DFT Foundations 1: Space and Time (part 2)

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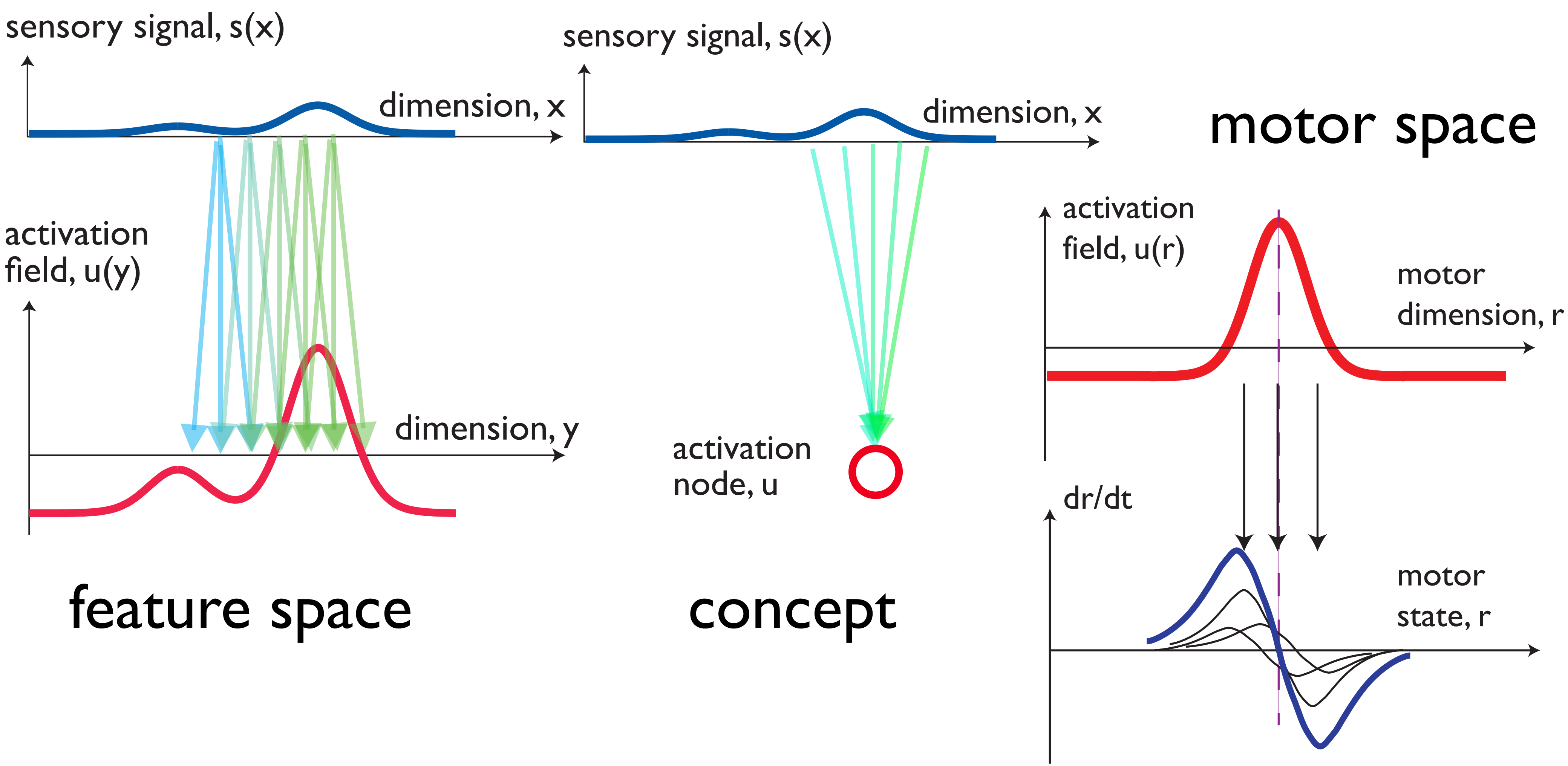
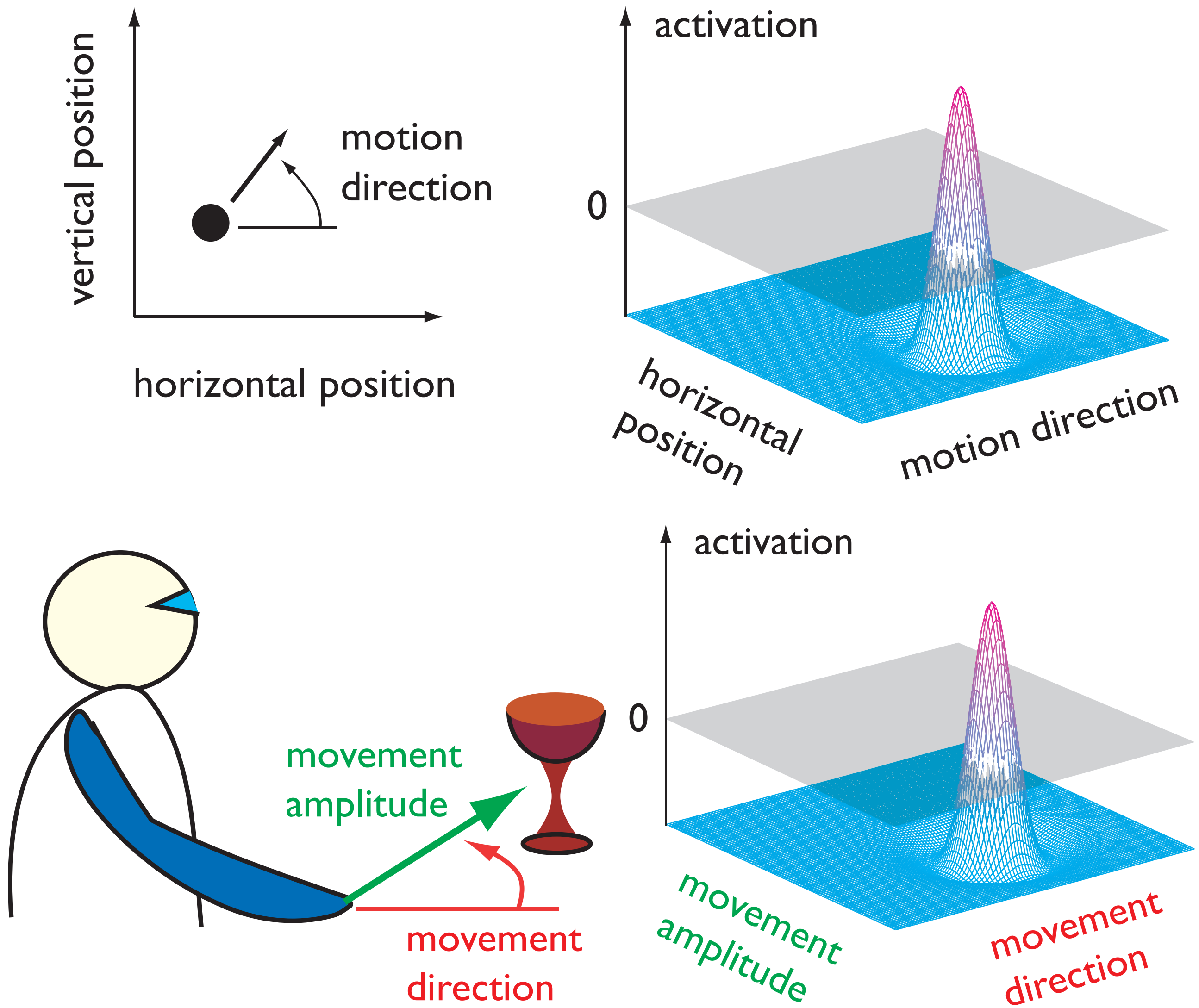
Recall…

Spaces arise through connectivity

- from sensory surfaces / to motor surfaces



=> mental states are localized in these low-dimensional spaces

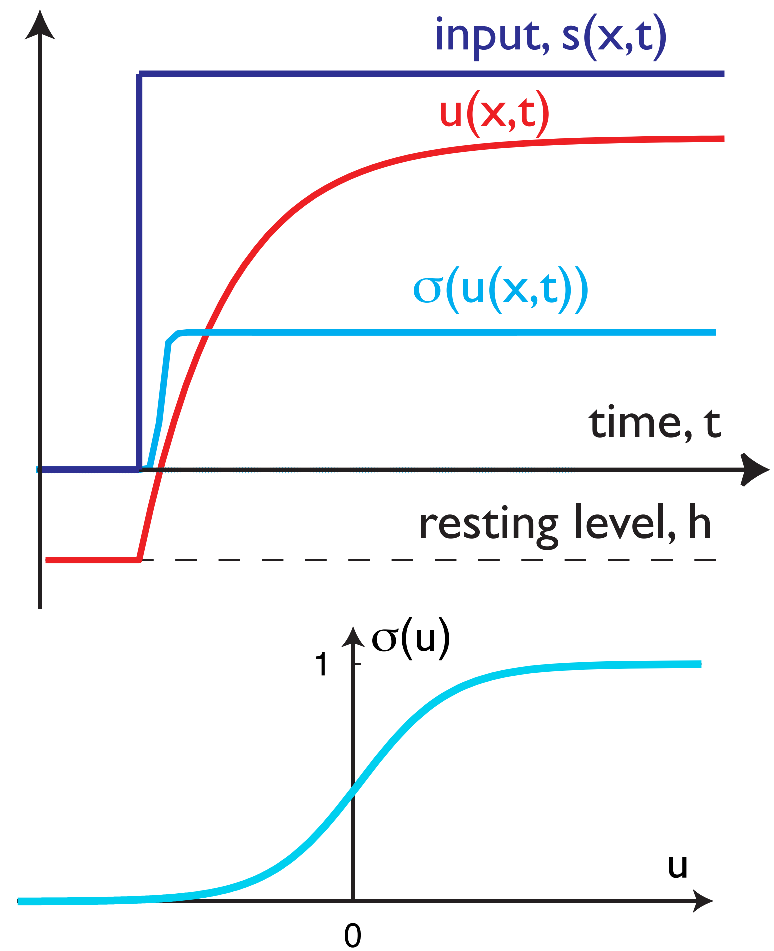


~ Gärdenfors

Neural dynamics

- activation u ~ population level membrane potential
- defined relative to sigmoid
  - above threshold: transmitted
  - below threshold: not transmitted

