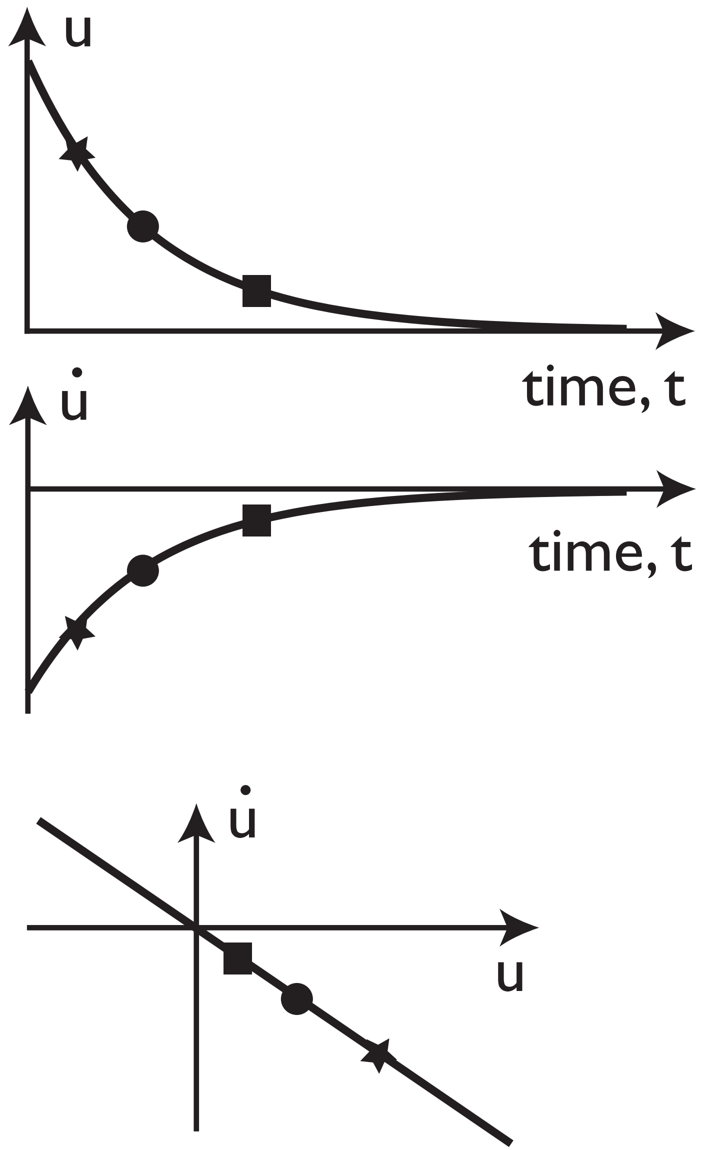
Summary: main conceptual points

Gregor Schöner, INI, RUB

Dynamical systems

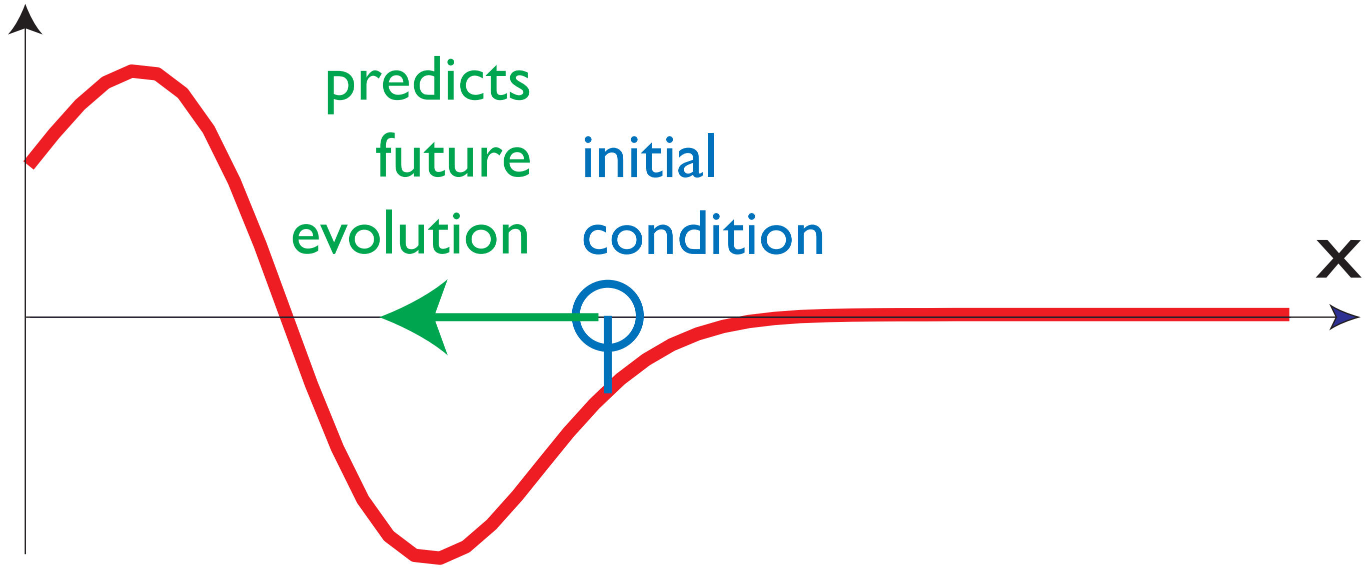
- functional link between state and its rate of change



Dynamical system

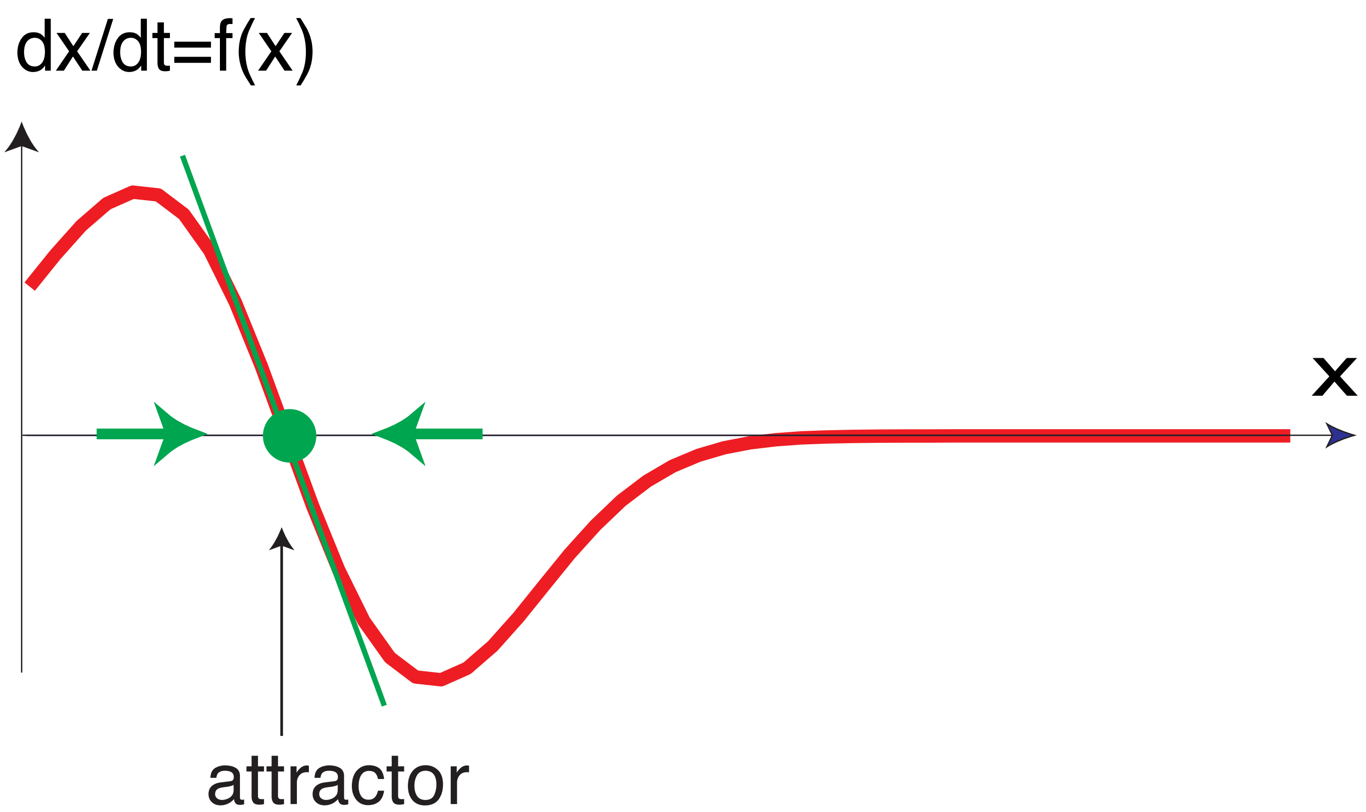
- present determines the future

$dx/dt=f(x)$



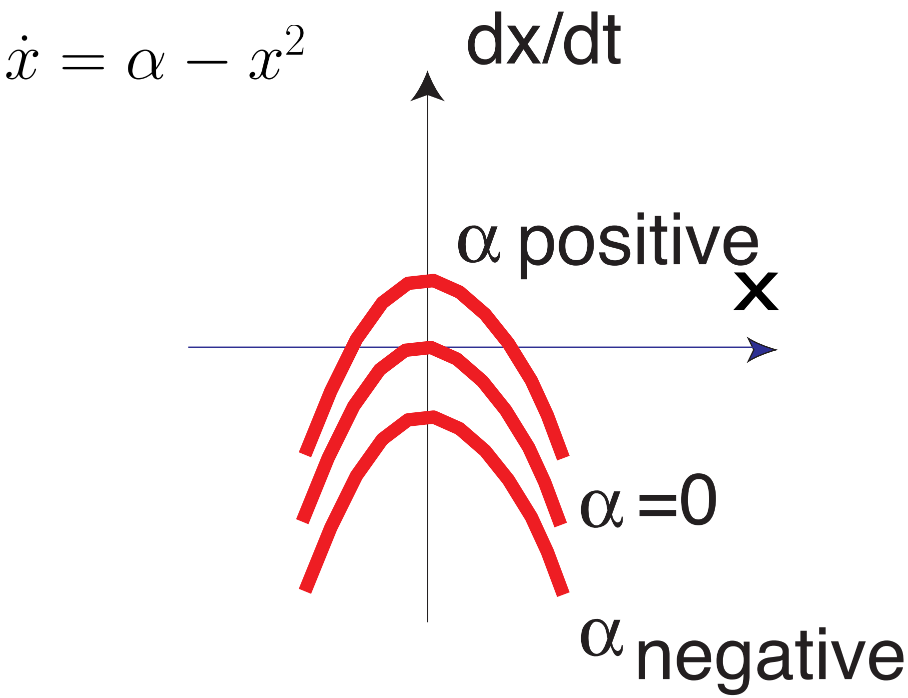
Dynamical systems

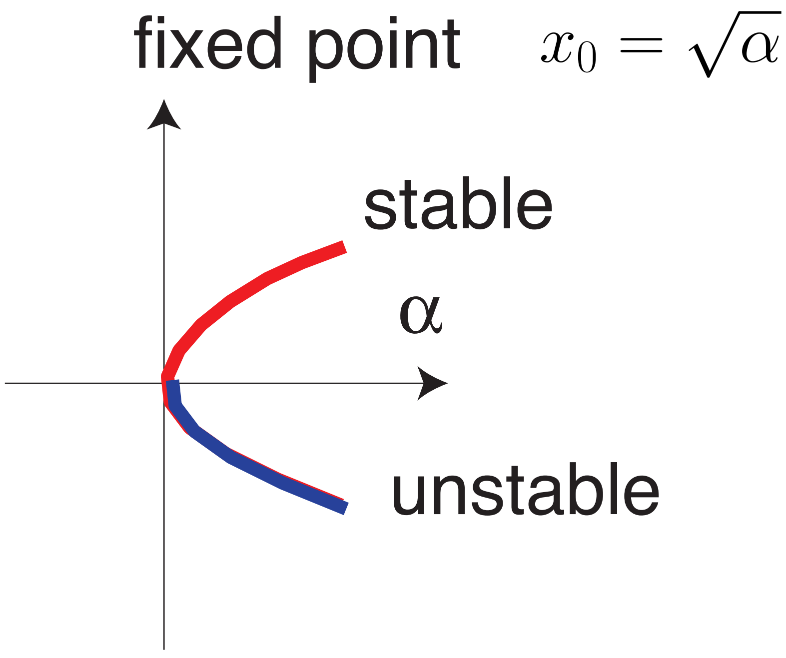
- fixed point = constant solution
- neighboring initial conditions converge = attractor



Bifurcations are instabilities

- In families of dynamical systems, which depend (smoothly) on parameters, the solutions change qualitatively at bifurcations
- at which fixed points change stability



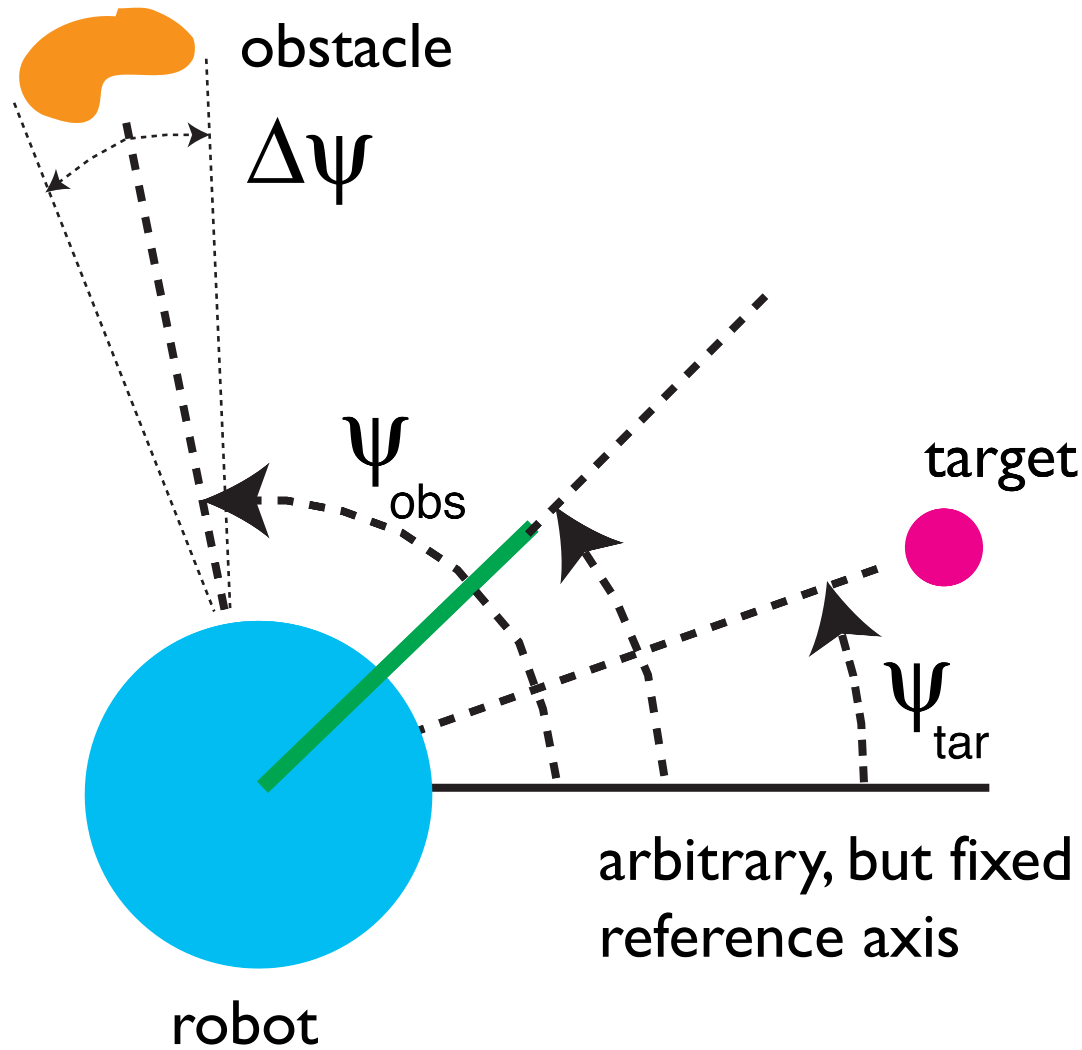


Basic ideas of attractor dynamics approach

- behavioral variables
- time courses from dynamical system: attractors
- tracking attractors
- bifurcations for flexibility

