Neural Dynamics

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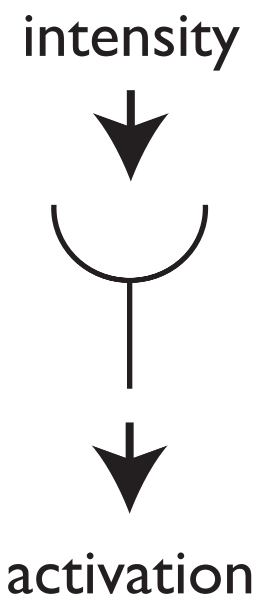
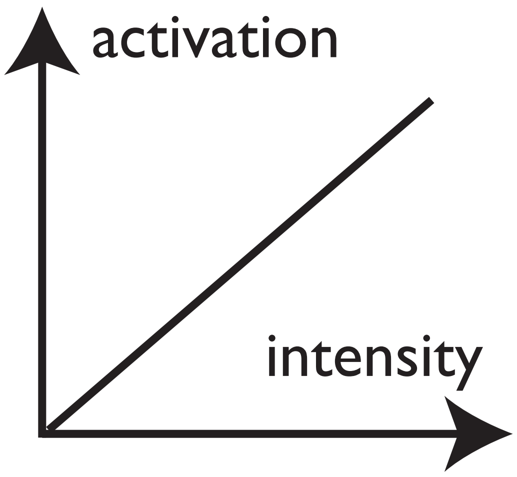
Sensors

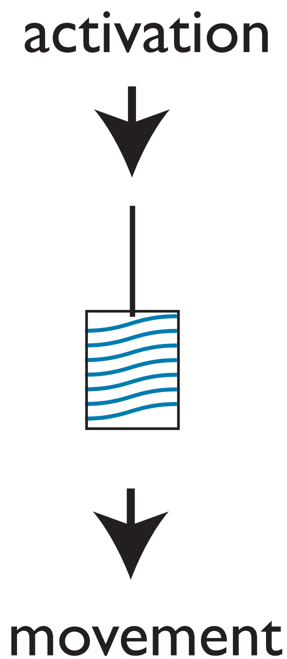
- transform a physical intensity into a neural activation
- intensity: light, sound, displacement
- neural activation: membrane potential, spike rate

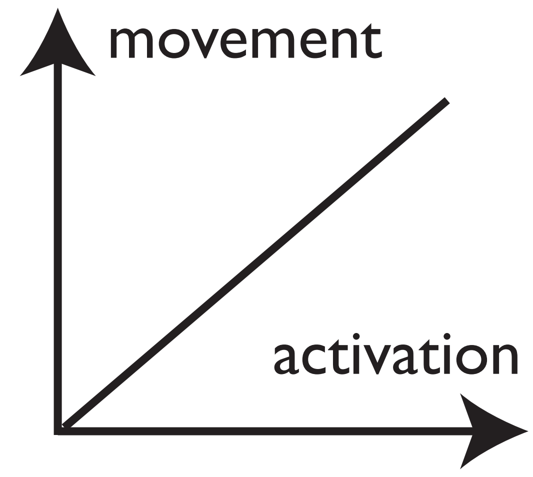




Motors

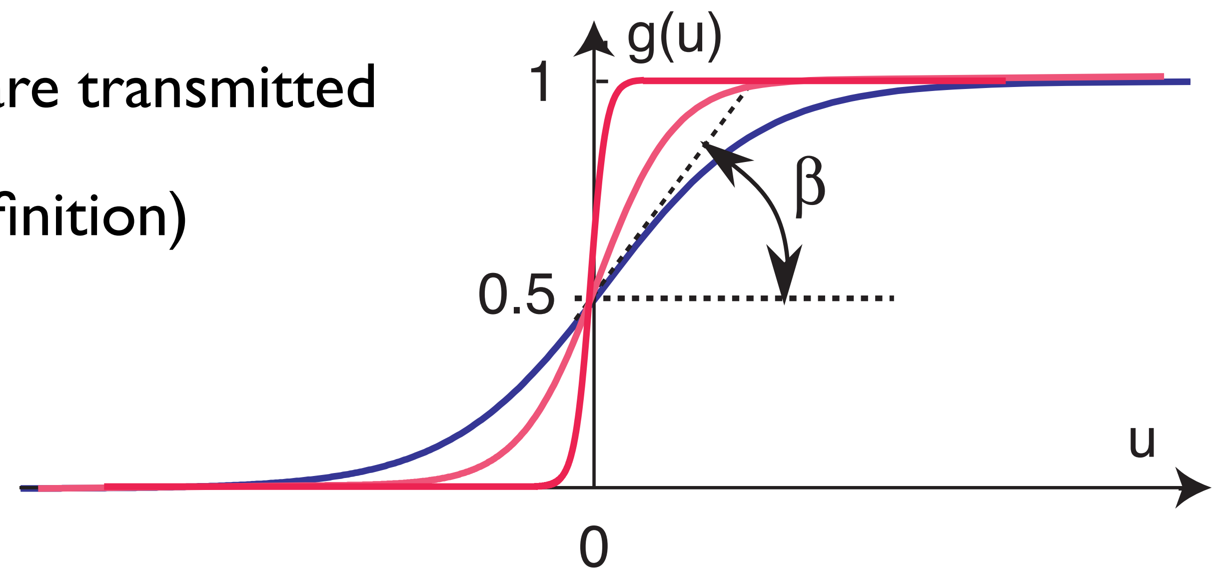
- transform activation into physical action
- … muscles





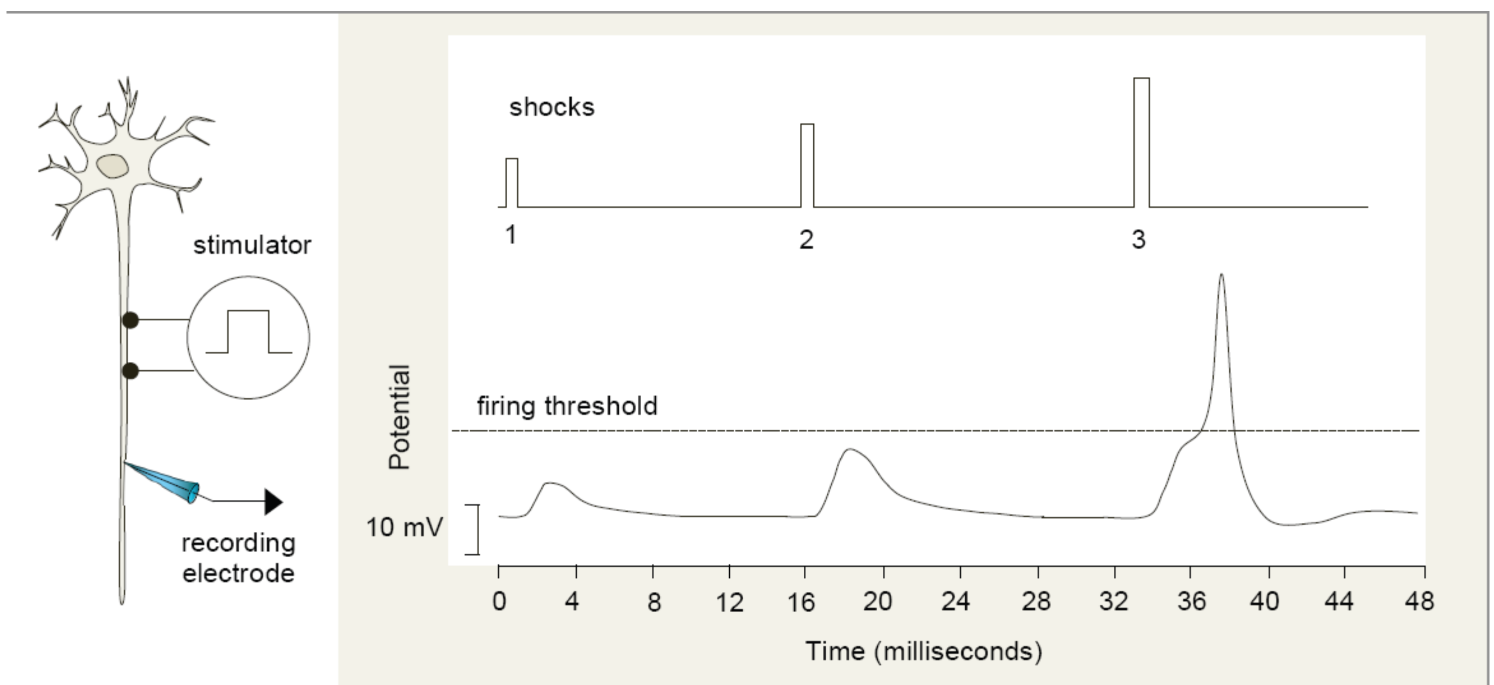
What is "activation"?

- activation is an abstraction of the state of neurons, defined relative to sigmoidal threshold function
  - low levels of activation are not transmitted (to other neural systems, to motor systems)
  - high levels of activation are transmitted
  - threshold at zero (by definition)



Origin of the activation concept in neurophysics

- activation, u, as a real number that reflects the (population) membrane potential

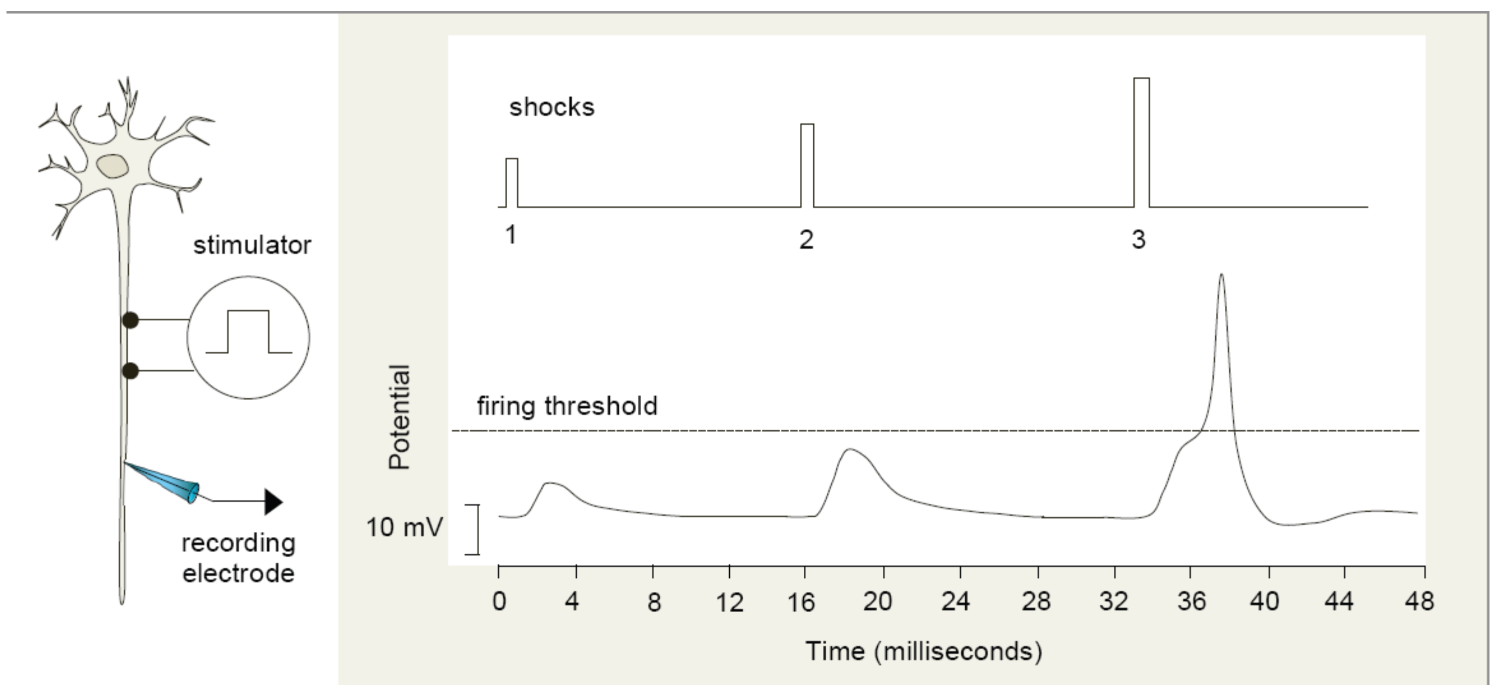


[from: Tresilian, 2012]

Grounding in neurophysics

- u(t) evolves as a dynamical system, characterized by a time scale, $\tau \approx 10\text{ms}$

