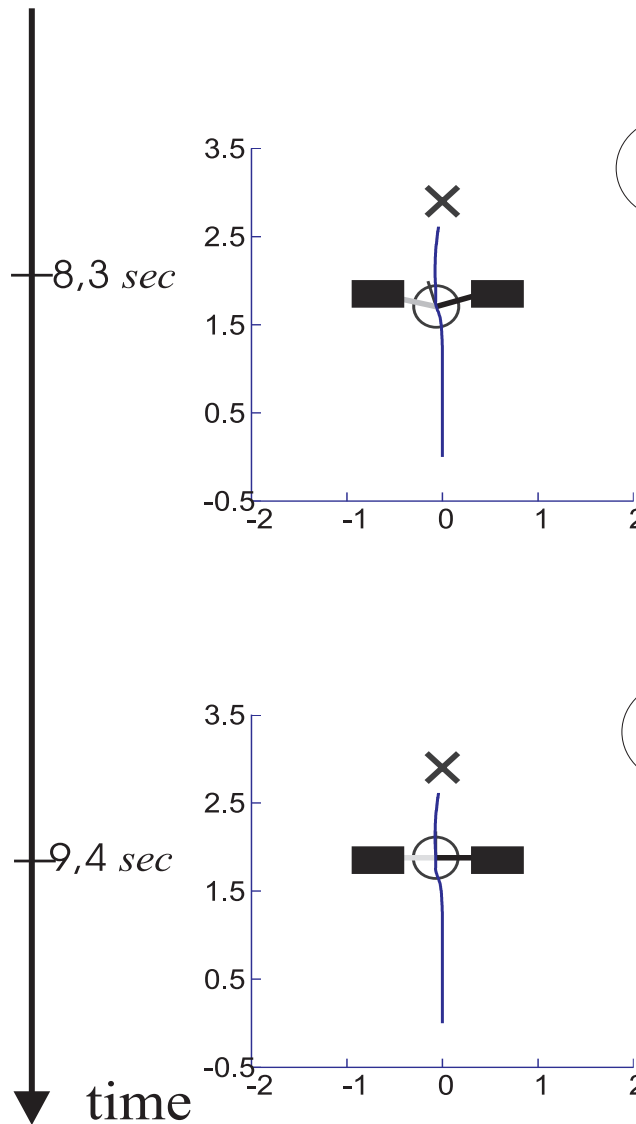
# Tracking attractor

as robot moves around obstacles, tracks the moving attractor

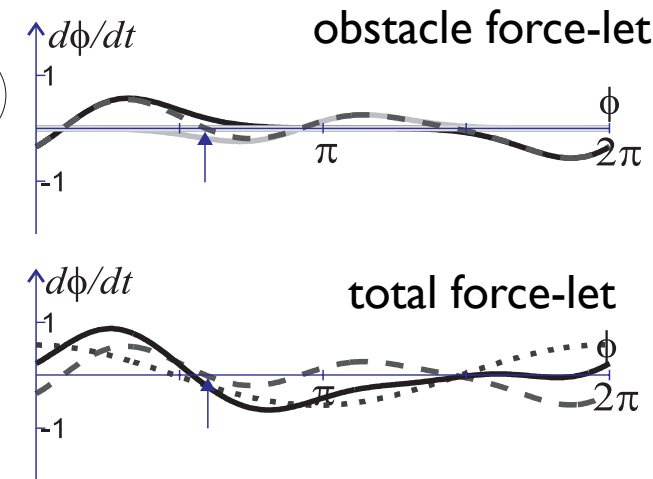


# Tracking attractor

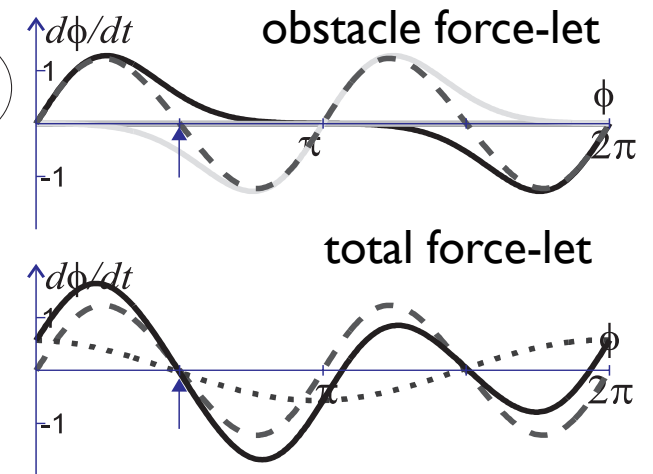
as robot moves in between obstacles, the dynamics changes but not the attractor