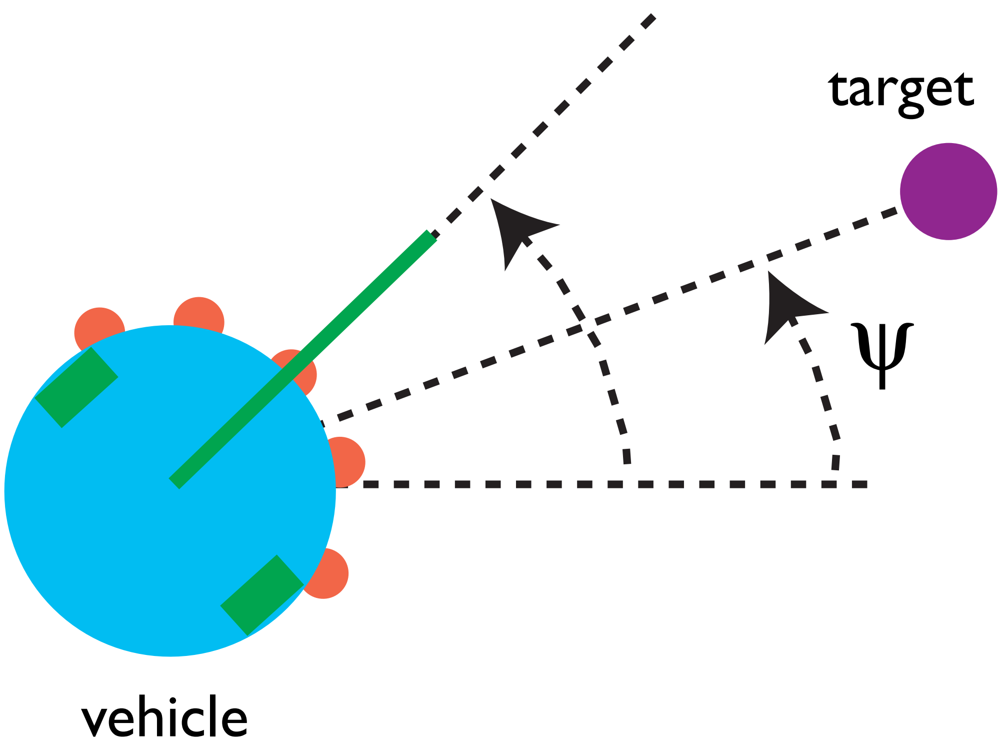
Behavioral variables: example

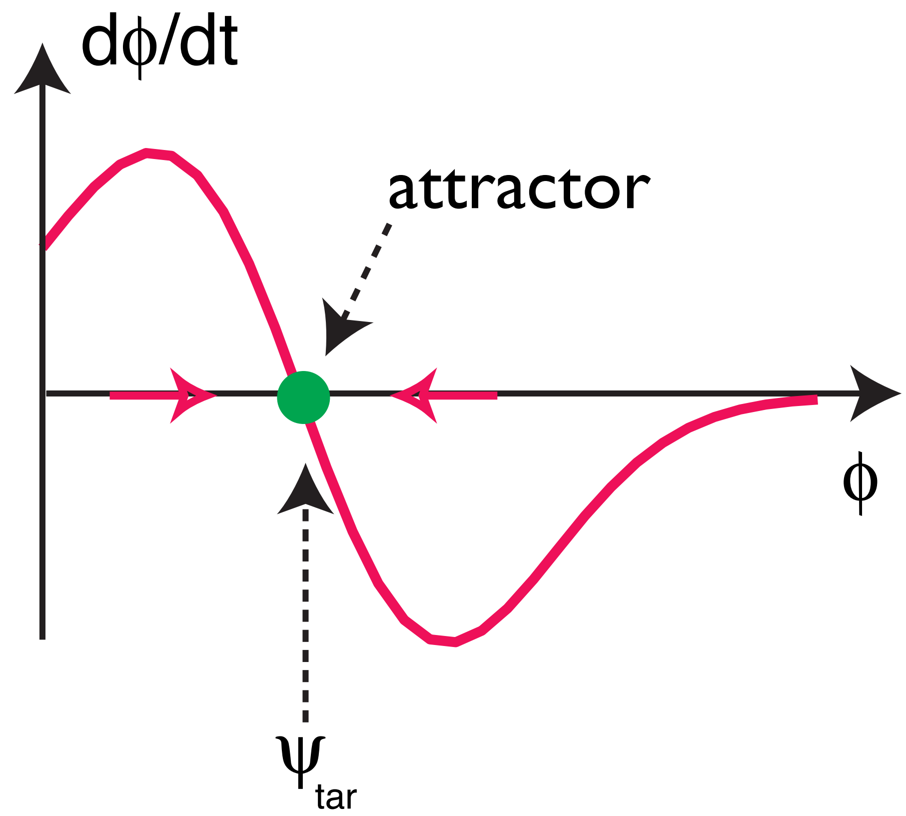
- vehicle moving in 2D: heading direction
- constraints: obstacle avoidance and target acquisition



Behavioral dynamics: example

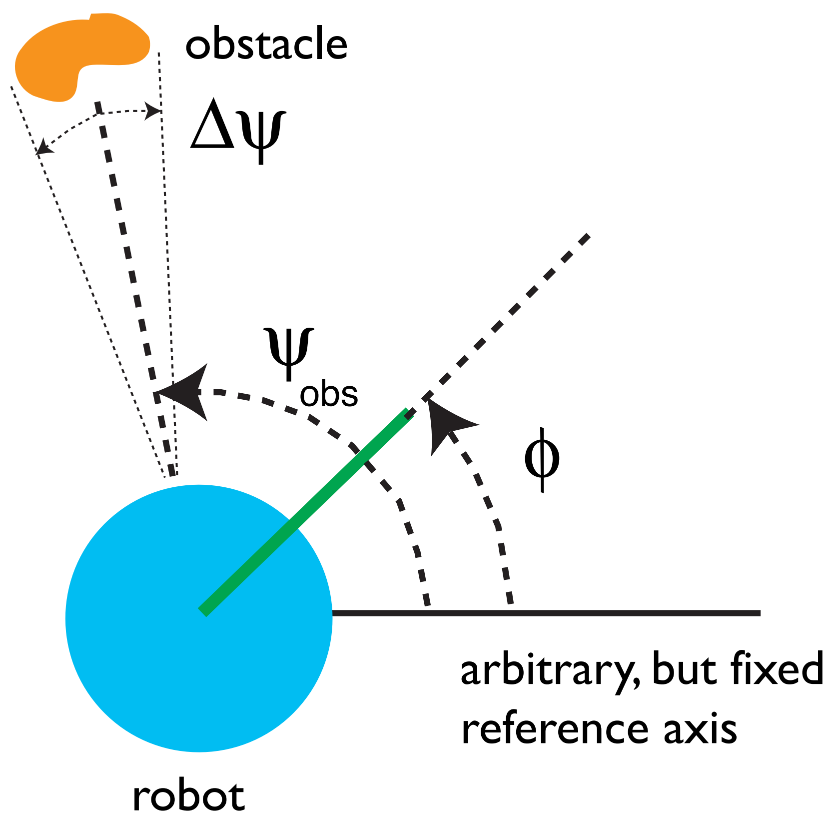
- behavioral constraint: target acquisition

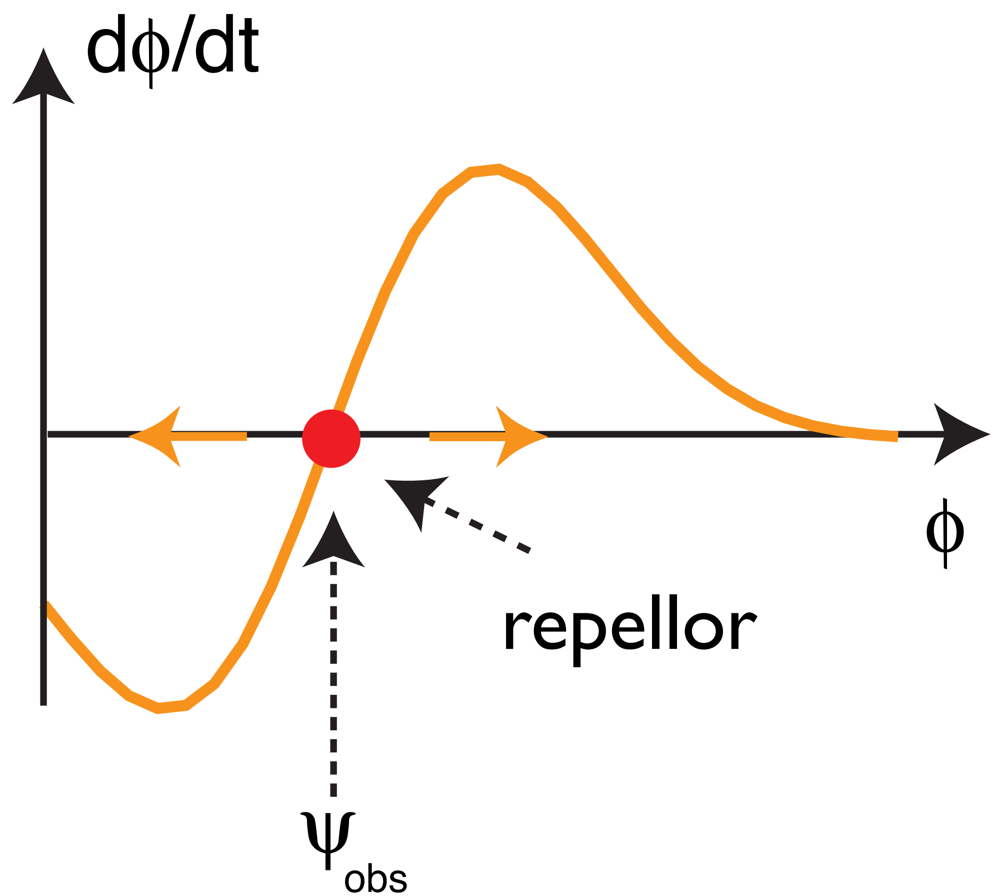




Behavioral dynamics: example

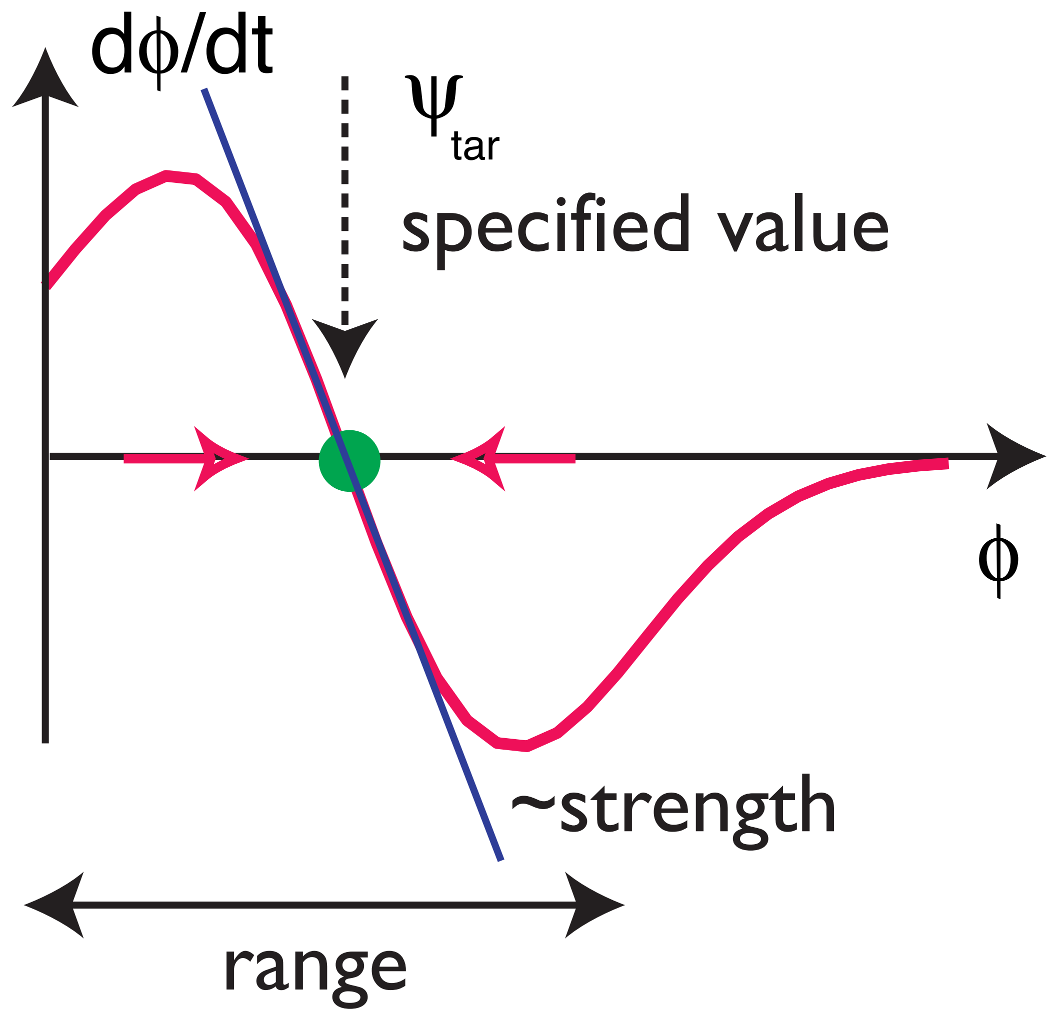
- behavioral constraint: obstacle avoidance