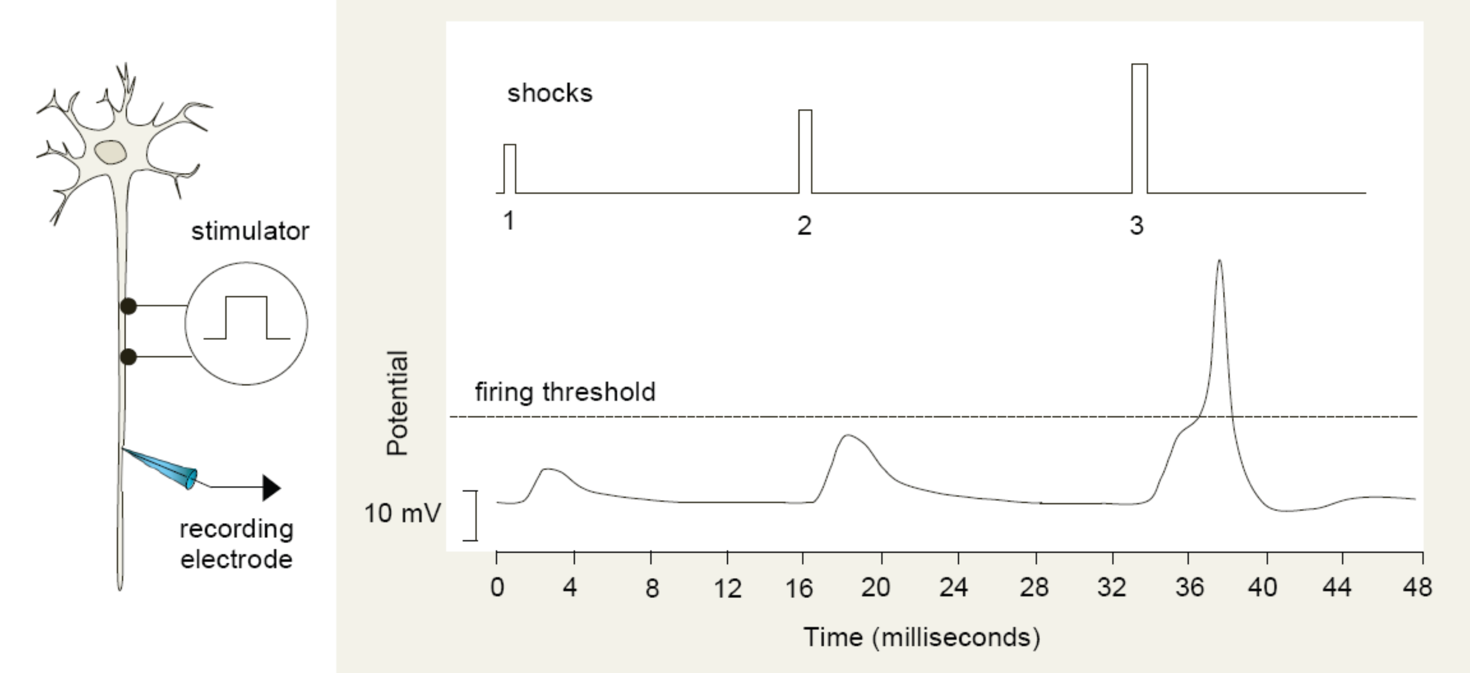
$$\tau \dot{u}(t) = -u(t) + h + \text{input}(t)$$



[from: Tresilian, 2012]

Grounding in neurophysics

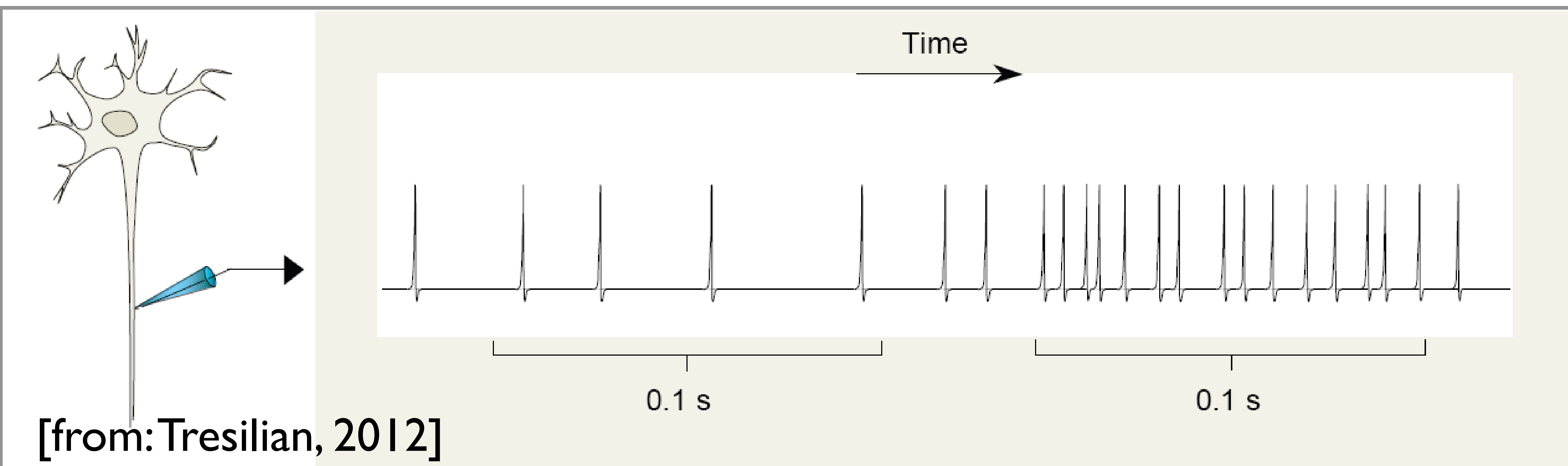
- spiking when membrane potential exceeds threshold….
- spike train is transmitted to downstream neurons



[from: Tresilian, 2012]

Grounding in neurophysics

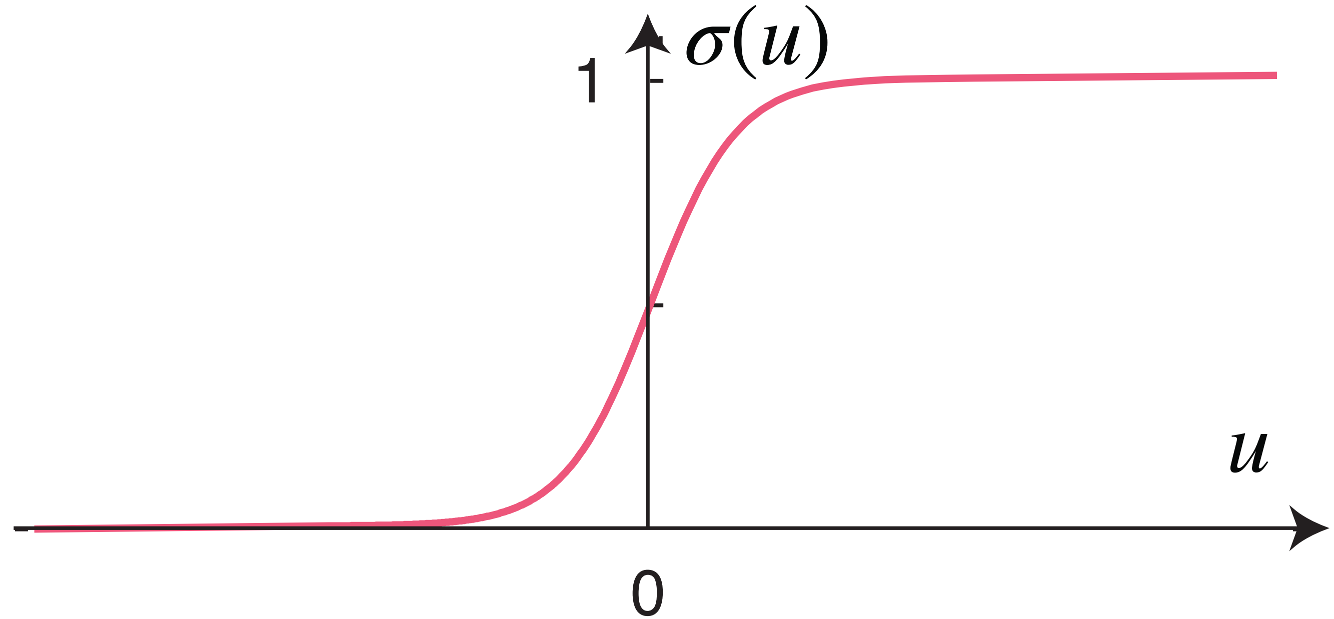
- activation captures different firing rates in a small population…



[from: Tresilian, 2012]

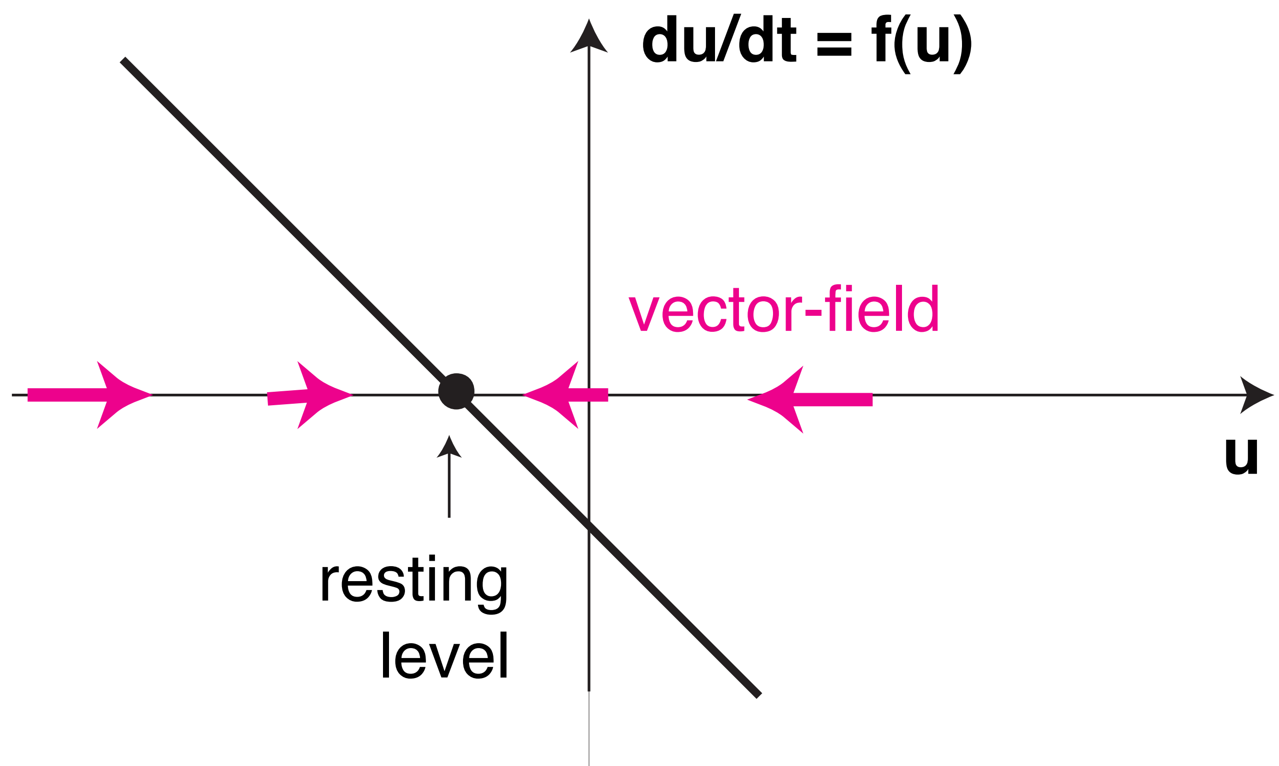
Grounding in neurophysics

- in neural dynamics, the spiking mechanism and associated firing rate is replaced by a statistical (population) description: threshold function



