# Attractor dynamics approach to vehicle movement generation

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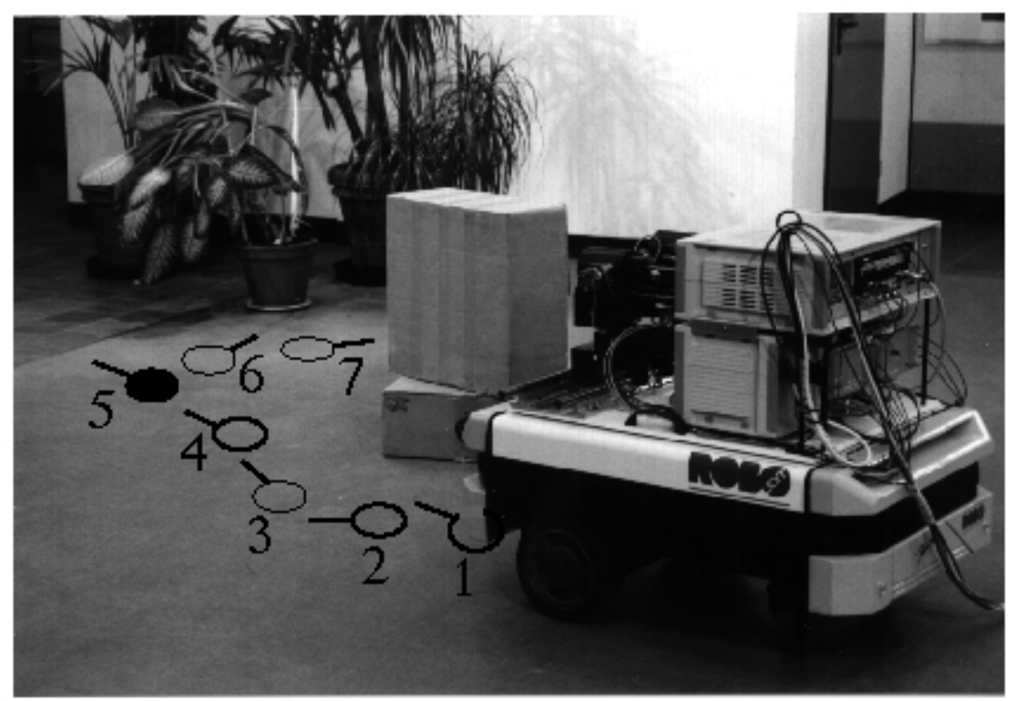
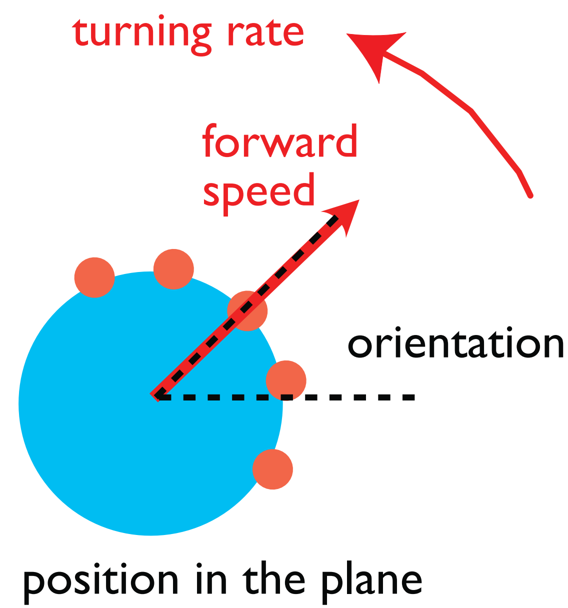
# The vehicle movement problem

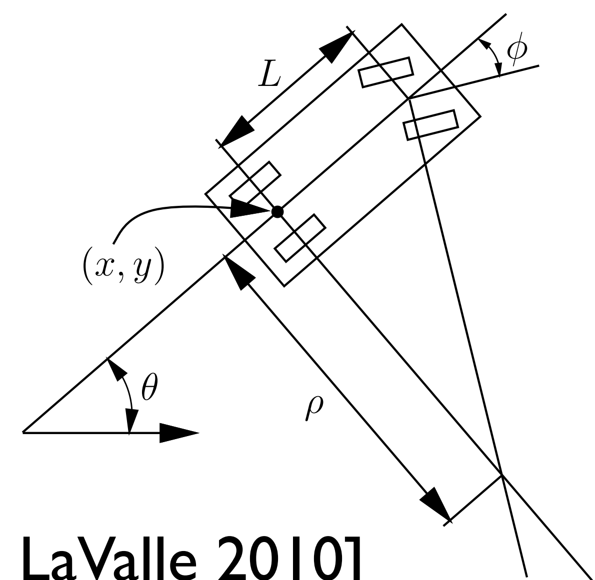
- move vehicle in a 2D world
- toward a target
- while avoiding collisions with obstacles
- potentially:
  - follow a road or a sequence of targets (via points)
  - docking: achieve a particular orientation



# Degree of Freedom (DoF)



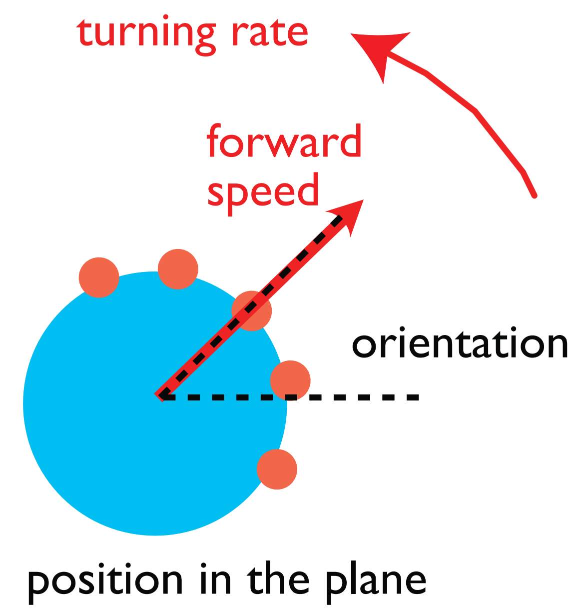

- Vehicles have 3 DoF
  - 2D position
  - Orientation

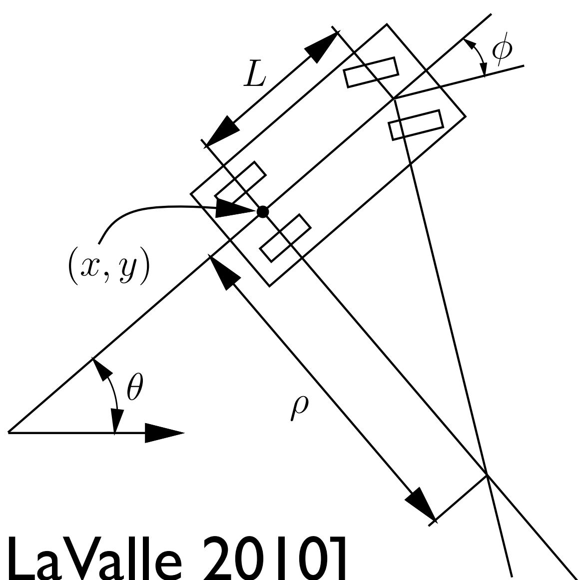



[from LaValle 2010]

# Non-holonomic constraints

- fewer variables than the number of DoF can be varied freely
  - robot with two active wheels: 2 wheel velocities
  - car: steering angle and speed
- state of the 3DoF depends on the history of movement
  - easy for robot with active wheels: turn on the spot
  - difficult for car: parking



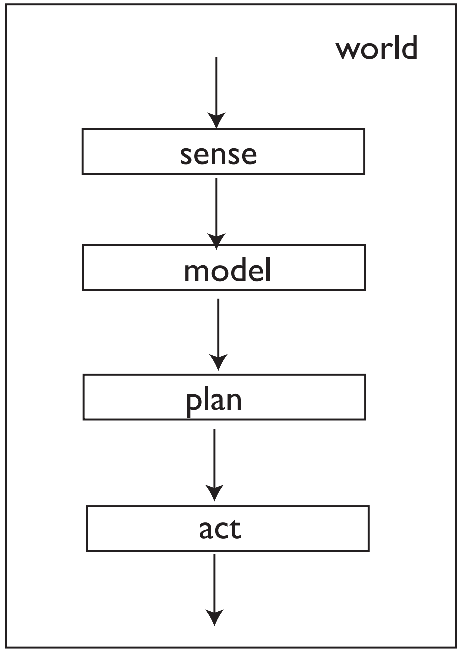

[from LaValle 2010]

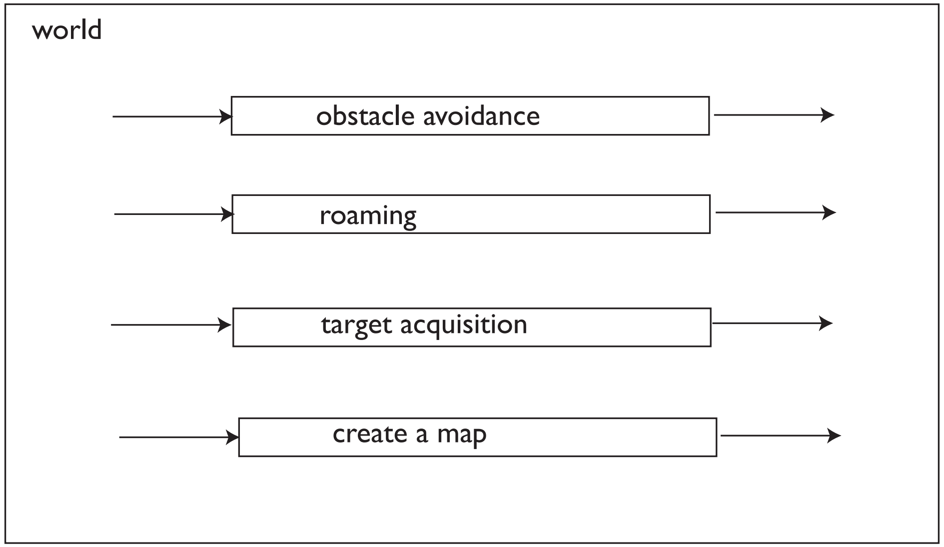
# Autonomous vehicle movement

- sense something about the environment or know about the environment (map)
- plan movement in the environment toward target that is collision-free
- control the vehicle to achieve the planned movement
- estimate what vehicle actually did: update the map

# Architectures

- sense-plan-act
  - planning based on a world world model
- behavior-based
  - low-level sensory information that is specific to each individual behavior
  - planning emerges from how behaviors interact



# Concepts for planning

- local vs. global
  - planning based on information only about the local environment of the robot
  - vs. based on global map information about the environment
- reactive vs. planning
  - motion planning "on the fly" in response to sensory inputs
  - vs. motion planning for an entire action from initial to goal state

# Concepts for planning

- exact vs. heuristic
  - exact: guarantee that a path that fulfills the constraints is found when one exists
  - vs. generate a plan based on ad hoc approach that is likely to fulfill constraints
- continuous vs. discrete:
  - continuous state space variables
  - vs. grid state spaces, graph state spaces

# Attractor dynamics approach

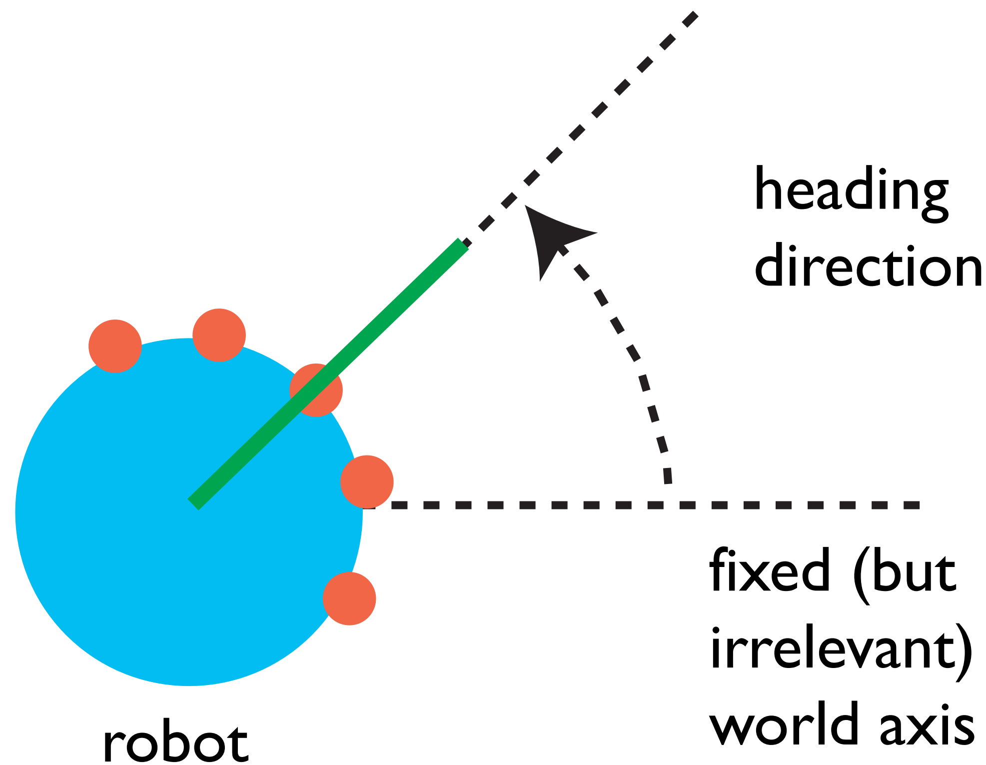
- developed by my team over many years
- a particular solution to the vehicle motion planning problem that is conceptually compatible with properties of the nervous system and human/animal behavior
- it can be used both in sense-plan-act ("symbolic") or behavior-based ("sub-symbolic") form
- it is local, reactive, heuristic, and continuous