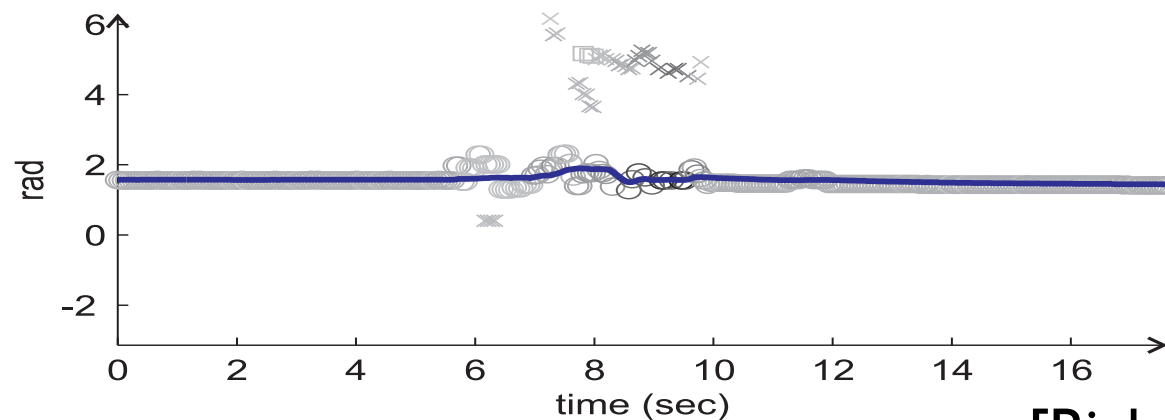
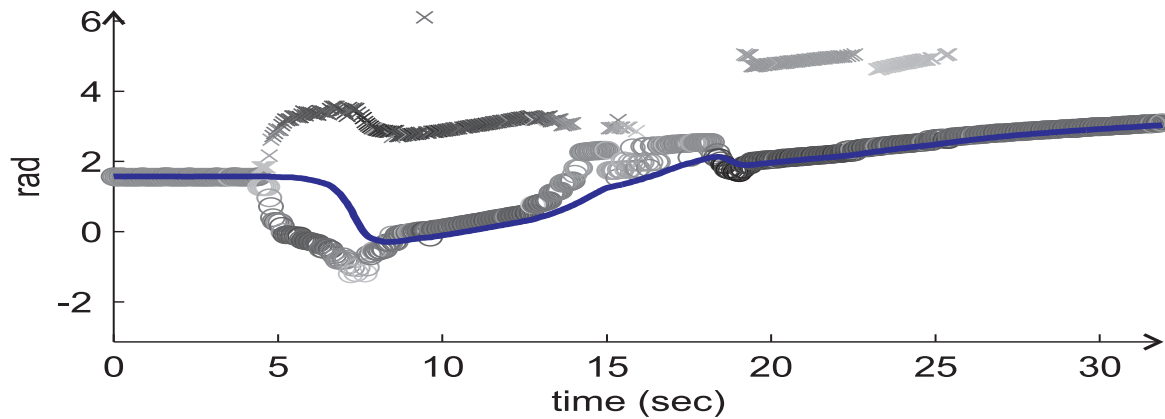
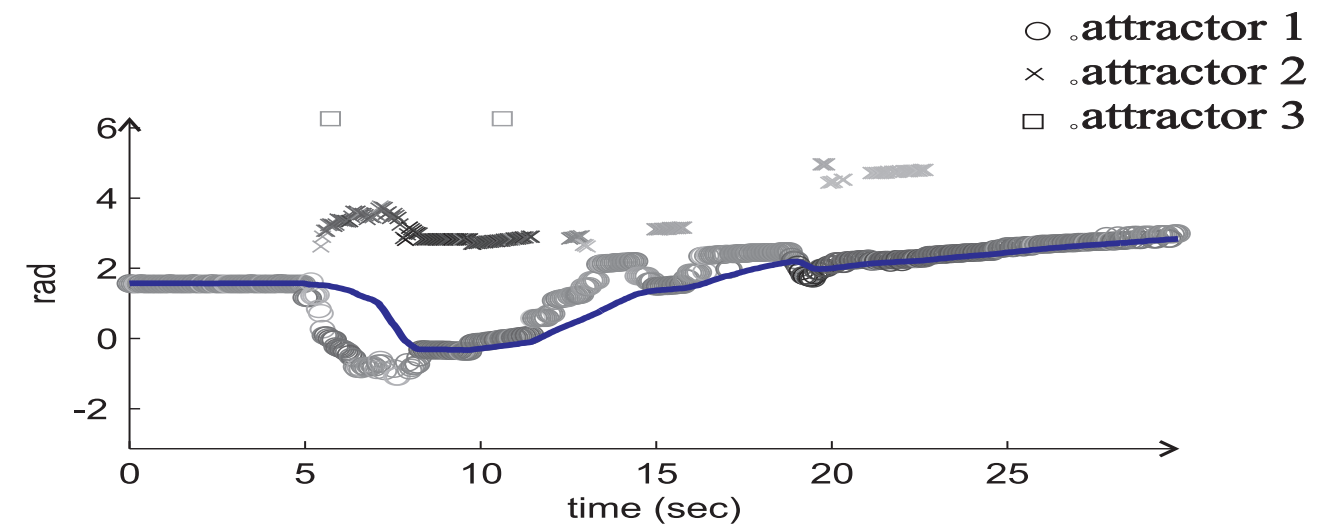
D