Neural dynamics

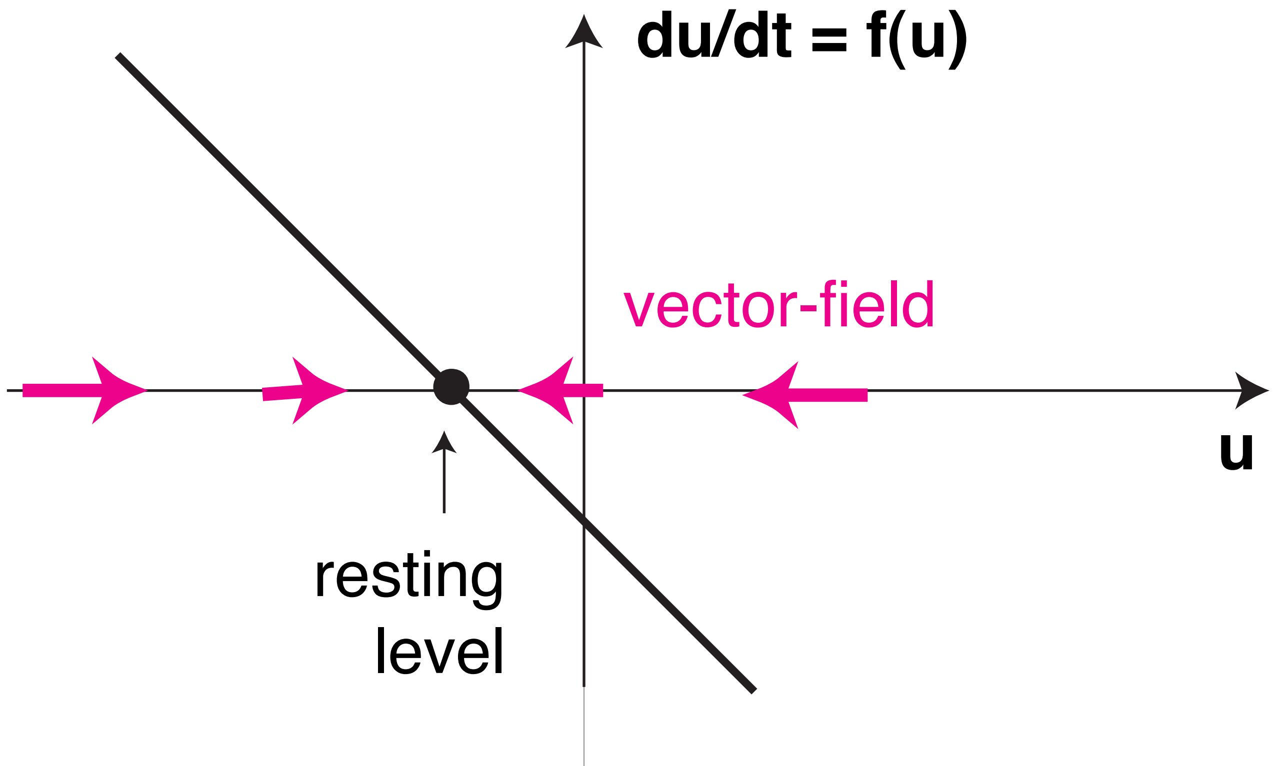
- dynamical system: the present predicts the future
- given a initial level of activation, u(0), the activation, u(t), at times t>0 is uniquely determined



$$\tau \dot{u}(t) = -u(t) + h$$

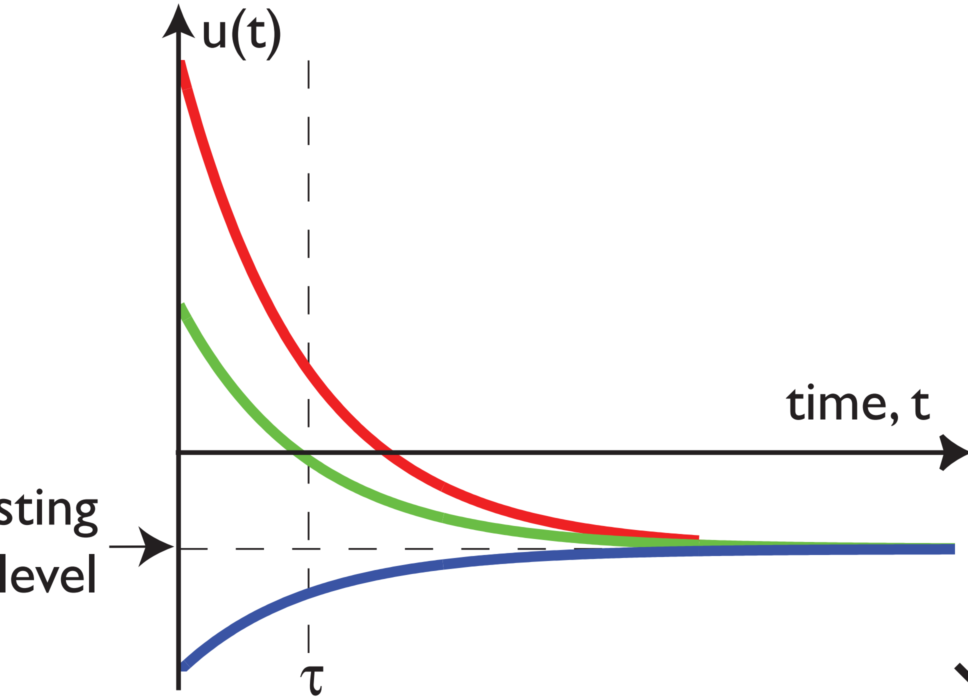
Neural dynamics

- fixed point = constant solution (stationary state)
- stable fixed point = attractor: nearby solutions converge to the fixed point

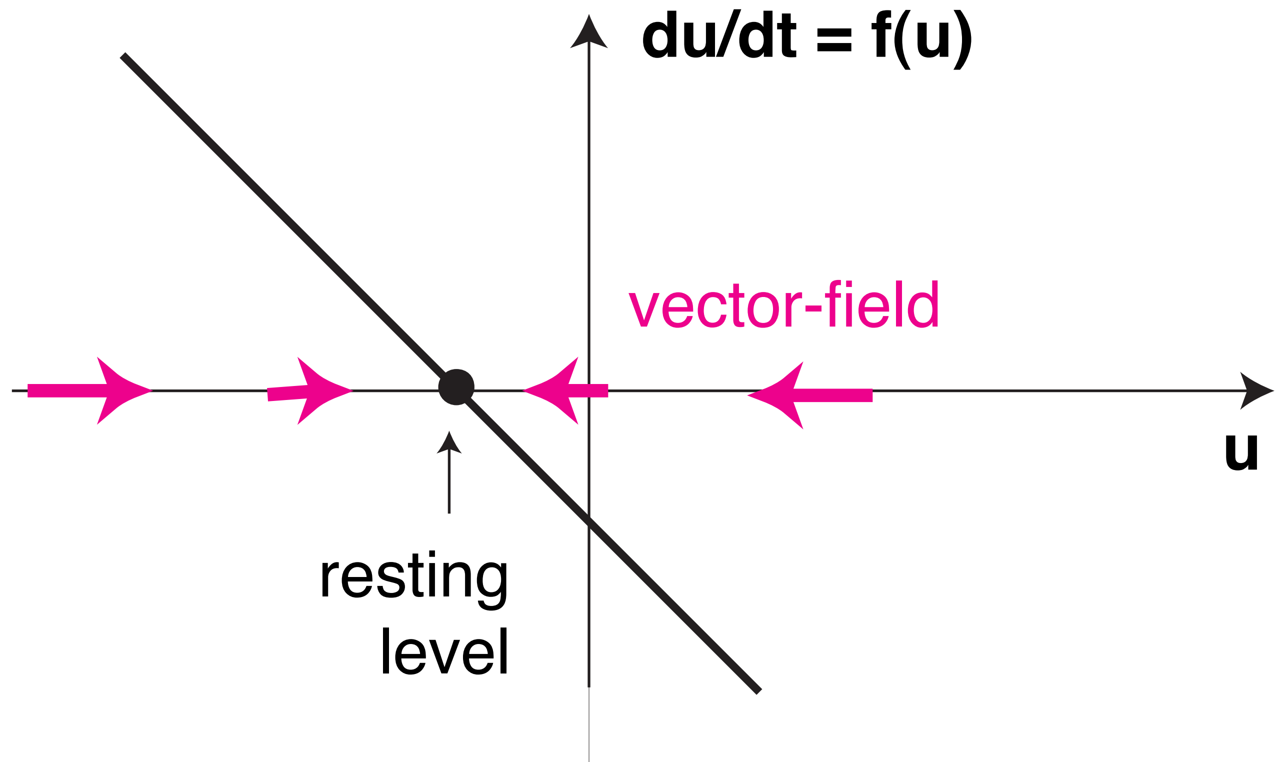


$$\tau\dot{u}(t) = -u(t) + h$$

Neural dynamics



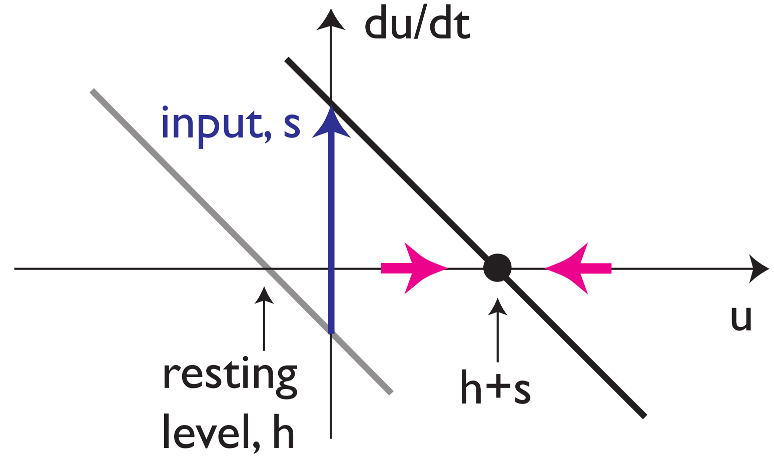
- attractors structure the ensemble of solutions (for all initial conditions) = flow

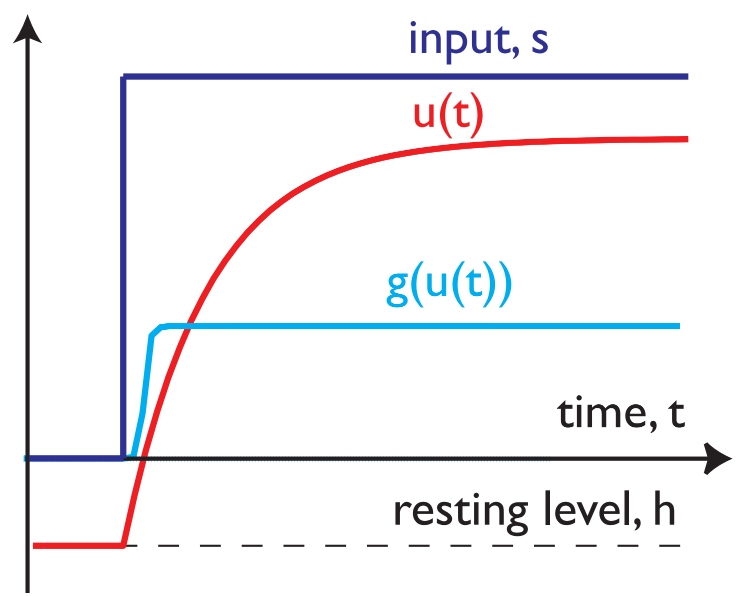


$$\tau \dot{u}(t) = -u(t) + h$$

Neuronal dynamics

- in neural dynamics, inputs are contributions to the rate of change
  - positive: excitatory
  - negative: inhibitory
- => shifts the attractor
- => activation tracks this shift due to stability



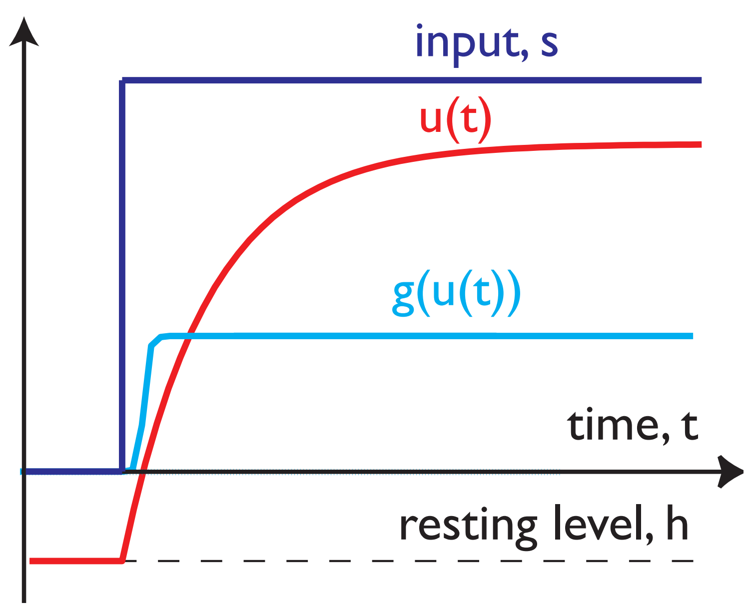


$$\tau \dot{u}(t) = -u(t) + h + s(t)$$

Neuronal dynamics

- what is transmitted is $\sigma(u(t))$
- (labelled g(t) in the book and in some figures)
- => neural dynamics as a low-pass filter of time varying input
- = input-driven solution