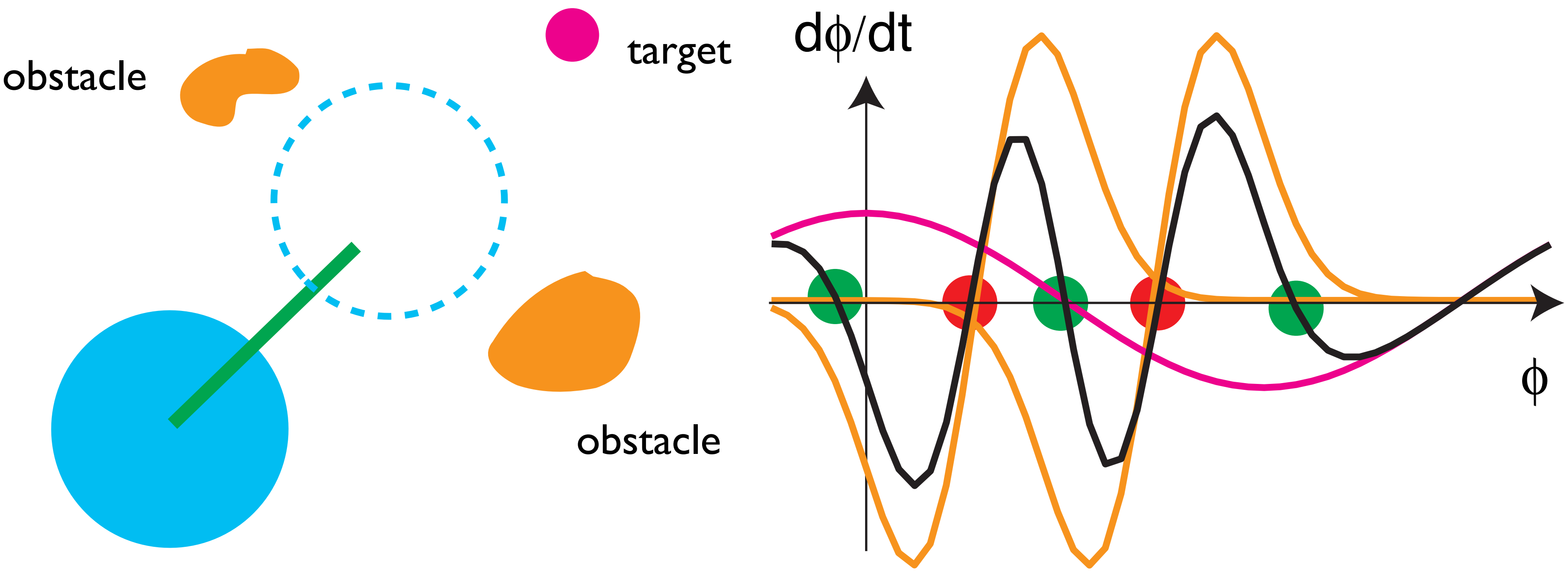
Behavioral dynamics

- each contribution is a “force-let” with
  - specified value
  - strength
  - range



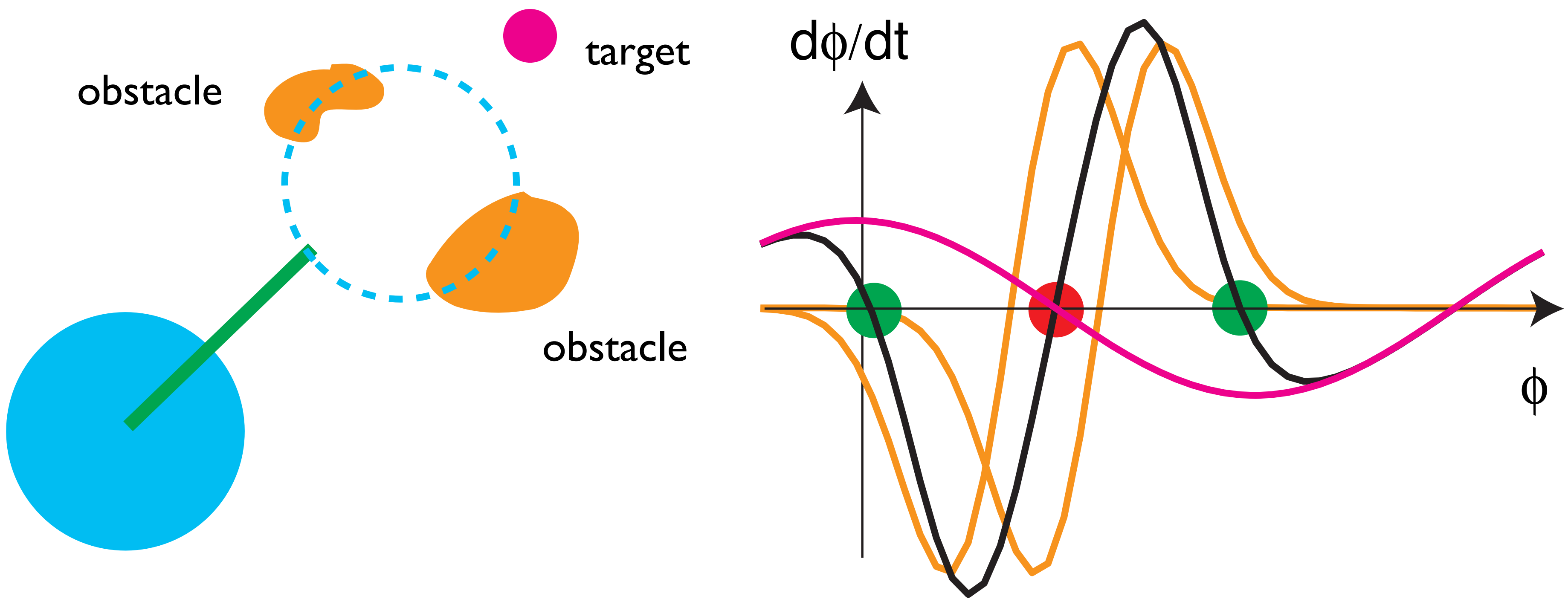
Behavioral dynamics: bifurcations

- constraints not in conflict



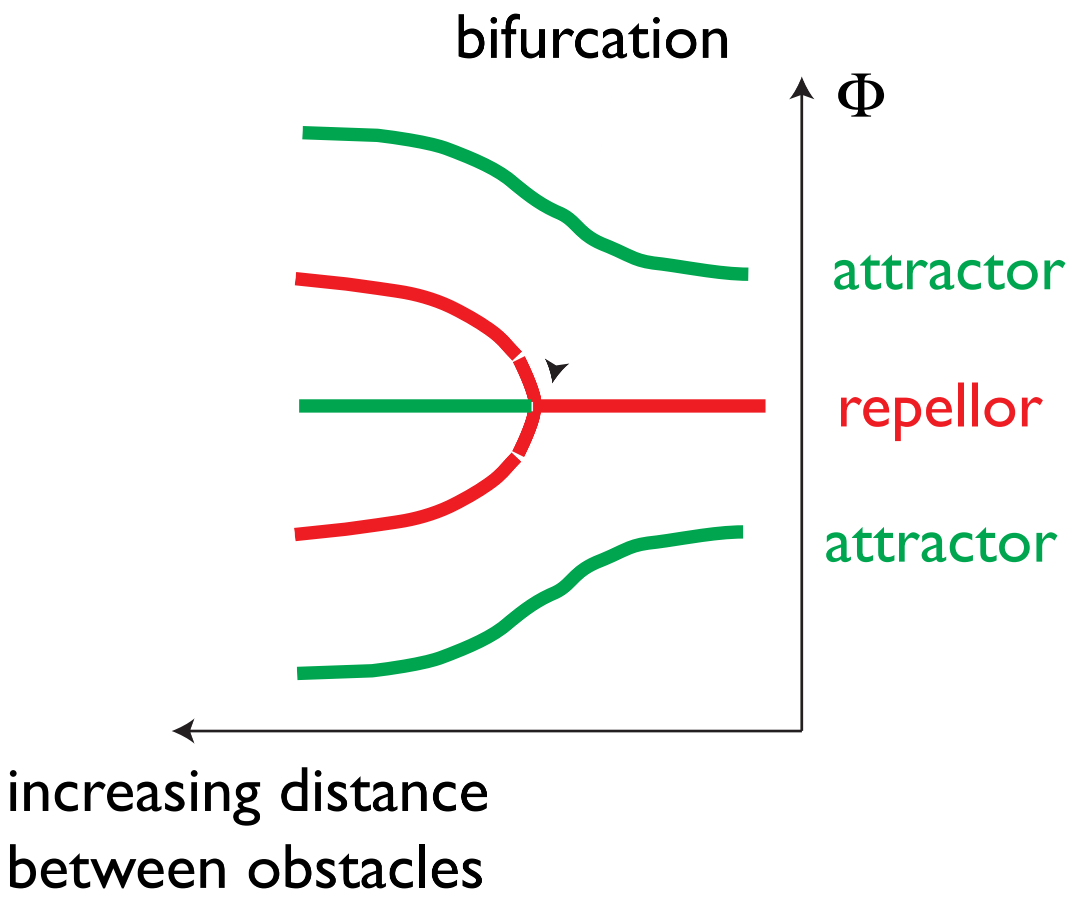
Behavioral dynamics

- constraints in conflict



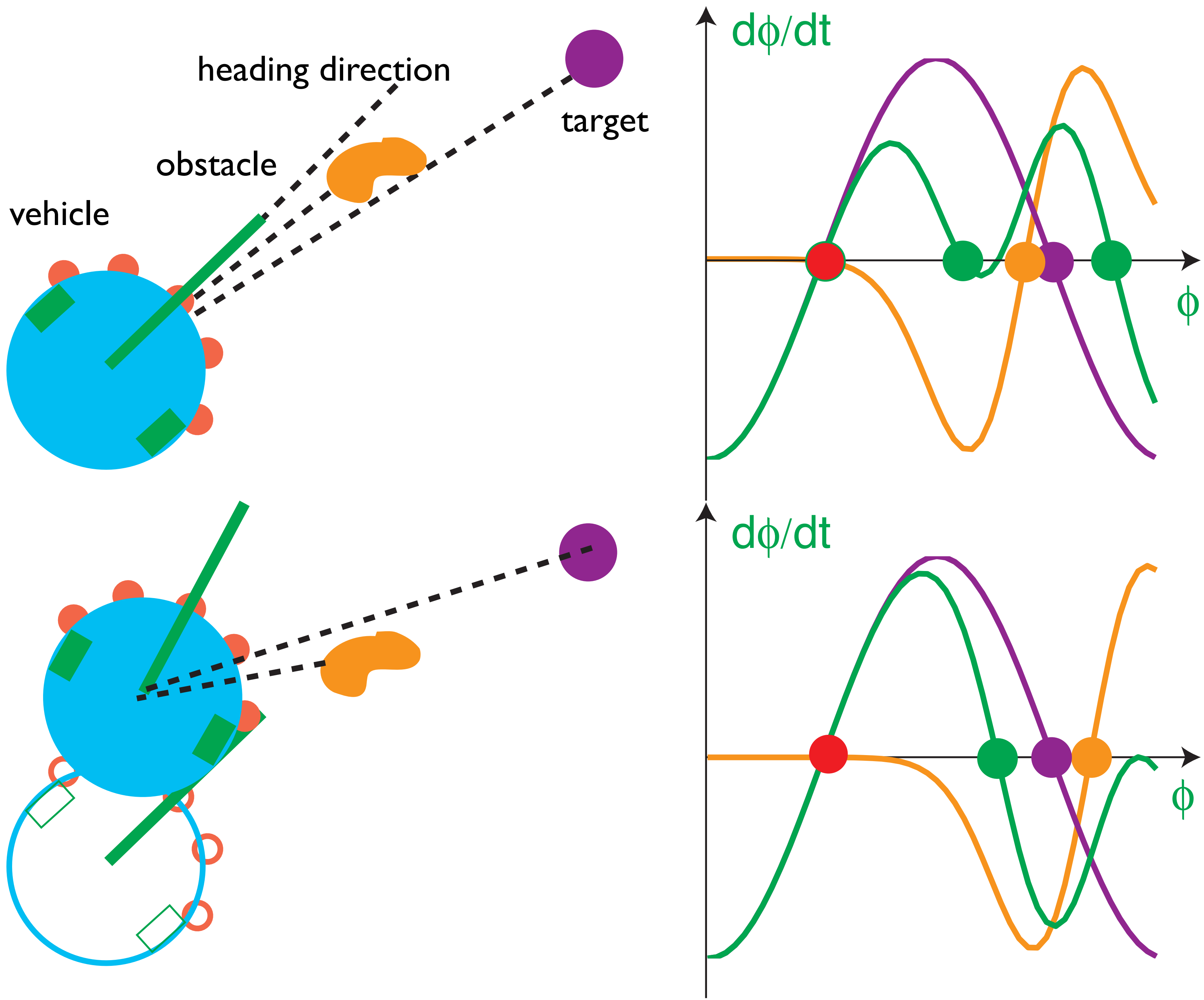
Behavioral dynamics

- transition from "constraints not in conflict" to "constraints in conflict" is a bifurcation



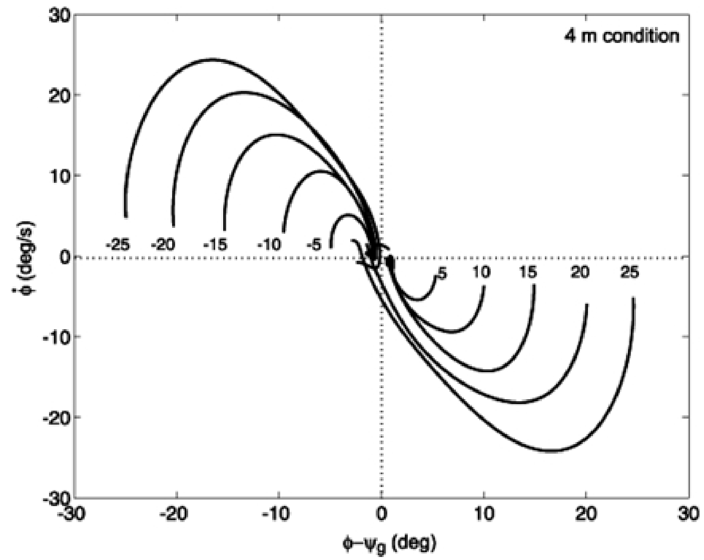
In a stable state at all times

3

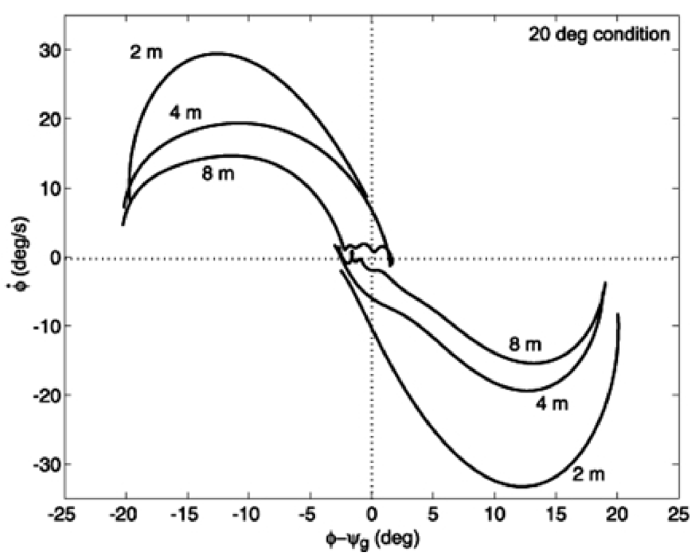


model-experiment match: goal

experiment

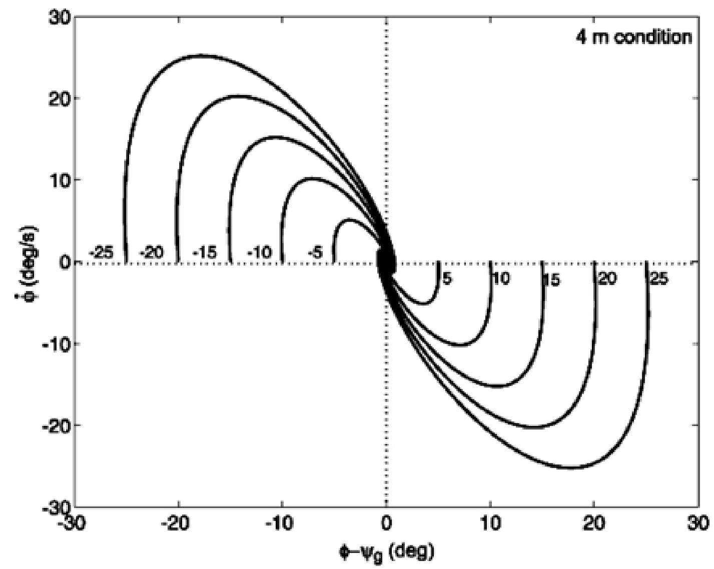


(a)