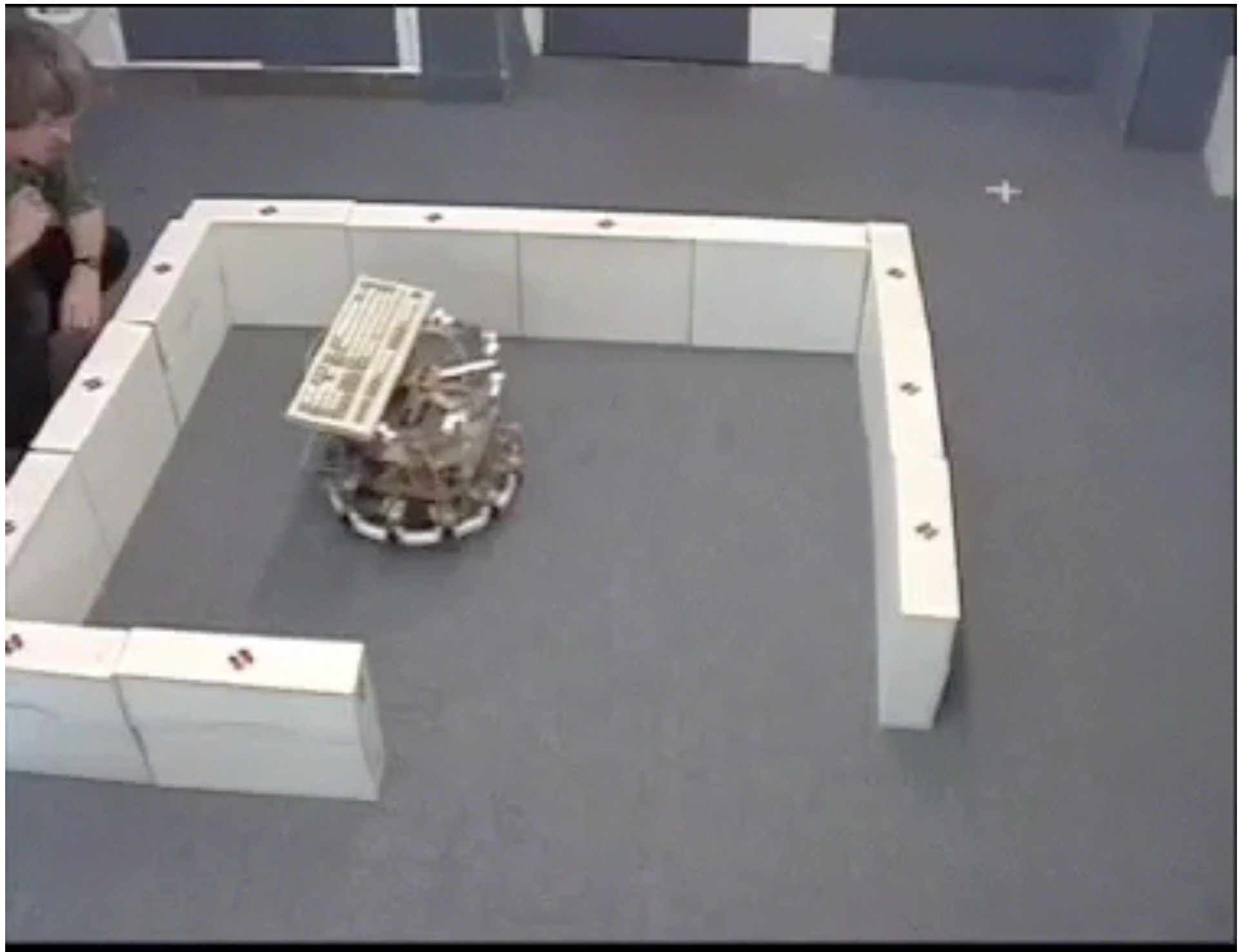
E

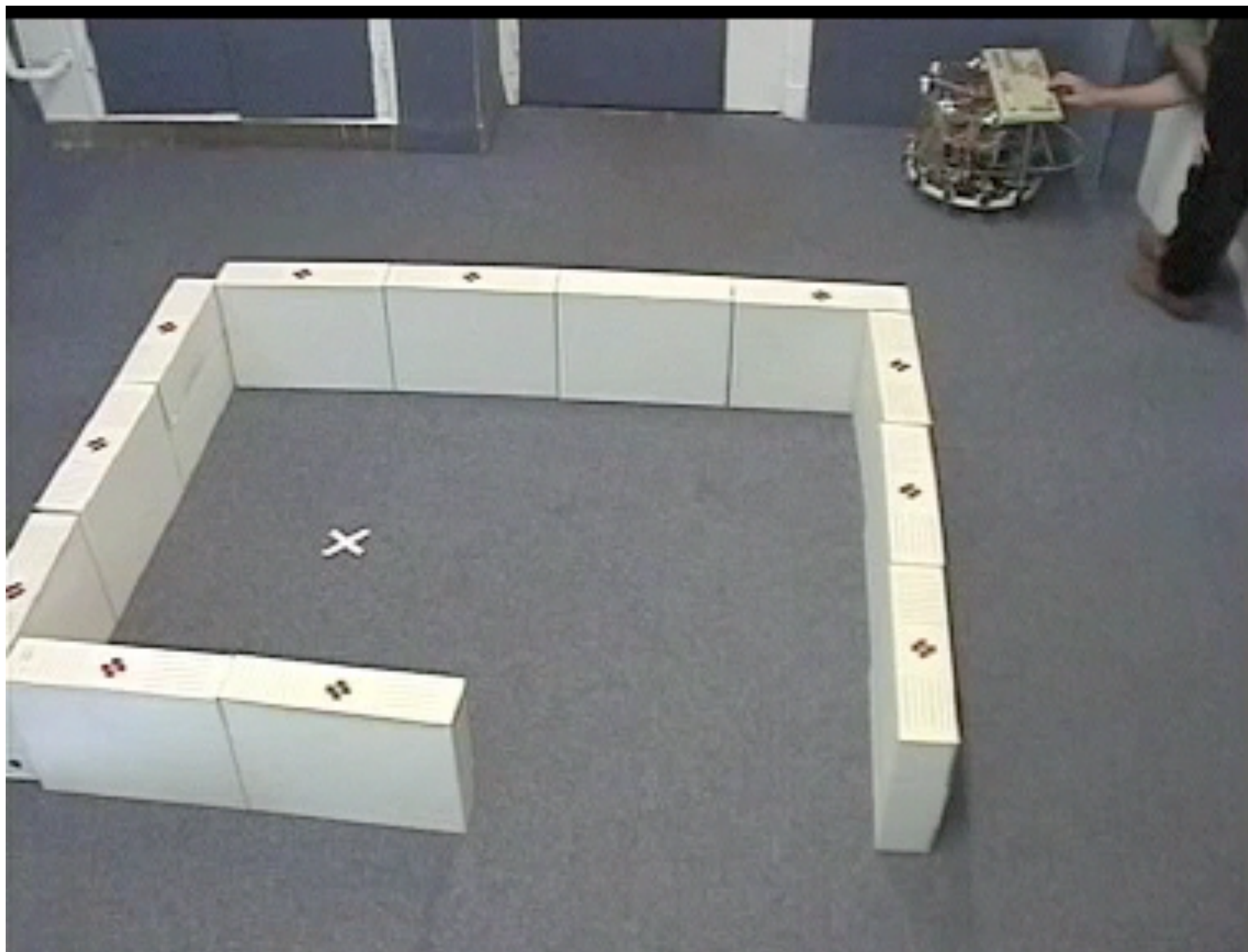


# Tracking attractors



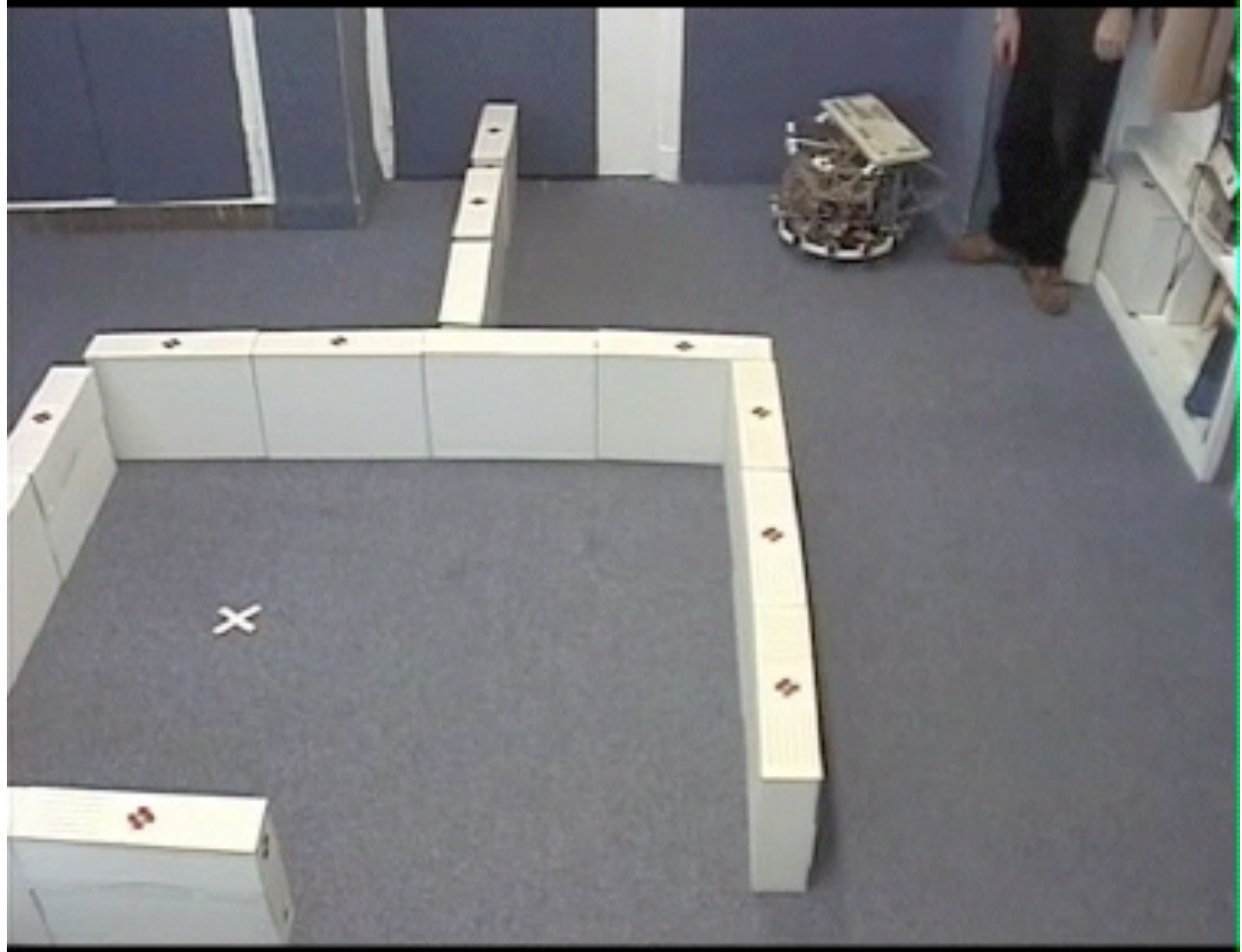
# Some implementations/demos





# Observation:

- even though the approach is purely local, it does achieve global tasks
- based on the structure of the environment!



# Observation

- different solutions may emerge depending on the environment...