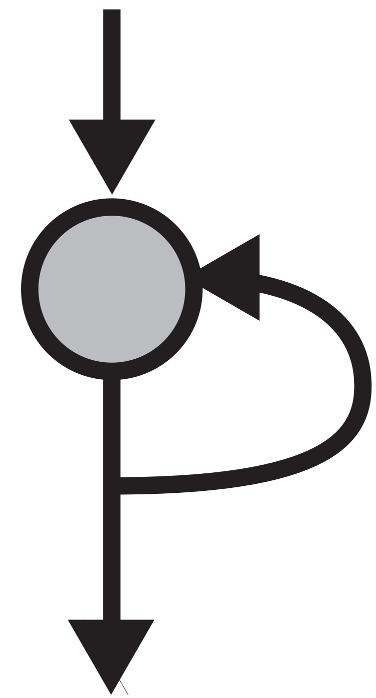
$$\tau \dot{u}(t) = -u(t) + h + s(t)$$



=> simulation

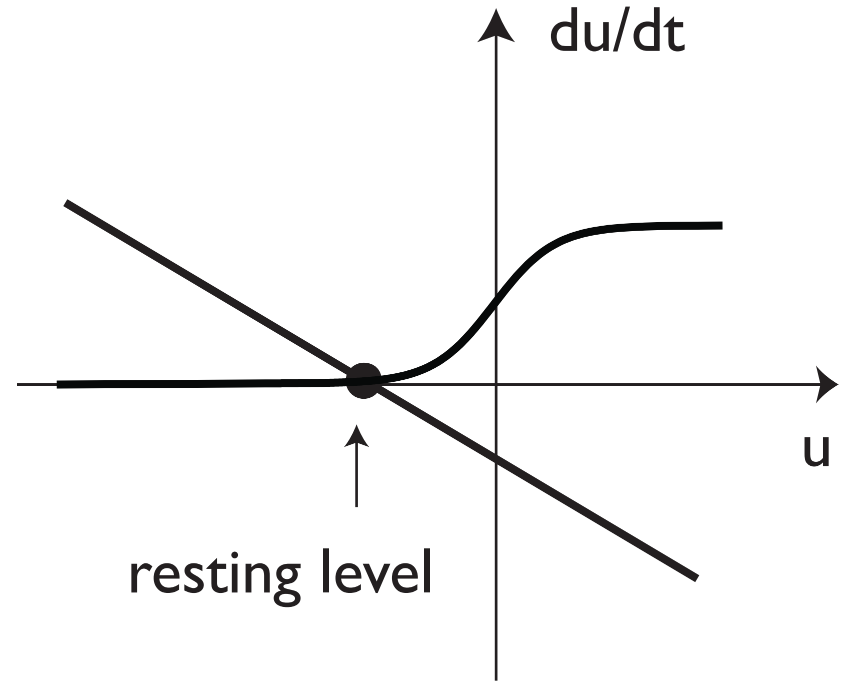
Neuronal dynamics with self-excitation

- single activation variable with self-excitation
- representing a small population with excitatory coupling

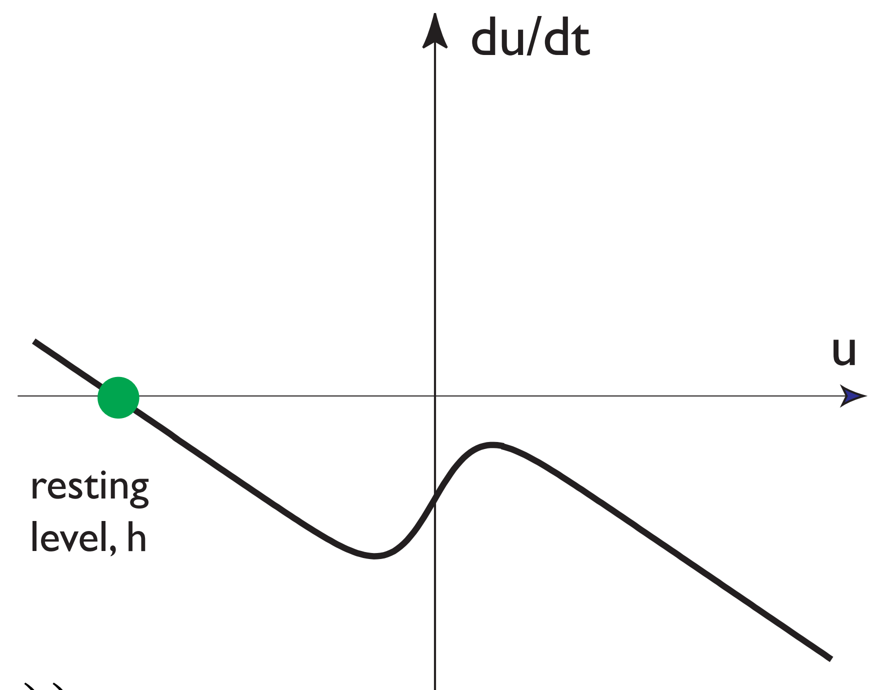


$$\tau \dot{u}(t) = -u(t) + h + s(t) + c\ \sigma(u(t))$$

Neuronal dynamics with self-excitation



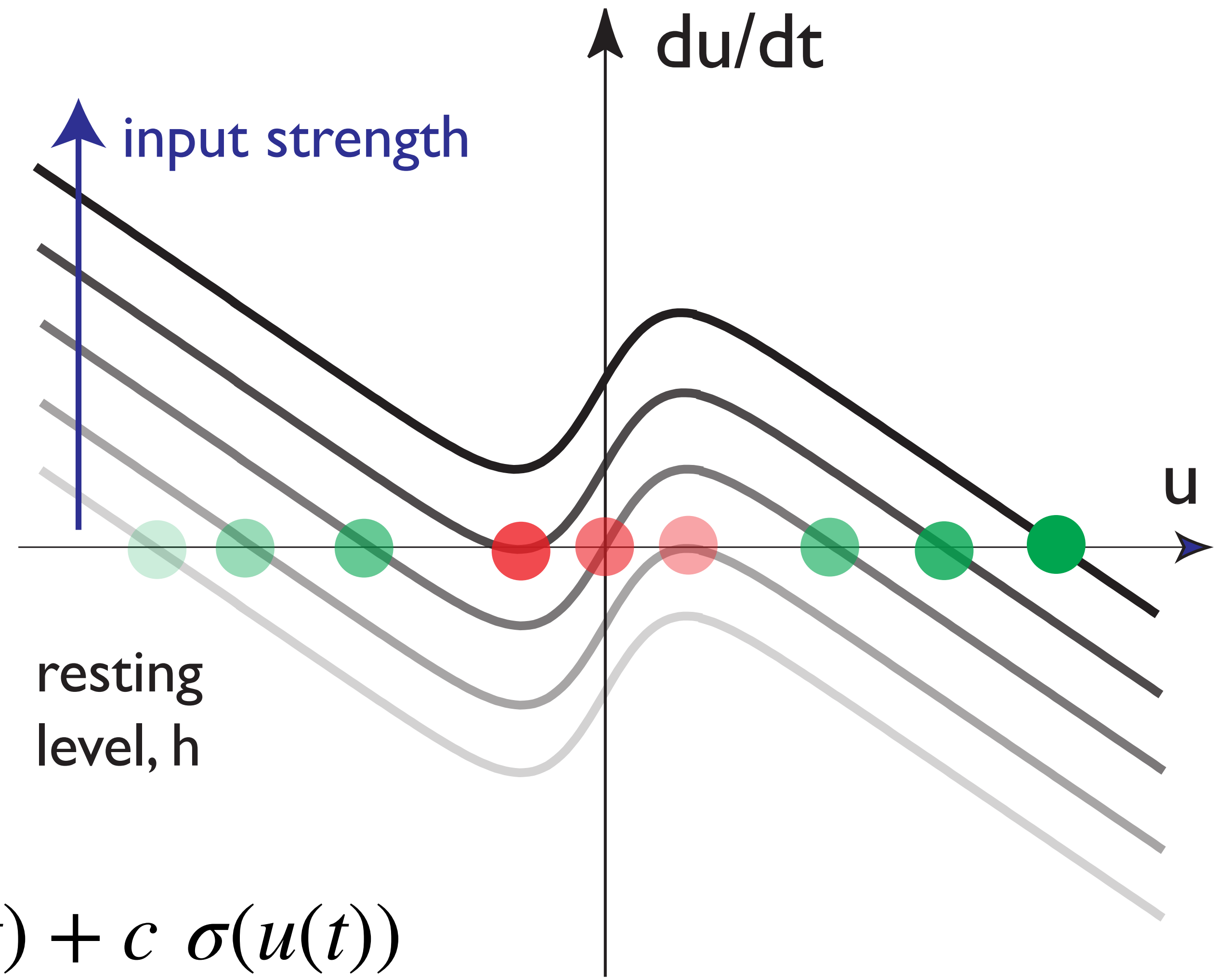
- => nonlinear dynamics!