$$\tau \dot{u}(x,t) = -u(x,t) + h + s(x,t)$$

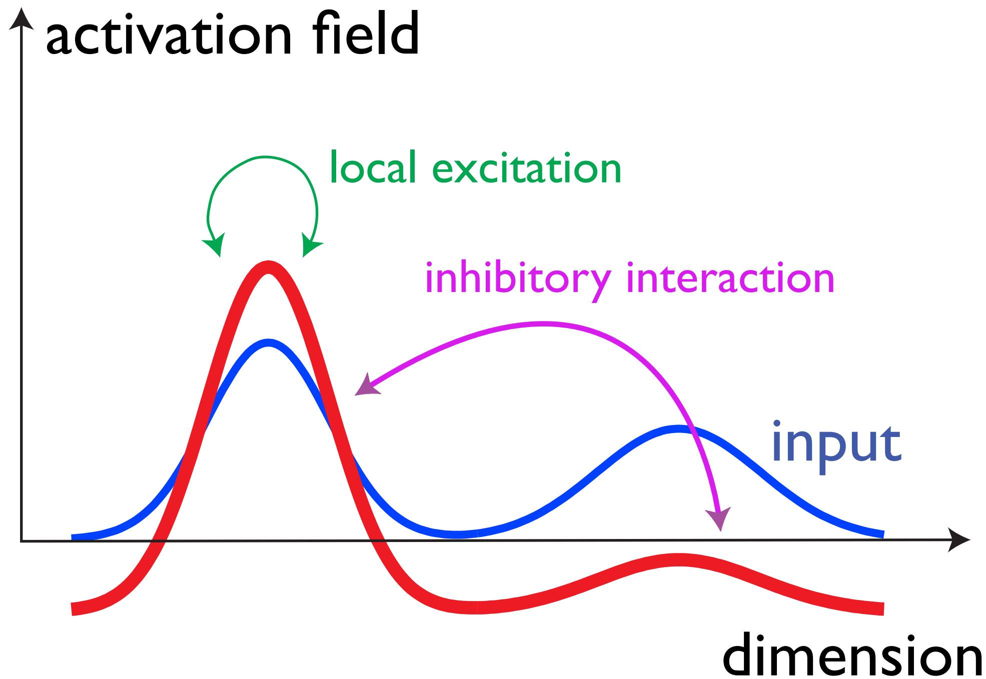


Neural dynamics with strong interaction

$$\tau\dot{u}(x,t) = -u(x,t) + h + s(x,t) + \int w(x-x')\sigma(u(x',t))dx'$$

interaction

- strong recurrent connectivity within populations
- excitatory for neighbors in space
- inhibitory for activation at a spatial distance



Attractors and their instabilities

- input driven solution (sub-threshold)
- self-stabilized solution (peak, supra-threshold)
- selection / selection instability
- working memory / memory instability
- boost-driven detection instability

↓ detection instability

↑ reverse detection instability

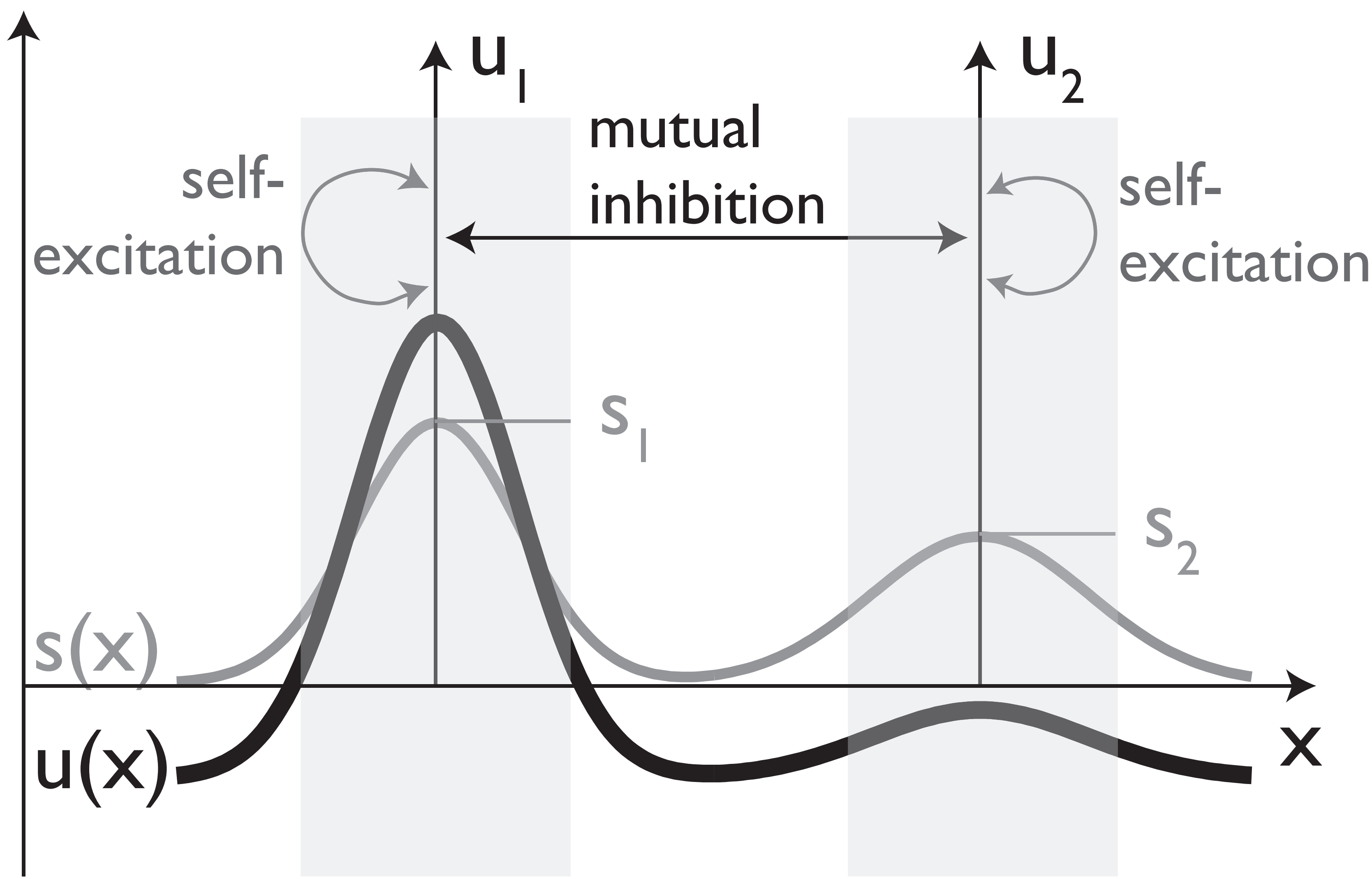
Noise is critical near instabilities

Goal: understanding the neural dynamics of fields more deeply

Foundations 1: Space and time

- Discretization of fields
- Self-excitation
- Inhibitory interaction
- Mathematical formalization
- … beyond 1D fields

Analysis for discrete activation variables



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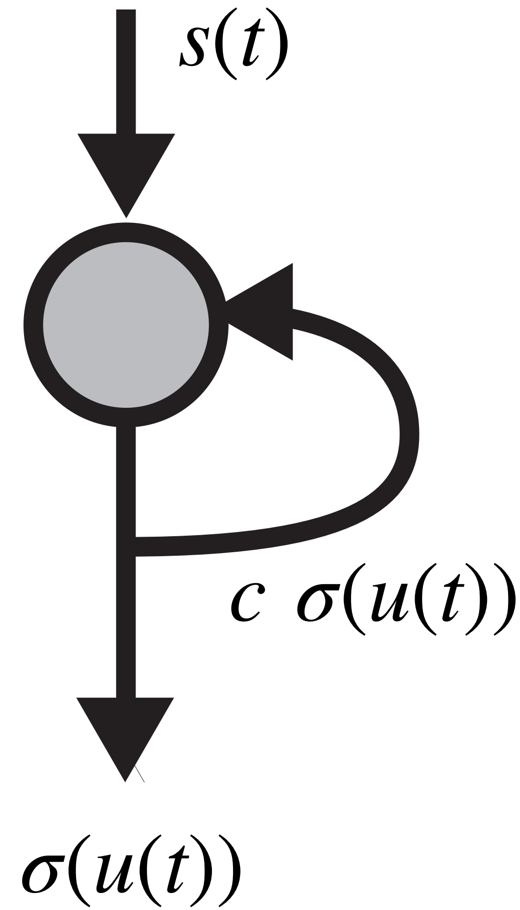
Excitatory interaction = self-excitation