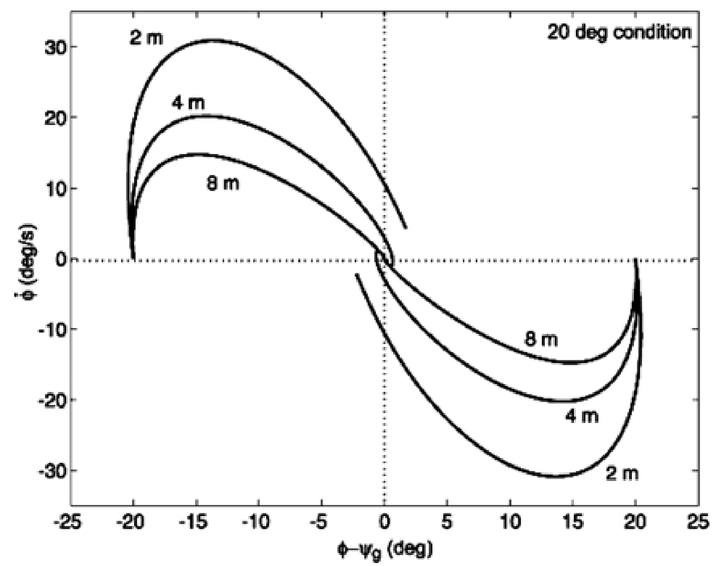


(b)

model



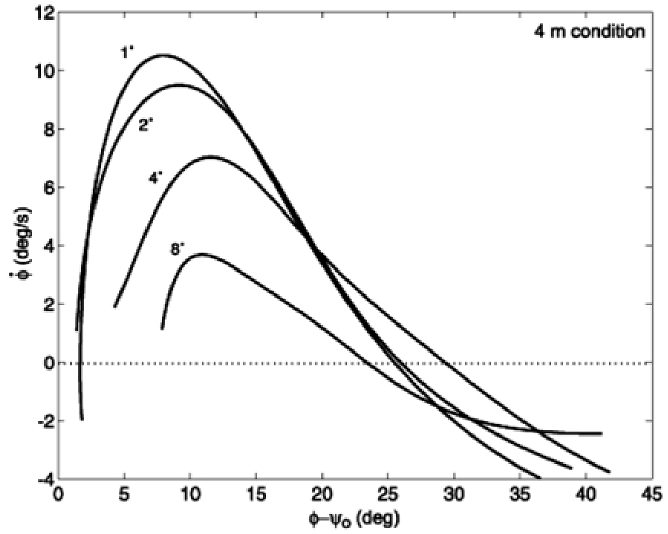
(a)



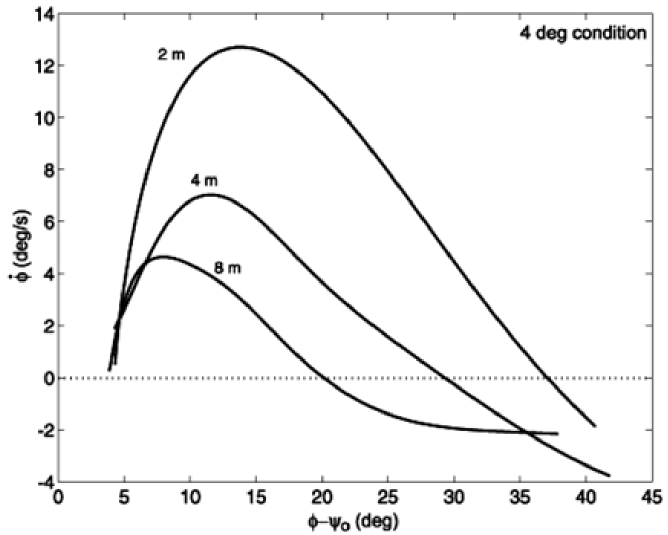
(b)

model-experiment match: obstacle

experiment

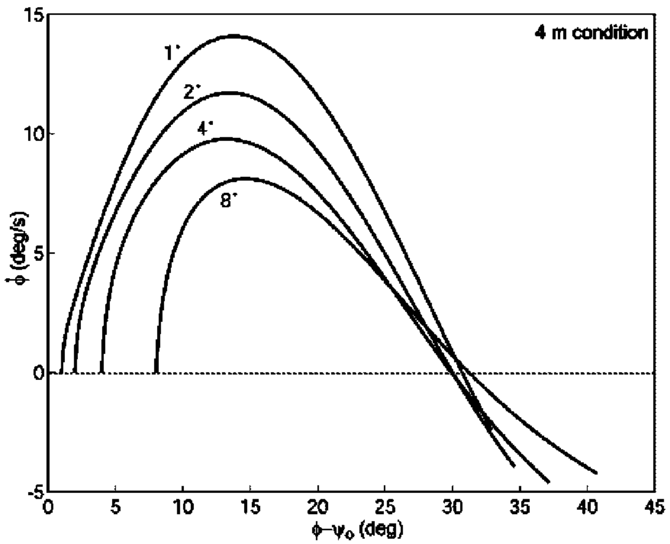


(a)

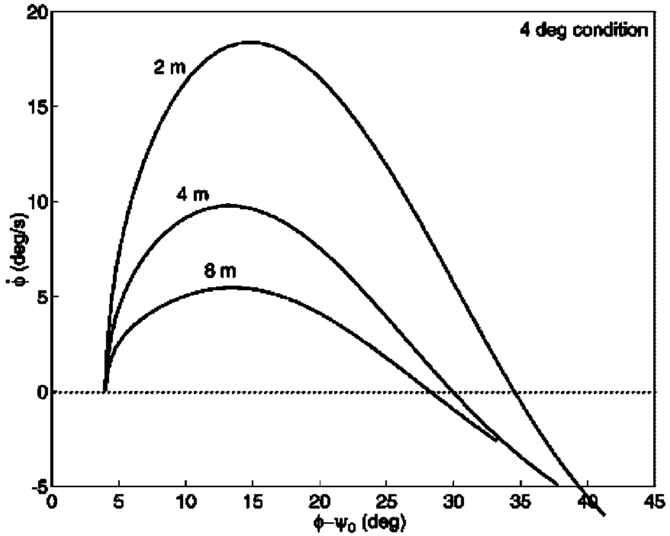


(b)

model



(a)



(b)

2nd order attractor dynamics to explain human navigation

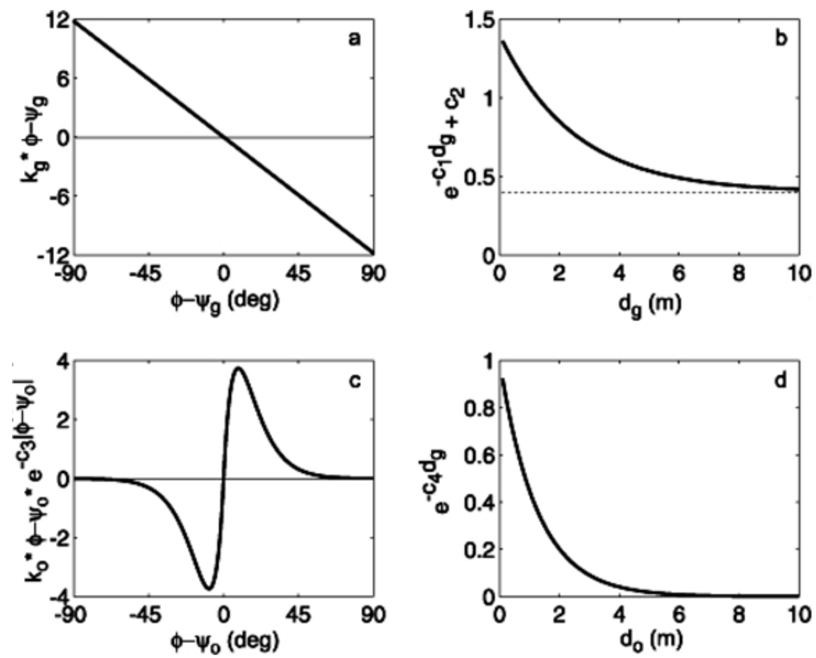
inertial term

damping term

attractor goal heading

$$\ddot{\phi} = -b\dot{\phi} - k_g(\phi - \psi_g)(e^{-c_1 d_g} + c_2) + k_o(\phi - \psi_o)\left(e^{-c_3|\phi - \psi_o|}\right)(e^{-c_4 d_o})$$

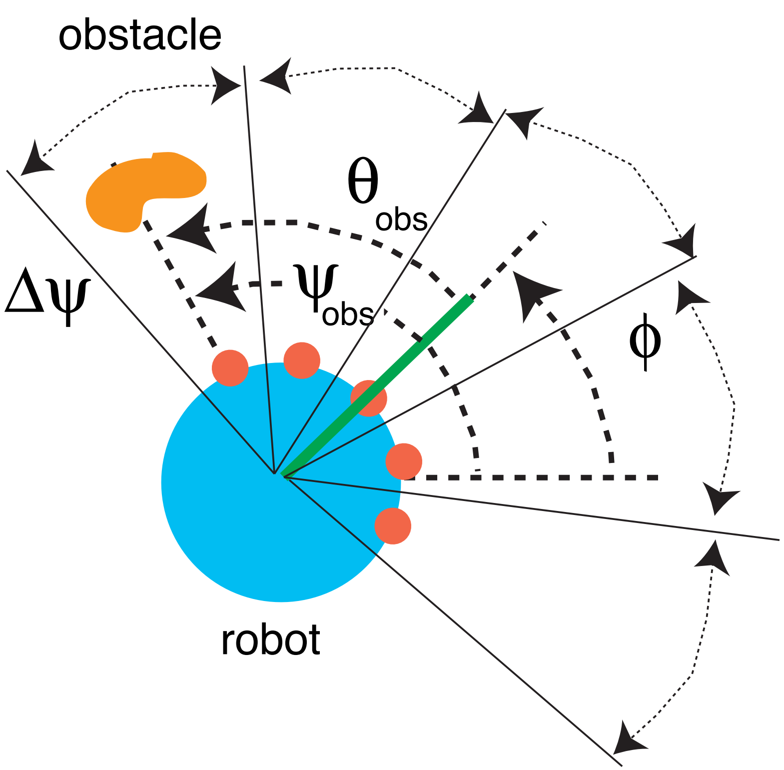
repellor obstacle heading

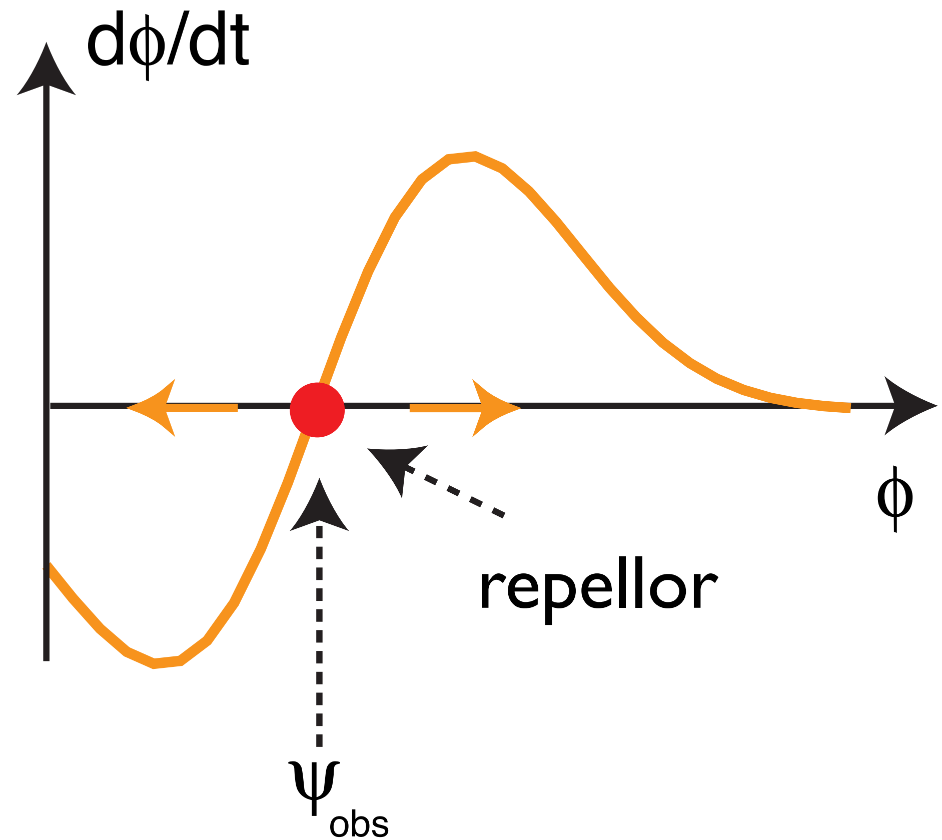


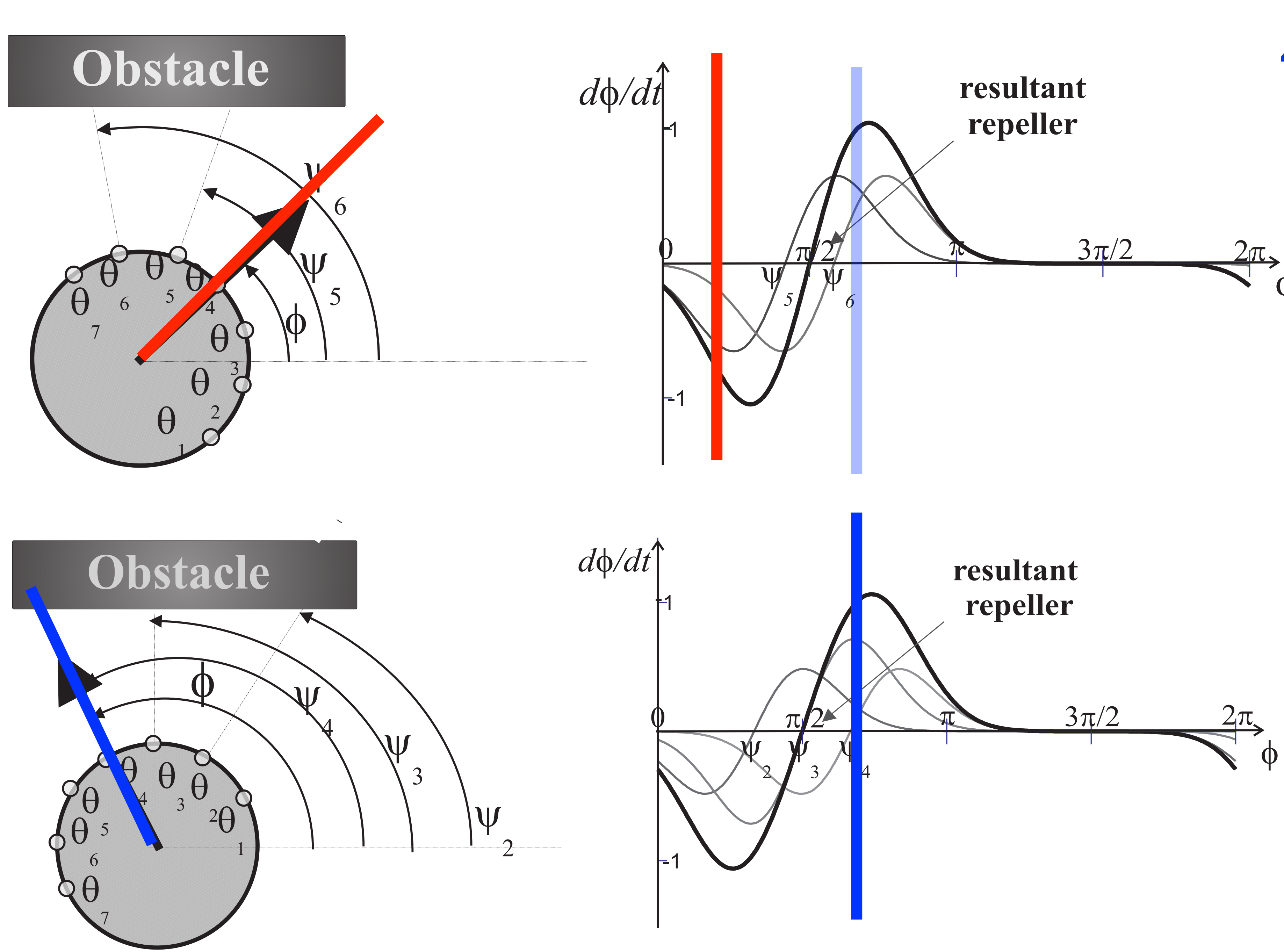
[Fajen Warren…]

