Timing and coordination

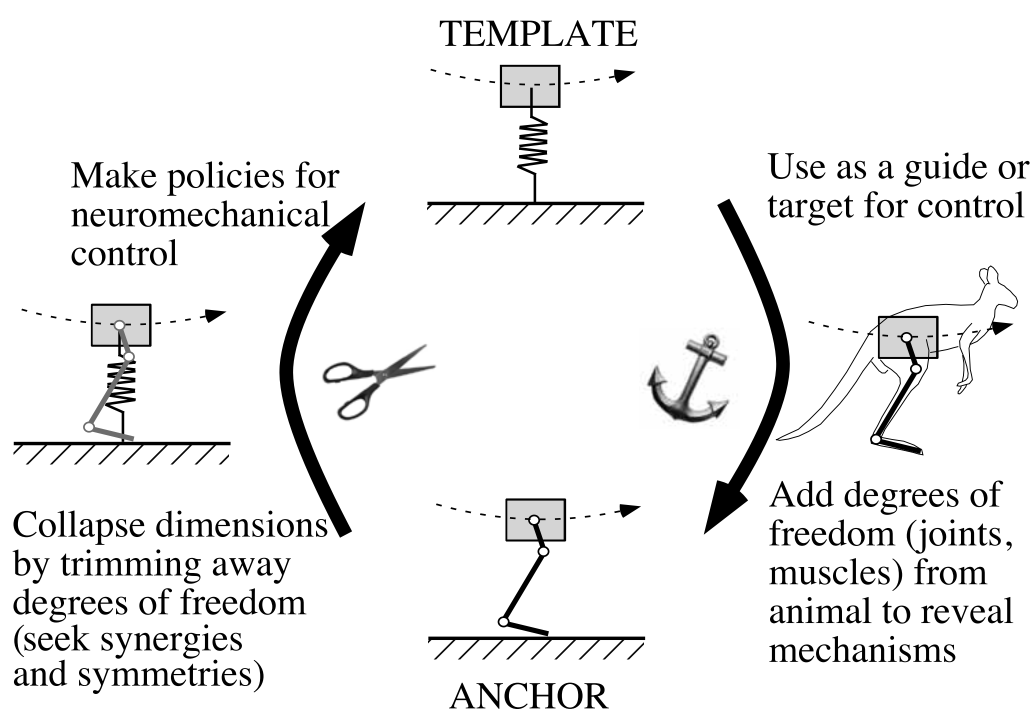
Gregor Schöner

Timed movement

- movements that is "timed":
  - end-effector arrives "on time"
  - movement coordinated across different effectors
  - movement coordinated with moving objects (e.g., catching)
- timing implies some form of anticipation…

Timing from a task/macro level

- template…oscillator at macro-level..
- anchor… kinematics at joint/actuator level