$$\tau \dot{u}(t) = -u(t) + h + s(t) + c\ \sigma(u(t))$$

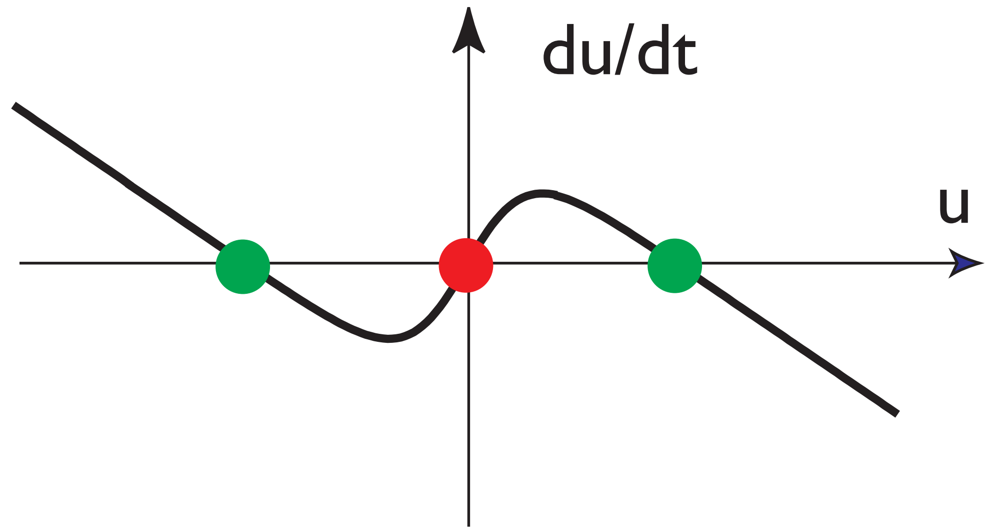
Neuronal dynamics with self-excitation

- varying input

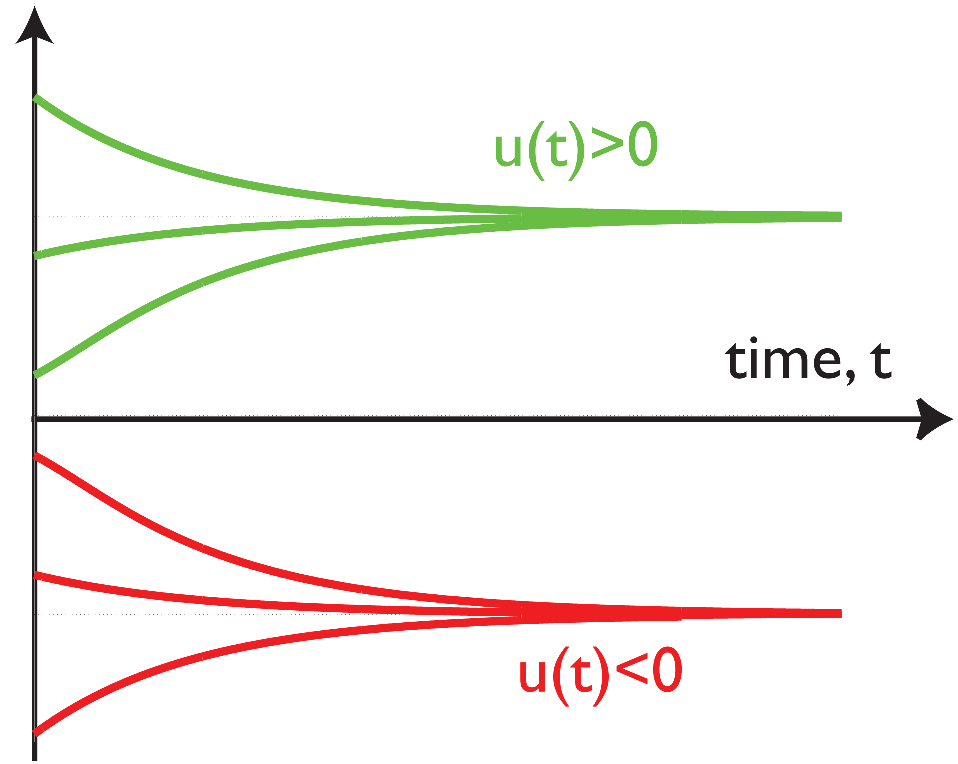


$$\tau \dot{u}(t) = -u(t) + h + s(t) + c\ \sigma(u(t))$$

Neuronal dynamics with self-excitation



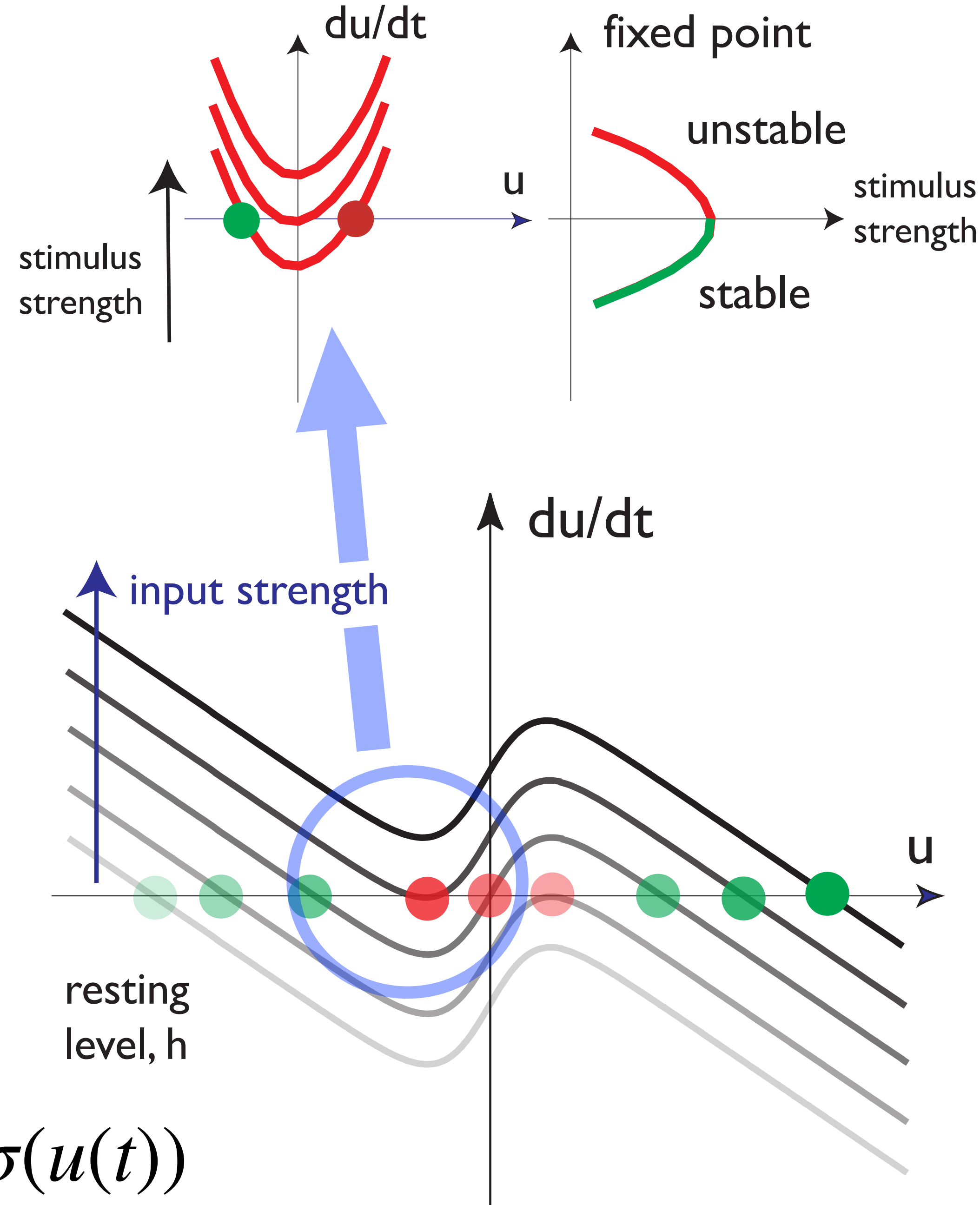
- at intermediate stimulus strength: bistable
- “on” vs “off” state



$$\tau \dot{u}(t) = -u(t) + h + s(t) + c\ \sigma(u(t))$$

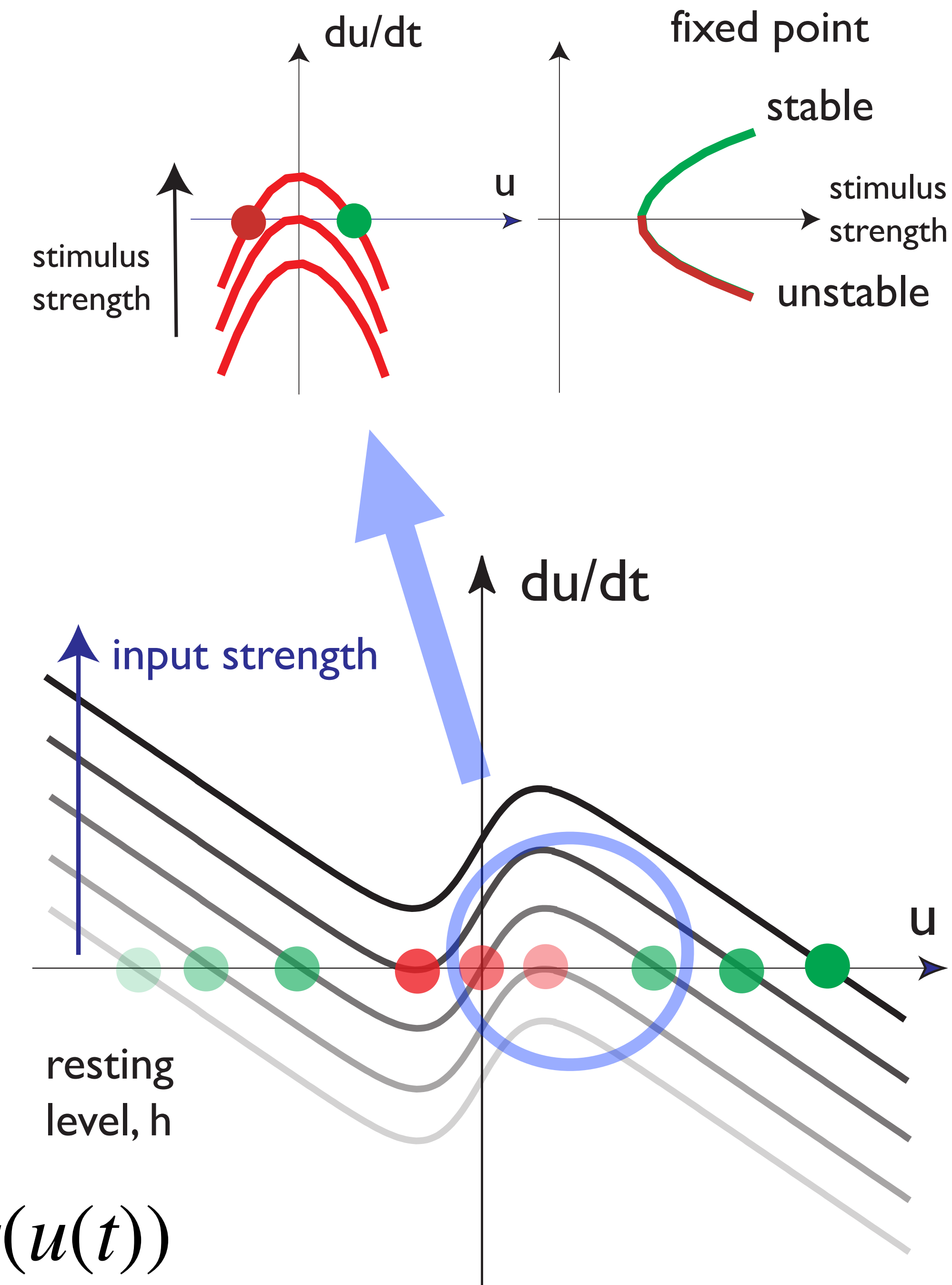
Neuronal dynamics with self-excitation

- increasing input strength => detection instability
- => the detection decision is stabilized



$$\tau \dot{u}(t) = -u(t) + h + s(t) + c\ \sigma(u(t))$$

Neuronal dynamics with self-excitation

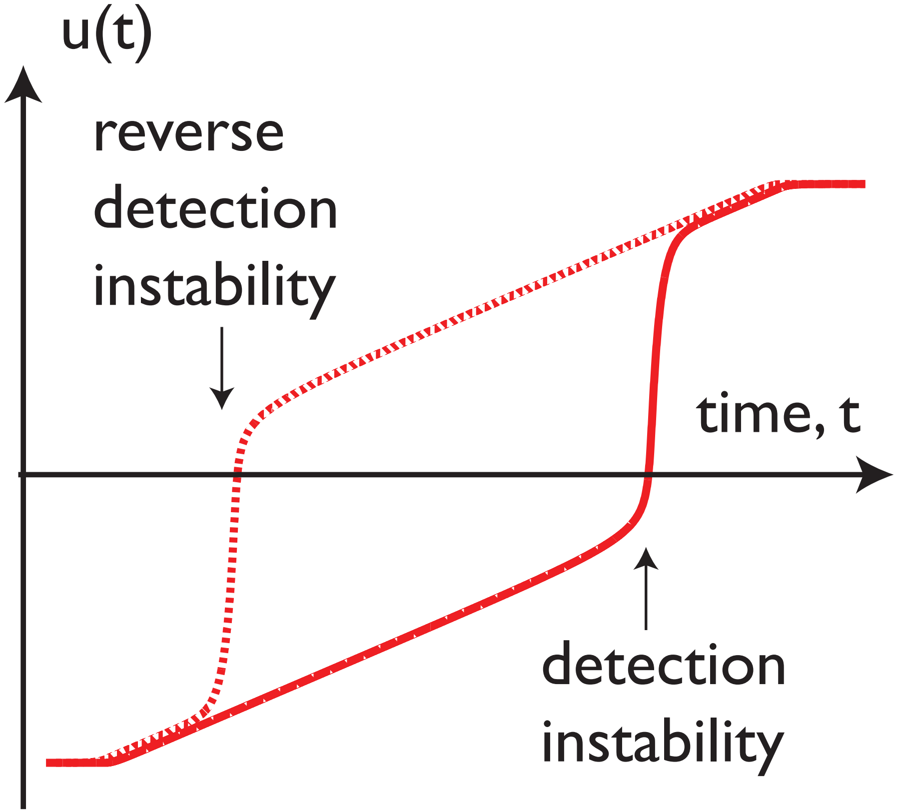


- decreasing input strength => reverse detection instability

$$\tau \dot{u}(t) = -u(t) + h + s(t) + c\ \sigma(u(t))$$

Neuronal dynamics with self-excitation

- the detection and its reverse => create discrete events from time-continuous changes

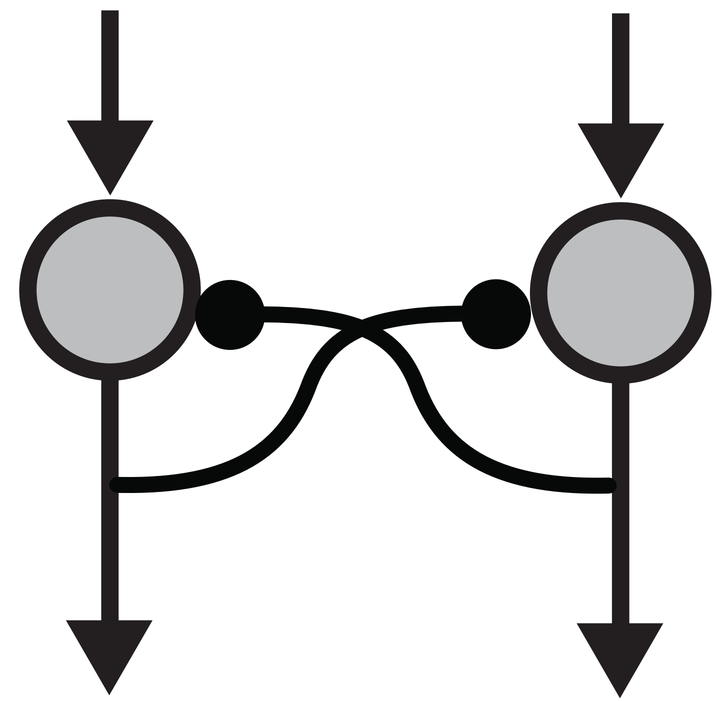


$$\tau \dot{u}(t) = -u(t) + h + s(t) + c\ \sigma(u(t))$$

=> simulation

Neuronal dynamics with competition

- two activation variables with reciprocal inhibitory coupling
- representing two small populations that are inhibitorily coupled