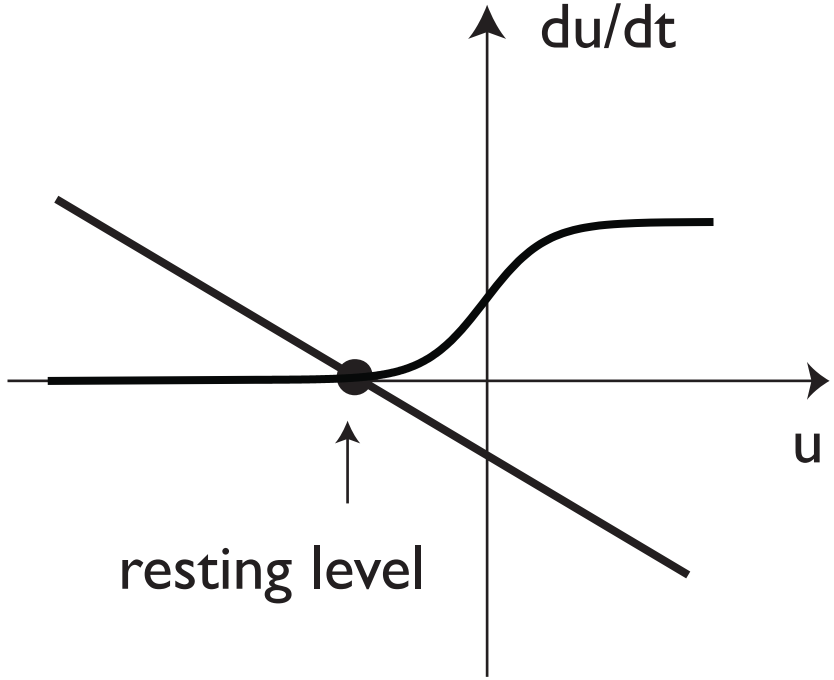
- a minimally recurrent network
- illustrates that "time" is conceptually necessary to understand these:
  - some inputs are outputs from the same neuron/population …
  - => not possible to frame as input-ouput systems
  - solution: time: past outputs are current inputs

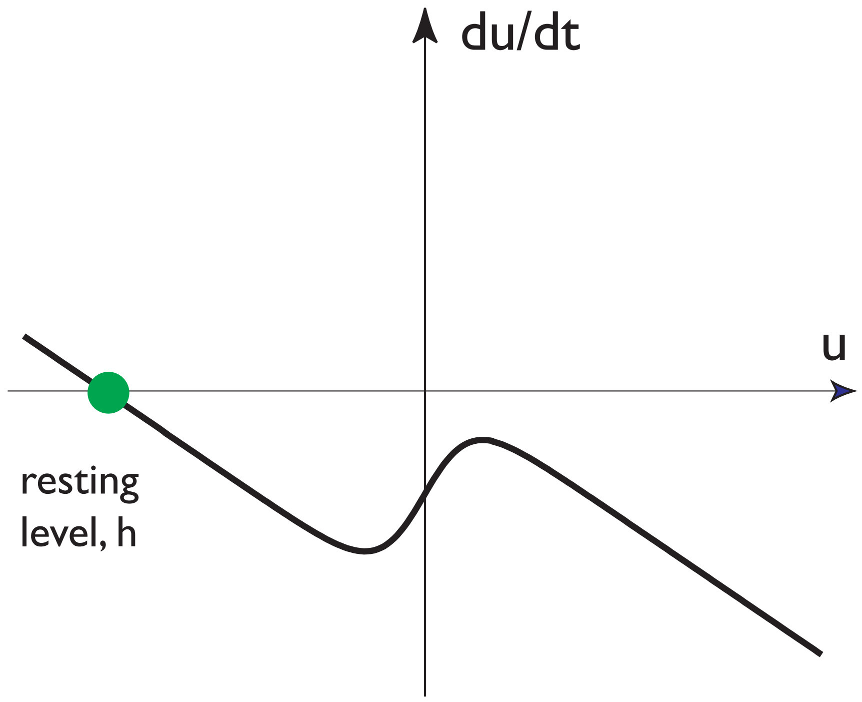


$$\tau \dot{u}(t) = -u(t) + h + s(t) + c\ \sigma(u(t))$$

Neuronal dynamics with self-excitation

- nonlinear dynamics!