Obstacle avoidance: sub-symbolic 4

- obstacles need not be segmented
- do not care if obstacles are one or multiple: avoid them anyway…





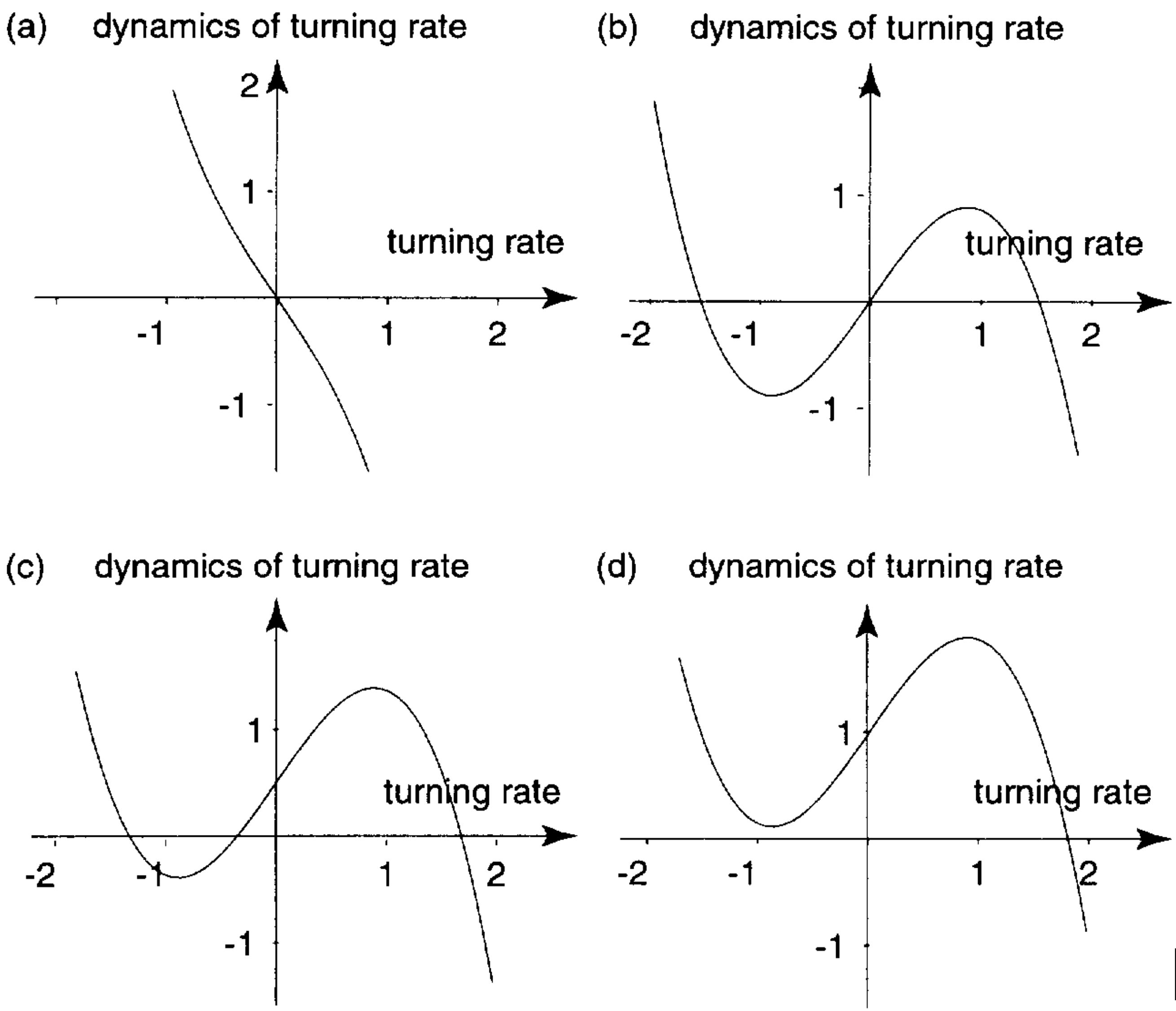


=> dynamics invariant!

[from: Bicho, Jokeit, Schöner]

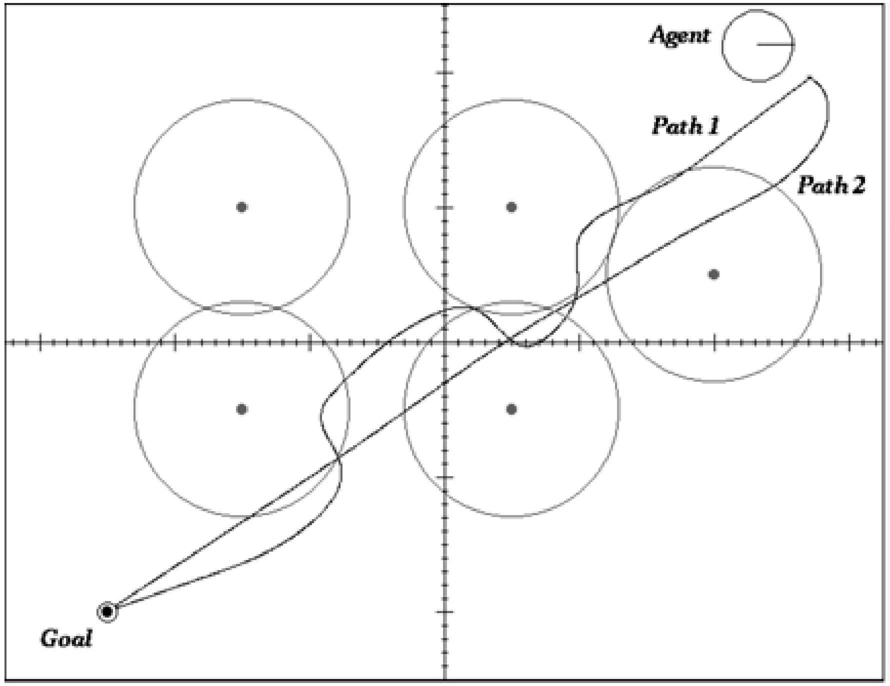
Alternative 2nd oder approach

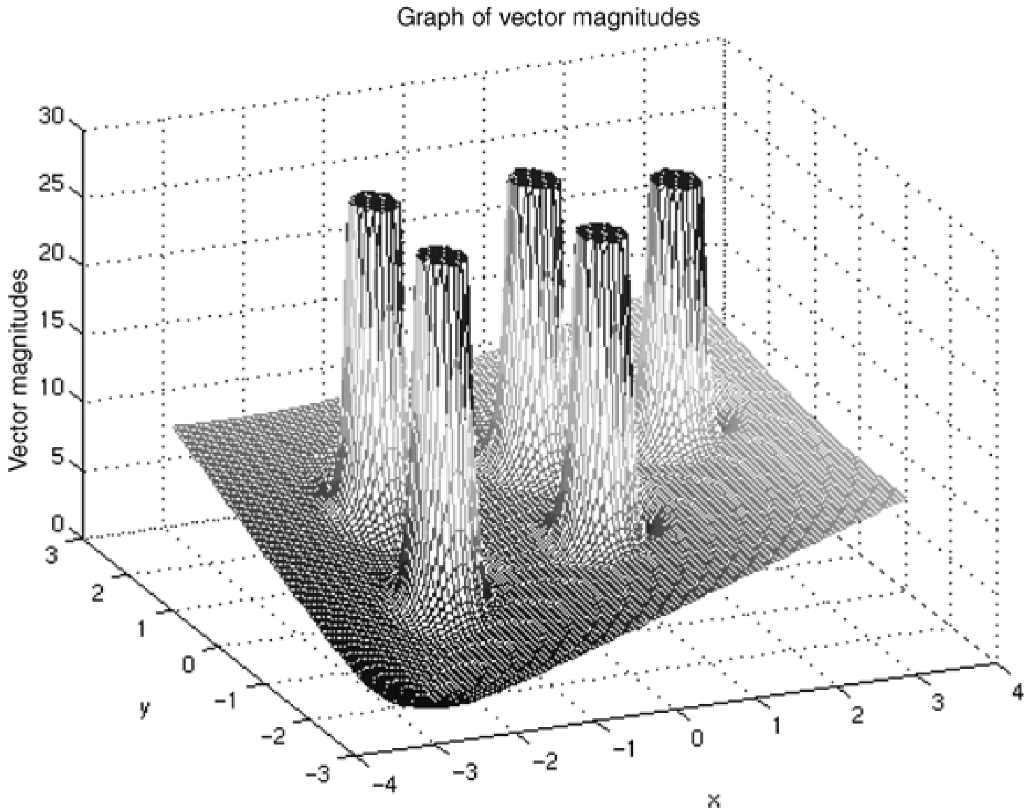
$$\dot{\omega} = (\alpha + \tfrac{1}{2}\pi)c_{\text{obs}}F_{\text{obs}} + \alpha\omega - \gamma\omega^3$$



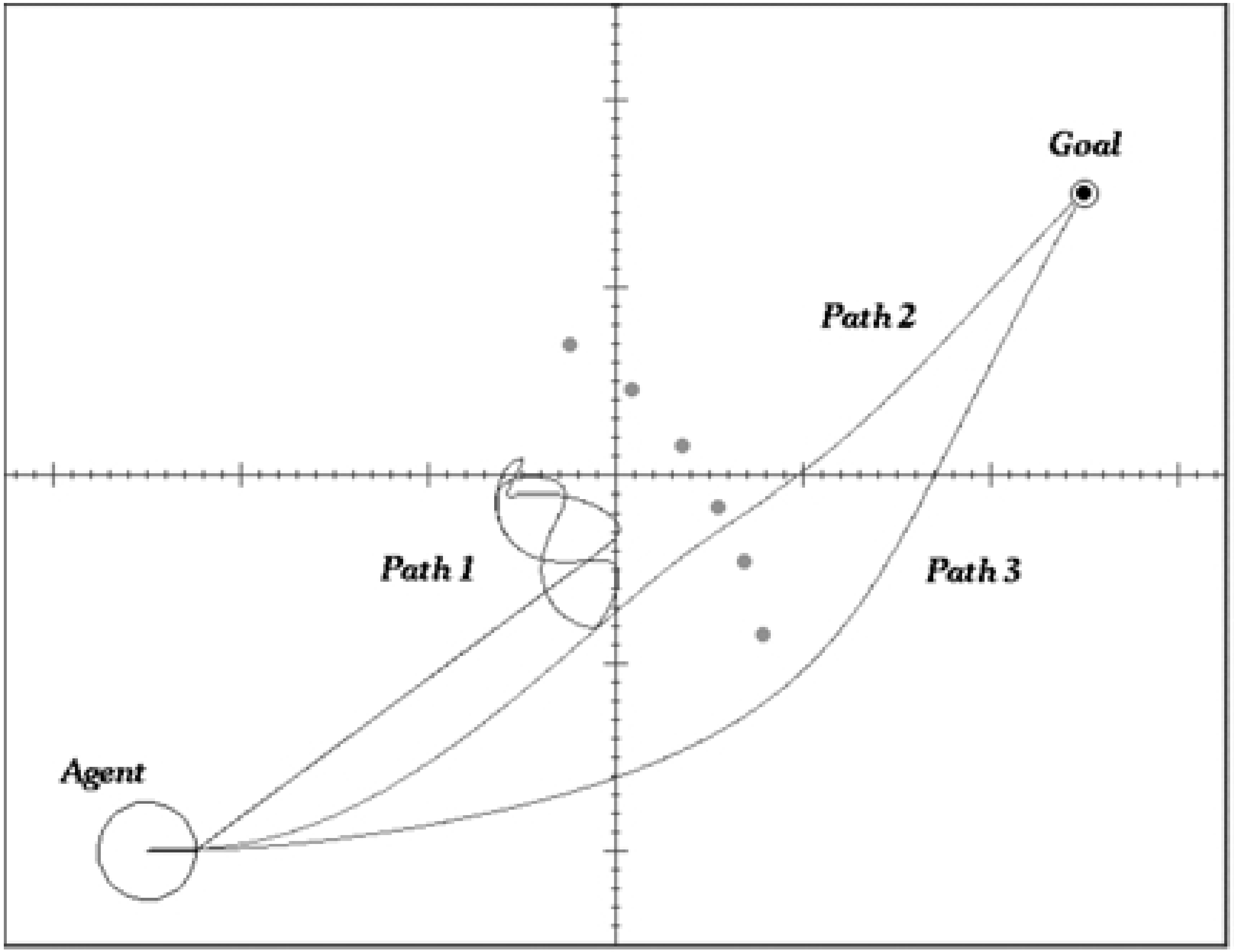
[Bicho, Schöner, 97]