# Basic ideas of the attractor dynamics approach

- plans are time courses of behavioral variables
- these time courses are generated by a dynamical system
- they are structured by attractor solutions of dynamical systems (which may change during the movement)
- decisions emerge from bifurcations of the attractor solutions

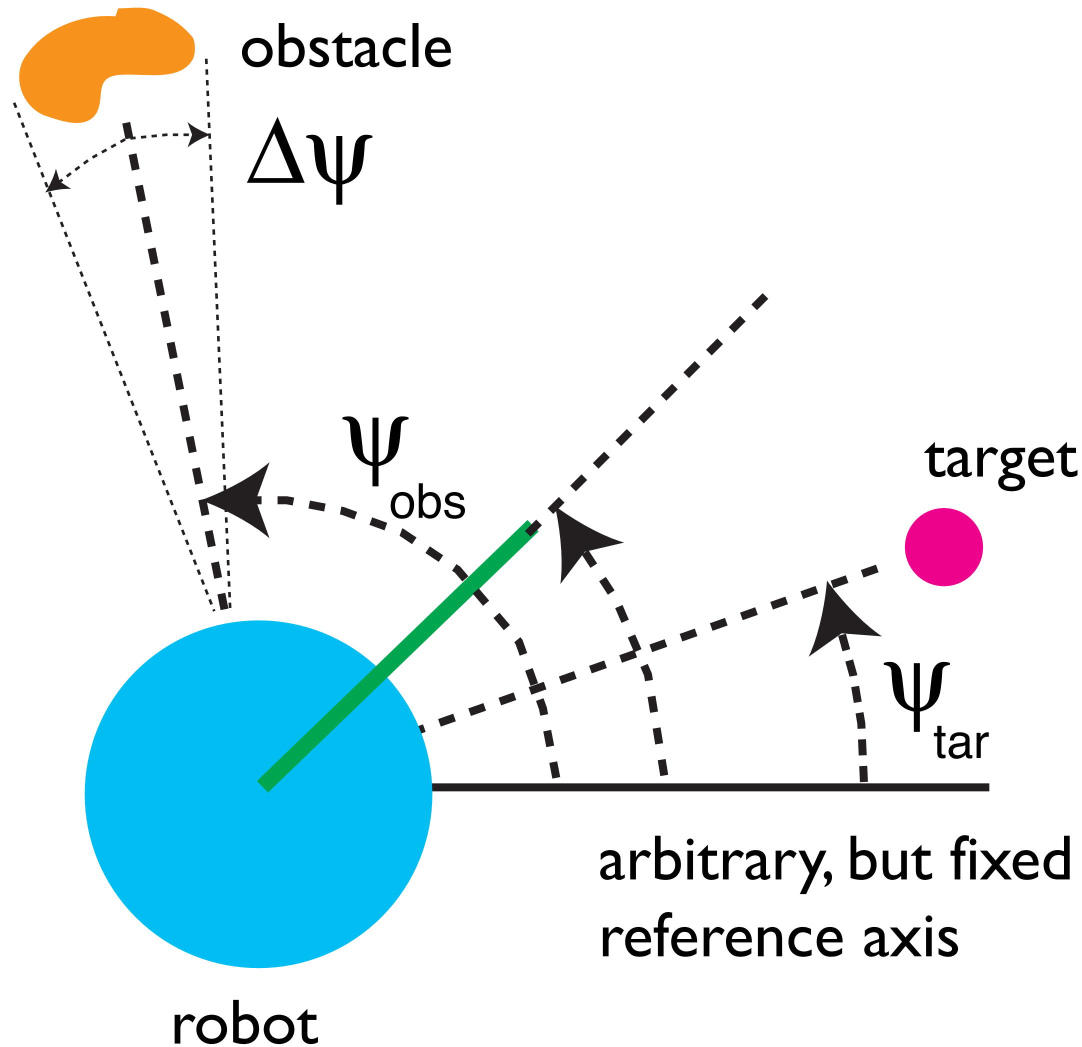
# Behavioral variables

- first behavioral variable: heading direction
- second behavioral variable: forward speed
  - (neglected in this lecture: constant speed)



# Constraints

- obstacle avoidance
- target acquisition



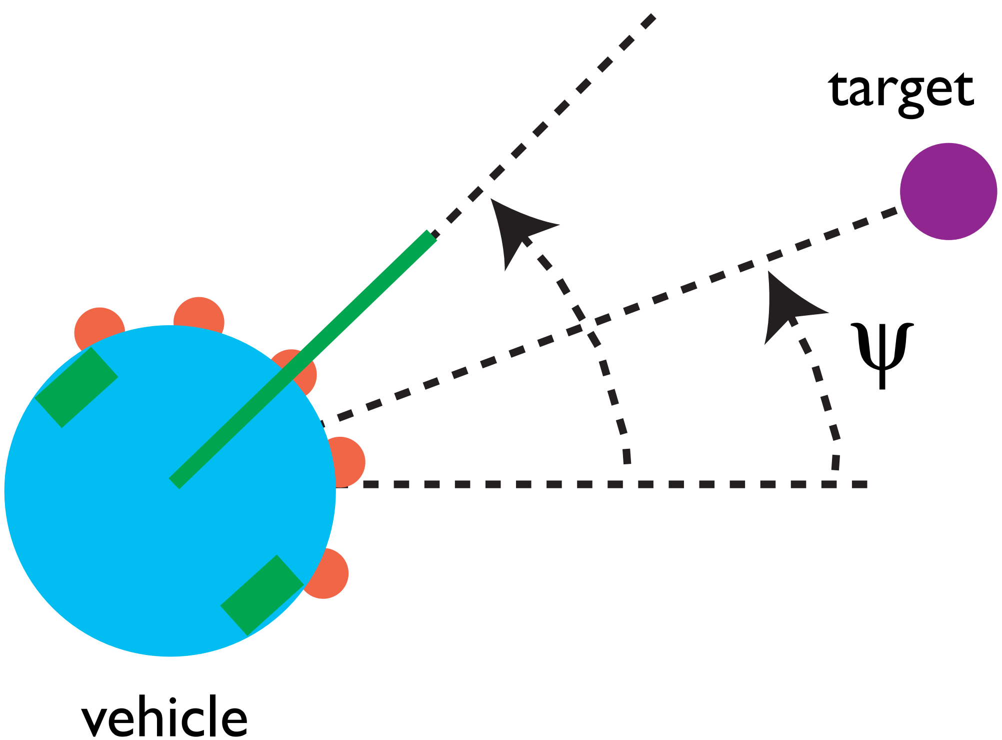
# Behavioral variables: properties

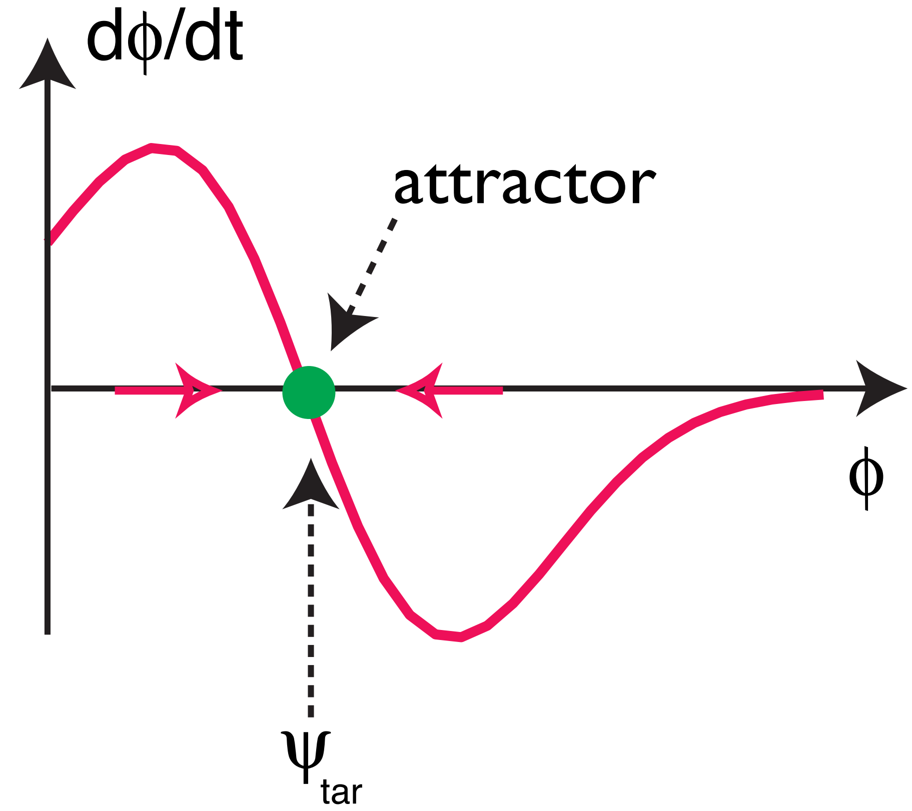
- uniquely describe the desired movement
- “enactable”: can be used to control the behavior
- constraints can be expressed as values/value ranges of the behavioral variables
- no calibration needed

# Behavioral dynamics: properties

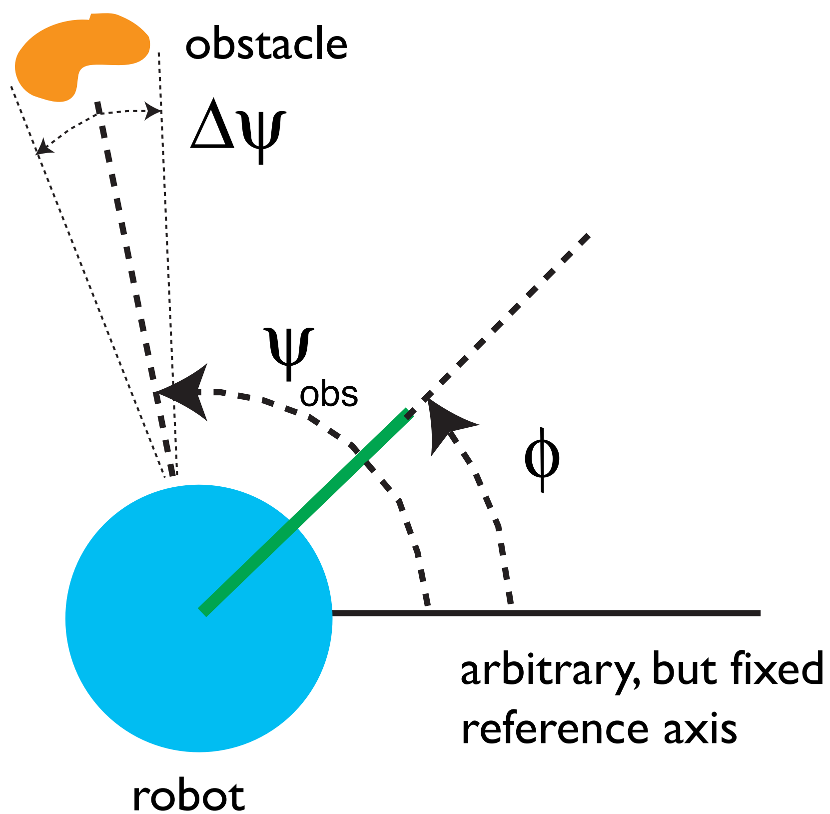
- plan (generate) movement by generating time courses of behavioral variables
- time course of behavioral variables emerge from attractor solutions of a (designed) dynamical system
- that dynamical system is constructed from contributions that express the behavioral constraints