# Other implementations

- autonomous wheel-chair by Pierre Mallet, Marseille





[Pierre Mallet, Marseille]

# other implementations

## ■ Bicho/Erlhagen cooperative robots

Attractor dynamics approach to joint transportation by autonomous robots: theory, implementation and validation on the factory floor

Toni Machado<sup>1</sup> · Tiago Malheiro<sup>1</sup> · Sérgio Monteiro<sup>1</sup> · Wolfram Erlhagen<sup>2</sup> · Estela Bicho<sup>1</sup> 

Autonomous Robots (2019) 43:589–610

<https://doi.org/10.1007/s10514-018-9729-2>

Video #4: Abrupt perturbations



# Conclusion

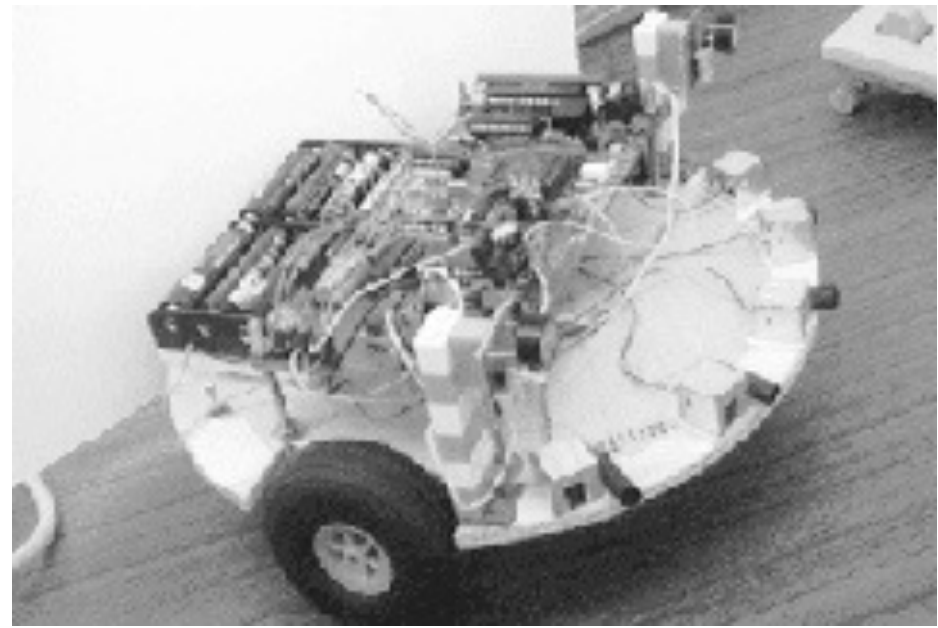
- attractor dynamics works on the basis low-level sensors information
- as long as the force-lets model the sensor-characteristics well enough to create approximate invariance of the dynamics under transformations of the coordinate frames

# Second order attractor dynamics

- source: Bicho, Schöner, Robotics and Autonomous Systems 21:23-35 (1997)

# Second order dynamics

- idea: go to even lower level sensory-motor systems:
  - a sensor that only knows there is a target or an obstacle on the left vs. on the right...
  - but is not able to estimate the heading of either
  - a motor system that is not calibrated well enough to steer into a given heading direction in the world



# behavior variable

- turning rate  $\omega$  rather than heading direction
- can be “enacted” by setting set-points for velocity servo controllers of each motor
- target: information about target being to the left, to the right, or ahead, but no calibrated bearing,  $\psi$ , to target
- obstacle: turning rate
  - to the right when obstacle close and to the left
  - to the left when obstacle close and to the right
  - zero when obstacle far

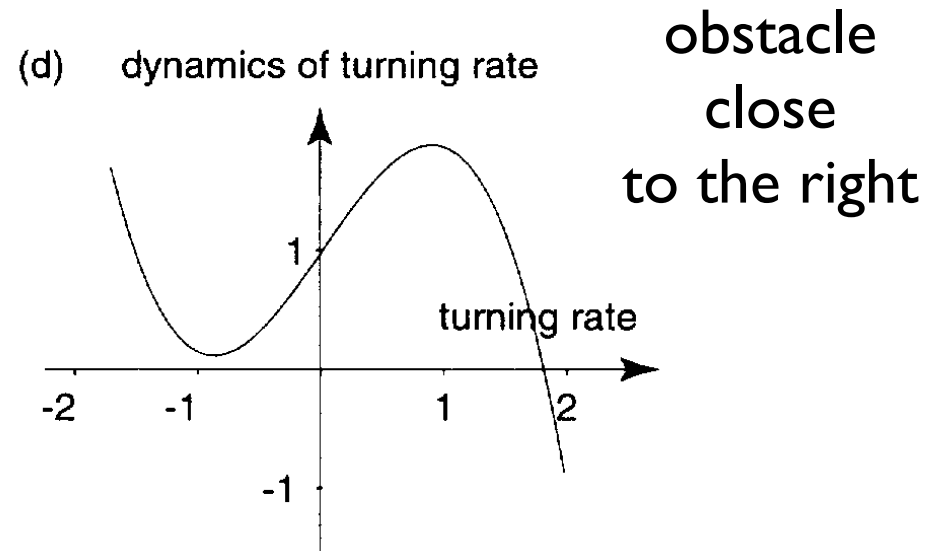
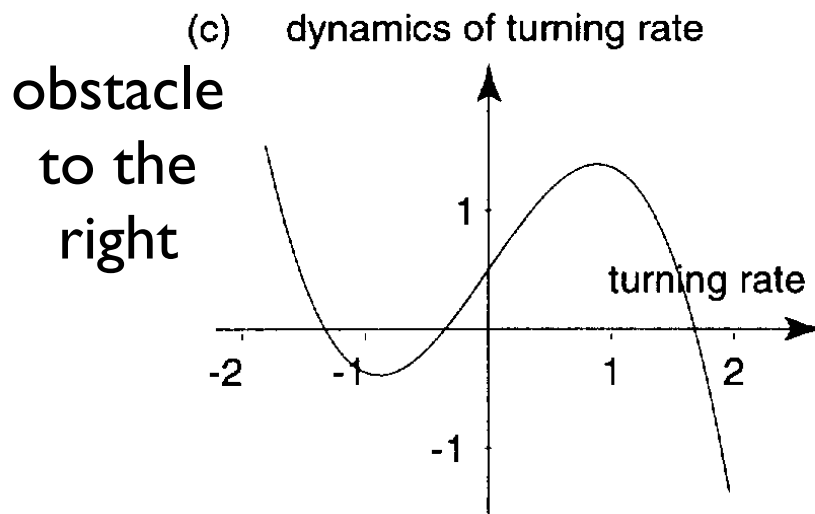
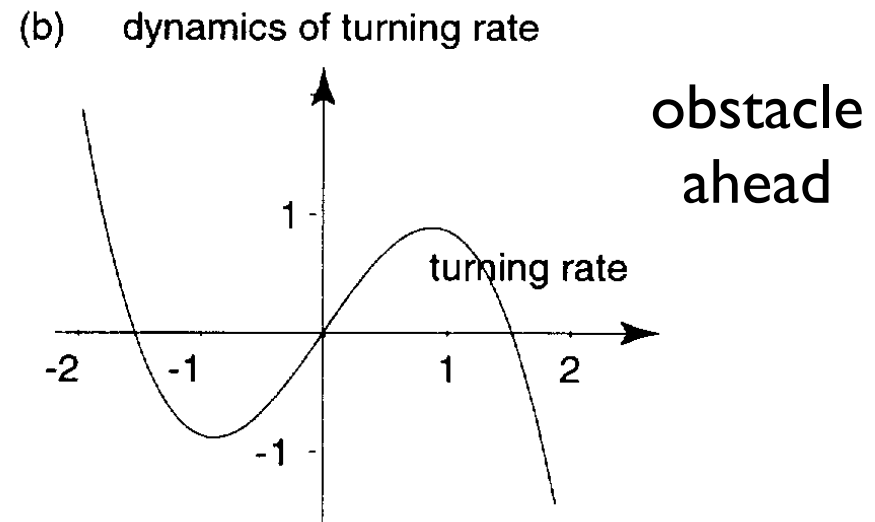
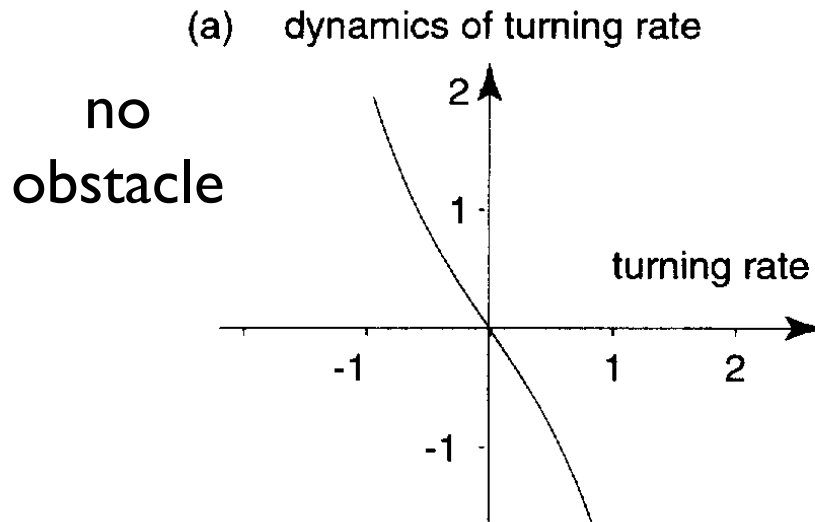
# dynamics of turning rate: obstacle avoidance