Potential field approach



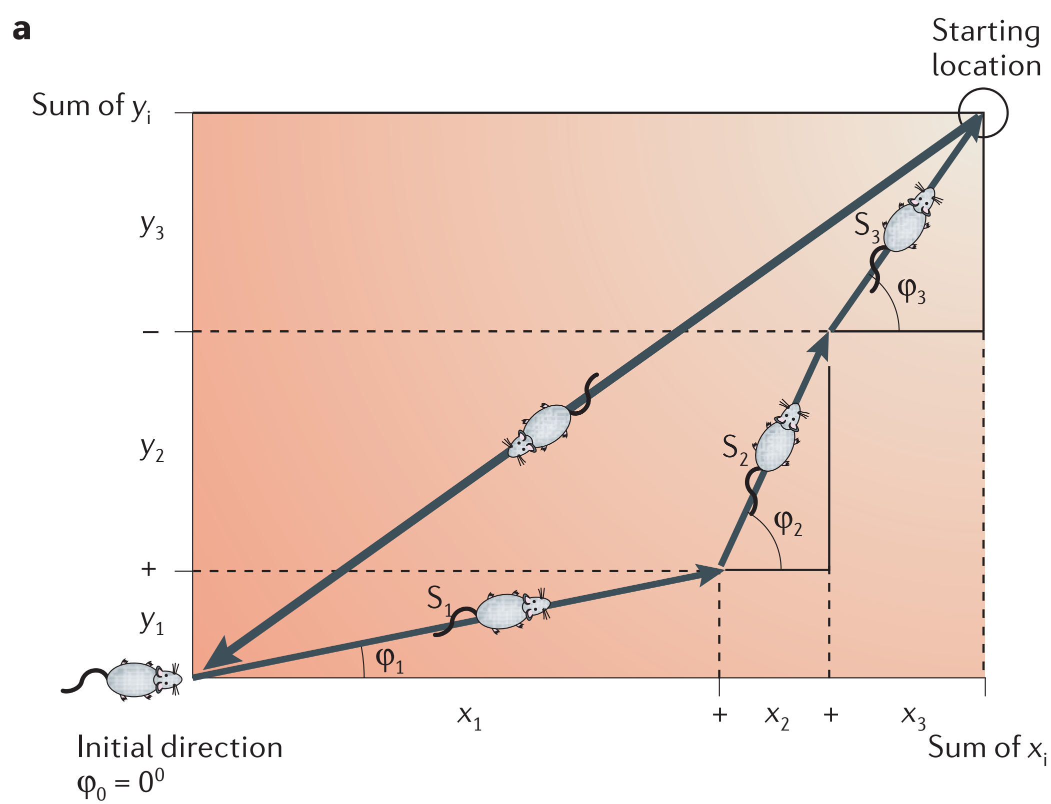


spurious attractors in potential field approach



Dead-reckoning/path integration

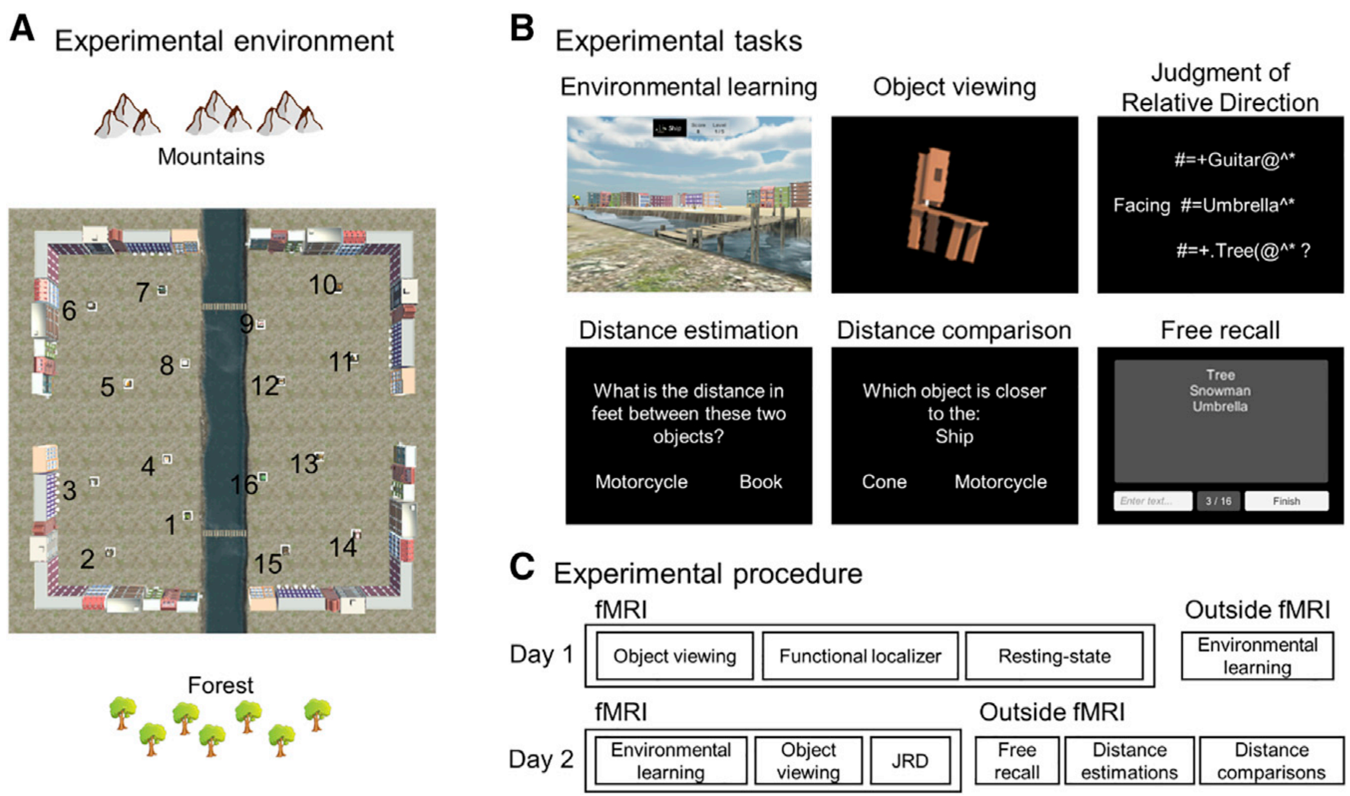
- if the agent knows its current velocity=heading direction + speed (and keeps track of time), it can estimate its change of position by integration



[McNaughton et al., *Nature reviews neuroscience* 2006]

Landmark recognition

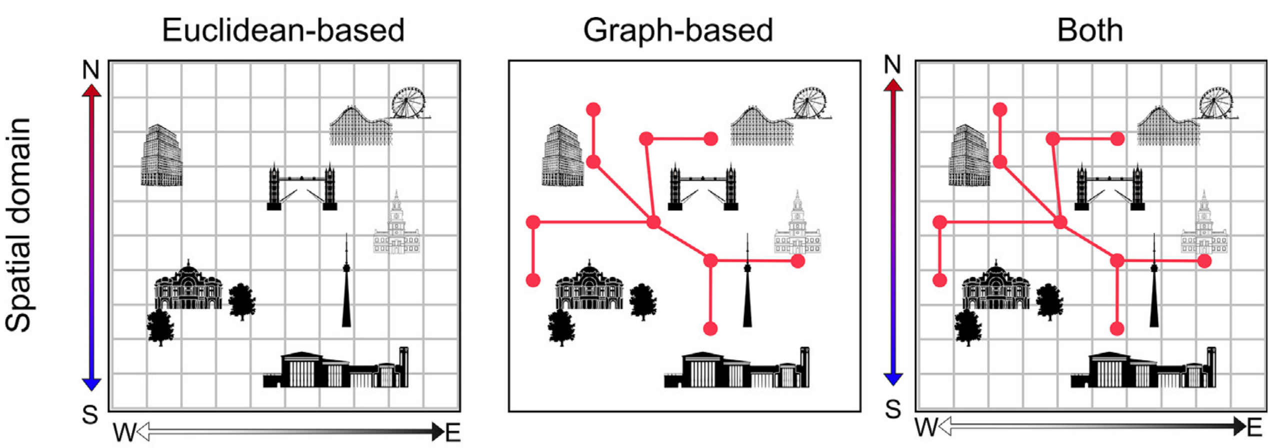
- landmarks are not necessarily objects…
- empirical evidence that views serve to estimate ego-position and pose
- evidence for views used from animal behavior and neural data



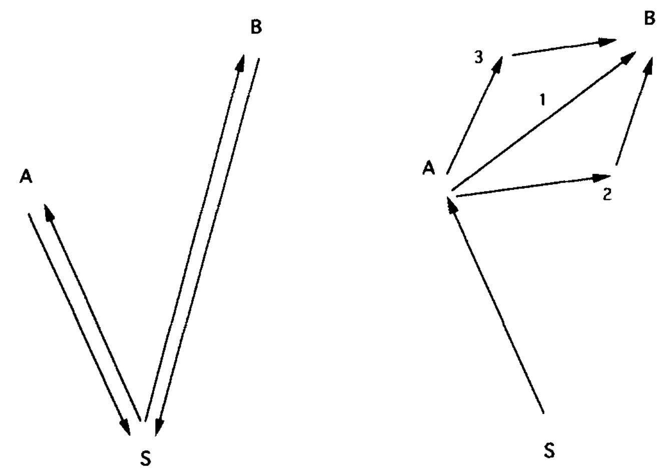
[Peer, Epstein, 2021]

Maps

- when can we say does an animal use a map?
- rather than use stimulus-response chaining
- => when it can take short-cuts



[Peer et al, 2020]

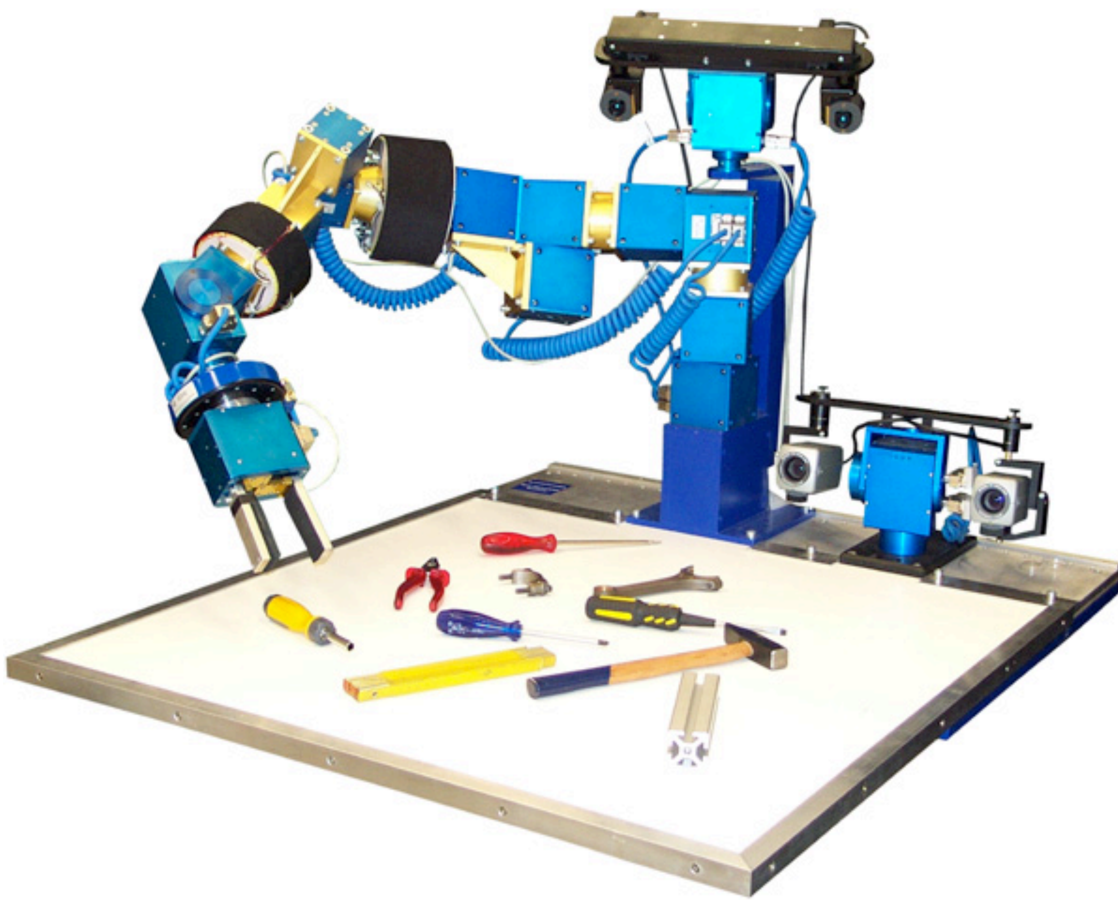


[Poucet, 1993]

Spaces for robotic motion planning

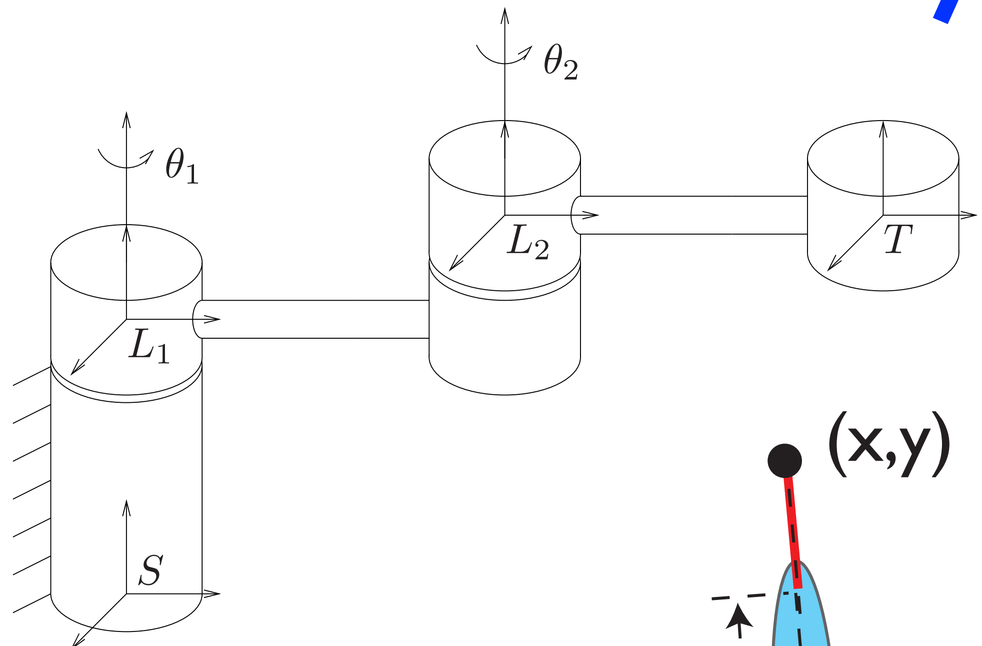
kinematic model	$\mathbf{x} = \mathbf{f}(\theta)$	$\dot{\mathbf{x}} = \mathbf{J}(\theta)\dot{\theta}$
inverse kinematic model	$\theta = \mathbf{f}^{-1}(\mathbf{x})$	$\dot{\theta} = \mathbf{J}^{-1}(\theta)\dot{\mathbf{x}}$

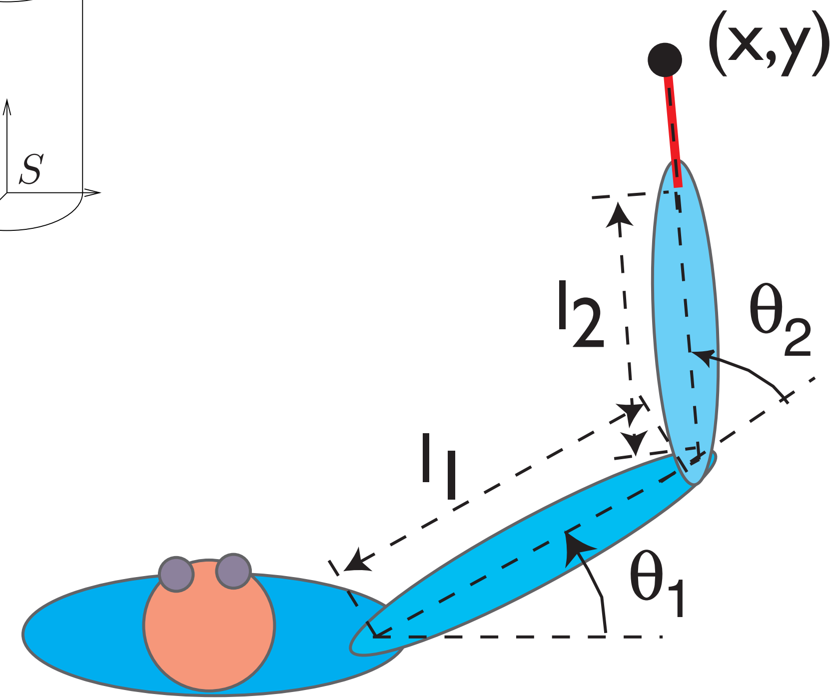
- transform end-effector to configuration space through inverse kinematics
- problems of singularities and multiple "leafs" of inverse…



Forward kinematics

[Murray, Li, Sastry 1994]





- where is the hand, given the joint angles..

