Neural Dynamics

Gregor Schöner
gregor.schoener@ini.rub.de
Sensors

- transform a physical intensity into a neural activation

- intensity: light, sound, displacement

- neural activation: membrane potential, spike rate

Diagram:

```
<table>
<thead>
<tr>
<th>intensity</th>
</tr>
</thead>
<tbody>
<tr>
<td>activation</td>
</tr>
</tbody>
</table>

activation

intensity
```
Motors

- transform activation into physical action
- ... muscles
What is “activation”?

Activation is an abstraction of the state of neurons, defined relative to sigmoidal threshold function.

- Low levels of activation are not transmitted (to other neural systems, to motor systems).
- High levels of activation are transmitted.
- Threshold at zero (by definition).

The diagram shows the sigmoidal function $g(u)$ with a threshold at zero. The function approaches 1 as $u$ increases, and the curvature is defined by the parameter $\beta$.
Origin of the activation concept in neurophysics

activation, $u$, as a real number that reflects the (population) membrane potential

[from: Tresilian, 2012]
Grounding in neurophysics

- $u(t)$ evolves as a dynamical system, characterized by a time scale, $\tau \approx 10\text{ms}$

$$\tau \dot{u}(t) = -u(t) + h + \text{input}(t)$$

[from: Tresilian, 2012]
Grounding in neurophysics

- spiking when membrane potential exceeds threshold....
- spike train is transmitted to downstream neurons

[from: Tresilian, 2012]
Grounding in neurophysics

activation captures different firing rates in a small population...

[from: Tresilian, 2012]
Grounding in neurophysics

In neural dynamics, the spiking mechanism and associated firing rate is replaced by a statistical (population) description: threshold function

\[ \sigma(u) \]
Neural dynamics

dynamical system: the present predicts the future

given a initial level of activation, $u(0)$, the activation, $u(t)$, at times $t > 0$ is uniquely determined

$$\tau \dot{u}(t) = -u(t) + h$$

$du/dt = f(u)$

vector-field
Neural dynamics

- **fixed point** = constant solution (stationary state)
- **stable fixed point** = attractor: nearby solutions converge to the fixed point

\[ \tau \dot{u}(t) = -u(t) + h \]
Neural dynamics

**attractors structure the ensemble of solutions (for all initial conditions) = flow**

\[ \tau \dot{u}(t) = -u(t) + h \]
Neuronal dynamics

- in neural dynamics, inputs are contributions to the rate of change
  - positive: excitatory
  - negative: inhibitory
- shifts the attractor
- activation tracks this shift due to stability

$$\tau \dot{u}(t) = -u(t) + h + s(t)$$
what is transmitted is $\sigma(u(t))$

(labelled $g(t)$ in the book and in some figures)

$\Rightarrow$ neural dynamics as a low-pass filter of time varying input

$= \text{input-driven solution}$

\[ \tau \dot{u}(t) = -u(t) + h + s(t) \]
=> simulation
Neuronal dynamics with self-excitation

- single activation variable with self-excitation
- representing a small population with excitatory coupling

\[ \tau \dot{u}(t) = -u(t) + h + s(t) + c \sigma(u(t)) \]
Neuronal dynamics with self-excitation

\[ \tau \dot{u}(t) = -u(t) + h + s(t) + c \sigma(u(t)) \]

**nonlinear dynamics!**
Neuronal dynamics with self-excitation

\[ \tau \frac{du(t)}{dt} = -u(t) + h + s(t) + c \sigma(u(t)) \]
Neuronal dynamics with self-excitation

- at intermediate stimulus strength: bistable
- “on” vs “off” state

\[ \tau \dot{u}(t) = -u(t) + h + s(t) + c \sigma(u(t)) \]
Neuronal dynamics with self-excitation

- Increasing input strength \( \Rightarrow \) detection instability
- \( \Rightarrow \) the detection decision is stabilized

\[
\tau \dot{u}(t) = -u(t) + h + s(t) + c \sigma(u(t))
\]
Neuronal dynamics with self-excitation

- Decreasing input strength => reverse detection instability

\[ \tau \dot{u}(t) = -u(t) + h + s(t) + c \sigma(u(t)) \]
Neuronal dynamics with self-excitation

- the detection and its reverse => create discrete events from time-continuous changes

\[
\tau \dot{u}(t) = -u(t) + h + s(t) + c \sigma(u(t))
\]
=> simulation
Neuronal dynamics with competition

- two activation variables with reciprocal inhibitory coupling
- representing two small populations that are inhibitorily coupled

\[
\tau \dot{u}_1(t) = -u_1(t) + h + s_1(t) - c_{12} \sigma(u_2(t)) \\
\tau \dot{u}_2(t) = -u_2(t) + h + s_2(t) - c_{21} \sigma(u_1(t))
\]
Neuronal dynamics with competition

Coupling: the rate of change of one activation variable depends on the level of activation of the other activation variable.

\[
\begin{align*}
\tau \dot{u}_1(t) &= -u_1(t) + h + s_1(t) - c_{12} \sigma(u_2(t)) \\
\tau \dot{u}_2(t) &= -u_2(t) + h + s_2(t) - c_{21} \sigma(u_1(t))
\end{align*}
\]
Neuronal dynamics with competition

To visualize, assume that $u_2$ has been activated by input to a positive level.

$\Rightarrow$ it inhibits $u_1$

$$\tau \dot{u}_1(t) = -u_1(t) + h + s_1(t) - c_{12} \sigma(u_2(t))$$

$$\tau \dot{u}_2(t) = -u_2(t) + h + s_2(t) - c_{21} \sigma(u_1(t))$$
Neuronal dynamics with competition

- why would $u_2$ be positive before $u_1$?
- more input to $u_2$ (better “match”) $\Rightarrow$ faster increase
- input advantage $\iff$ time advantage $\iff$ competitive advantage

\[
\tau \dot{u}_1(t) = -u_1(t) + h + s_1(t) - c_{12} \sigma(u_2(t))
\]
\[
\tau \dot{u}_2(t) = -u_2(t) + h + s_2(t) - c_{21} \sigma(u_1(t))
\]
Neuronal dynamics with competition

vector-field in the absence of input

\[ \frac{du}{dt} = f(u) \]

ID cut through vector-field
Neuronal dynamics with competition

vector-field (without interaction) when both neurons receive input

\[
du/dt = f(u)
\]

ID cut through vector-field
Neuronal dynamics with competition

only activated neurons participate in interaction!
Neuronal dynamics with competition

- vector-field of mutual inhibition

site 1 inhibits site 2

site 2 inhibits site 1

interaction combined
Neuronal dynamics with competition

vector-field with strong mutual inhibition: bistable
Neuronal dynamics with competition

before input is presented

after input is presented
Neuronal dynamics with competition

stronger input to $u_1$ => attractor with positive $u_1$ stronger, attractor with positive $u_2$ weaker => closer to instability
Neuronal dynamics with competition

- decision made at detection instability!

before input is presented

after input is presented
=> simulation
The neural dynamics of fields

- the same underlying math
- coupling among continuously many activation variables
- local excitatory coupling ("self-excitation")
- global inhibitory coupling ("mutual inhibition")

\[ \tau \dot{u}(x, t) = -u(x, t) + h + s(x, t) + \int dx' w(x - x') \sigma(u(x', t)) \]
field vs. activation variables

- self-excitation
- mutual inhibition
- self-excitation

$s(x)$
$u(x)$
$u_1$
$u_2$
$s_1$
$s_2$