- pitch-fork normal form (to get left-right symmetry)
- but symmetry potentially broken by additive constant: biases bifurcation toward left or toward right

$$\dot{\omega} = (\alpha + \frac{1}{2}\pi)c_{\text{obs}}F_{\text{obs}} + \alpha\omega - \gamma\omega^3$$

# obstacle avoidance

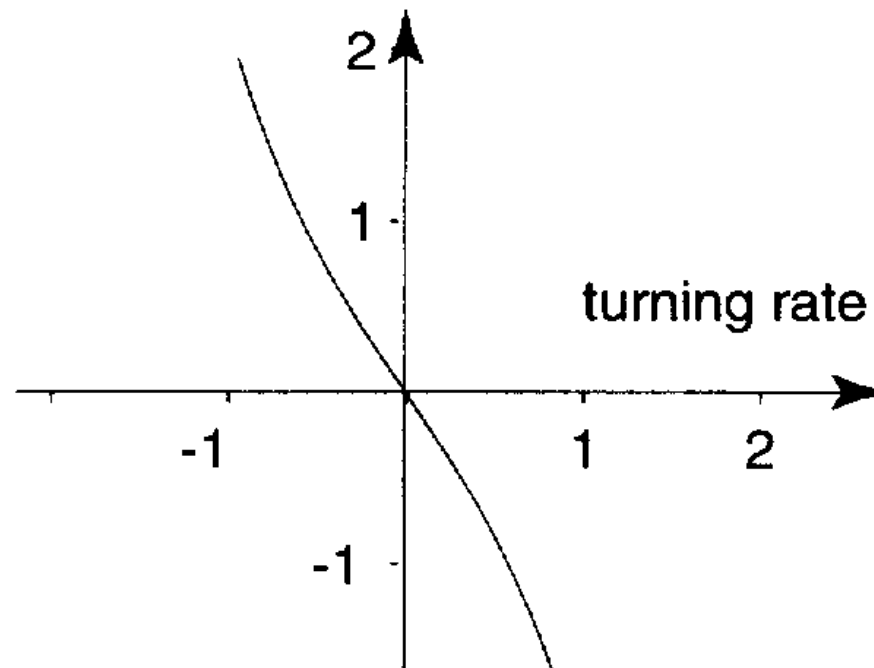
$$\dot{\omega} = (\alpha + \frac{1}{2}\pi)c_{\text{obs}}F_{\text{obs}} + \alpha\omega - \gamma\omega^3$$



# obstacle avoidance

- in absence of obstacle in forward direction (distance large):  $\alpha$  negative, constant zero

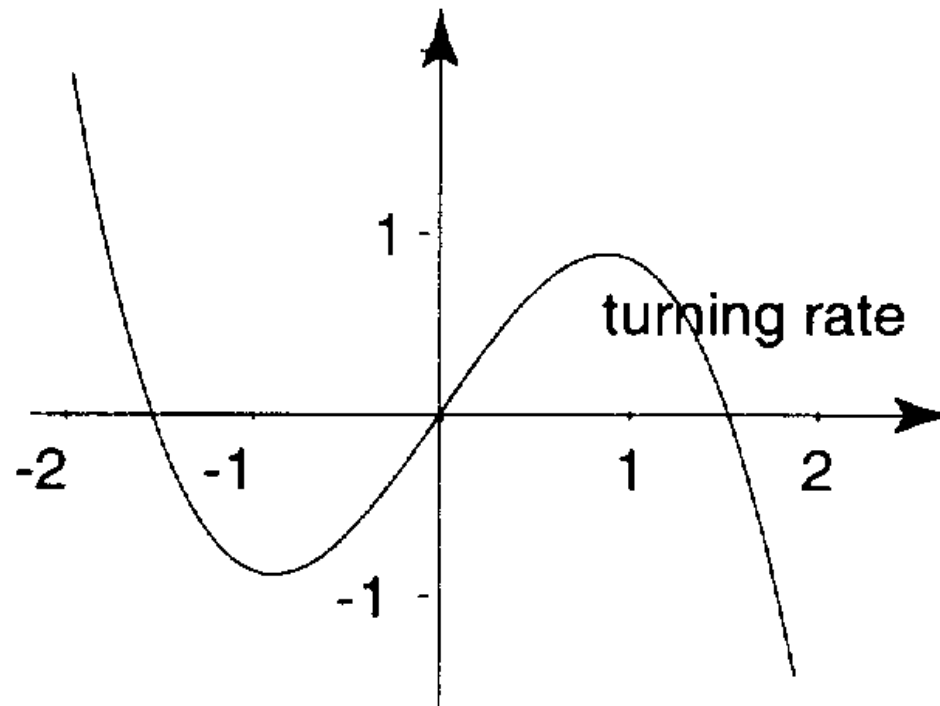
(a) dynamics of turning rate



# obstacle avoidance

- in presence of obstacle in forward direction, symmetric bifurcation to desired avoidance rotations:  $\alpha$  positive, constant zero