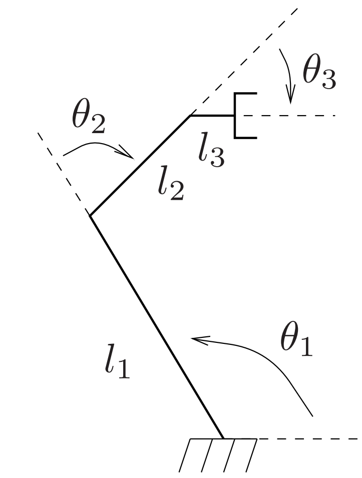
$$\mathbf{x} = \mathbf{f}(\theta)$$

$$x = l_1 \cos(\theta_1) + l_2 \cos(\theta_1 + \theta_2)$$

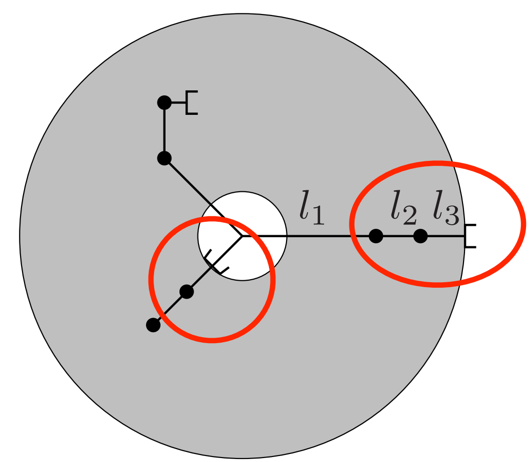
$$y = l_1 \sin(\theta_1) + l_2 \sin(\theta_1 + \theta_2)$$

Workspace / Singularities

- where the Eigenvalue of the Jacobian becomes zero (real part)…
- so that movement in a particular direction is not possible…
- typically at extended postures or inverted postures
- at limits of workspace



(a)



(c)

Redundant kinematics

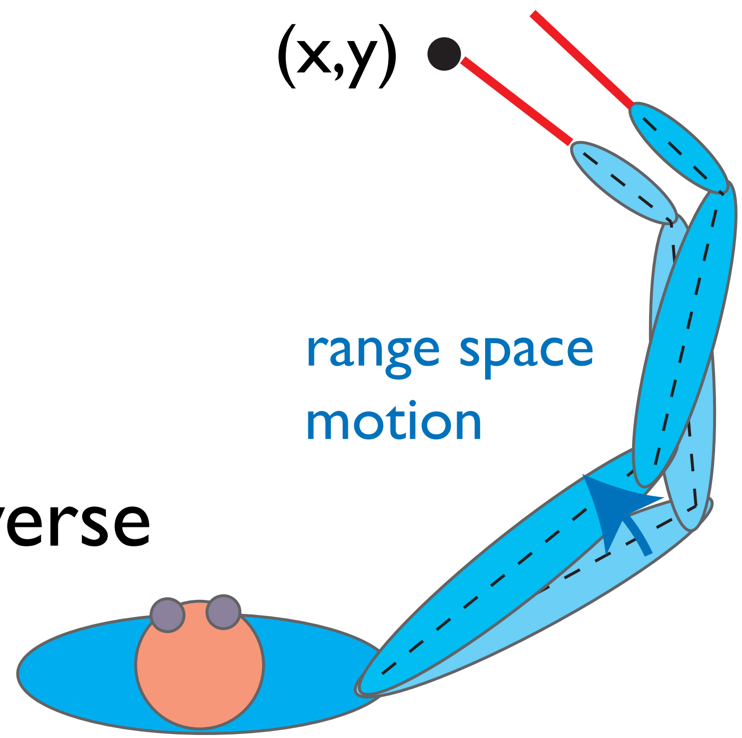
- use pseudo-inverses that minimize a functional (e.g., total joint velocity or total momentum)

$$\dot{\mathbf{x}} = \mathbf{J}(\theta)\dot{\theta}$$

$$\dot{\theta} = \mathbf{J}^{+}(\theta)\dot{\mathbf{x}}$$

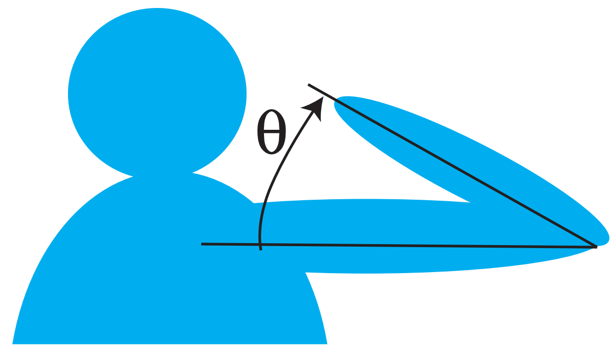
$$\mathbf{J}^{+}(\theta) = \mathbf{J}^{T}(\mathbf{J}\mathbf{J}^{T})^{-1}$$ pseudo-inverse

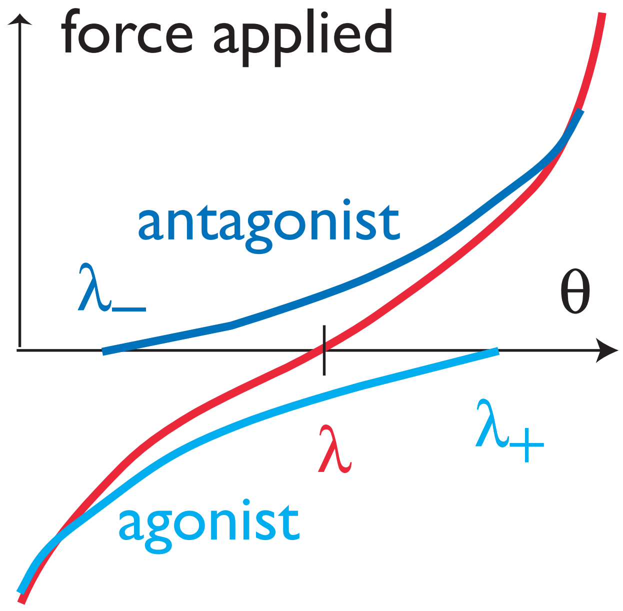
minimizes $\dot{\theta}^2$



Human motor control

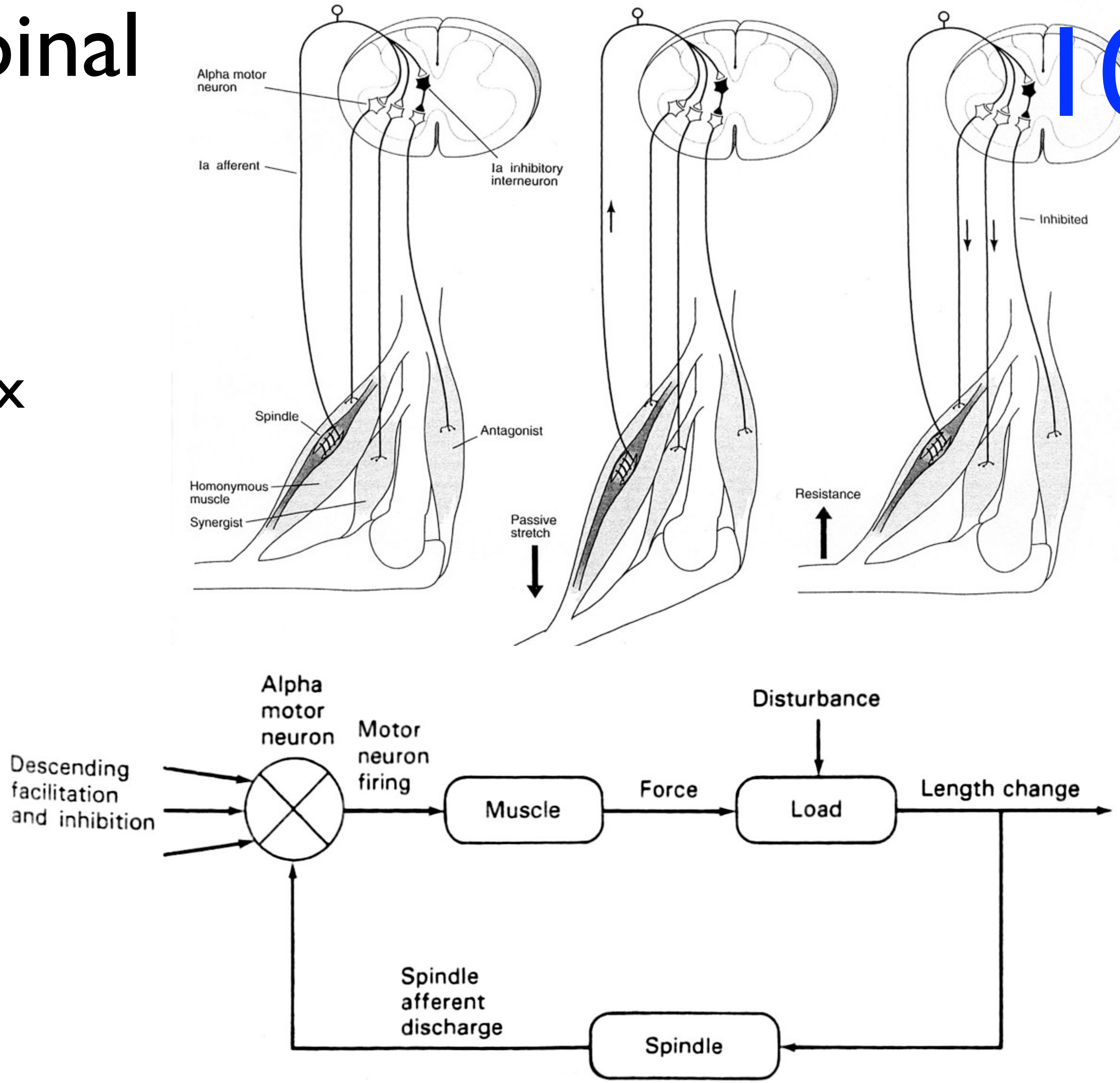
- posture resists when pushed => is actively controlled = stabilized by feedback
- invariant characteristic
  - one lambda per muscle
  - co-contraction controls stiffness





based on spinal reflexes

- stretch reflex



[Kandel, Schartz, Jessell, Fig. 37-11]

Timing

- generate movements that are "timed", that is,
  - they arrive "on time"
  - the are coordinated across different effectors
  - the are coordinated with moving objects (e.g., catching)
- timing implies some form of anticipation…

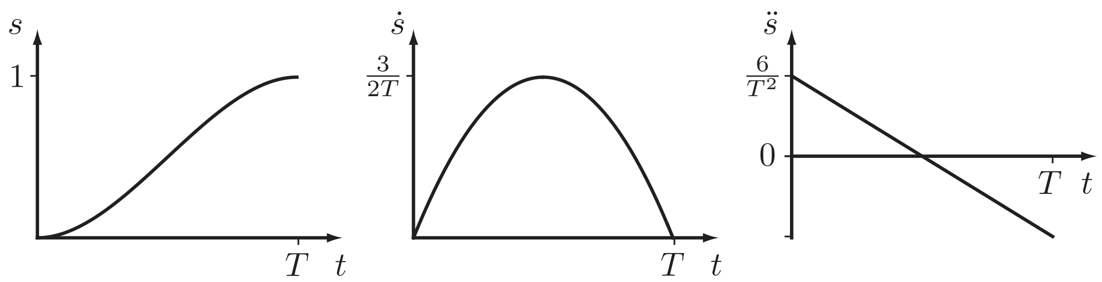
Conventional robotic timing

- time scaling

$$s(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3.$$

$$X(s) = X_{\text{start}} + s(X_{\text{end}} - X_{\text{start}}), s \in [0, 1].$$

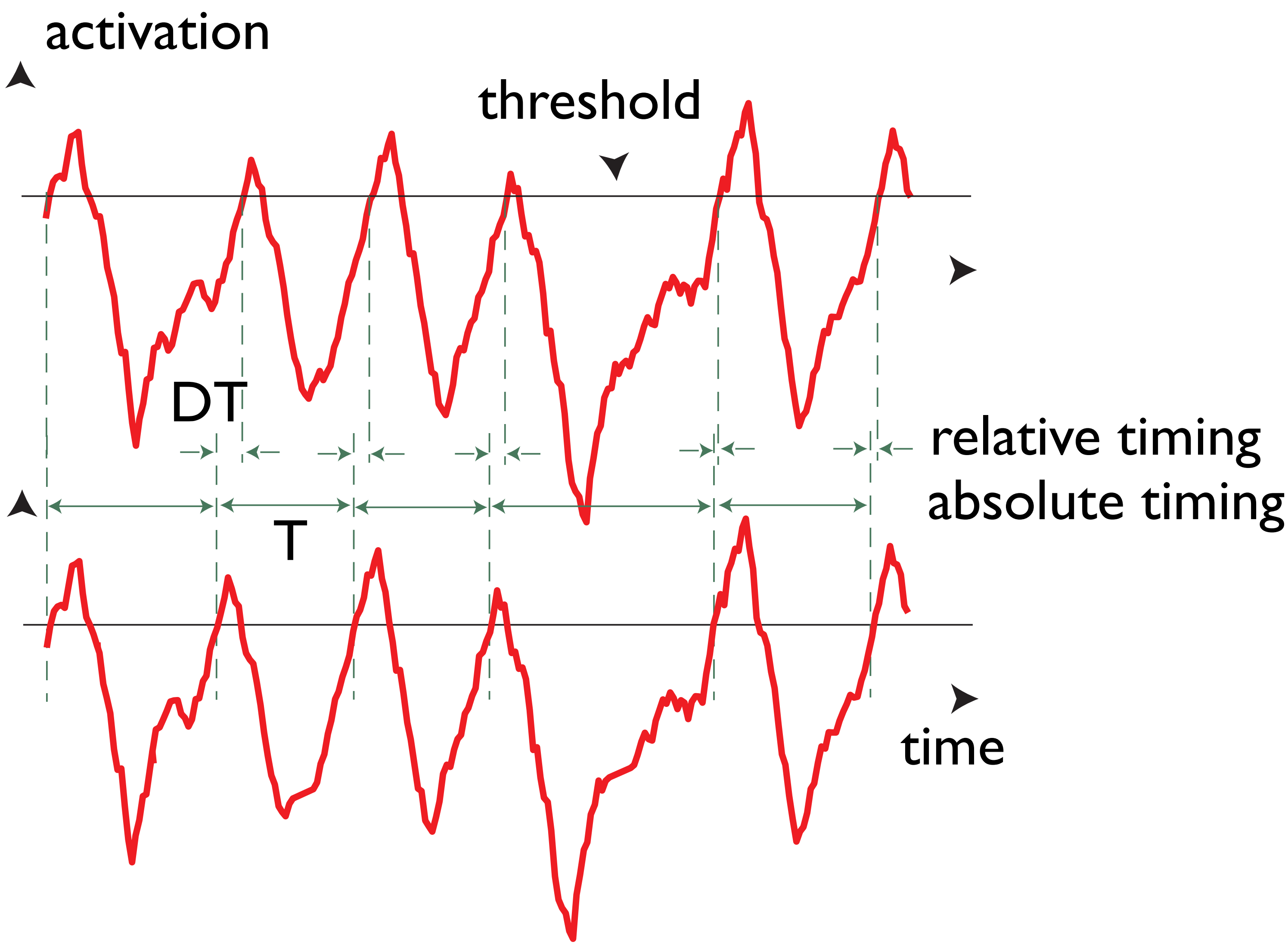
$$\theta(s) = \theta_{\text{start}} + s(\theta_{\text{end}} - \theta_{\text{start}}),$$



