Prof. Dr. G. Schöner, Institut für Neuroinformatik June 23, 2022

Autonomous robotics exercise 6: Kinematics of robot arms

Consider a two-point planar arm with a "shoulder" and an "elbow" joint, controlling an end-effector, the tip of the "hand" that is rigidly attached to the elbow.

1. Make a drawing of the arm and of coordinate axes for the end-effector position (horizontal: x, vertical: y) inspired by the drawings used in the lecture. Mark the two joint angles, θ_1 and θ_2 (counting the shoulder angle from the horizontal, the elbow angle from the continuation of the upper arm). Denote the segment lengths, l_1 and l_2.
2. Derive the forward kinematics that links the joint angles to the end-effector. The result was given in the lecture. [Make use of the geometrical meaning of sin and cos.]
3. Determine the Jacobian by computing the partial derivates of these equations. Write down the differential forward kinematics as a vector-valued equation with the Jacobian as a Matrix.
4. Compute the end-effector velocity vector for a movement when only the shoulder joint moves. Evaluate that function when the arm is initially full extended along the horizontal axis and do the same when the arm is initially extended along the vertical axis. Interpret the geometrical meaning of that vector in each case.
5. Make a sketch of the reachable workspace of the arm (assume the upper arm is longer than the lower arm, no joint limits).
6. Compute the inverse of the differential forward kinematics by inverting the Jacobian matrix.
7. Insert the end-effector velocity vectors of $(0, 1)$ and $(1, 0)$ onto this inverse differential kinematics to compute $\dot{\theta}_1$ and $\dot{\theta}_2$. Look at this for the fully extended arm posture ($\theta_2 = 0$) along the horizontal and the vertical posture of the arm. [Caution, something goes wrong in certain cases!]

The book listed below give more background about kinematics. You won't really need them for the simple tasks of this exercise sheet, however.

- A Mathematical Introduction to Robotic Manipulation by RM Murray, Z Li, SS Sastry, CRC Press, Boca Raton FL USA, available from RM Murray at `http://www.cds.caltech.edu/~murray/mlswiki/`
- KM Lynch, FC Park: Modern Robotics — Mechanics, planning, and control. Cambridge Univ. Press, 2017. `http://hades.mech.northwestern.edu/index.php/Modern_Robotics`