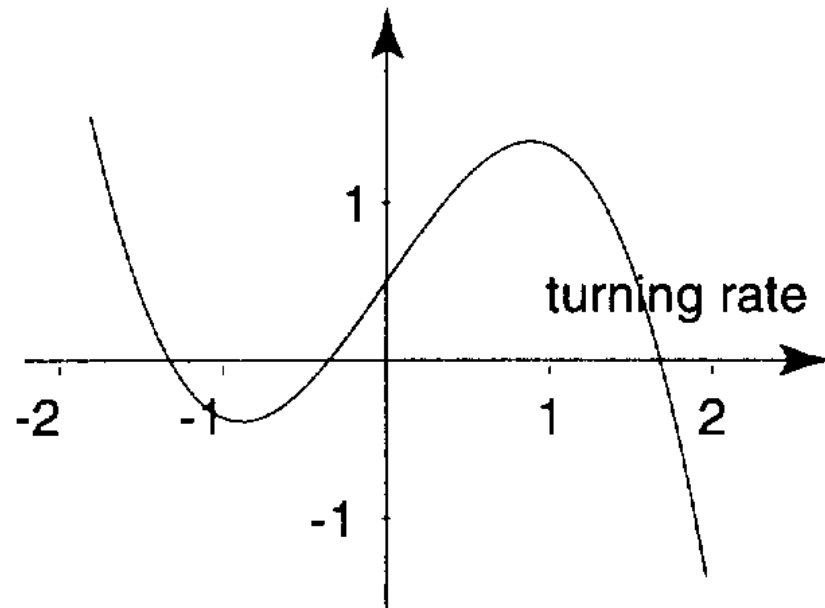
(b) dynamics of turning rate



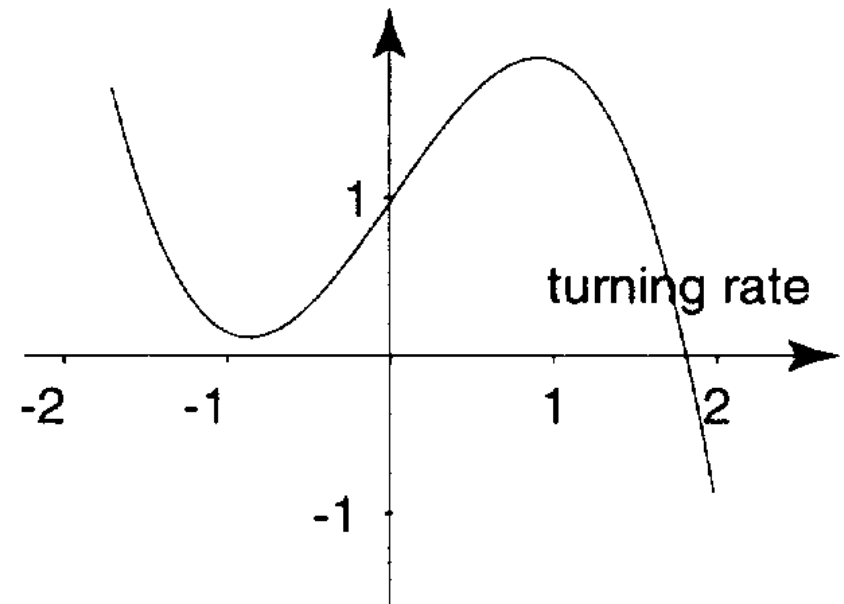
# obstacle avoidance

- in presence of obstacle to the right of current heading: tangent bifurcation removes attractor at negative  $\omega$ ,  $\alpha$  negative, constant negative

(c) dynamics of turning rate



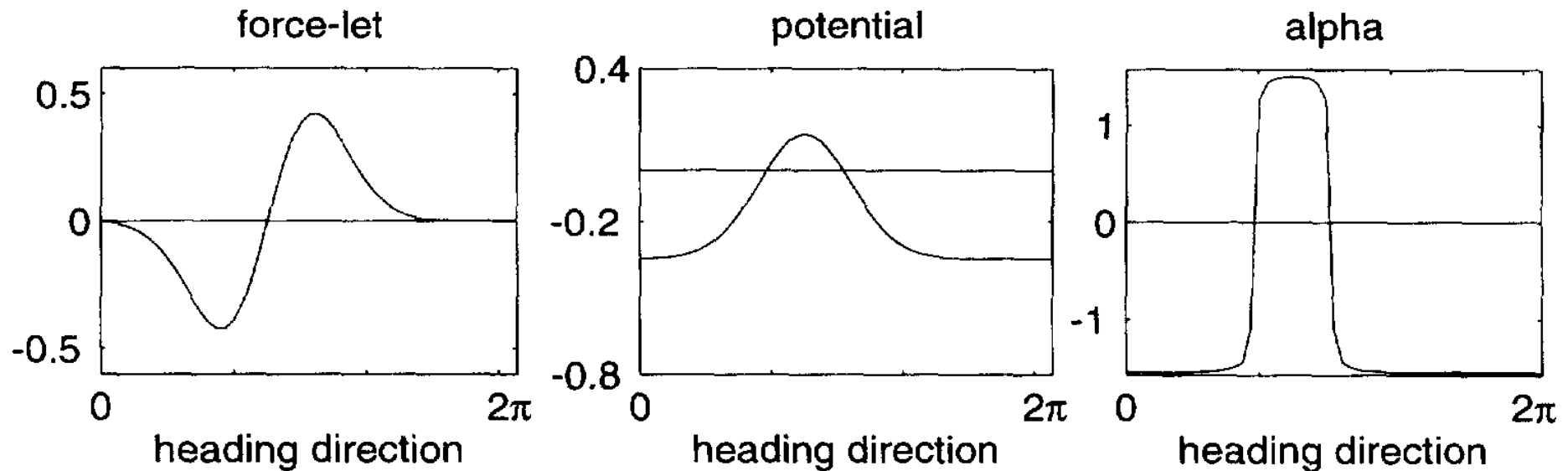
(d) dynamics of turning rate



# mathematical form

■ compute constant and alpha from obstacle force lets

$$\dot{\omega} = (\alpha + \frac{1}{2}\pi)c_{\text{obs}}F_{\text{obs}} + \alpha\omega - \gamma\omega^3$$



$$F_{\text{obs}} = \sum_i \lambda_i (\phi - \psi_i) \exp\left[-\frac{(\phi - \psi_i)^2}{2\sigma_i^2}\right]$$