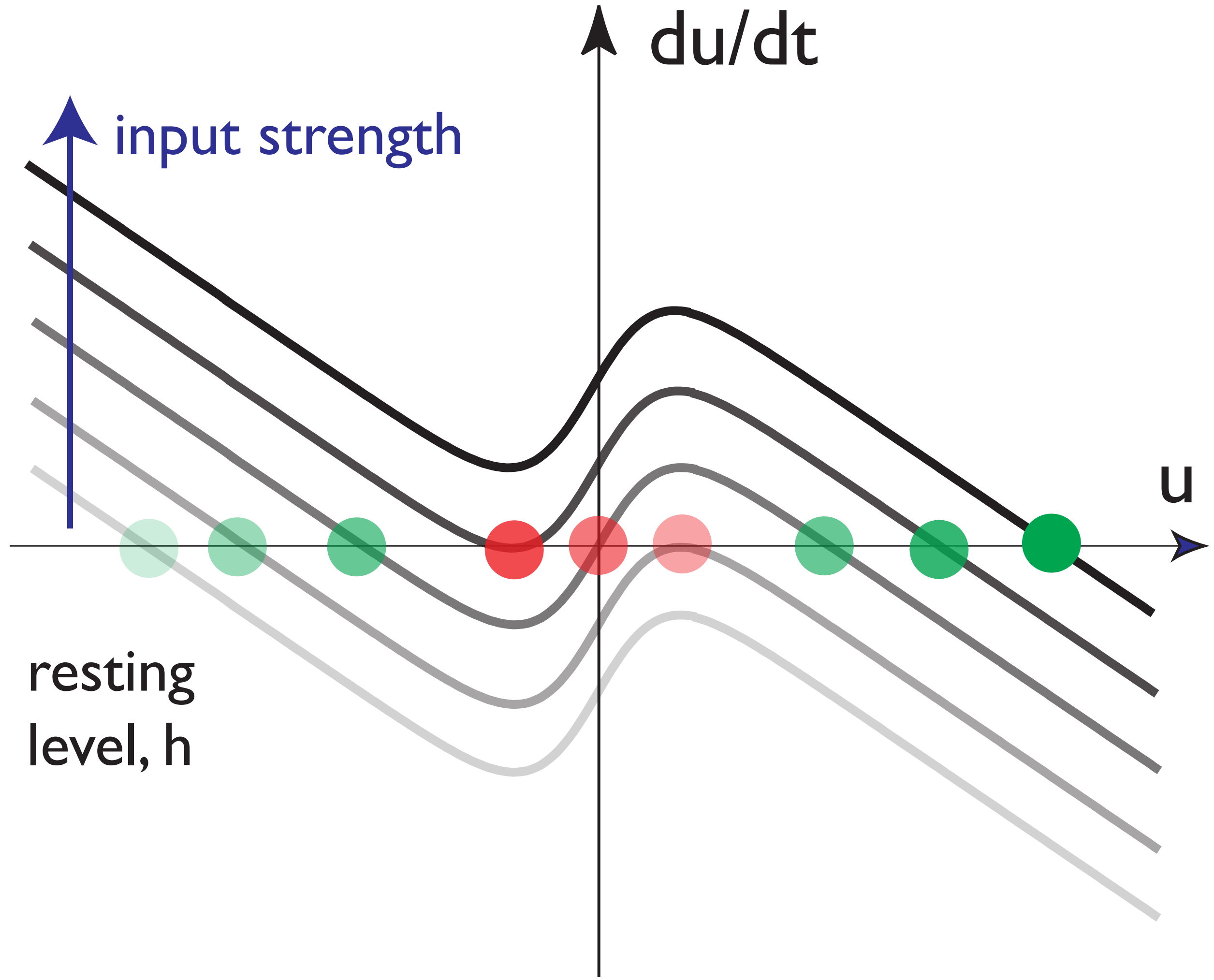




$$\tau \dot{u}(t) = -u(t) + h + s(t) + c\ \sigma(u(t))$$

Neuronal dynamics with self-excitation

- varying input
- => number of attractors changes



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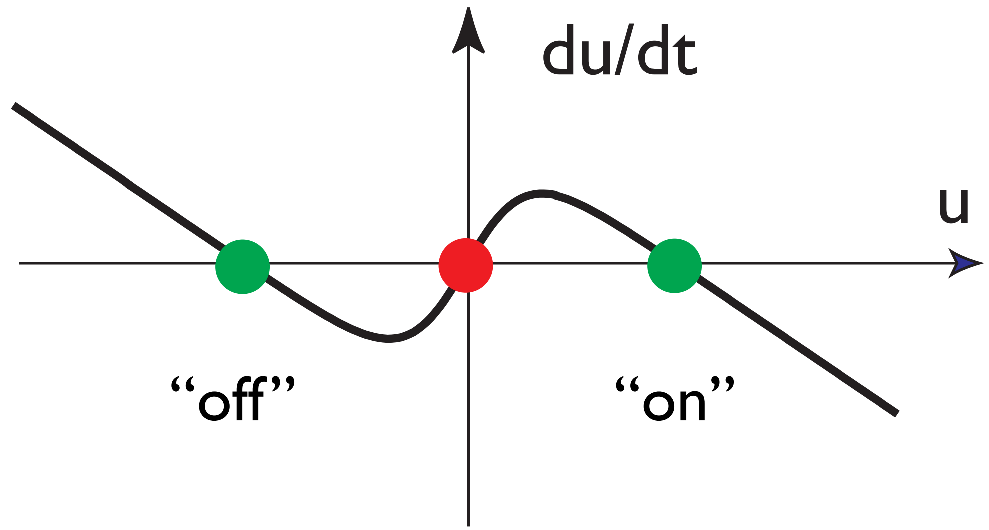
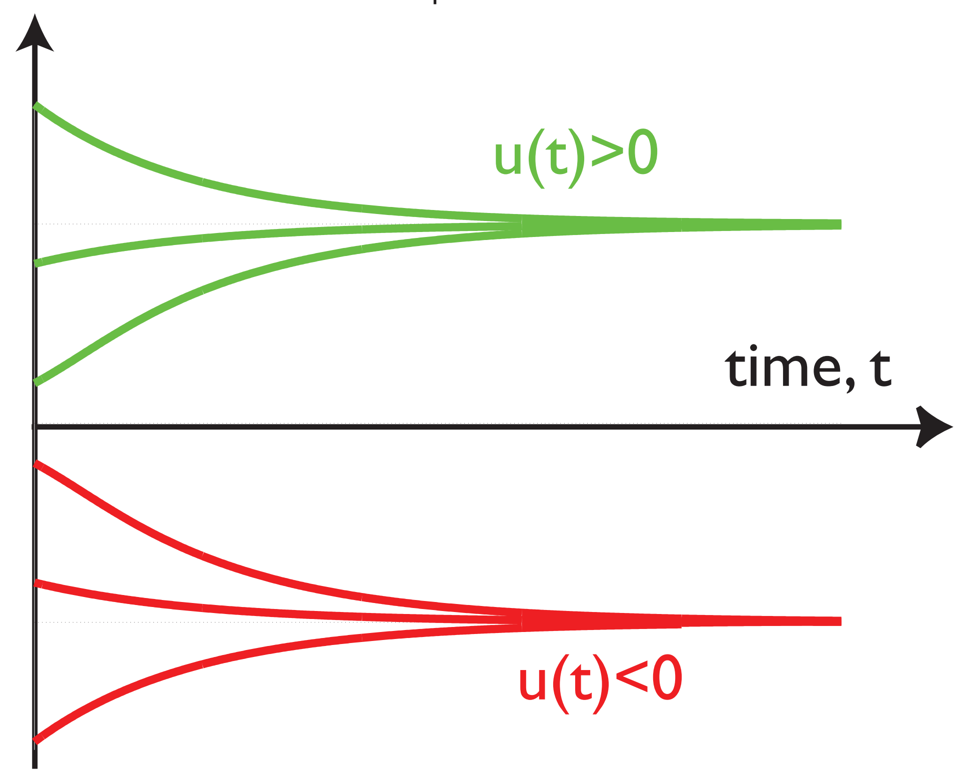
Neuronal dynamics with self-excitation



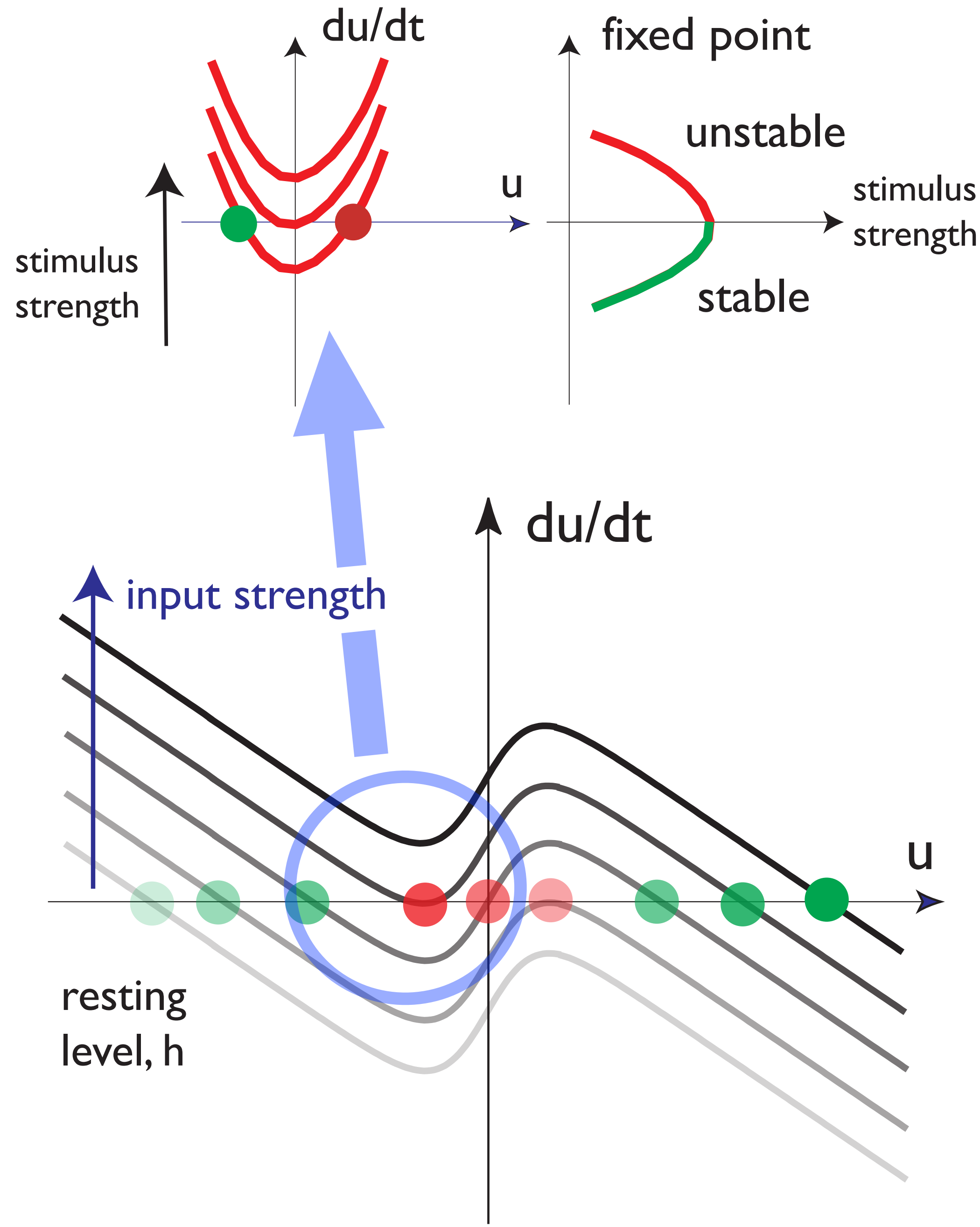
- at intermediate input levels: bistable dynamics
- "on" vs "off" state



$$\tau \dot{u}(t) = -u(t) + h + s(t) + c\ \sigma(u(t))$$

Neuronal dynamics with self-excitation

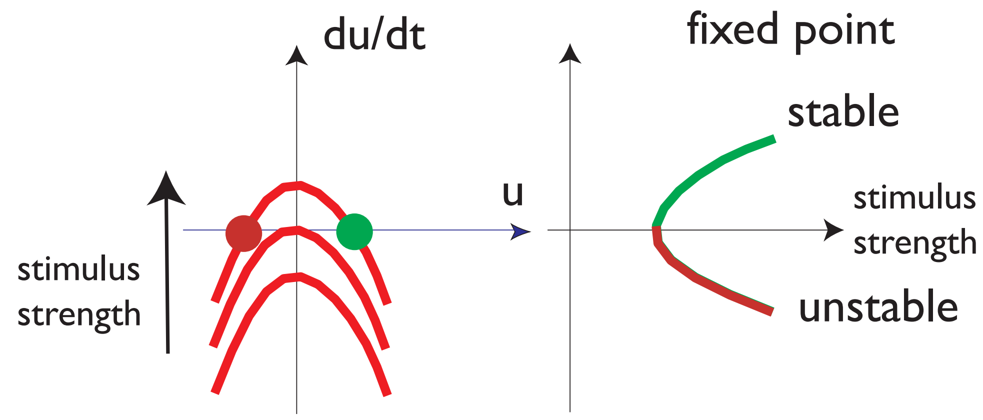
- increasing input strength => detection instability