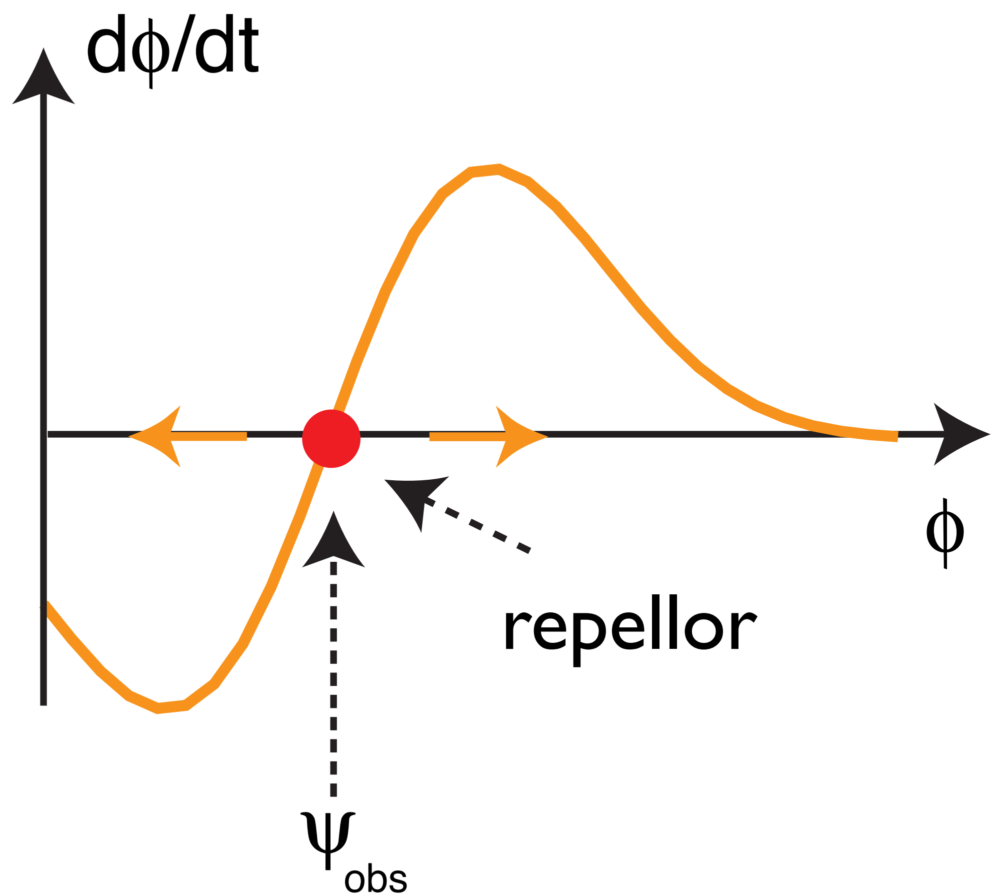
# Behavioral dynamics: target constraint



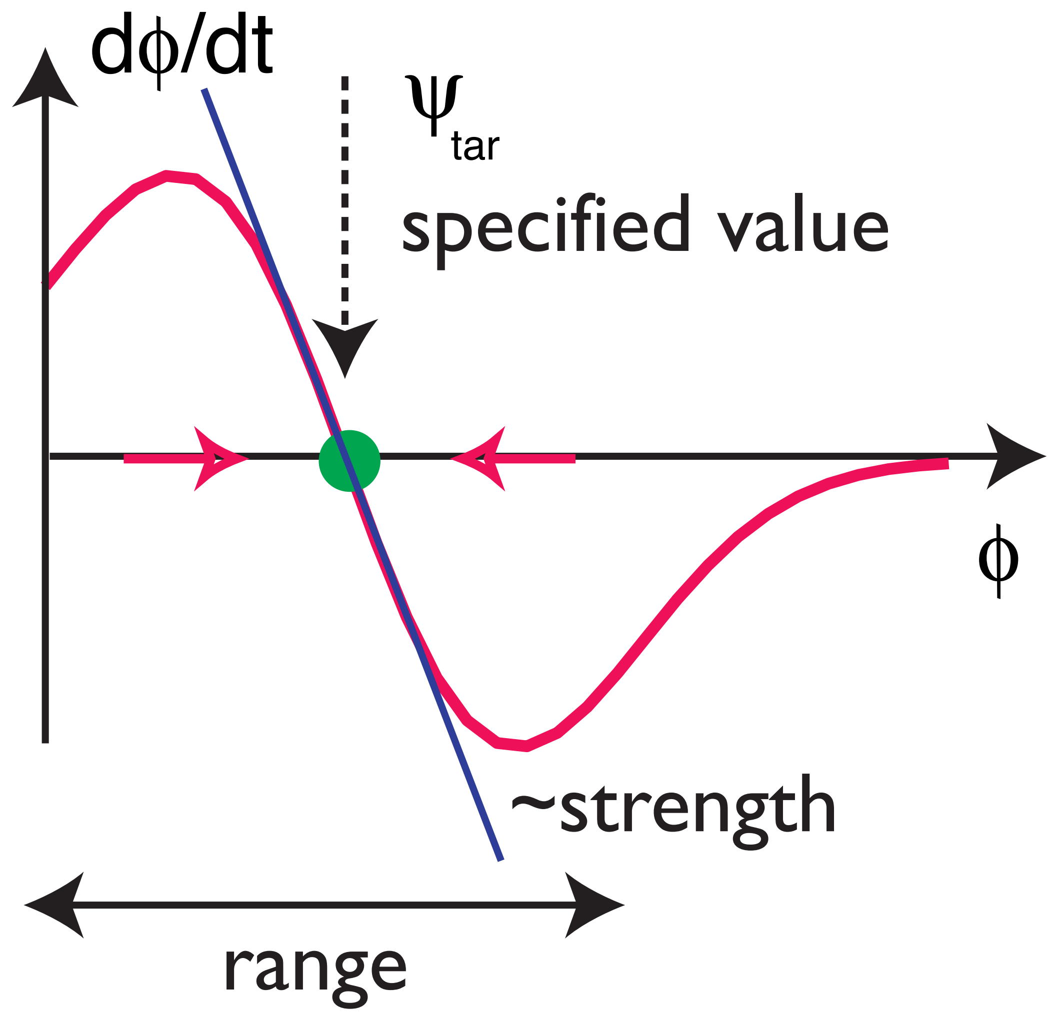
# Behavioral dynamics: obstacle constraint



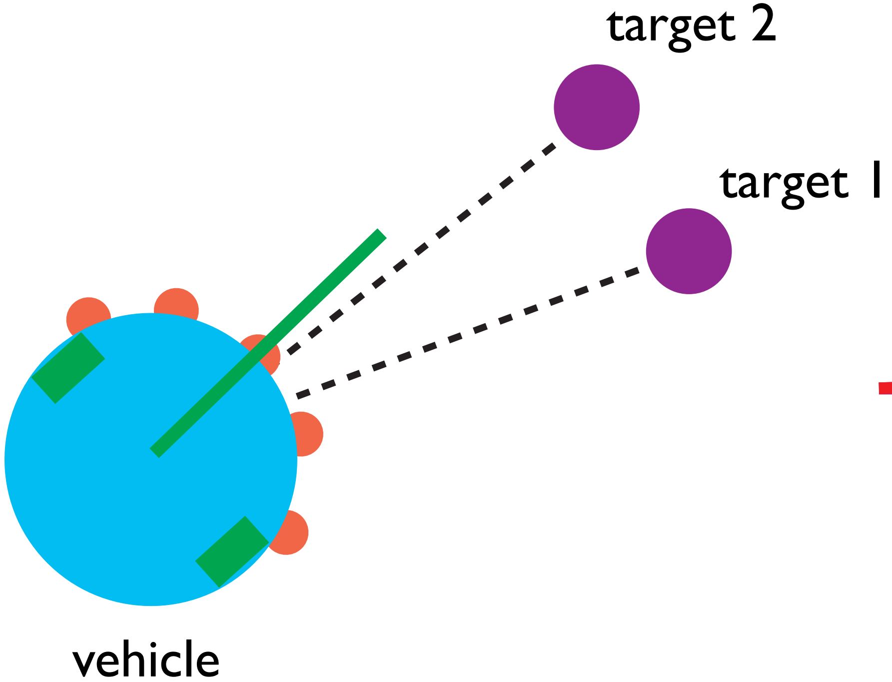
# Behavioral dynamics

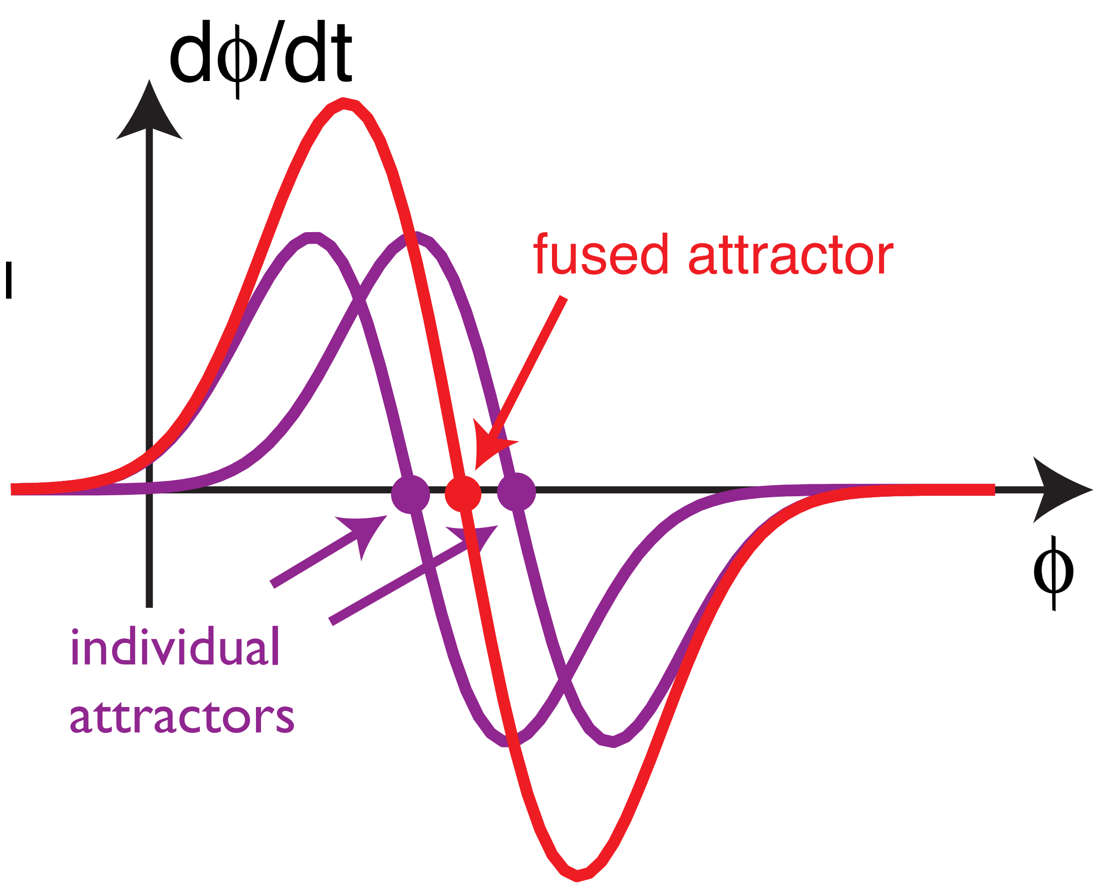
- each contribution is a "force-let" with
  - specified value
  - strength
  - range



# Behavioral dynamics

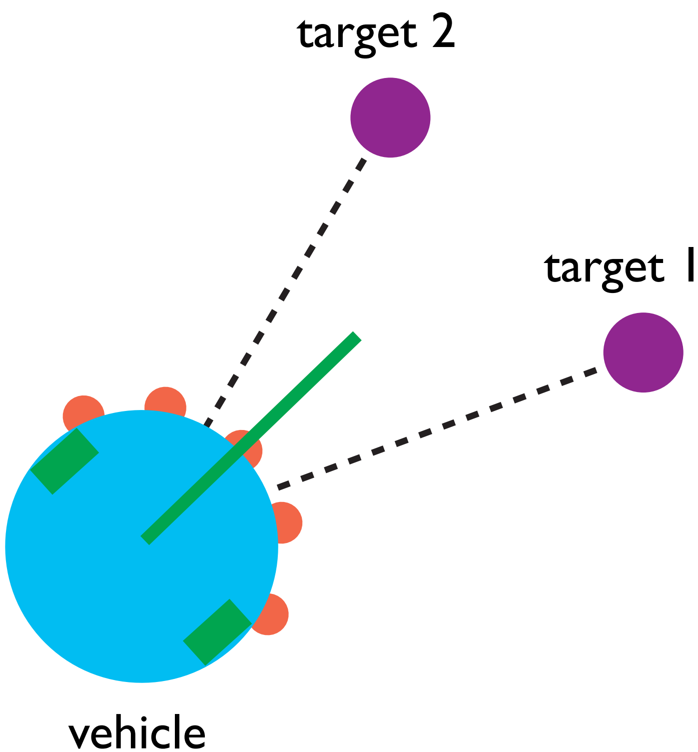
- multiple constraints: superpose "force-lets"
- e.g. fuse two target constraints