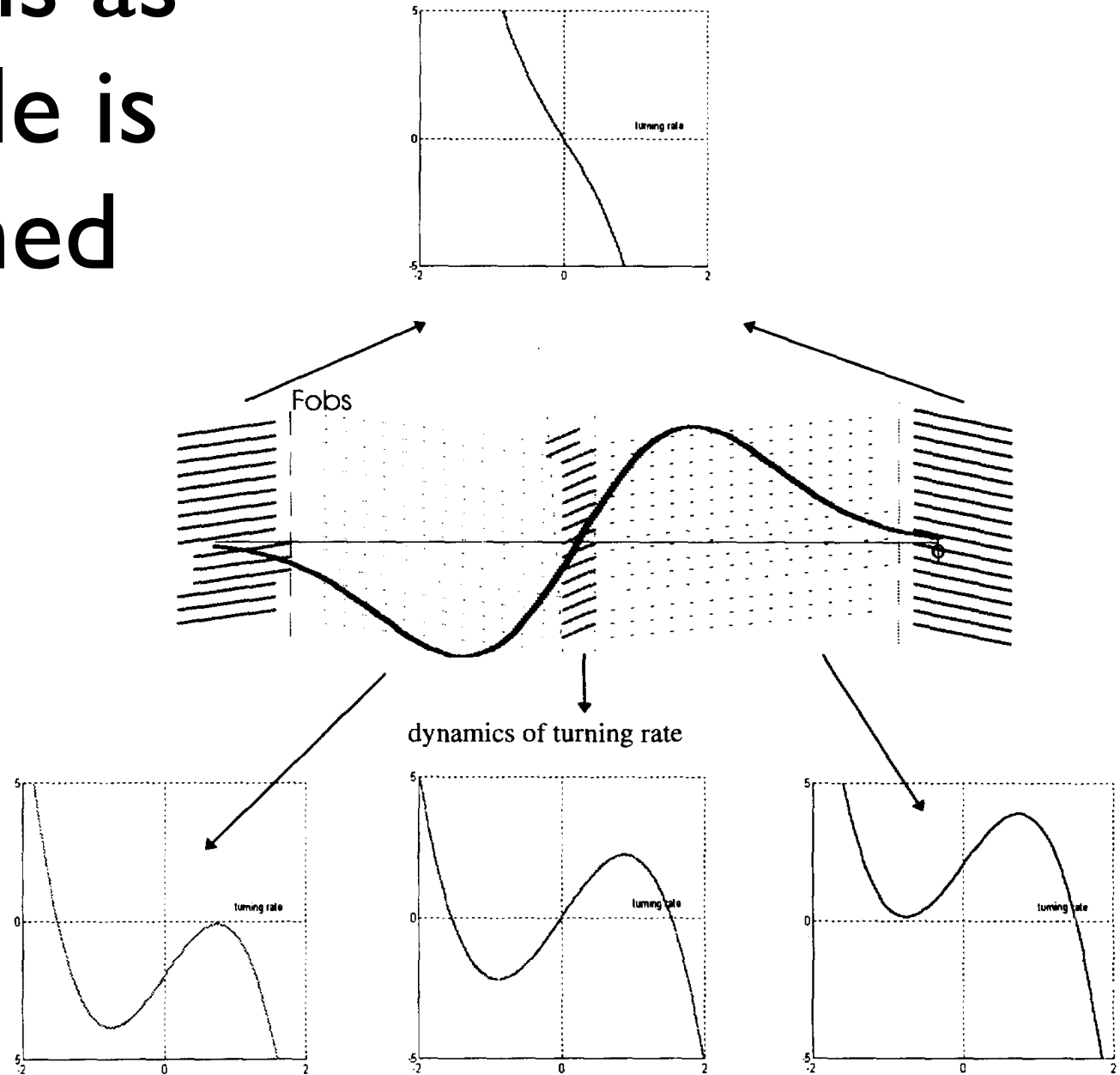
$$\lambda_i = \beta_1 \exp[-d_i / \beta_2]$$

$$\sigma_i = \arctan\left[\tan\left(\frac{\Delta\theta}{2}\right) + \frac{R_{\text{robot}}}{R_{\text{robot}} + d_i}\right]$$

$$V = \sum_i \left( \lambda_i \sigma_i^2 \exp\left[-\frac{\theta_i^2}{2\sigma_i^2}\right] - \frac{\lambda_i \sigma_i^2}{\sqrt{e}} \right)$$

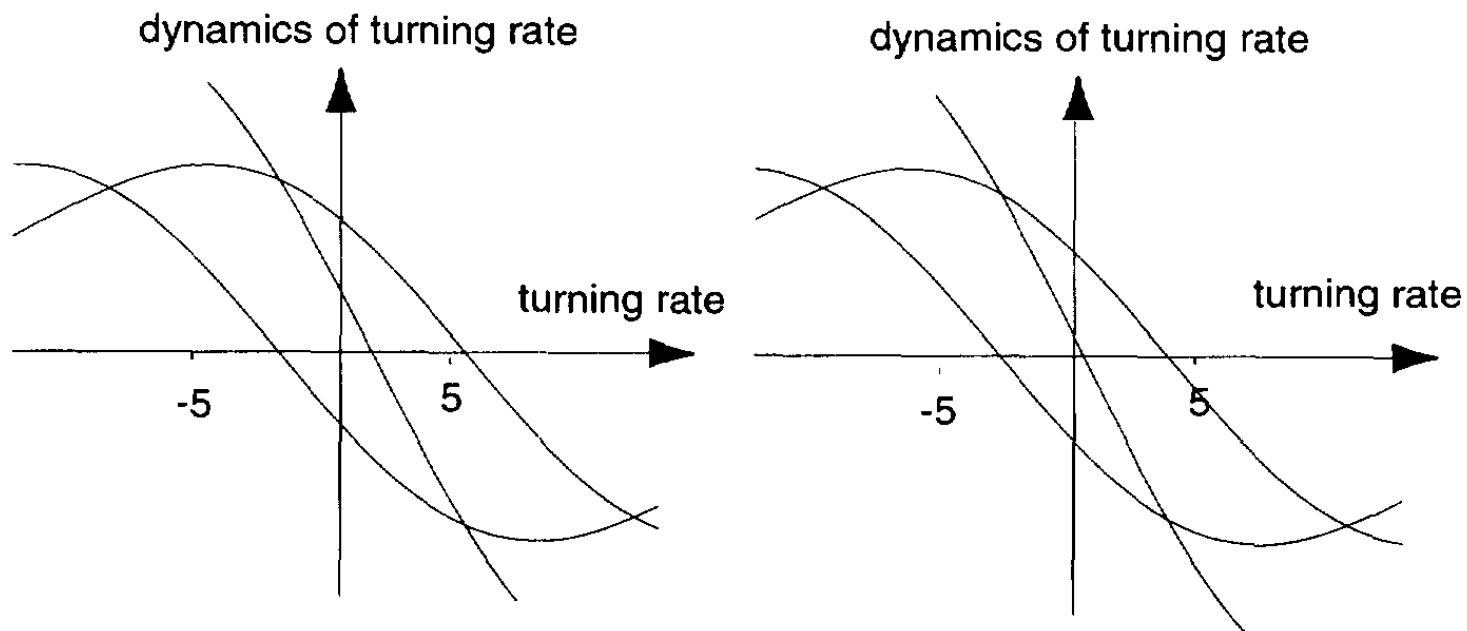
$$\alpha = \arctan[c \ V]$$

# bifurcations as an obstacle is approached



# dynamics: target acquisition

- a sensor for a target on the left sets an attractor at positive turning rate, strength graded with intensity
- a sensor for a target on the right sets an attractor at negative turning rate, strength graded with intensity



# mathematical formulation

■ force-let of  
each target  
sensor

$$g_i(\omega) = -\frac{1}{\tau_\omega}(\omega - \omega_i) \exp\left[-2\frac{(\omega - \omega_i)^2}{\Delta\omega^2}\right].$$