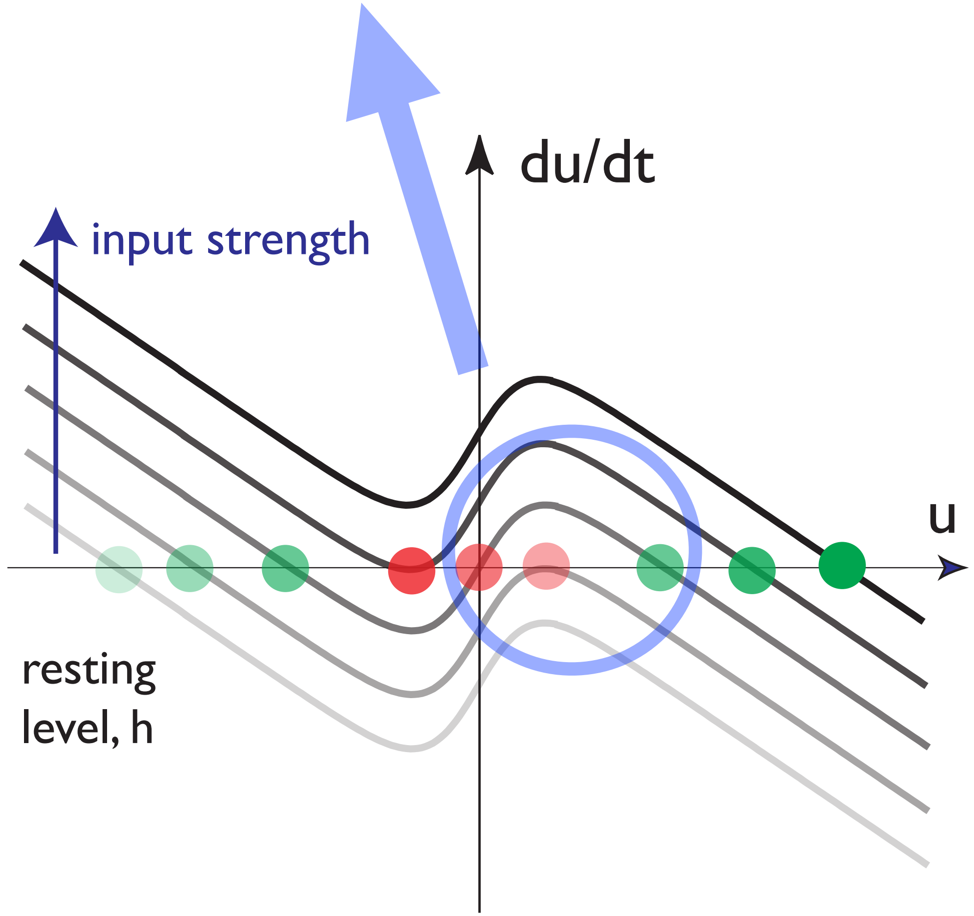


$$\tau\dot{u}(t) = -u(t) + h + s(t) + c\ \sigma(u(t))$$

Neuronal dynamics with self-excitation

- decreasing input strength => reverse detection instability

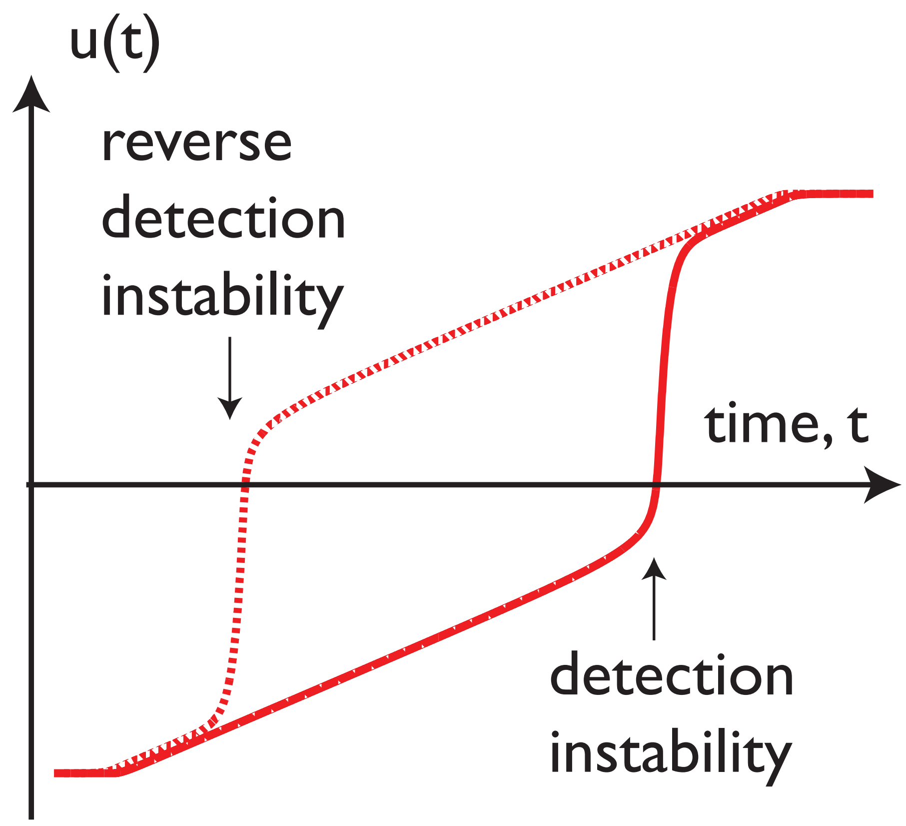




$$\tau \dot{u}(t) = -u(t) + h + s(t) + c\ \sigma(u(t))$$

Neuronal dynamics with self-excitation

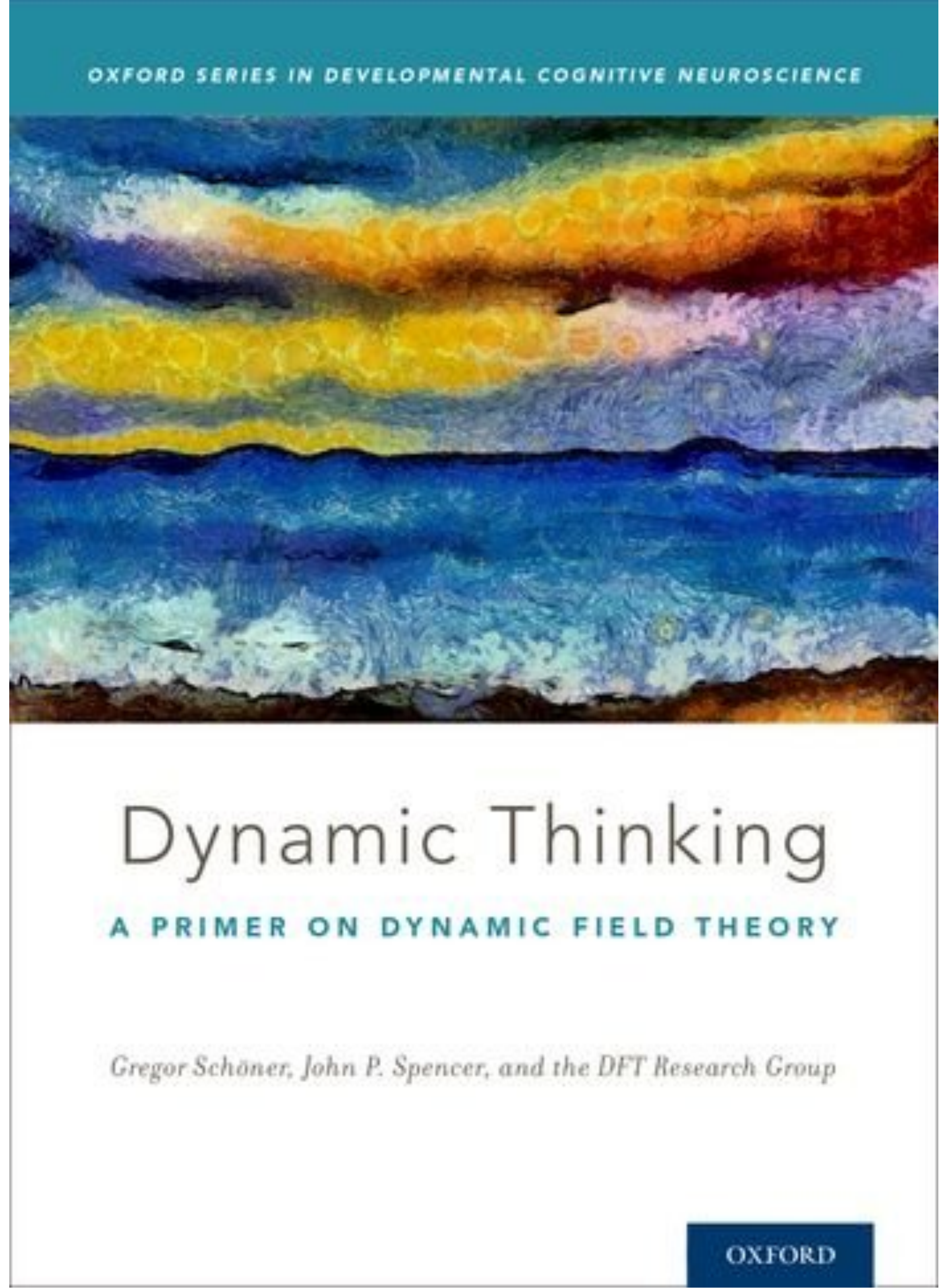
- the detection and its reverse create events at discrete times from time-continuous changes



