Prof. Dr. G. Schöner, Institut für Neuroinformatik gregor.schoener@rub.de

Autonomous robotics

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Exercise 7 The mass-spring model of reflex-controlled muscles

Consider a single joint (the elbow, let us say), with two muscles, which oppose each other. When the elbow is flexed, the flexor (e.g. biceps) shortens and thus acts as agonist of a flexion movement. The extensor (e.g. triceps) lengthens and thus acts as the antagonist of a flexion movement. The roles change for an extension movement.

The relation between mucles lengths and elbow joint angle is modelled in linear and symmetric approximation: $l_f = a - b\,\Theta$ and $l_e = a + b\,\Theta$, where the index refers to f:flexor and e:extensor. The joint angle, Θ, is defined to be zero when the elbow is fully extended and to increase when the elbow is flexed. (To get an idea for numbers: the constants may be approximately: $a = 0.3m$, $b = 0.02m$ when the joint angle is measured in radians.)

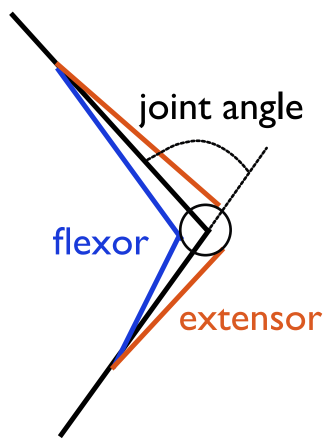

Figure 1: Schematic of the elbow with joint angle, Θ, and flexor (biceps) and extensor (triceps) muscles.

1. Model each muscle as a spring, so that muscle force, f_f and f_e (indices f and e as above), depends linearly on the length. (Compared to the lecture, this neglects viscous forces that depend on the rate of change of length.) Note that a muscle only pulls, never pushes, so only positive forces are admitted. The force is zero at the resting length, λ_f and λ_e. Write down these equations introducing notation for the spring constant and the damping coefficient. Assume these constants are the same for the two muscles. Make a plot of these forces as a function of muscle length.

2. The forces combine to the elbow torque via $T = T_f + T_e$, where the flexor torque is $T_f = -bf_f$ and the extensor torque is $T_e = bf_e$. Compute each torque as a function of joint angle, Θ. Determine the joint angles at which flexor torque is zero as a function of λ_f, and the same for the extensor.

3. Plot the two torques into the same plot, keeping track of the signs of the contributions, for two different configurations of λ_e and λ_f. In one, torque should become zero for both contributions at the same joint angle. In the other, the two torque contributions should overlap.

4. At zero total torque, compute the slope the torque-joint angle curve. This is the stiffness of the joint at equilibrium (when there is no external torque). Does stiffness differ for the two cases you just plotted?

5. Take a copy of your plot of the two torque contributions and sketch graphically a nonlinear torque-joint angle curve that is tangential to the linear torque function near zero torque. The lecture used such nonlinear torque functions (see also the paper cited below). Does the stiffness of this nonlinear system (slope of total torque at the zero crossing) differ for the two cases you considered earlier?

If you want to read up on some of these ideas, look at this paper available fron the course web page: Mark L Latash: Evolution of Motor Control: From Reflexes and Motor Programs to the Equilibrium-Point Hypothesis. *Journal of Human Kinetics* **19**:3-24 (2009).