

April 9, 2025

## Exercise 1 Dynamical Systems Tutorial

Please upload solutions on the web page before midnight on April 16, 2025 (Wednesday).

Please look through the slides of the dynamical systems tutorial (on the webpage). The first 10 pages are critical for this exercise, but you also need pages 21-23. We will come back to later parts in the next lecture/exercise.

1. Solve the linear differential equation

$$\dot{x} = -\alpha x \tag{1}$$

analytically. If unfamiliar, get help from a text book or online resource.

2. Plot the solution as a function of time by setting  $\alpha = 1/\text{second}$ , using a few discrete times,  $t = 0, 1, 2, \dots, 10$  seconds. Or evaluate the function numerically on the computer in a program of your choice. Make this plot for two values of the initial condition,  $x(0)$ .
3. From the analytical solution, determine the moment in time,  $\tau$  at which  $x(t)$  has fallen to  $1/e$  of its initial value,  $x(0)$ . Does this time depend on  $x(0)$ ? Answer the same question for the hyperbolic decay,  $x(t) = x(0)/(1 + t)$ .
4. Plot this dynamics Eq (1) and designate the fixed point. Make the same drawing for a dynamics with the same slope and a fixed point at  $x_0 > 0$ . Write down the equation for that dynamics.
5. Make a plot of this nonlinear differential equation

$$\dot{x} = \beta - x^2. \tag{2}$$

Compute its fixed points as a function of  $\beta$  and plot this function. Mark the fixed points in the dynamics as attractors or repellers.