$$\tau \dot{u}(t) = -u(t) + h + s(t) + c\ \sigma(u(t))$$

simulating discrete activation variables with self-excitation

- dynamicfieldtheory.org



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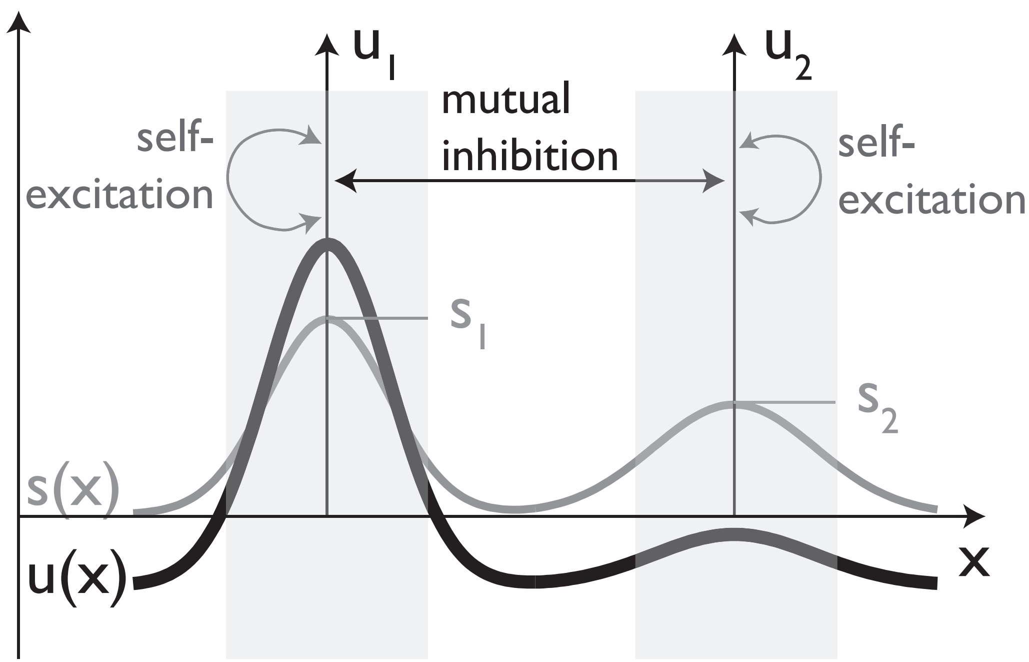
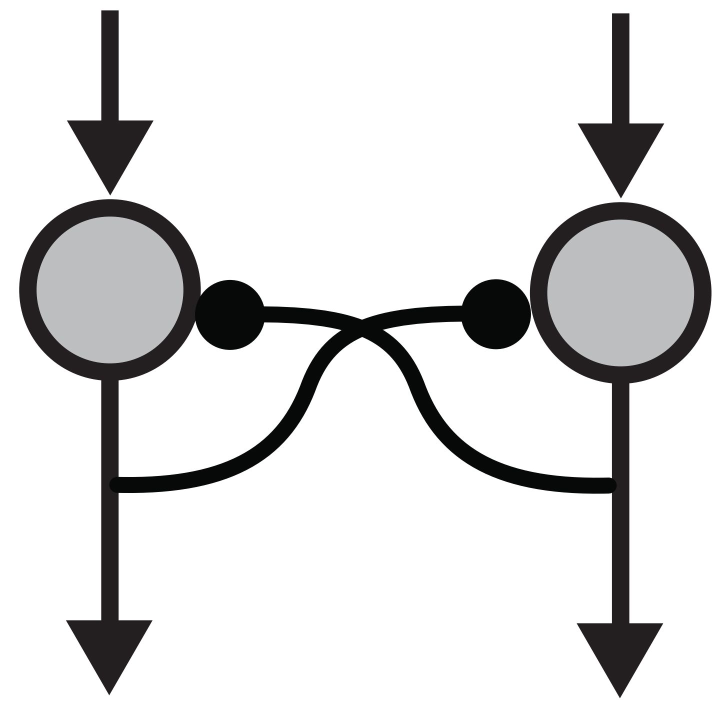
Inhibitory interaction: inhibitory recurrent connectivity





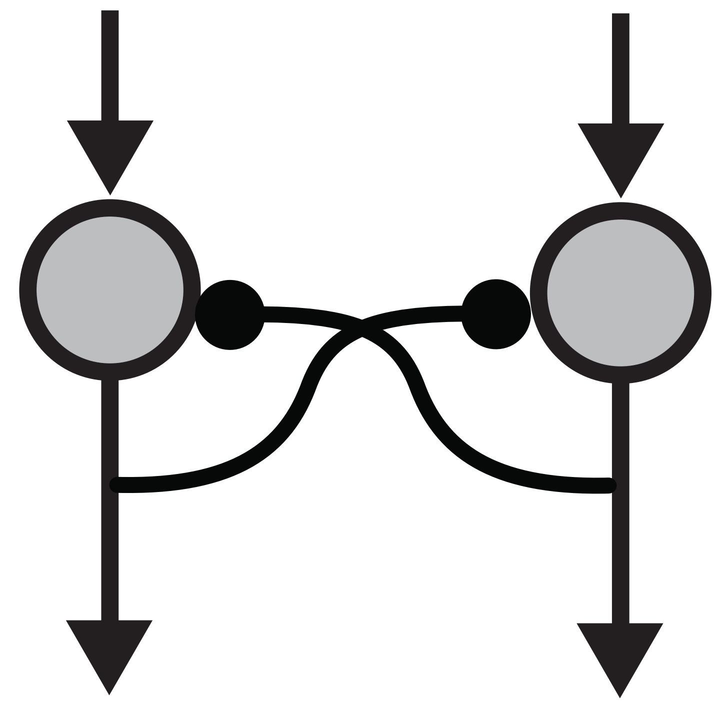
coupling/interaction

$$\tau \dot{u}_1(t) = -u_1(t) + h + s_1(t) - c_{12}\sigma(u_2(t))$$

$$\tau \dot{u}_2(t) = -u_2(t) + h + s_2(t) - c_{21}\sigma(u_1(t))$$

Inhibitory coupling

- two possible attractor stats
  - $u_2 > 0$ and $u_1 < 0$
  - $u_2 < 0$ and $u_1 > 0$
- => competition/selection



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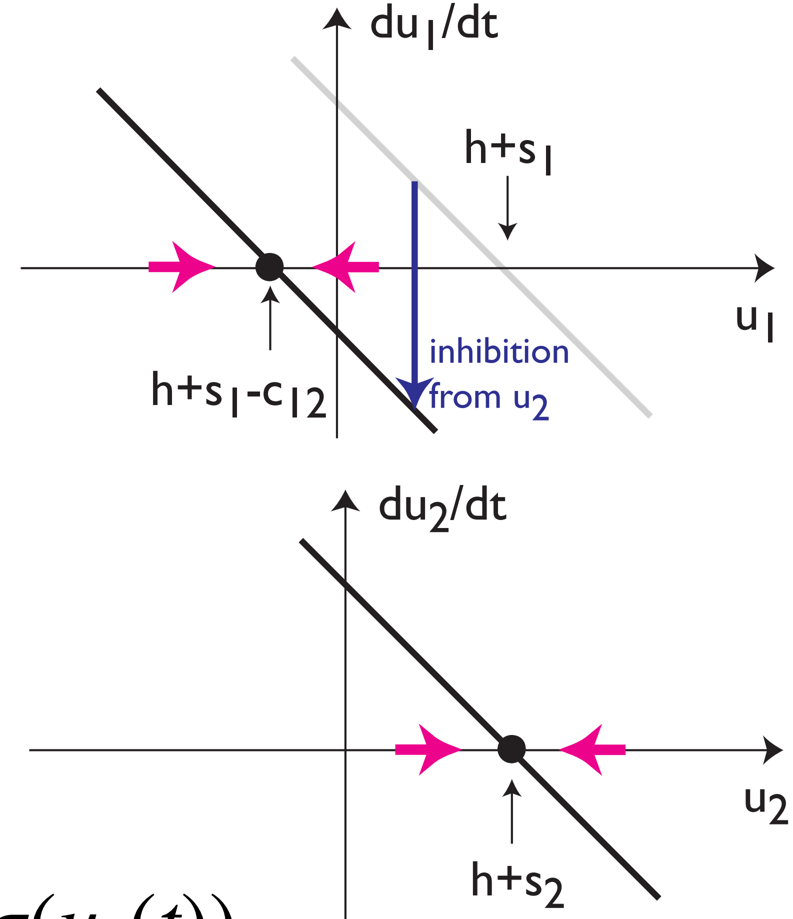
$$\tau \dot{u}_2(t) = -u_2(t) + h + s_2(t) - c_{21}\sigma(u_1(t))$$

Inhibitory coupling

- to visualize, assume that u_2 has been activated by input to a positive level
- => it inhibits u_1



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$$\tau \dot{u}_2(t) = -u_2(t) + h + s_2(t) - c_{21}\sigma(u_1(t))$$