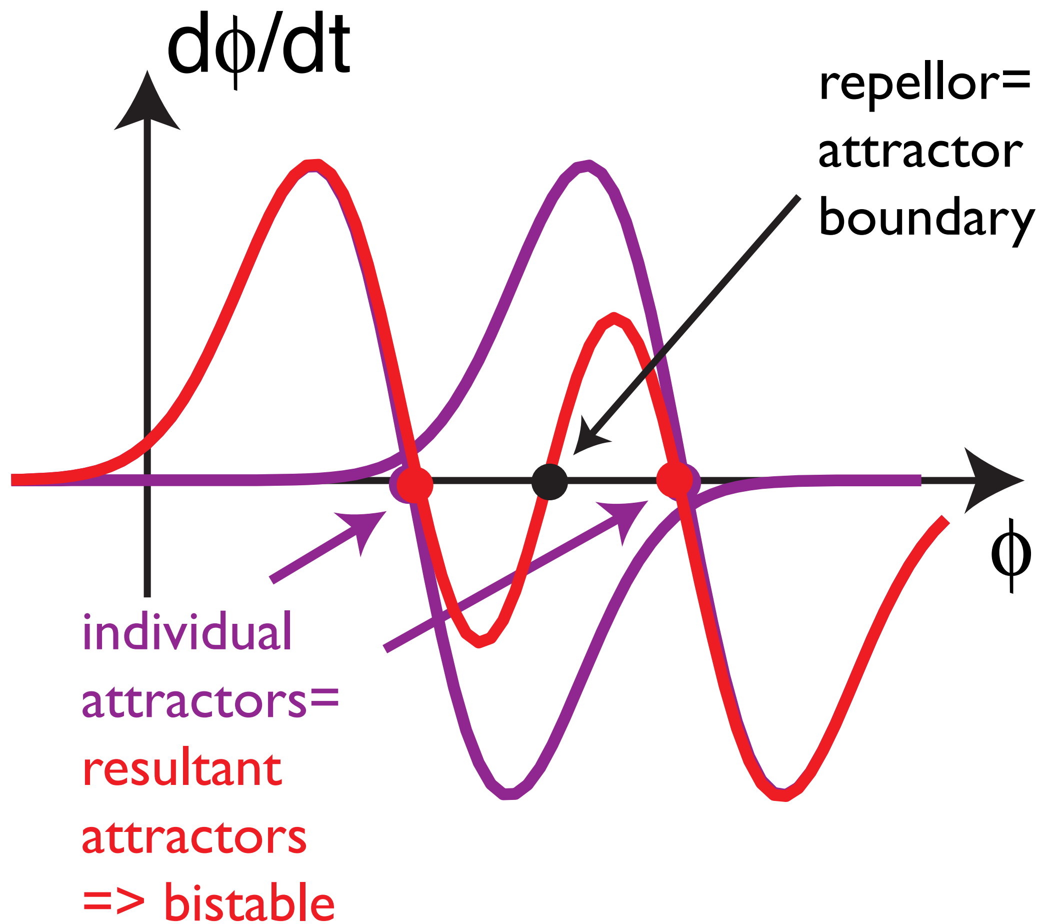


# Behavioral dynamics

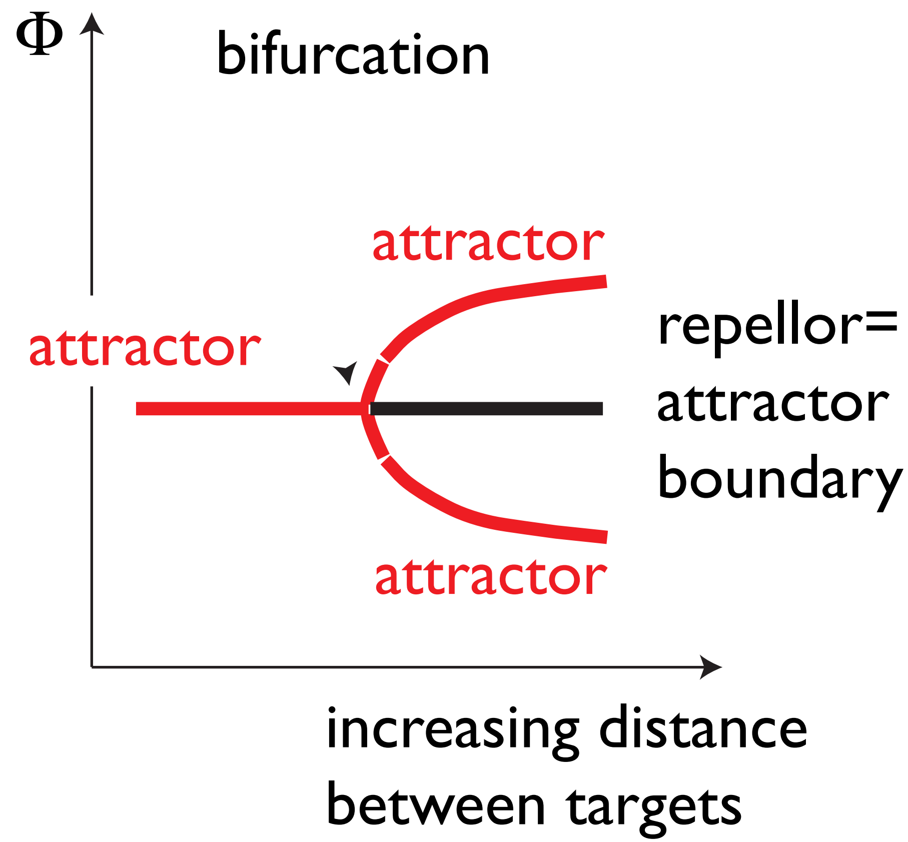
- example: select between two targets… decision



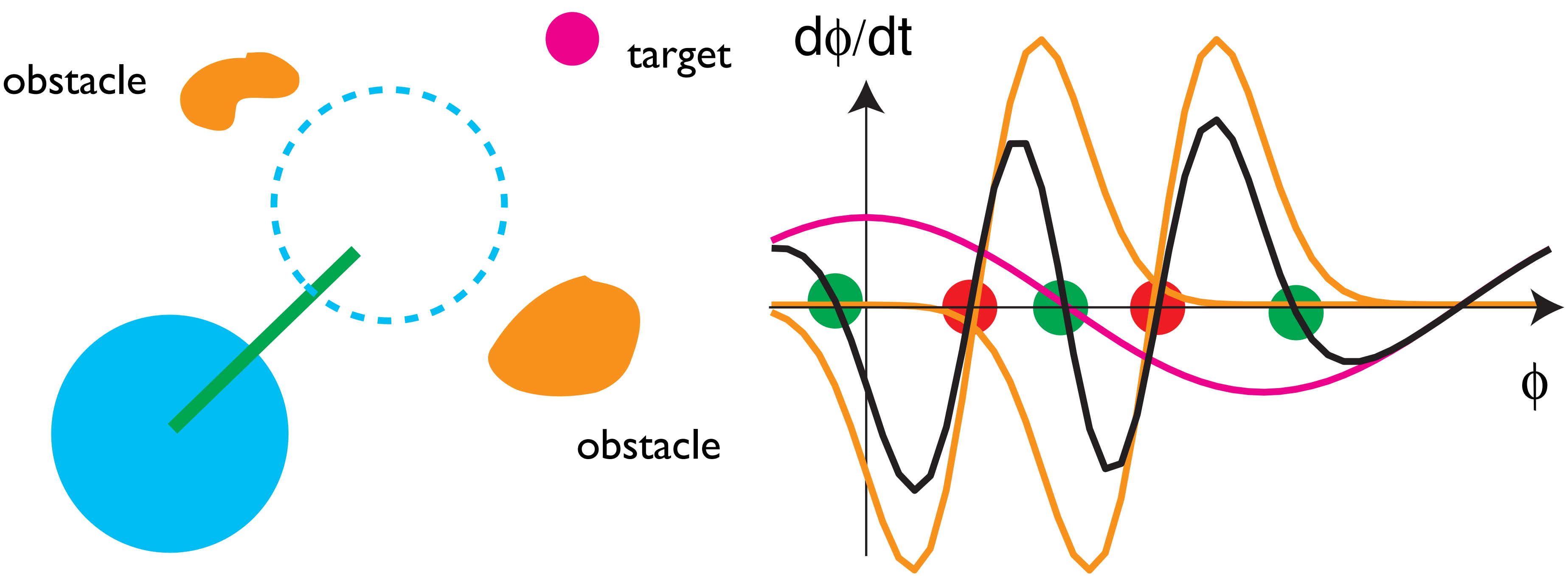
# Behavioral dynamics

- capacity to make decision comes from bifurcation



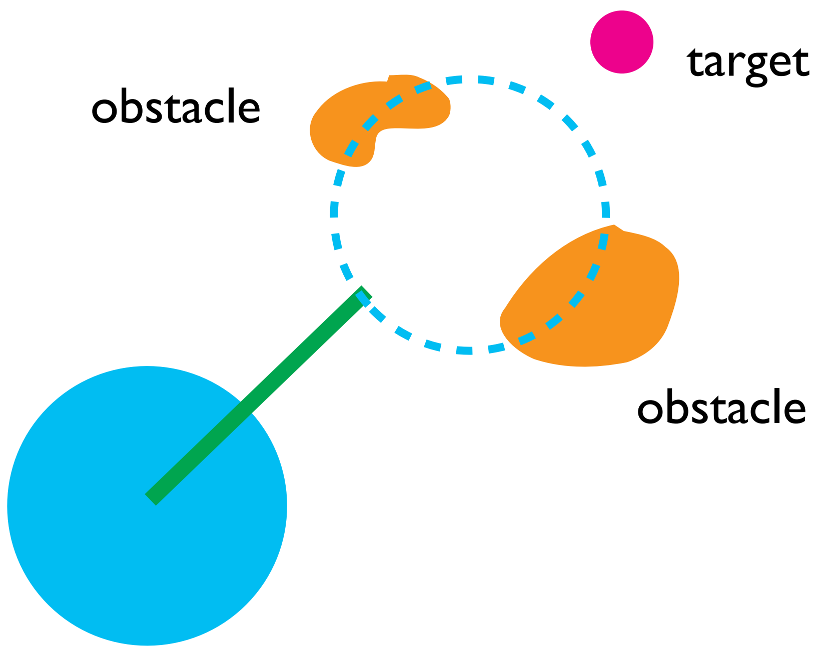
# Behavioral dynamics

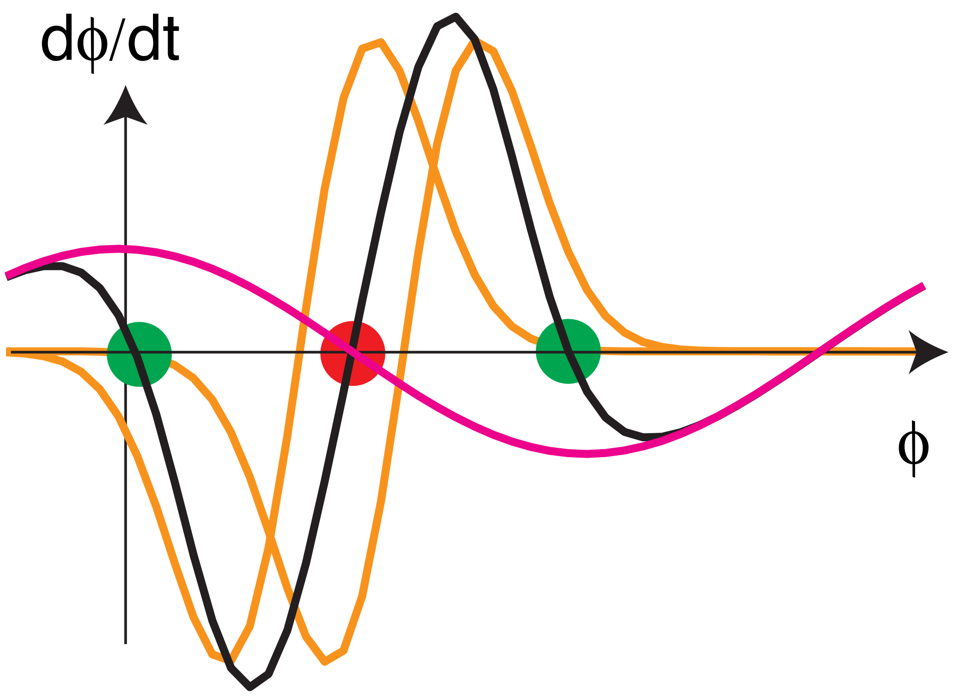
- closer to "real life": bifurcations in obstacle avoidance and target acquisition
- one regime: constraints not in conflict