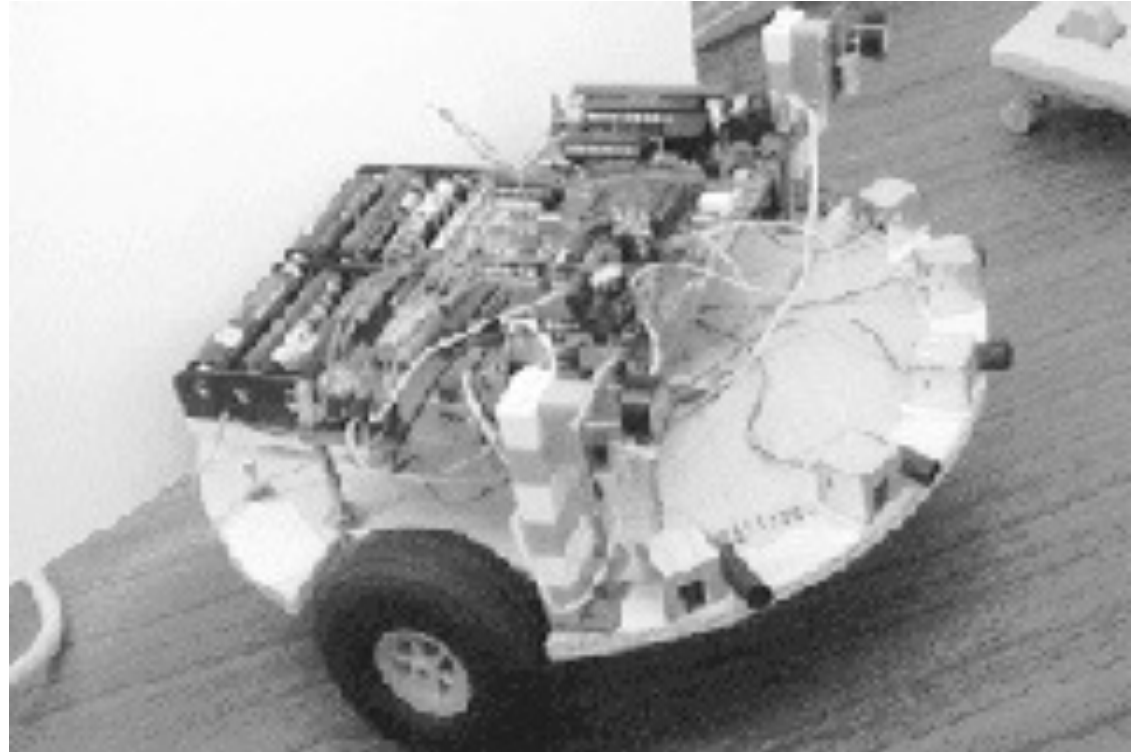
( $i = \text{right or left}$ )

■ summed to  
total dynamics

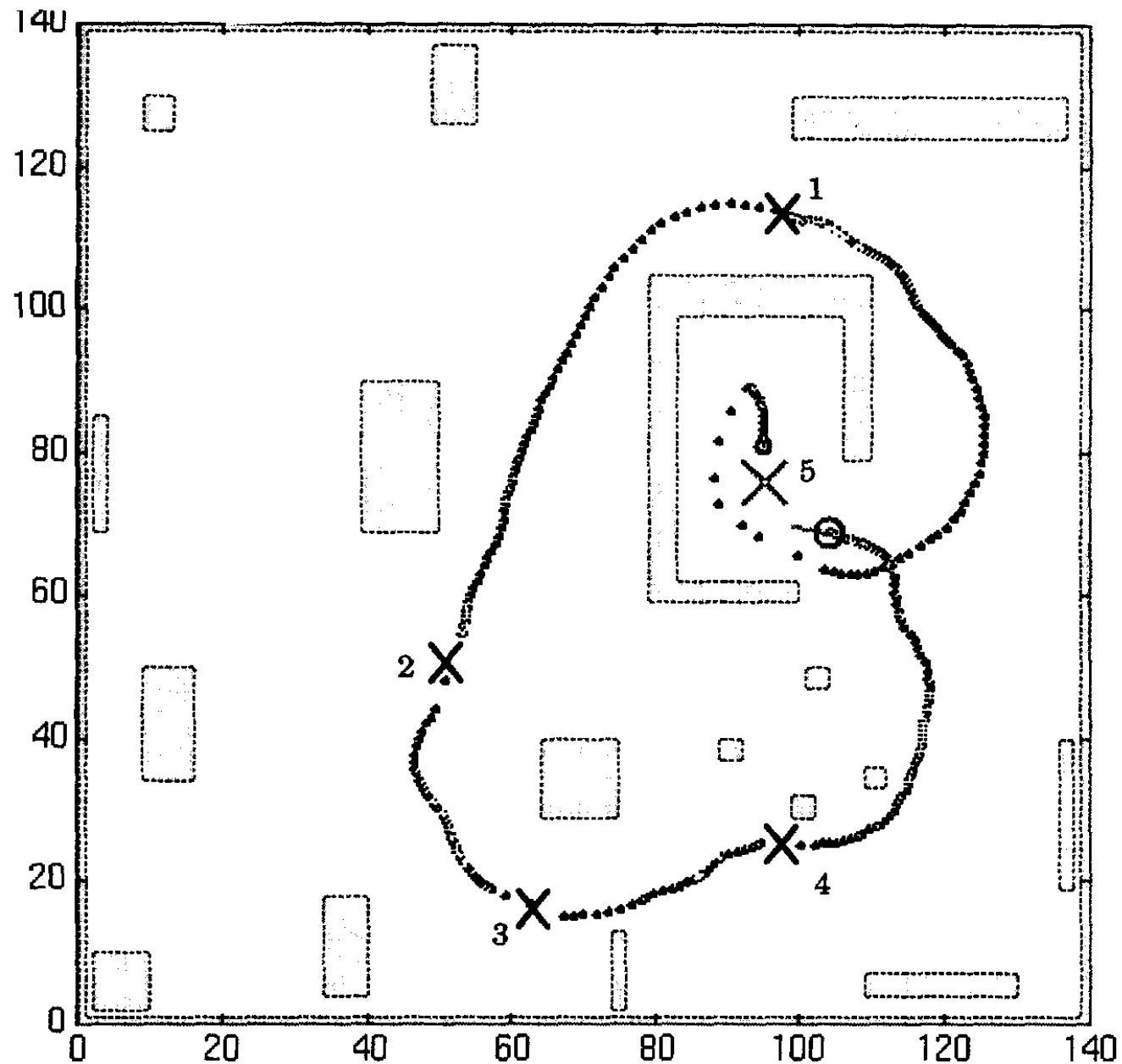
$$g_{\text{left}}(\omega) + g_{\text{right}}(\omega)$$

# putting it to work on a simple platform

- Rodinsky!
- circular platform with passive caster wheel
- two (unservoed) motors
- 5 IR sensors
- 2 LDR's
- microcontroller  
MC68HCA11A0  
Motorola (32 K RAM),  
8 bit



# example trajectories



# demonstration



# why does it work?

- here the dynamics exists instantaneously while vehicle is heading in a particular direction
- while the vehicle is turning under the influence of the corresponding attractor for turning rate, the dynamics is changing!
- typically undergoing an instability as vehicle's heading turns away from an obstacle...

# Summary

- behavioral variables
- attractor states for behavior
- attractive force-let: target acquisition
- repulsive force-let: obstacle avoidance
- bistability/bifurcations: decisions
- can be implemented with minimal requirements for perception