Inhibitory coupling

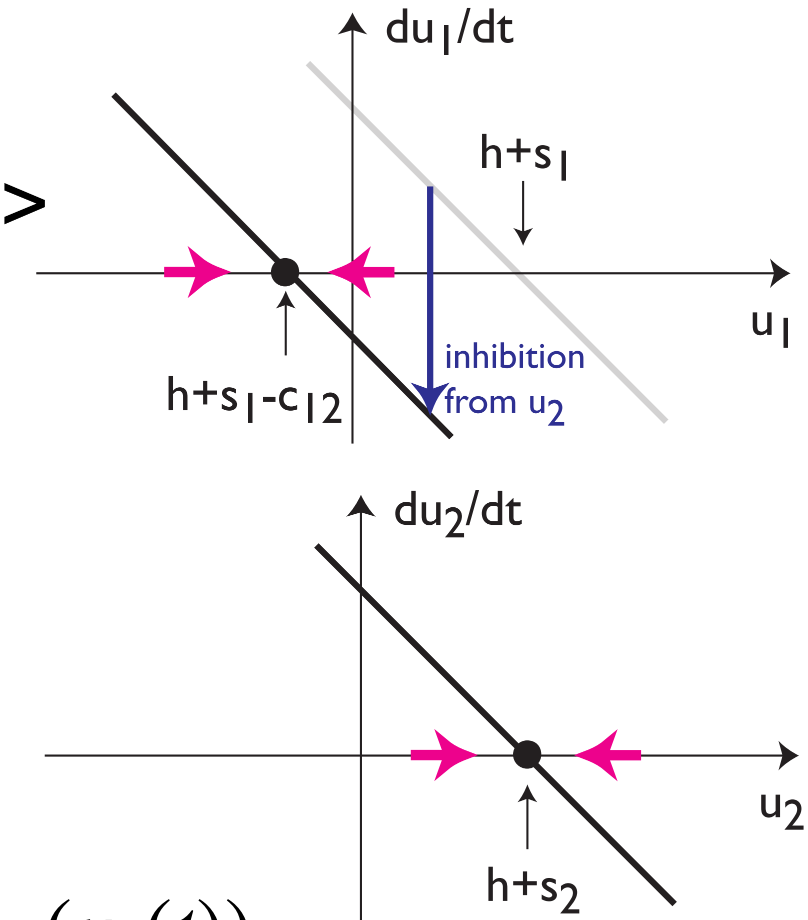
- symmetry: same logic if u_1 was initially activated it would prevent u_2 from activating
- => bistable selection of either u_1 or u_2

$$\tau \dot{u}_1(t) = -u_1(t) + h + s_1(t) - c_{12}\sigma(u_2(t))$$

$$\tau \dot{u}_2(t) = -u_2(t) + h + s_2(t) - c_{21}\sigma(u_1(t))$$

Inhibitory coupling

- asymmetric case: e.g. more input to u_2 (better "match") => faster increase => u_2 selected
- => input advantage => time advantage => competitive advantage



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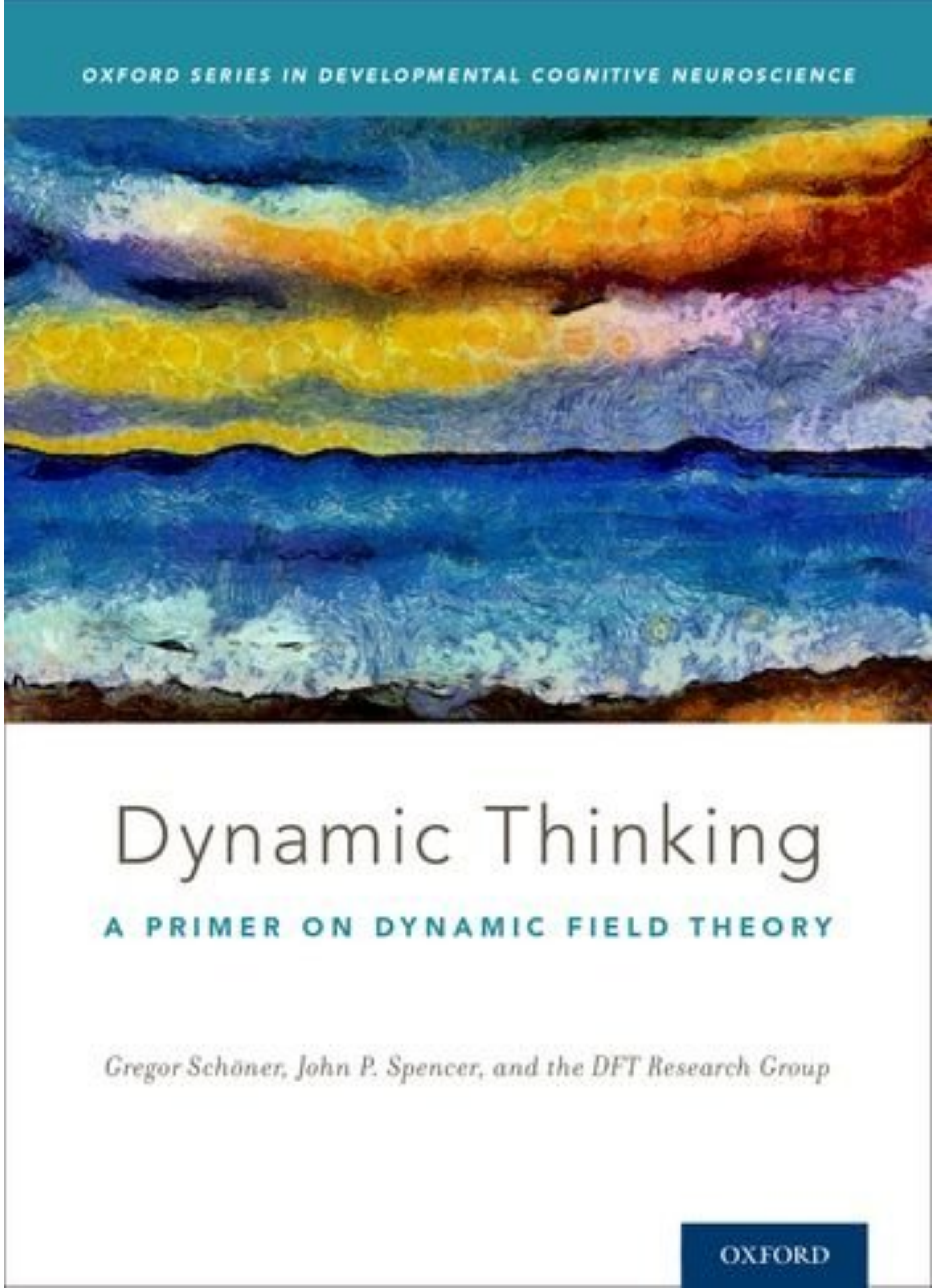
simulating inhibitorily coupled activation variables

- dynamicfieldtheory.org



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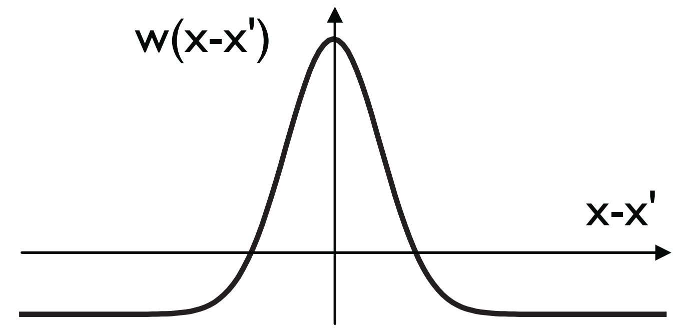
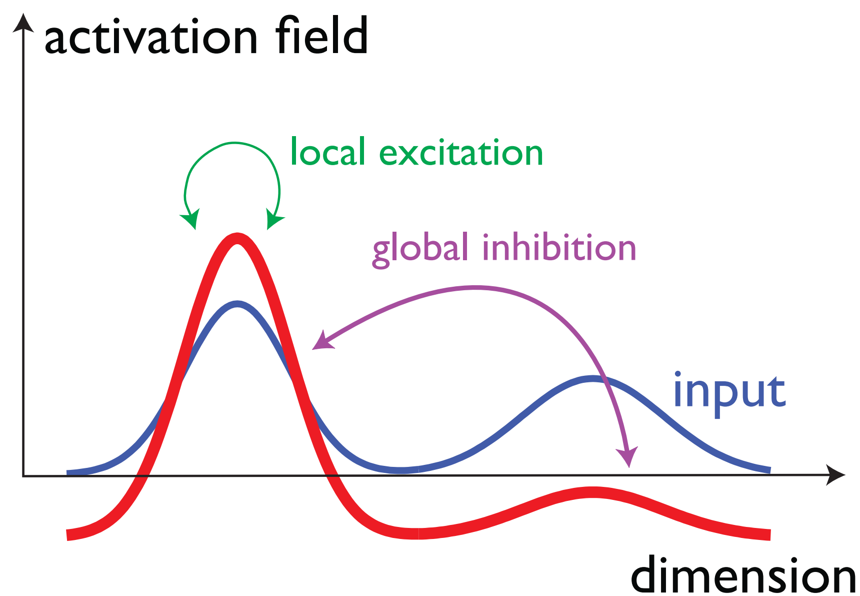
Mathematical formalization