# Behavioral dynamics

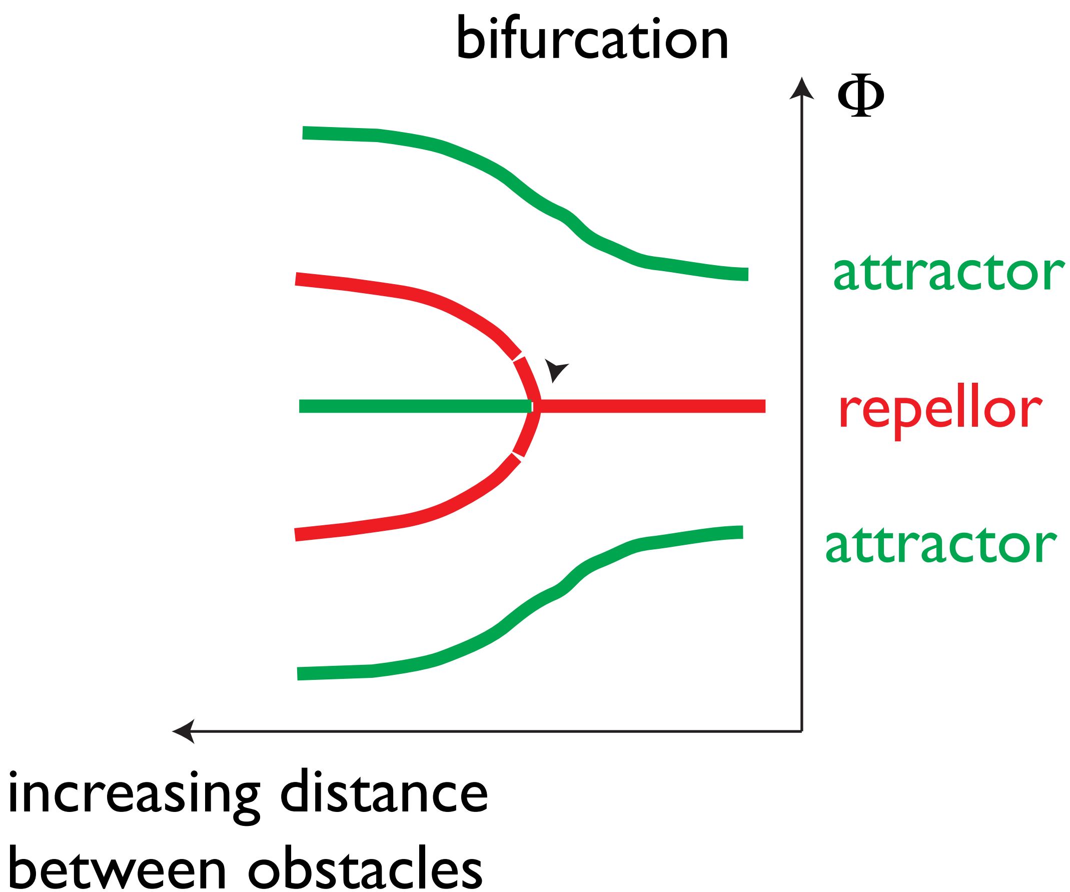
- other regime: constraints are in conflict



# Behavioral dynamics

- transition from "constraints not in conflict" to "constraints in conflict" is a bifurcation

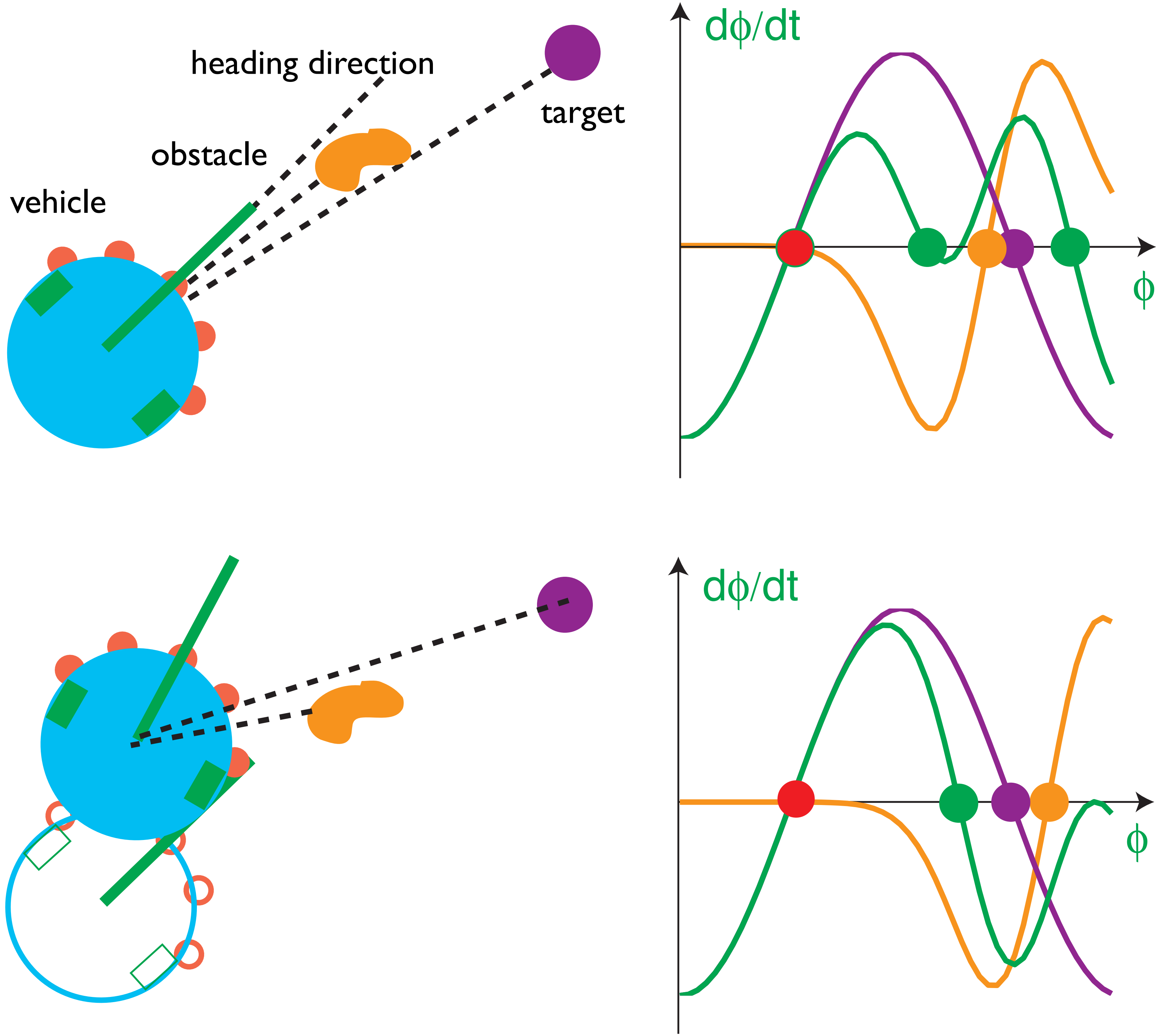


# Behavioral dynamics

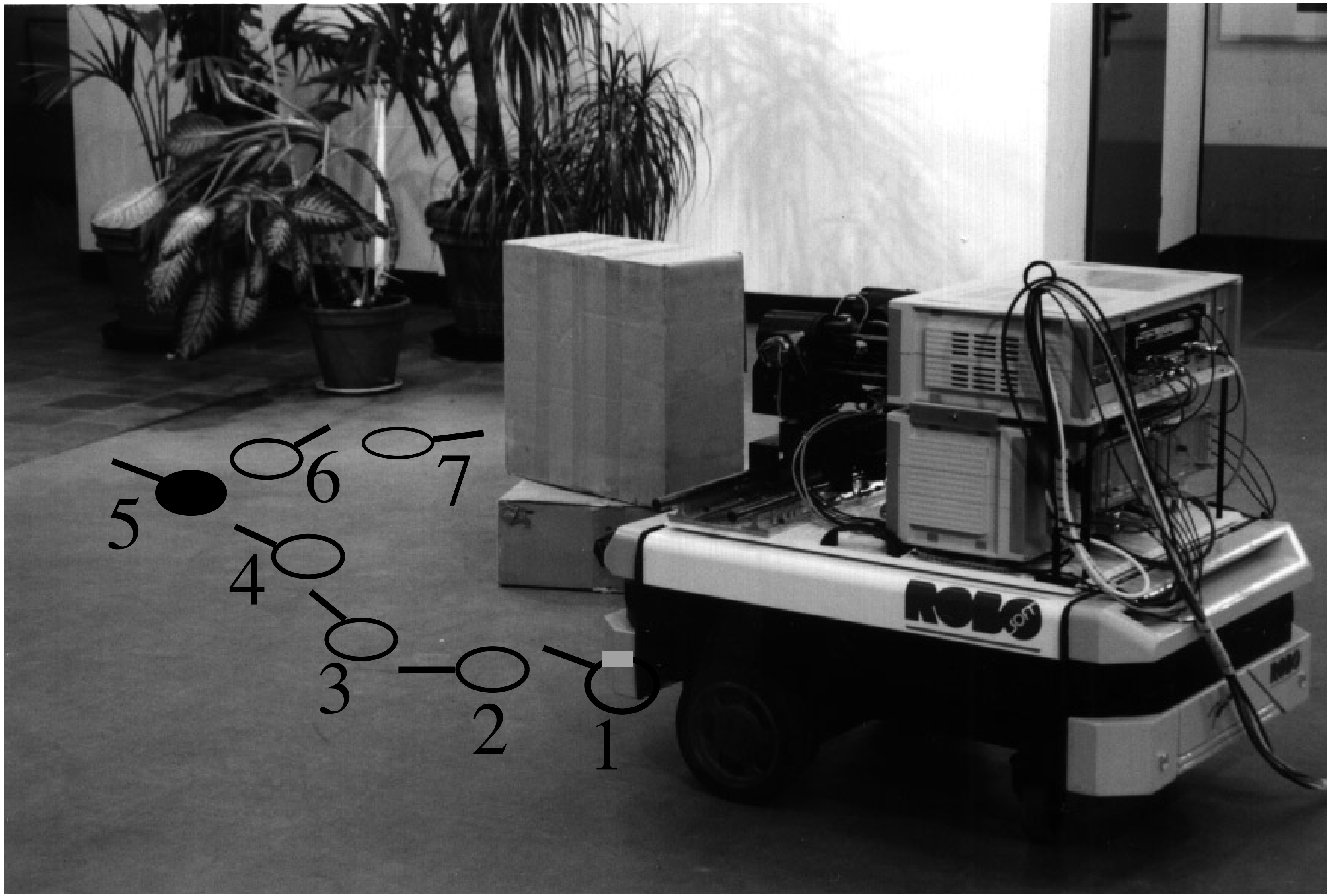
- Such design of decision making is only possible because system “sits” in attractor.
- This reduces the difficult design of the full flow (ensemble of all transient solutions) of non-linear dynamical systems to the easier design of attractors (bifurcation theory).