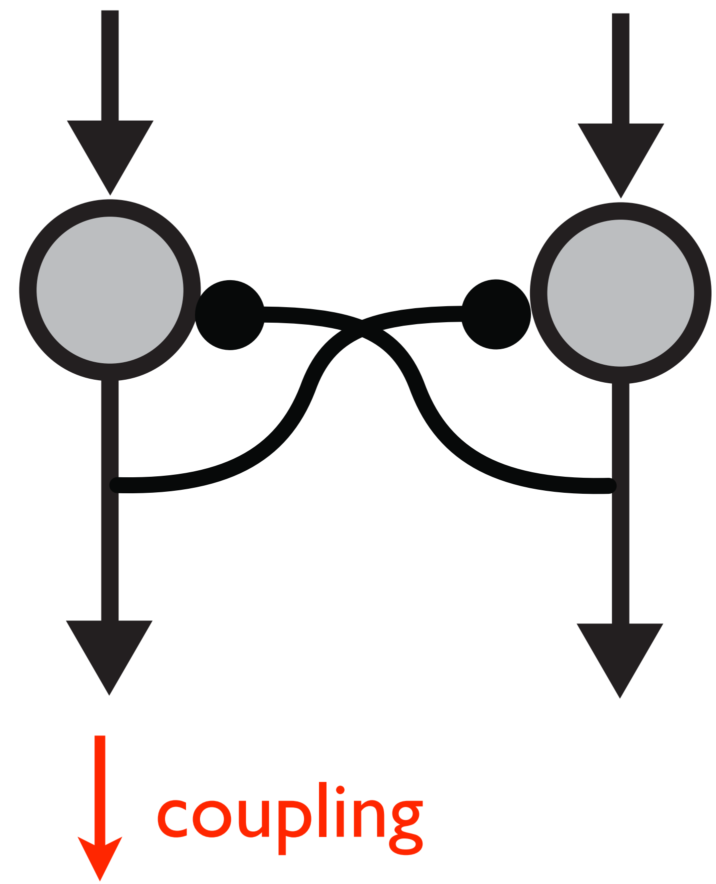


$$\tau \dot{u}_1(t) = -u_1(t) + h + s_1(t) - c_{12}\sigma(u_2(t))$$

$$\tau \dot{u}_2(t) = -u_2(t) + h + s_2(t) - c_{21}\sigma(u_1(t))$$

Neuronal dynamics with competition

- Coupling: the rate of change of one activation variable depends on the level of activation of the other activation variable

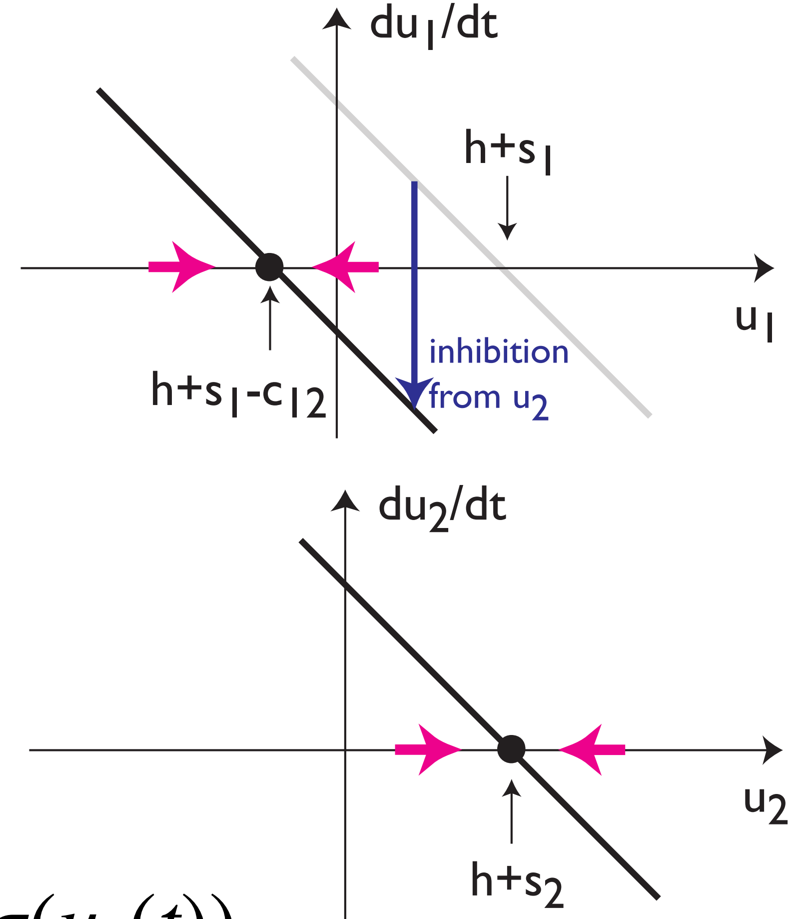


$$\tau \dot{u}_1(t) = -u_1(t) + h + s_1(t) - c_{12}\sigma(u_2(t))$$

$$\tau \dot{u}_2(t) = -u_2(t) + h + s_2(t) - c_{21}\sigma(u_1(t))$$

Neuronal dynamics with competition

- to visualize, assume that u_2 has been activated by input to a positive level
- => it inhibits u_1

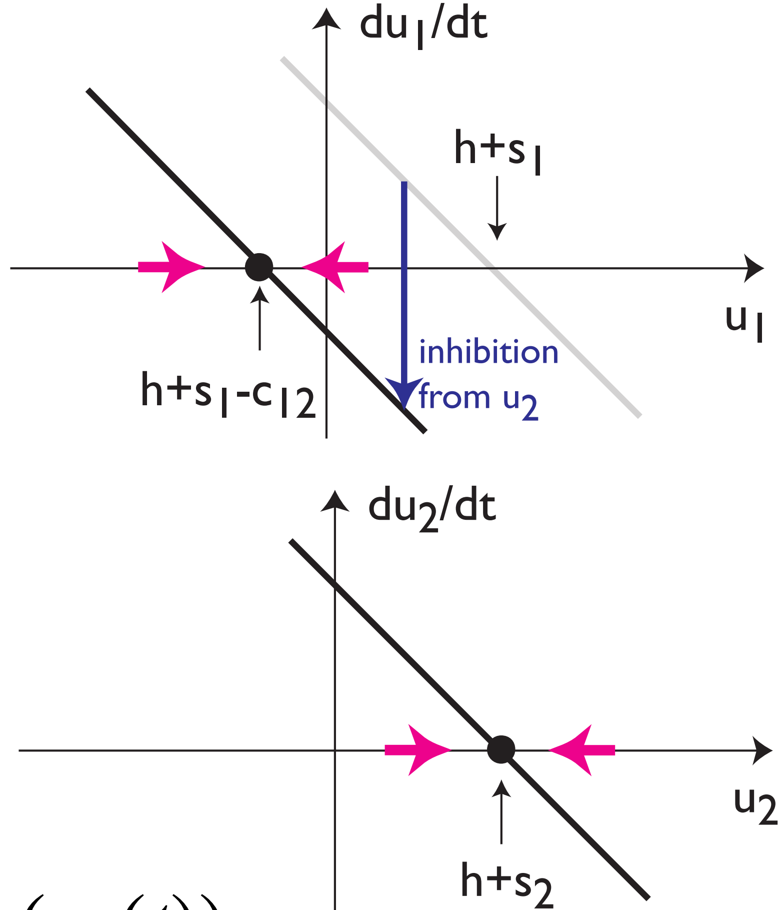


$$\tau \dot{u}_1(t) = -u_1(t) + h + s_1(t) - c_{12}\sigma(u_2(t))$$

$$\tau \dot{u}_2(t) = -u_2(t) + h + s_2(t) - c_{21}\sigma(u_1(t))$$

Neuronal dynamics with competition

- why would u_2 be positive before u_1?
- more input to u_2 (better "match") => faster increase
- input advantage <=> time advantage <=> competitive advantage

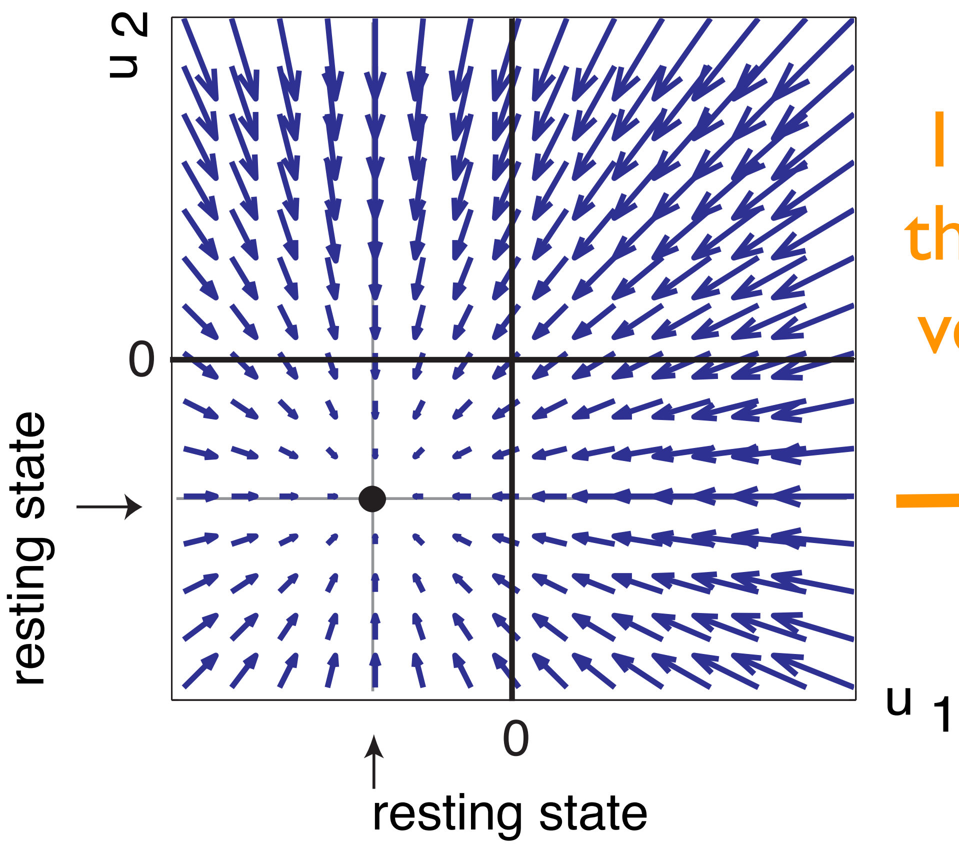


$$\tau \dot{u}_1(t) = -u_1(t) + h + s_1(t) - c_{12}\sigma(u_2(t))$$

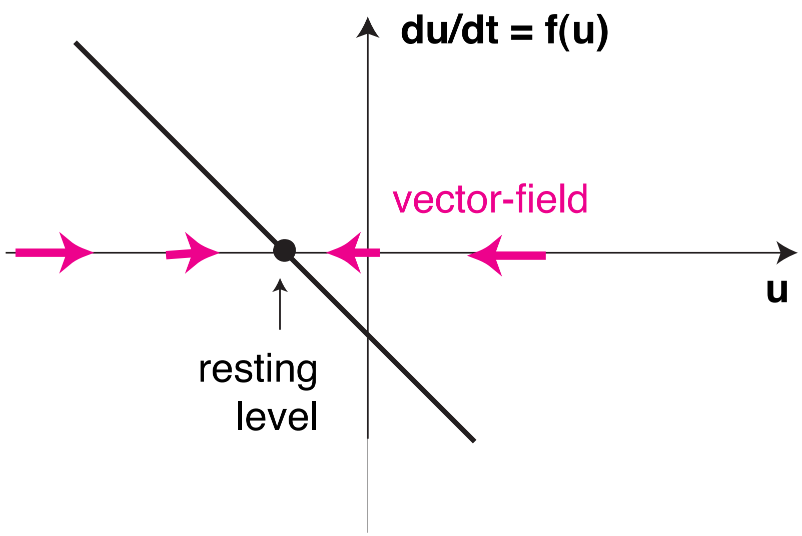
$$\tau \dot{u}_2(t) = -u_2(t) + h + s_2(t) - c_{21}\sigma(u_1(t))$$