- kernel: local excitatory interaction/ global inhibitory interaction

$$w(x-x') = w_{\text{exc}} e^{-\frac{(x-x')^2}{2\sigma^2}} - w_{\text{inh}}$$





$$\tau \dot{u}(x,t) = -u(x,t) + h + s(x,t) + \int dx'\ w(x-x')\ \sigma(u(x'))$$

Mathematical formalization

Amari equation

$$\tau \dot{u}(x,t) = -u(x,t) + h + S(x,t) + \int w(x-x')\sigma(u(x',t))\;dx'$$

where

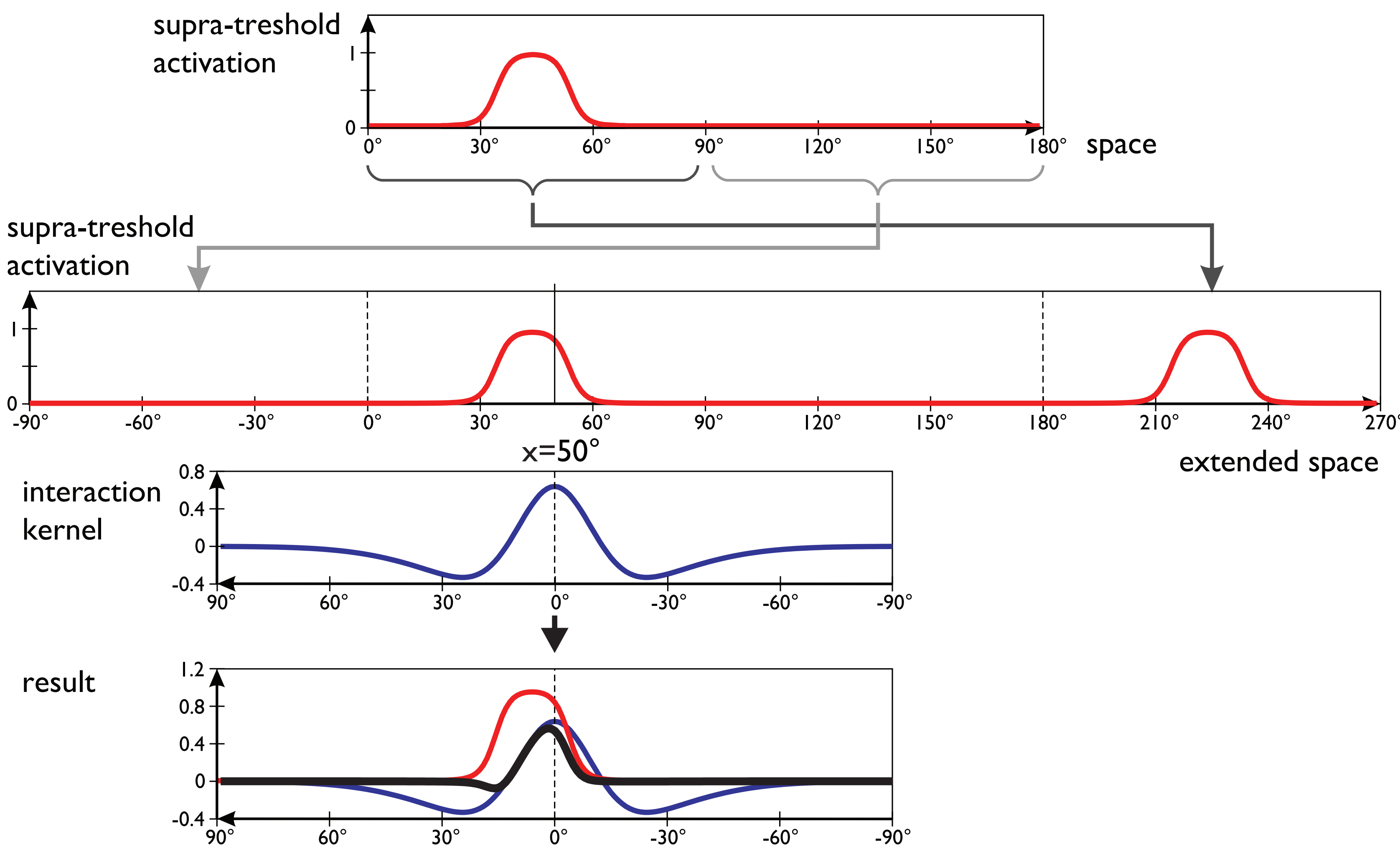
- time scale is τ
- resting level is $h < 0$
- input is $S(x,t)$
- interaction kernel is

$$w(x-x') = w_i + w_e \exp\left[-\frac{(x-x')^2}{2\sigma_i^2}\right]$$

- sigmoidal nonlinearity is

$$\sigma(u) = \frac{1}{1+\exp[-\beta(u-u_0)]}$$

Interaction: convolution



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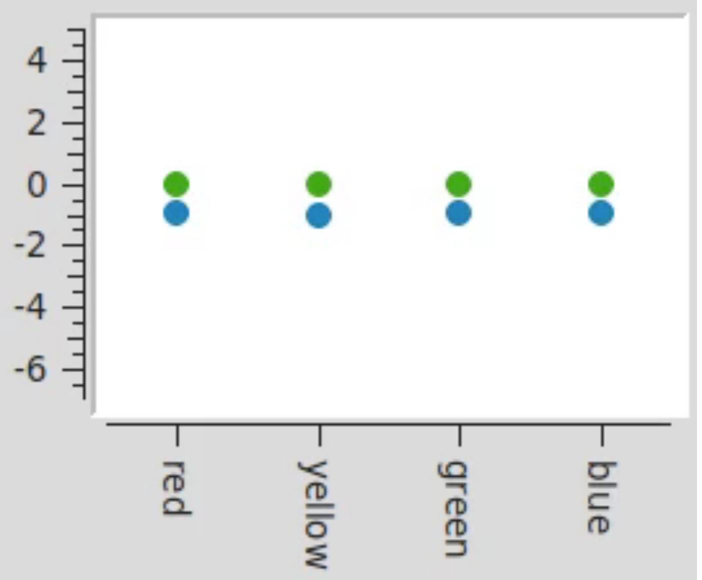
Neural dynamic nodes

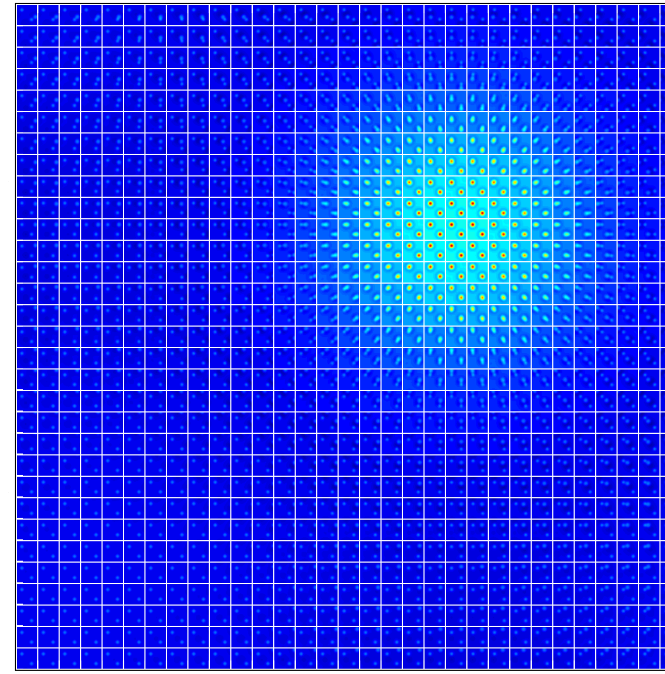
- sets of discrete activation variables as "nodes"
  - self-excitatory: "on" vs "off" states, detection instability, sustained activation
  - all nodes coupled inhibitorily: selection
  - => discretely sampled fields



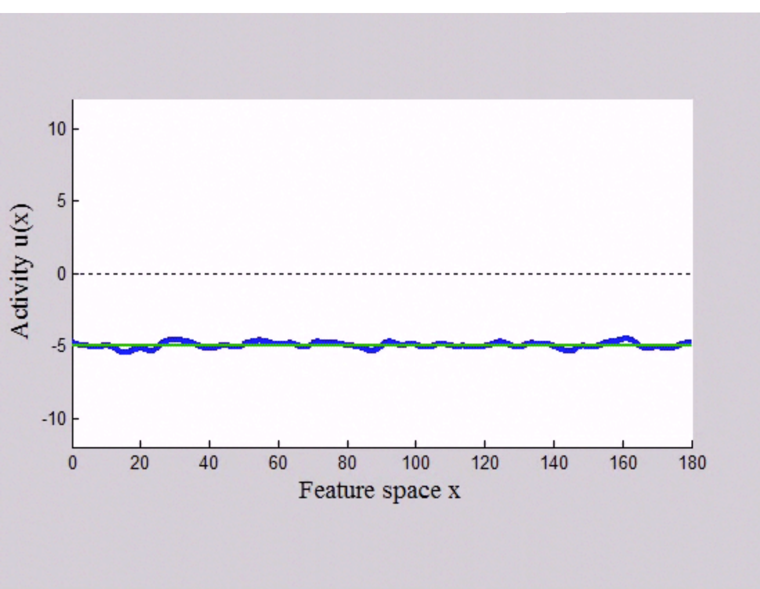
Field dynamics in different dimensions

- 1, 2, 3, 4… dimensions: peaks/ blobs as attractors

4-dimensional

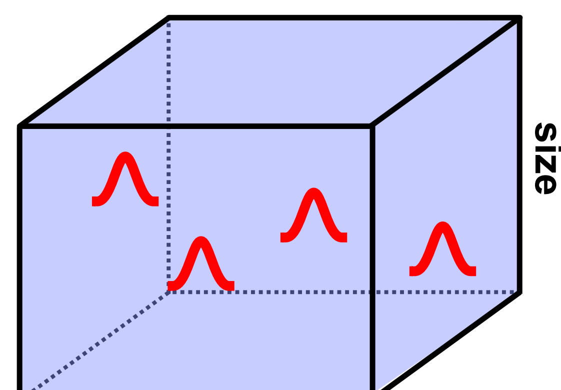


1-dimensional



2-dimensional

3-dimensional



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