# Behavioral dynamics

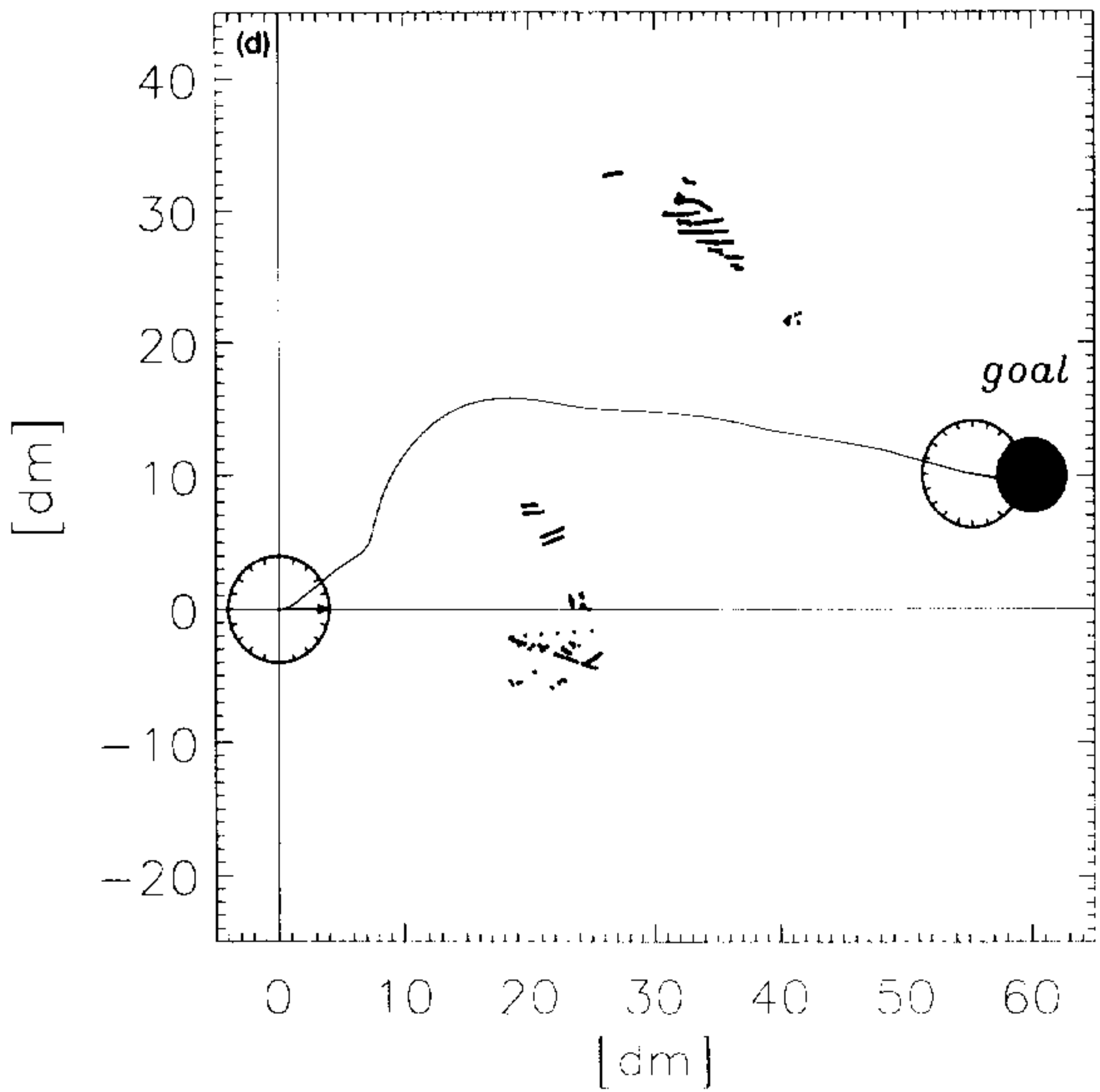
- But how may complex behavior be generated while “sitting” in an attractor?
- Answer: force-lets depend on sensory information and sensory information changes as the behavior unfolds



heading direction
target
obstacle
vehicle
dϕ/dt
ϕ
dϕ/dt
ϕ




[Schöner, Dose, 1992]




[Schöner, Dose, Engels, 1995]

# The “symbolic” approach

- “obstacles” and “targets” are objects, that have identity, preserved over time…
- implies demands on perceptual systems to deliver such objects and their parameters consistently across time
- next week we’ll look at how a “sub-symbolic” attractor dynamics approach may work directly off low-level sensory information

# Attractor dynamics model of human locomotory movement

- Fajen et al, International Journal of Computer Vision 54(1/2/3), 13–34, 2003 2003

# human locomotion

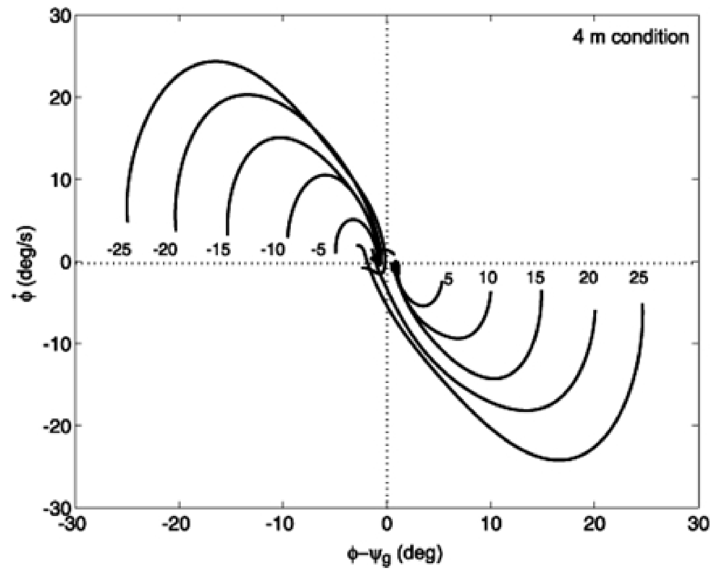
- Bill Warren and Bret Fajen have used the attractor dynamics approach to account for how humans locomote in virtual reality

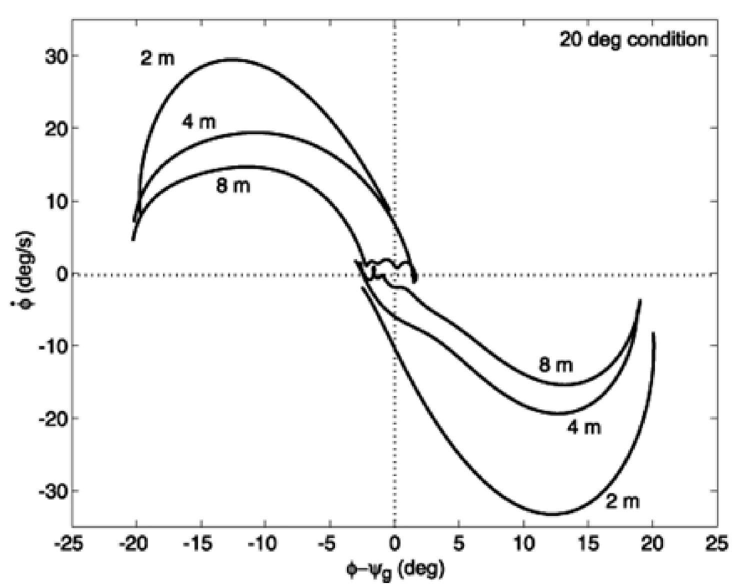
# human locomotion to goal

- participants begins to walk
- after walking 1 m, a goal appears at 5, 10, 15, 20, or 25 deg from the straight heading at a distance of 2, 4, or 8 m from participant...
- participants are asked to walk toward the goal

# human locomotion to a goal

- => turning rate increased with increasing goal angle
- => turning rate decreased with increasing distance form goal



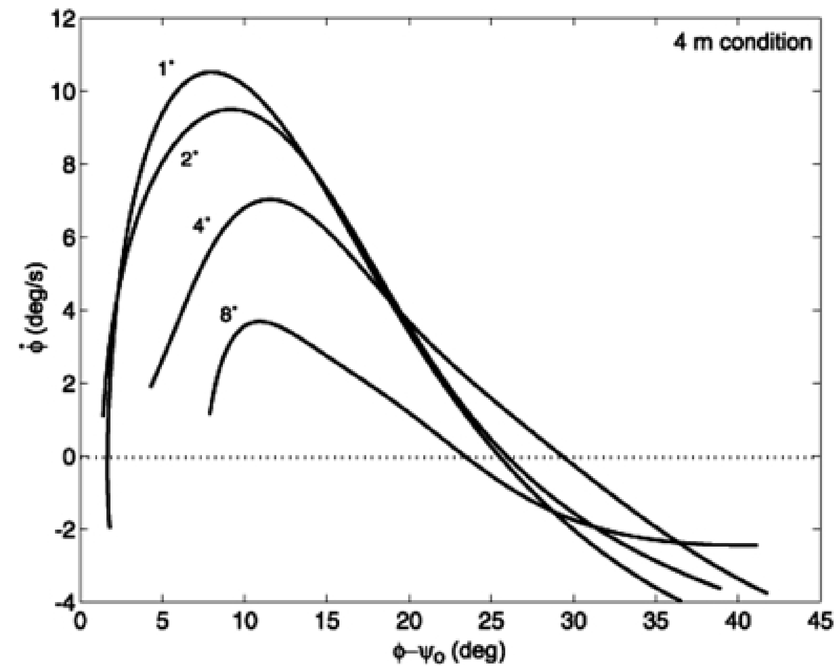



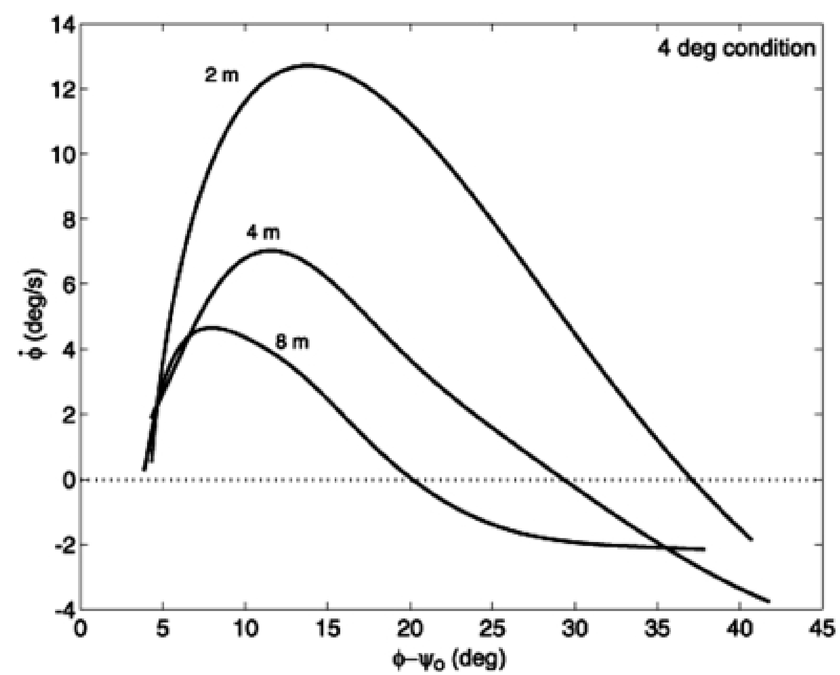
# human locomotion: obstacle

- humans walk toward goal at 10 m distance
- after walking 1 m, an obstacle appears at 1, 2, 4, or 8 deg from heading and a distance of 3, 4, or 5 m

# human locomotion: obstacle

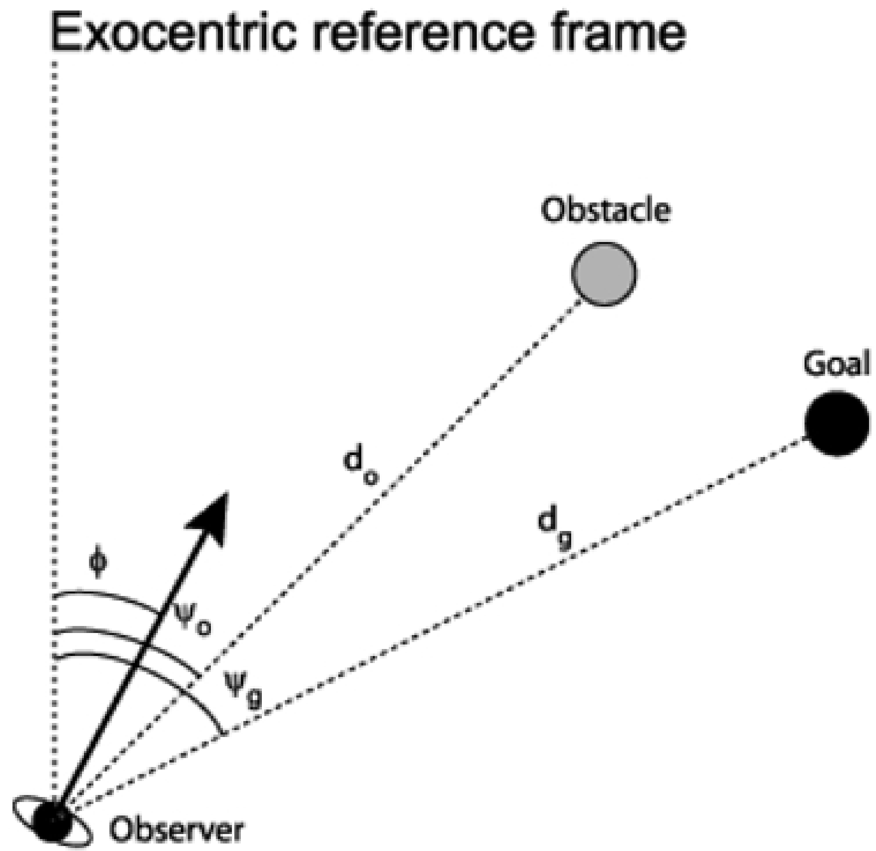
- => turning rate away from obstacle decreased with obstacle angle
- => and with obstacle distance



