Exam

Preparatory Course
Computer Science and Mathematics

Programming

1. What is the output of the following script?

```
x = -3.5
if x > 0:
    print("Yes")
else:
    print("No")
print("Goodbye")
```

2. What is the output of the following script? What value does a have at the end of the script?

```
a = 3
while a <= 10:
    print(a)
    a = a + 2
print("Hello")

a = a + 1
```

3. Assume that x is a variable that has a floating point number as its value. Write a script that prints "x is greater than 2" if x is greater than 2, "x is smaller than 2" if x is smaller than 2, and "x is equal to 2" if x is equal to 2.

4. What is the output of the following script?

```
numbers = [1,2,3,5,8]
numbers[2] = 4
for number in numbers:
    number2 = number*number
    print(number2)
```

5. Write a Python function that receives two arguments and returns the product of the two arguments. Call that function with the arguments 2 and 3.

Functions

6. Let $f(x) = 3(x-2)^2$. Which of the following is true? Just list the letters of the true statements.

   (a) f is the result of translating the function $g(x) = 3x^2$ by 3 along the y axis

   (b) f is the result of translating the function $h(x) = (x-2)^2$ by 3 along the y axis

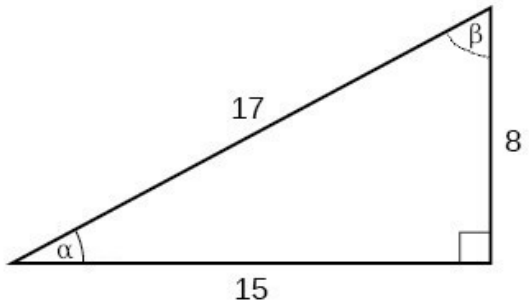
   (c) f is the result of translating the function $i(x) = 3x^2$ by 2 along the x axis

   (d) f is the result of compressing the function $j(x) = (x-2)^2$ by a factor of 3

   (e) f is the result of scaling the function $k(x) = (x-2)^2$ by a factor of 3

Linear Algebra

7. Calculate the angle β in the following right triangle:



   Note: You can calculate the inverse sine of a number x by entering $arcsin(x)$ into wolframalpha.com or google.com. Same for $arccos(x)$ and $arctan(x)$.

8. Let $\boldsymbol{a} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$. Calculate the angle between the two vectors using the formula $\alpha = cos^{-1}\left(\frac{<\boldsymbol{a},\boldsymbol{b}>}{|\boldsymbol{a}||\boldsymbol{b}|}\right)$.

9. Calculate $2 \cdot \begin{pmatrix} 2 \\ 3 \end{pmatrix} + \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

10. (bonus) Calculate $\begin{pmatrix} 2 & 3 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

11. (bonus) Calculate $\begin{pmatrix} -1 & 3 & -1 \\ 2 & 1 & -1 \\ 2 & -1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix}$

Derivatives

12. Calculate the derivative and local extremum of $f(x) = 2x^2 + x + 3$

13. Calculate the derivative of $h(x) = 2e^{2x}$

14. Calculate the derivative of $j(x) = x^2(5x^2 + x)$ using the product rule.

Integration

15. The antiderivative of the function $f(x) = 2x+3$ is $F(x) = x^2+3x$. Calculate the integral $\int_1^3 f(x)dx$.

16. Assume that there is a function s such that $s(x) = 2$ in the interval $[2, 3)$ and $s(x) = 3$ in the interval $[3, 4]$. Calculate $\int_2^4 s(x)dx$.