- compute parameters to achieve a particular movement time T, with zero velocity at target

[Lynch, Park, 2017 (Chapter 9)]

Relative vs. absolute timing



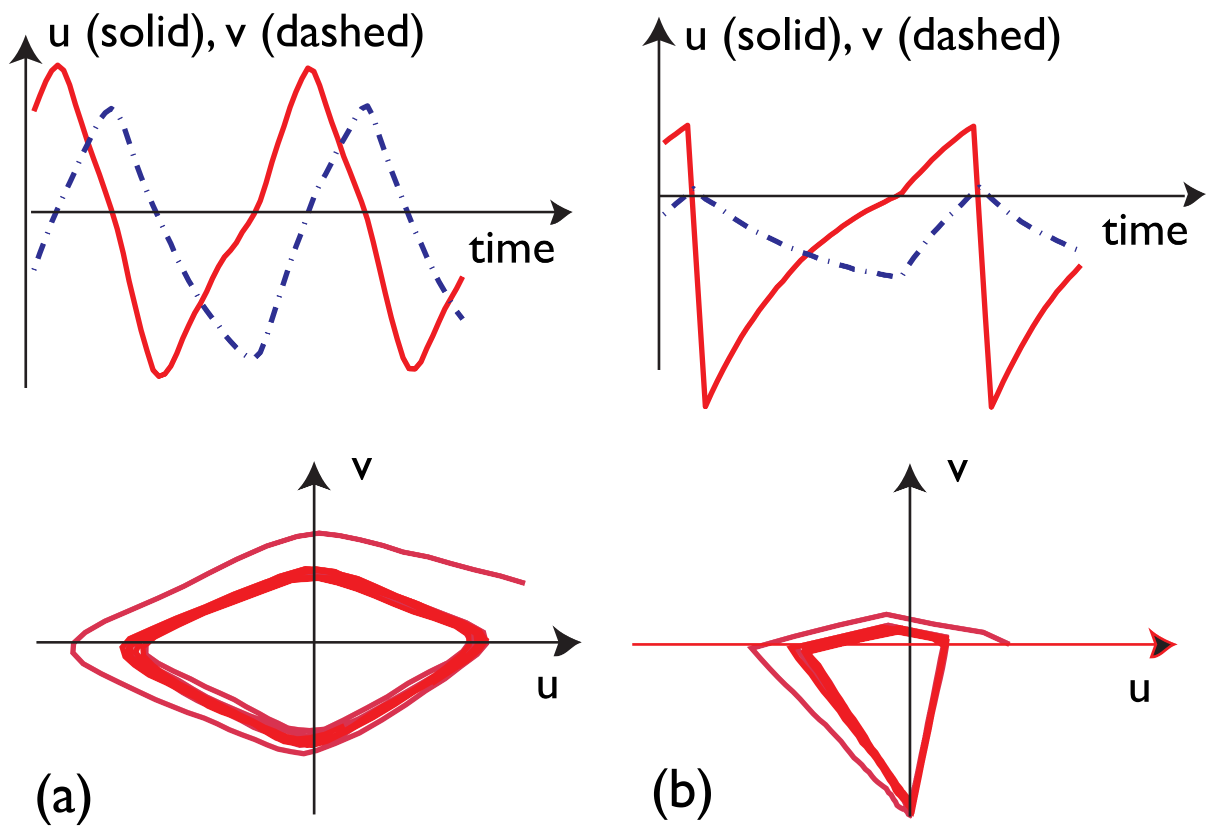
relative phase=DT/T

Theoretical account for absolute timing

- (neural) oscillator autonomously generates timing signal, from which timing events emerge
- => limit cycle oscillators
- = clocks

Neural oscillator

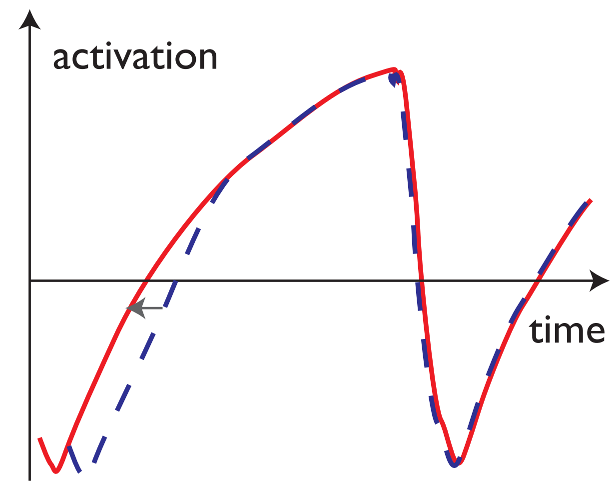
$$\tau \dot{u} = -u + h_u + w_{uu} f(u) - w_{uv} f(v)$$
$$\tau \dot{v} = -v + h_v + w_{vu} f(u),$$

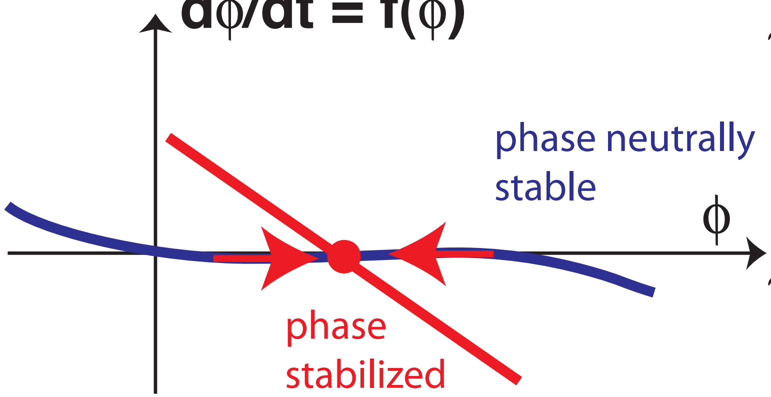


[Amari 77]

Coordination from coupling

- coordination=stable relative timing emerges from coupling of neural oscillators



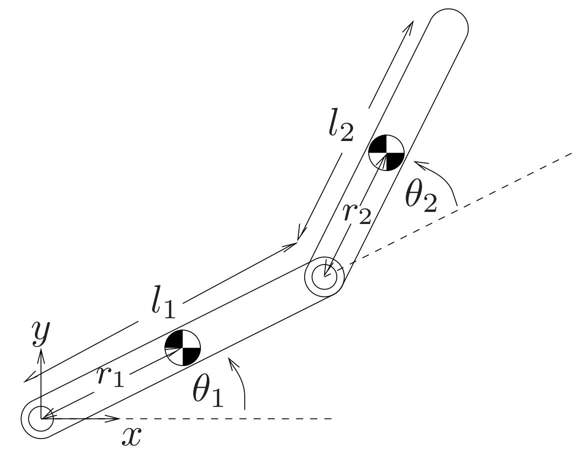


$$\tau \dot{u}_1 = -u_1 + h_u + w_{uu} f(u_1) - w_{uv} f(v_1)$$
$$\tau \dot{v}_1 = -v_1 + h_v + w_{vu} f(u_1) \boxed{+\ c f(u_2)}$$
$$\tau \dot{u}_2 = -u_2 + h_u + w_{uu} f(u_2) - w_{uv} f(v_2)$$
$$\tau \dot{v}_2 = -v_2 + h_v + w_{vu} f(u_2) \boxed{+\ c f(u_1)}$$

[Schöner: Timing, Clocks, and Dynamical Systems. *Brain and Cognition* 48:31-51 (2002)]

Rigid bodies: constraints

- constraints reduce the effective numbers of degrees of freedom. . .



$$F_i = m_i \ddot{r}_i \qquad r_i \in \mathbb{R}^3, i = 1, \ldots, n.$$

$$g_j(r_1, \ldots, r_n) = 0 \qquad j = 1, \ldots, k.$$

Open-chain manipulator

$$\boxed{M(\theta)\ddot{\theta} + C(\theta, \dot{\theta})\dot{\theta} + N(\theta, \dot{\theta}) = \tau}$$

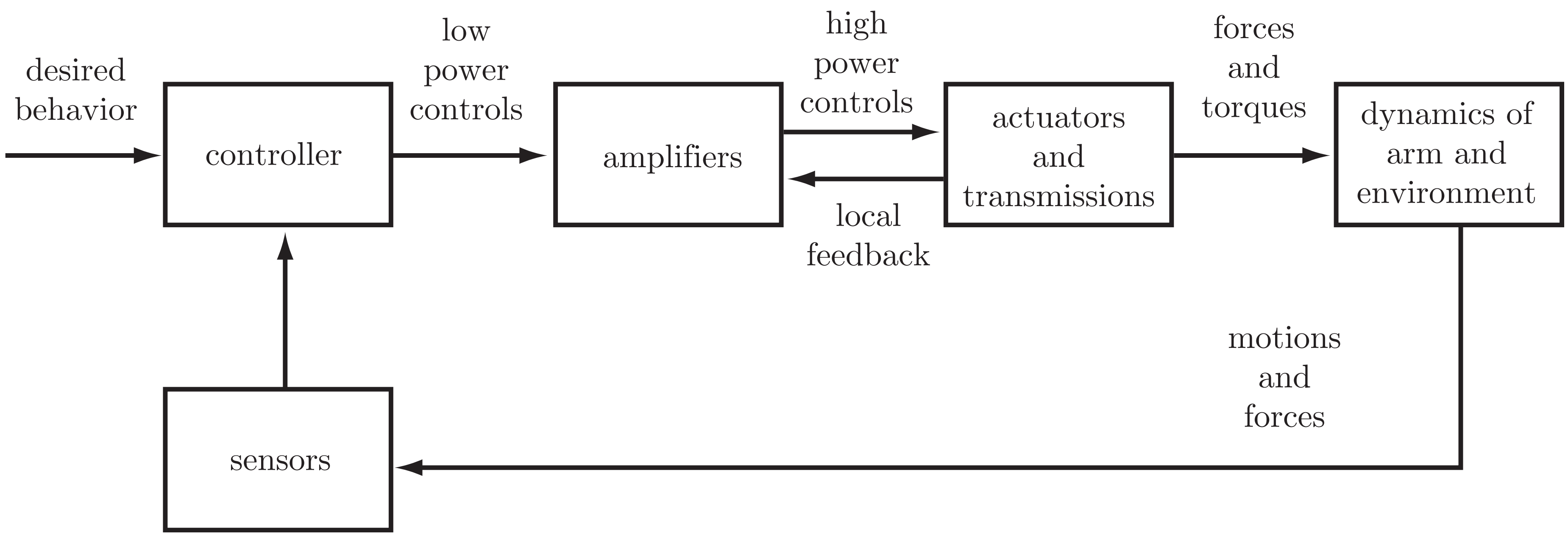
inertial

centrifugal/ coriolis

gravitational

active torques

Robotic control



(a)

[Lunch, Park, 2017]

Motion control single joint

- $\tau = M\ddot{\theta} + mgr\cos(\theta) + b\dot{\theta}$
- feedback PID controller
- $\tau = K_p\theta_e + K_d\dot{\theta}_e + K_i\int\theta(t')dt'$

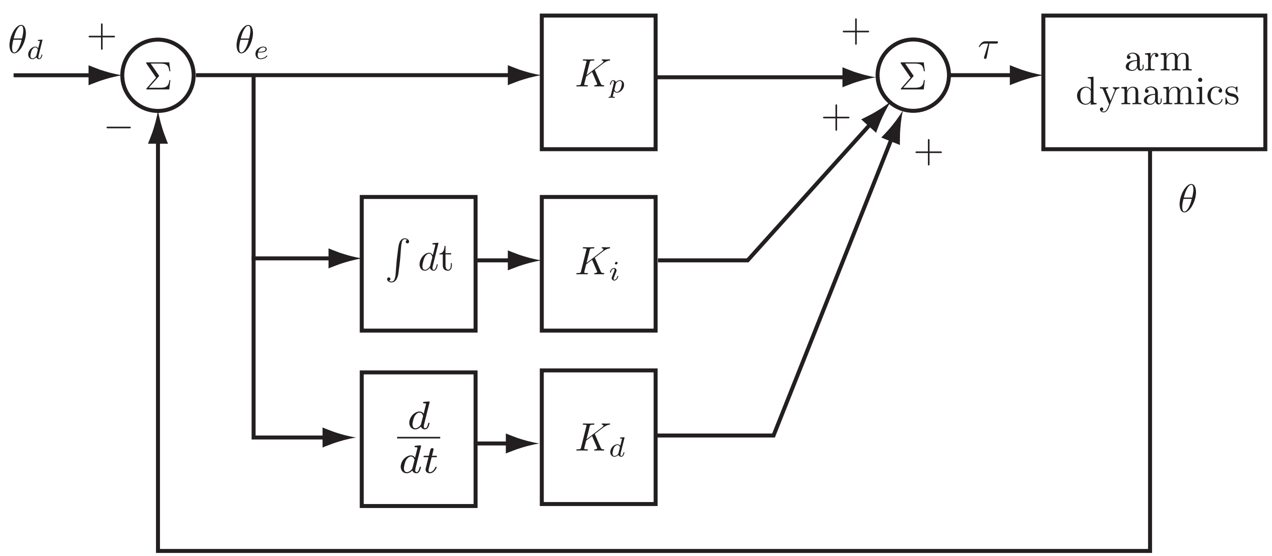


Figure 11.12: Block diagram of a PID controller.

[Lunch, Park, 2017]

Control of multi-joint arm

- generate joint torques that produce a desired motion…θ_d
- error $\theta_e = \theta - \theta_d$
- PID control $\tau = K_p\theta_e + K_e\dot{\theta}_d + K_i\int \theta_e(t')dt'$
- => controlling joints independently

$$M(\theta)\ddot{\theta} + C(\theta,\dot{\theta})\dot{\theta} + N(\theta,\dot{\theta}) = \tau$$