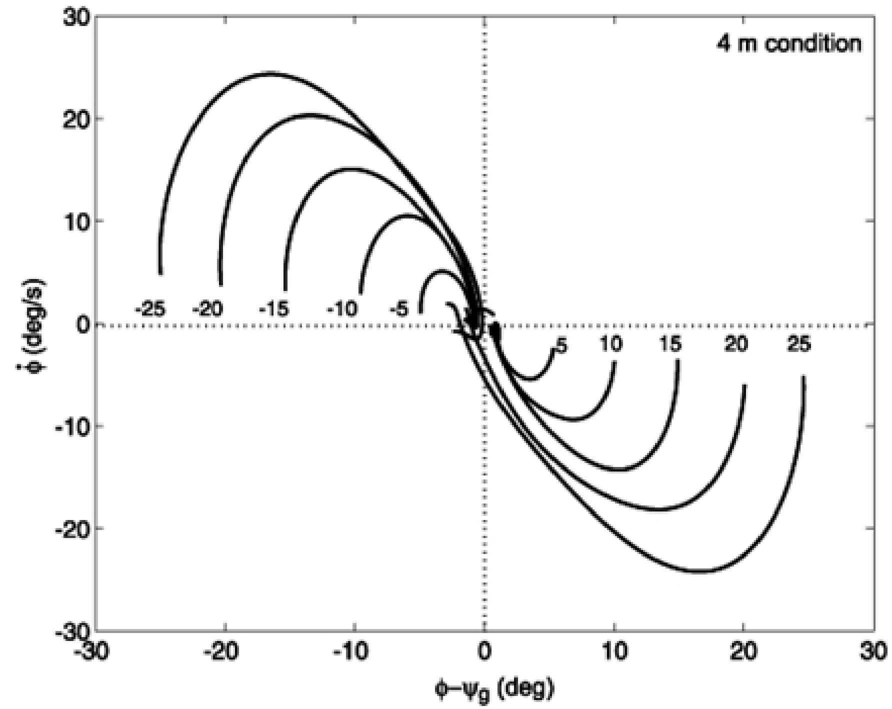
# model

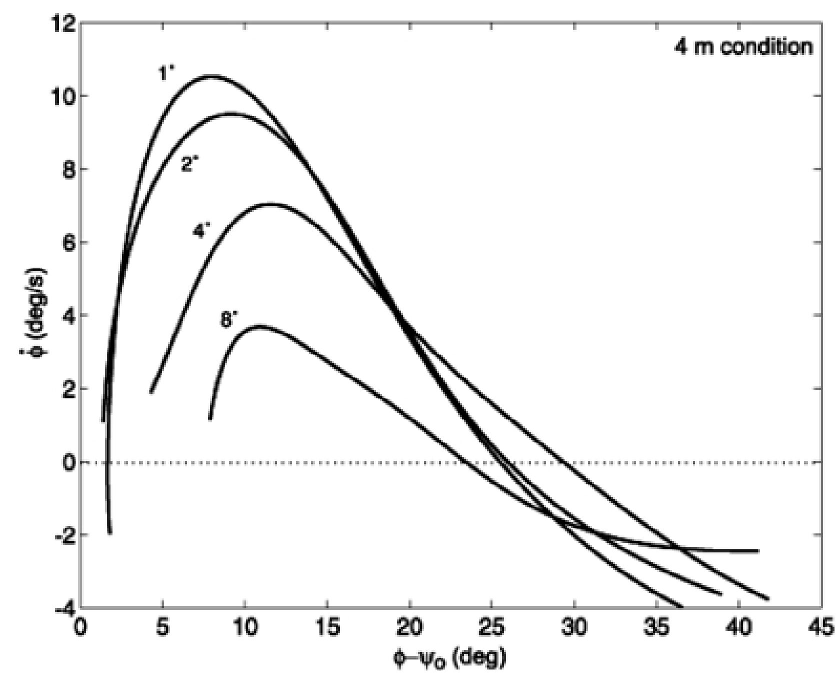
- heading direction as dynamical variable



# model

- first order dynamics dot phi = f(phi) not quite consistent with dependence on initial heading…
- but overall shape of phidot vs phi and distance dependence consistent with attractor dynamics approach to heading direction



# attractor dynamics model

- solution: 2nd order dynamics in heading

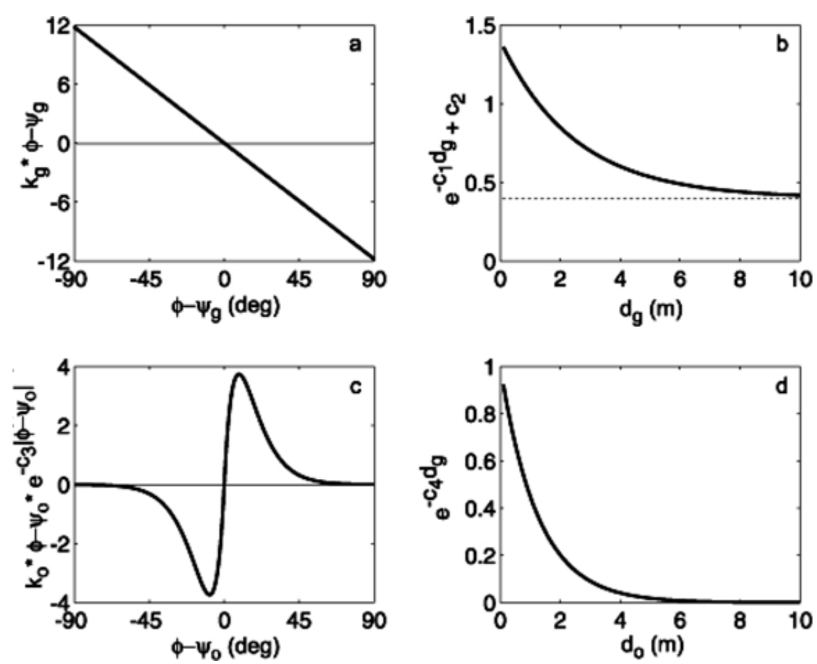
inertial term

damping term

attractor goal heading

$$\ddot{\phi} = -b\dot{\phi} - k_g(\phi - \psi_g)(e^{-c_1 d_g} + c_2) + k_o(\phi - \psi_o)\left(e^{-c_3|\phi - \psi_o|}\right)(e^{-c_4 d_o})$$

repellor obstacle heading



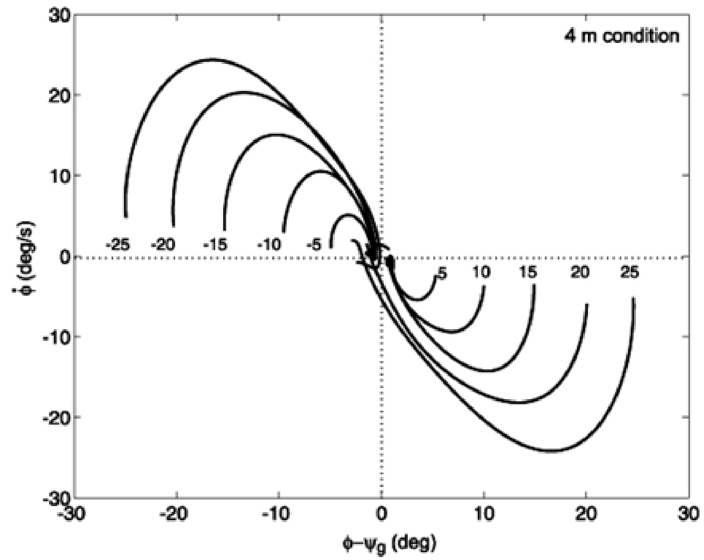
# attractor dynamics model

- approximation: inertia to zero: find first order dynamics with time scale b
- compute fixed points and stability: fixed points of first order dynamics are fixed points too and have the matching stability

$$
\begin{aligned}
\ddot{\phi} = -b\dot{\phi} &- k_g(\phi - \psi_g)(e^{-c_1 d_g} + c_2) \quad \text{attractor goal heading} \\
&+ k_o(\phi - \psi_o)\left(e^{-c_3|\phi-\psi_o|}\right)(e^{-c_4 d_o}) \quad \text{repellor obstacle heading}
\end{aligned}
$$

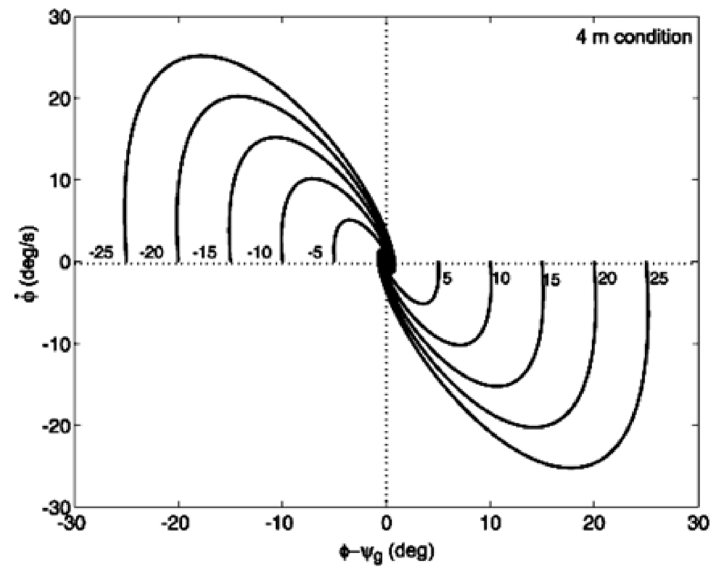
# model-experiment match: goal

## experiment

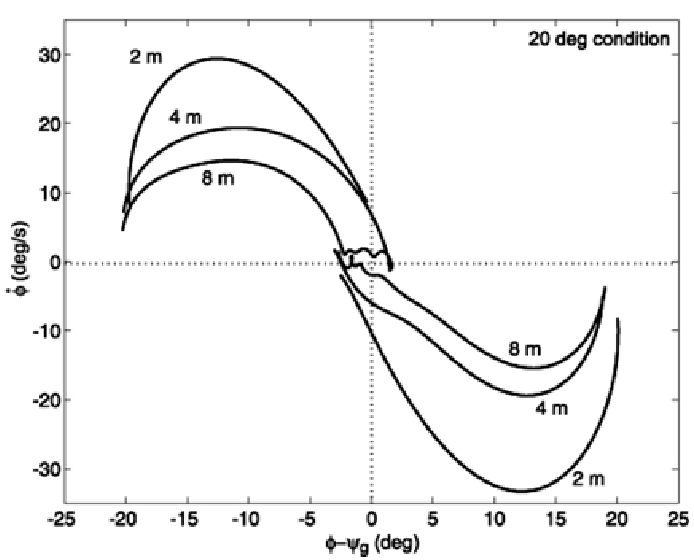



(a)

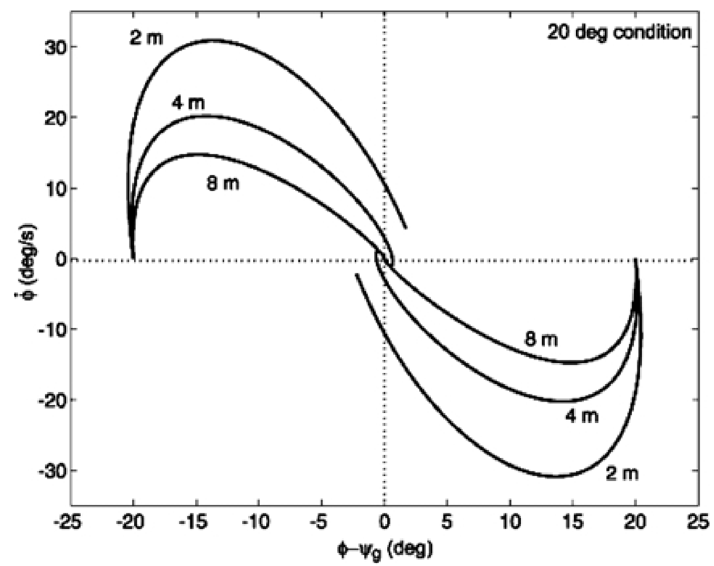
## model




(a)




(b)