Neuronal dynamics with competition

vector-field in the absence of input

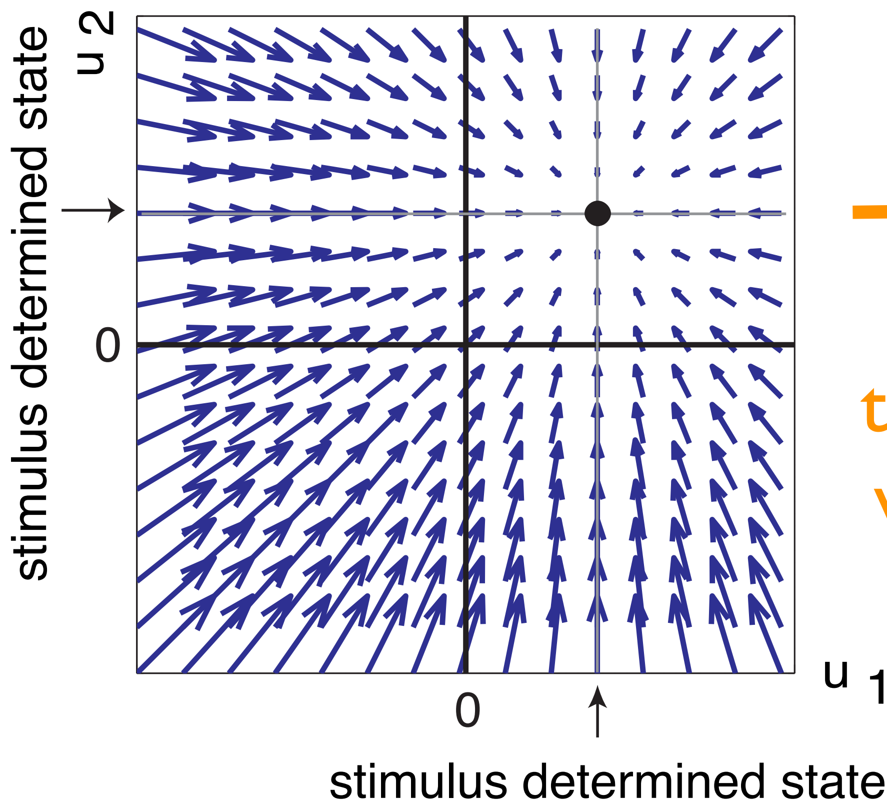


1D cut through vector-field

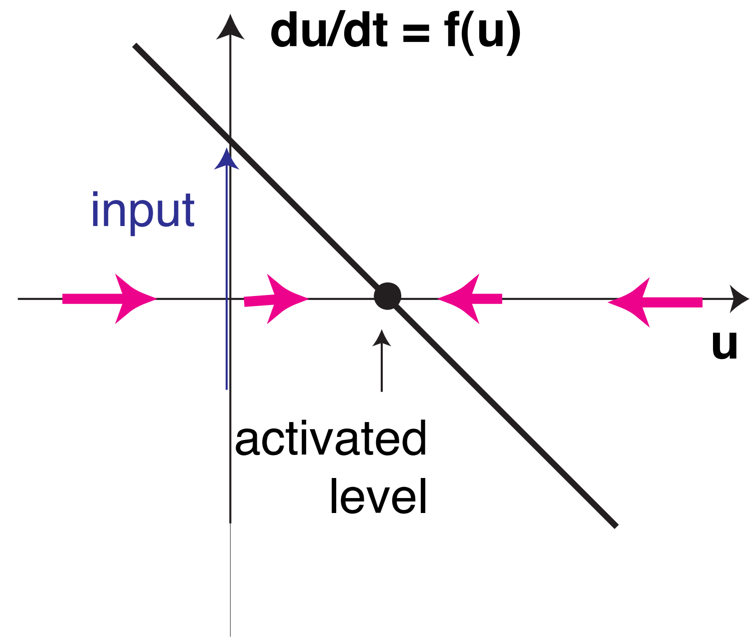


Neuronal dynamics with competition

vector-field (without interaction) when both neurons receive input

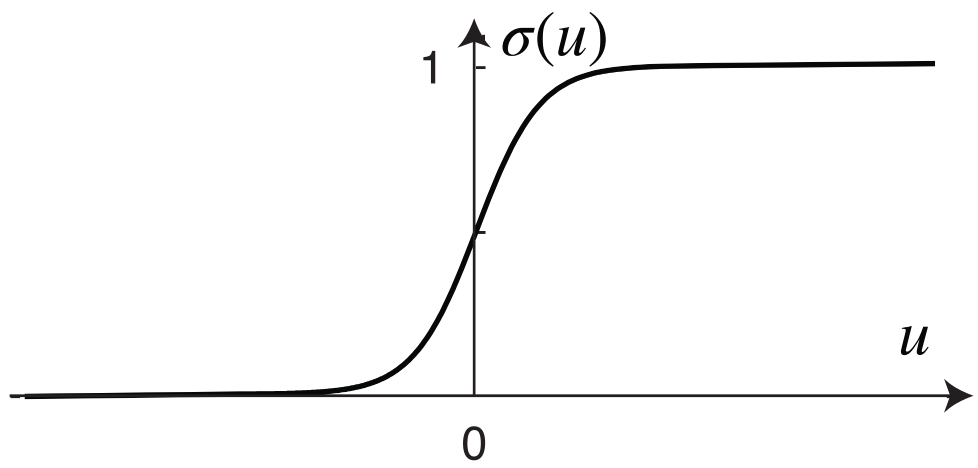


1D cut through vector-field



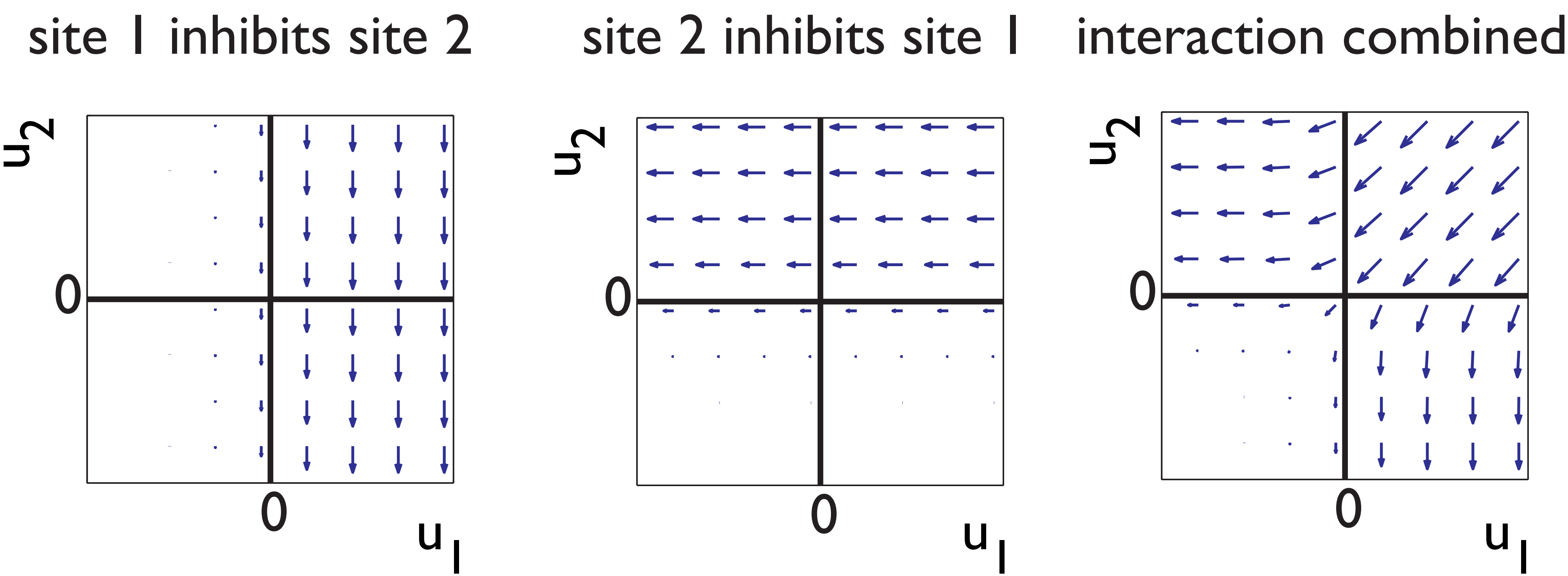
Neuronal dynamics with competition

- only activated neurons participate in interaction!



