

Computational Neuroscience: Neural Dynamics**Exercise 3, hand in by October 24, 2024**

Use complete sentences where textual answers are requested. Explain symbols when using mathematical notation. In graphs that you provide, label the axes.

1. The linear dynamical system

$$\dot{x} = -\alpha x$$

governs the temporal evolution of a real-valued dynamical variable, x , where α is a parameter.

- (a) Plot this equation for $\alpha > 0$ and $\alpha < 0$. Label the axes and write a short caption.
- (b) Write down its solution as a formula and verify that this solution solves the equation by computing its derivative.
- (c) Plot the solution for $\alpha > 0$ and at least two initial conditions. (A qualitative plot is sufficient, but you can also choose specific values for α and the initial values and do the plot numerically.)
- (d) (more advanced) For $\alpha > 0$, compute the times, t_n , at which the solution reaches $x(0)/e^n$ (where $e = 2.7\dots = \exp(1)$). Compute $t_{n+1} - t_n$ (called “relaxation time” in physics). Does it depend on n ? Explain your answer!
- (e) Based on the last two results, how does the solution change as α becomes larger. Plot or describe.

2. The non-linear dynamical system

$$\dot{x} = a - x^2$$

governs the temporal evolution of a real-valued dynamical variable, x , where a is a parameter.

- (a) Plot this equation for $a > 0$ and $a < 0$ and $a = 0$. Label the axes and write a short caption.
- (b) Determine the fixed points of this dynamics by solving for $\dot{x} = 0$.
- (c) By “mental simulation” guess the asymptotic behavior when time goes to infinity for initial conditions below and above zero for $a < 0$ vs $a > 0$.
- (d) (more advanced) At $a = 0$, solve the equation analytically. [Hint: use separation of variables leading to $\int_{x_0}^{x(t)} dx/x^2 = -t$ and solve the integral.]
- (e) Examine what happens when time goes to infinity for $x_0 > 0$ and compare to your “mental simulation”.
- (f) (advanced) More complex is to understand what happens for $x_0 < 0$. Can you explain that?