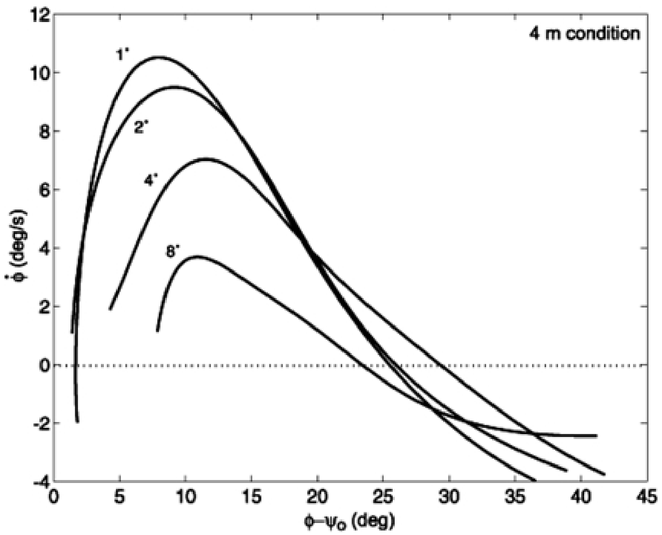



(b)

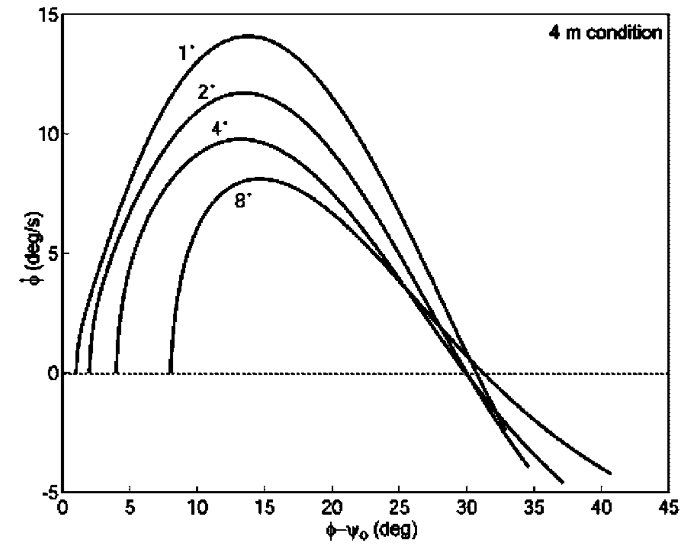
# model-experiment match: obstacle

## experiment

## model




(a)

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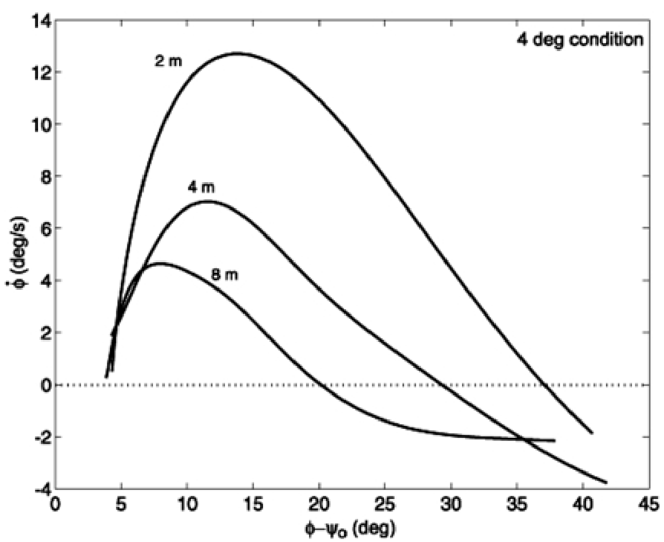


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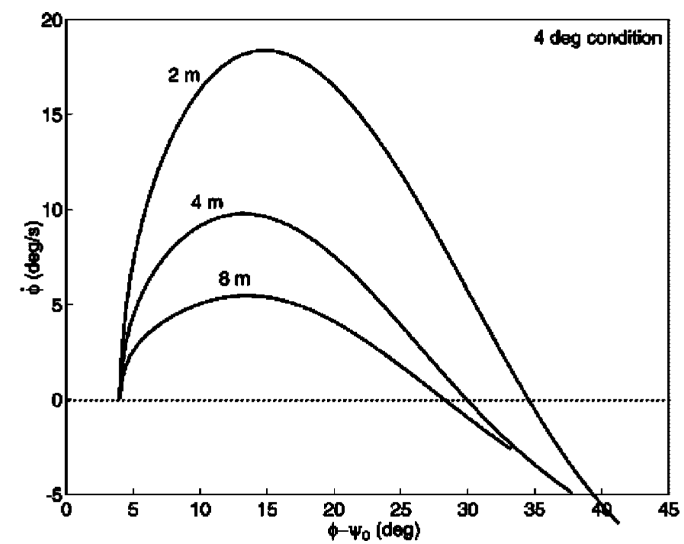
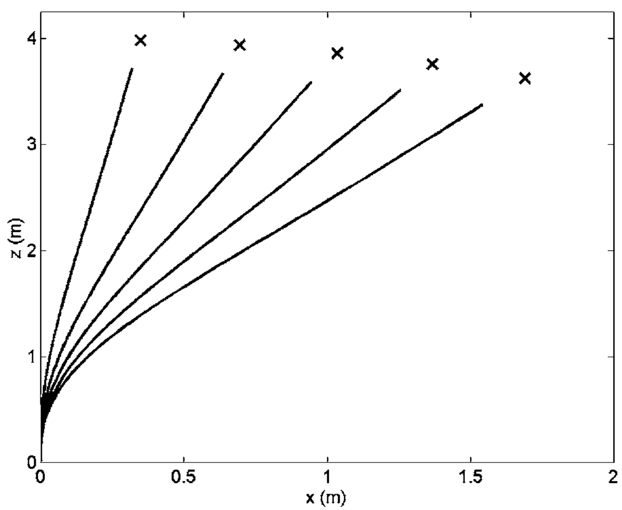



(b)

# model: paths



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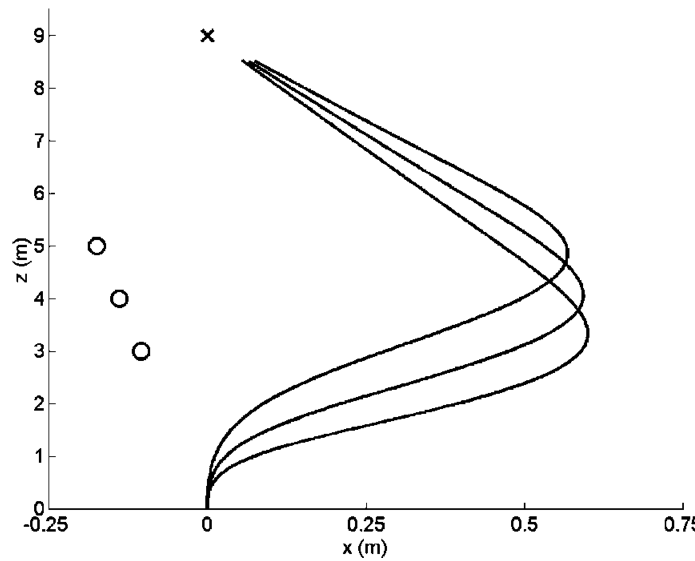


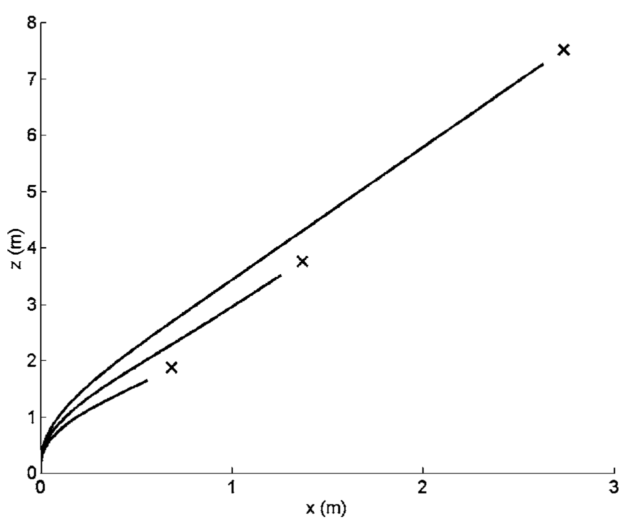

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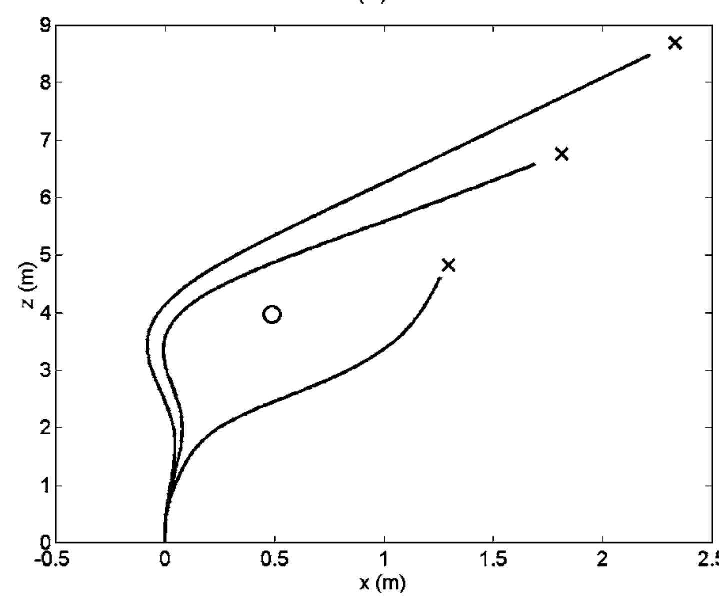
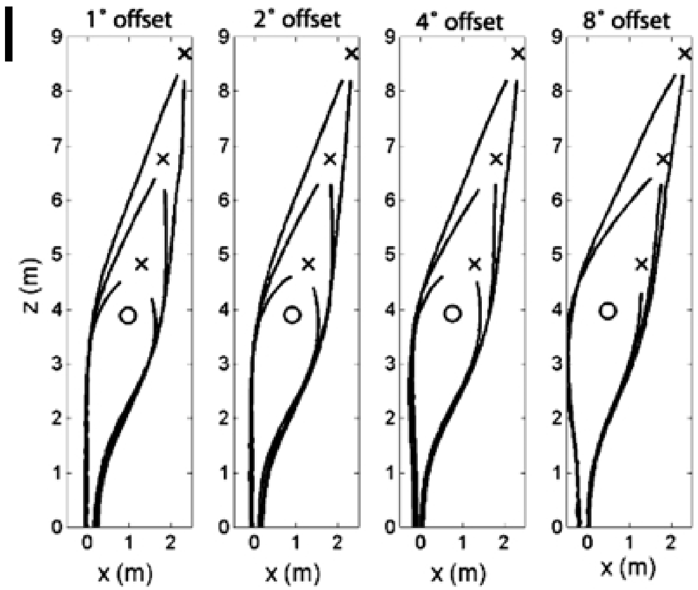


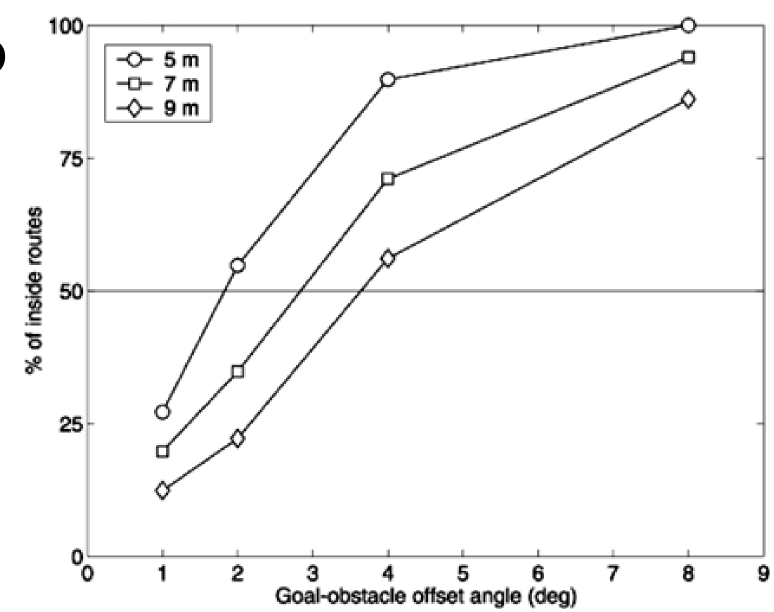
# model-exp: decision making

model




(a)

exp




(b)

- inside vs. outside path

# Conclusion

- (symbolic) attractor dynamic approach
    - plans are time courses of behavioral variables
    - generated at attractor solutions of a dynamical system
    - target and obstacle constraints (symbolic) contribute “force-lets’ to the dynamical system
    - decisions emerge from bifurcations of the attractor solutions
- the (symbolic) attractor dynamic account captures for human locomotory behavior in target acquisition and obstacle avoidance