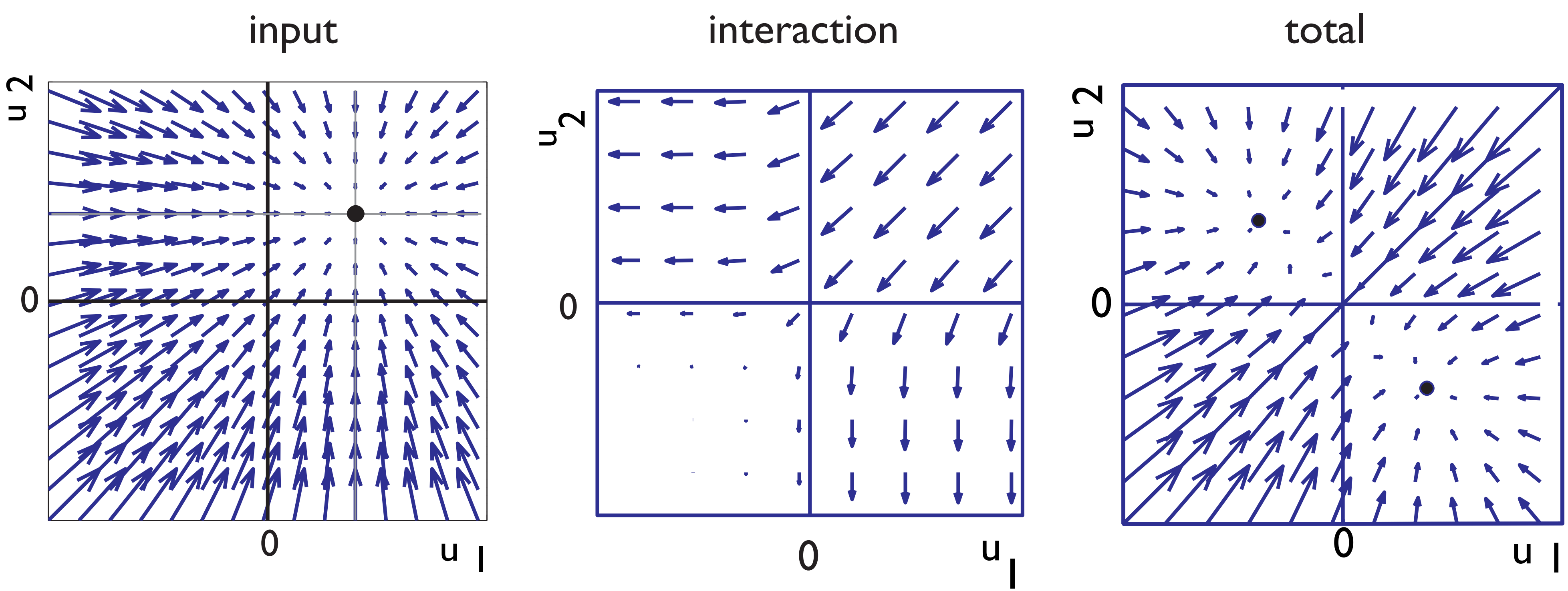
Neuronal dynamics with competition

- vector-field of mutual inhibition



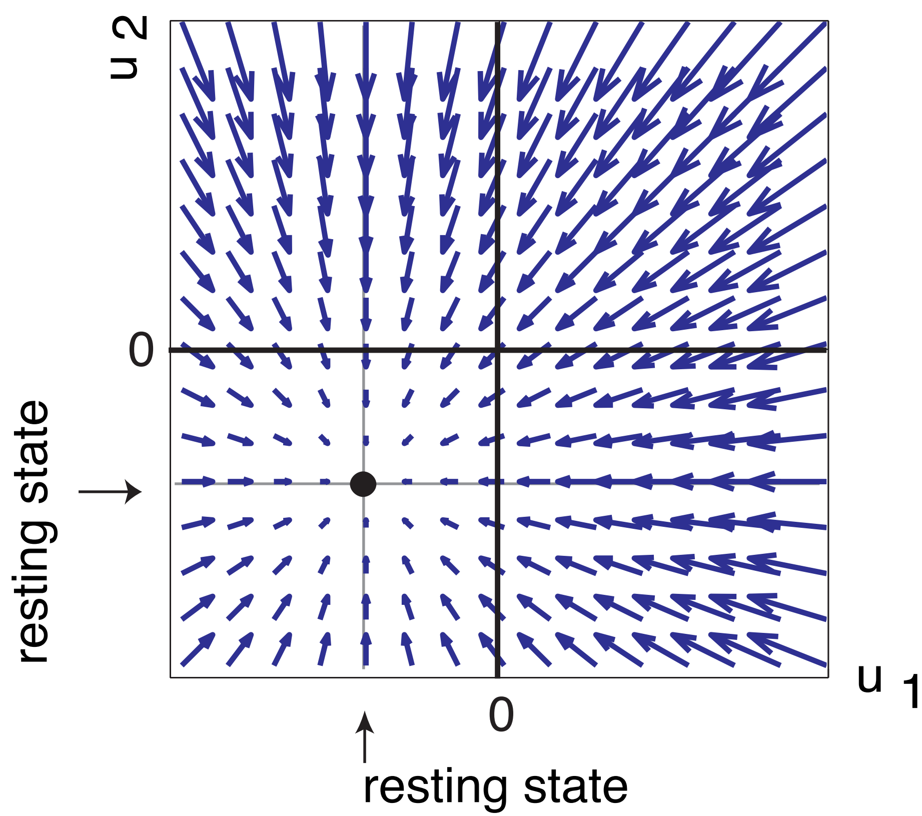
Neuronal dynamics with competition

vector-field with strong mutual inhibition: bistable

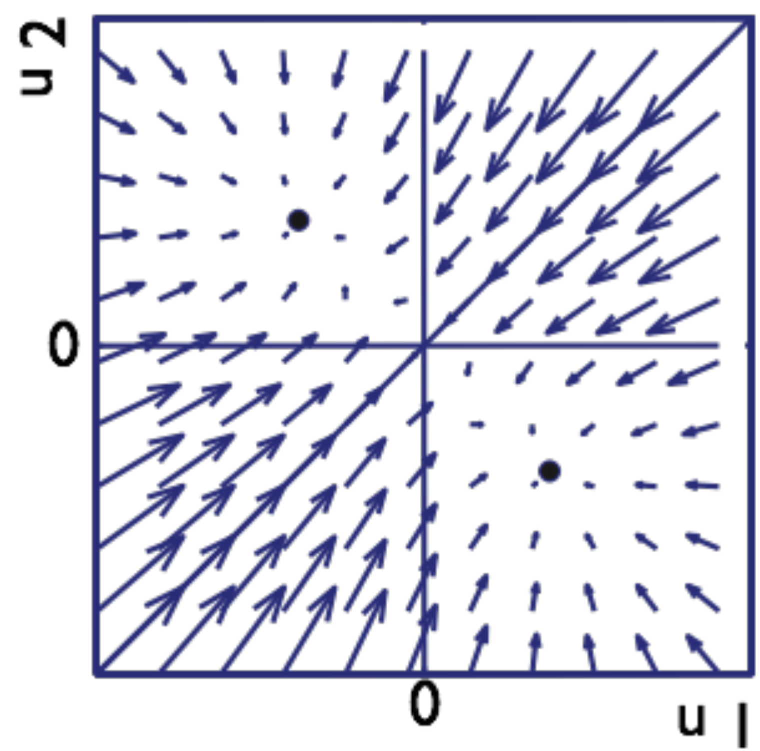


Neuronal dynamics with competition

before input is presented

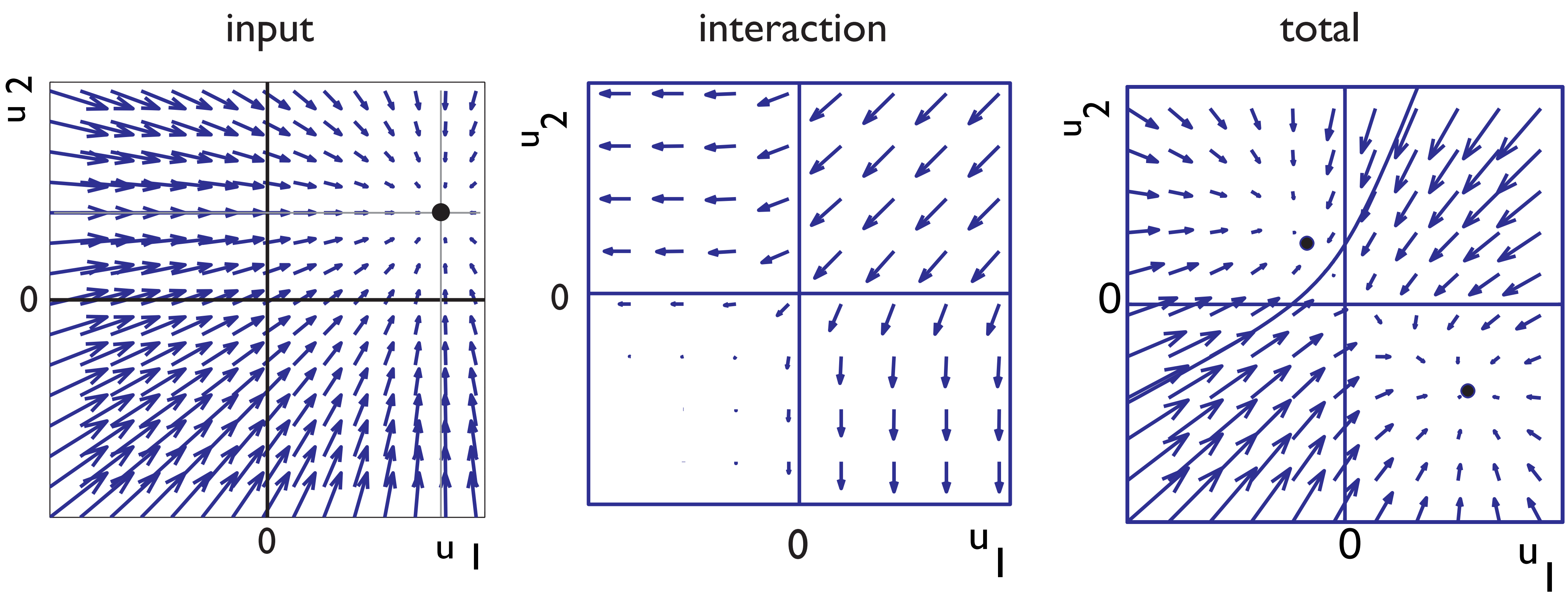


after input is presented



Neuronal dynamics with competition

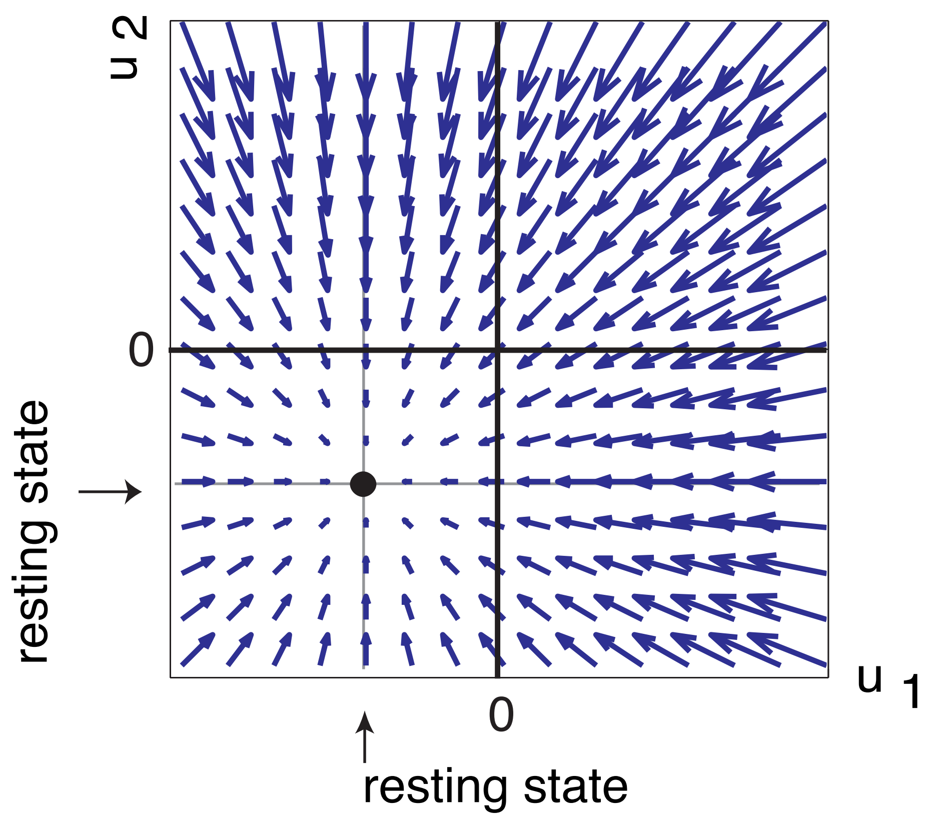
stronger input to u_1 => attractor with positive u_1 stronger, attractor with positive u_2 weaker => closer to instability



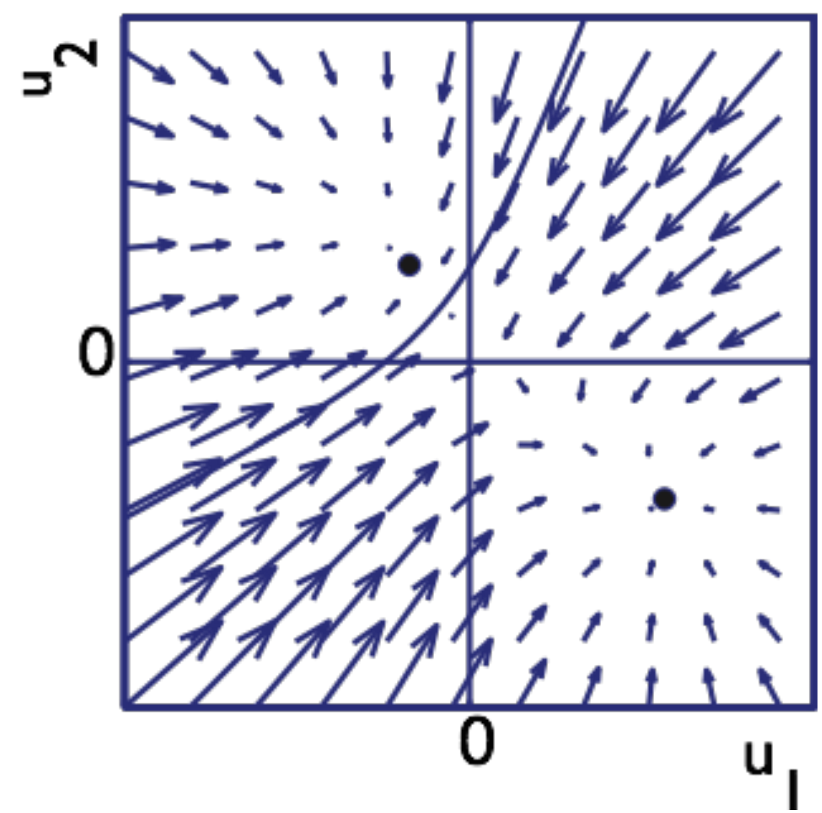
Neuronal dynamics with competition

- decision made at detection instability!

before input is presented



after input is presented

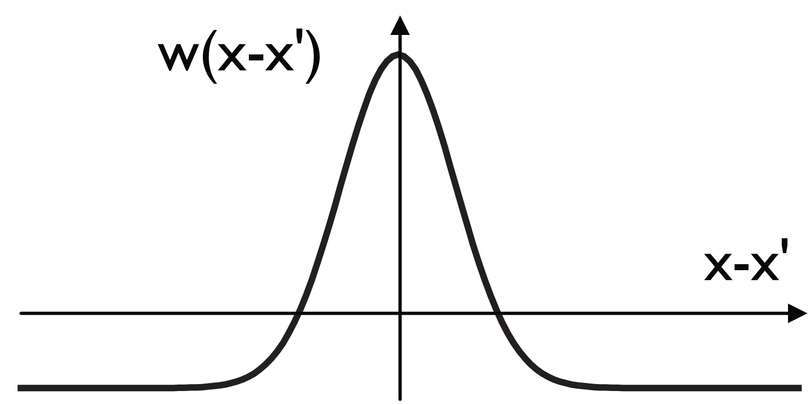


=> simulation

The neural dynamics of fields

- … the same underlying math
- coupling among continuously many activation variables
- local excitatory coupling (“self-excitation”)
- global inhibitory coupling (“mutual inhibition”)

$$\tau \dot{u}(x,t) = -u(x,t) + h + s(x,t) + \int dx' w(x-x')\ \sigma(u(x',t))$$



field vs. activation variables