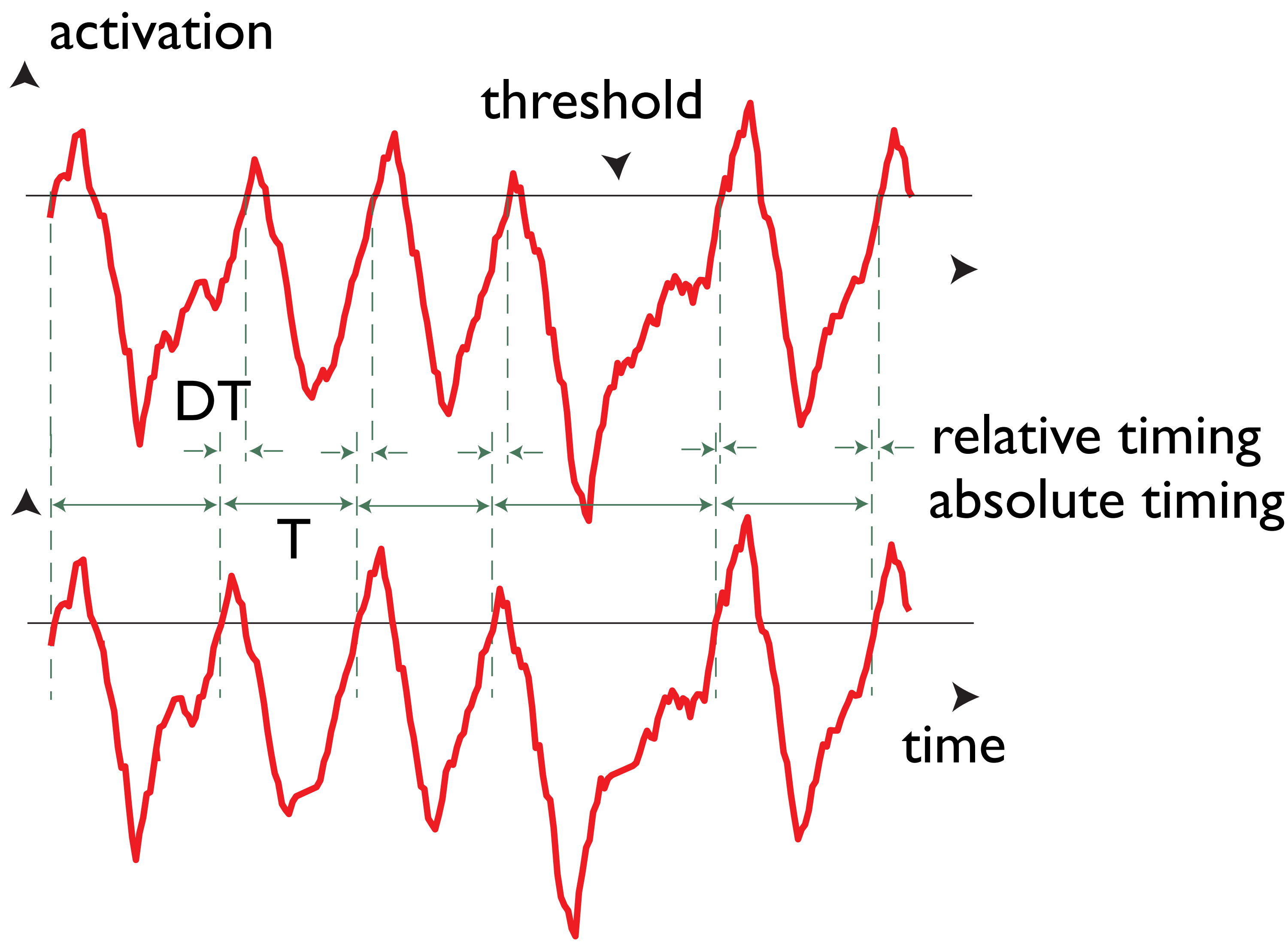


[Full Koditschek 99]

Timing in human movement

- timing
- absolute vs relative timing
- coordination
- coupled oscillators

Relative vs. absolute timing



relative phase=DT/T

Theoretical account for absolute timing

- (neural) oscillator autonomously generates timing signal, from which timing events emerge
- => limit cycle oscillators = clocks

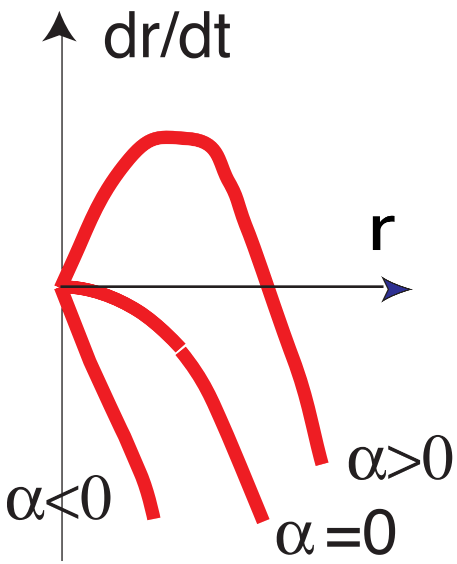
Limit cycle oscillator: Hopf

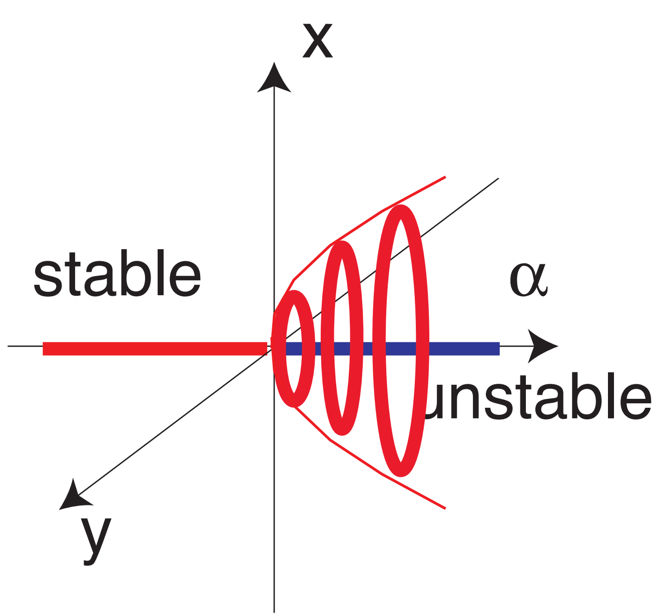
- normal form

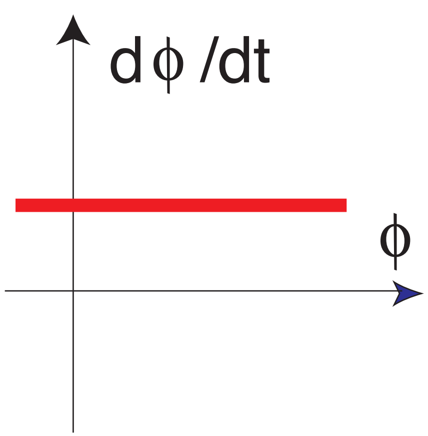
$$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} \alpha & -\omega \\ \omega & \alpha \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} - (x^2 + y^2) \begin{pmatrix} x \\ y \end{pmatrix}$$

$$x = r\cos(\phi) \qquad \dot{r} = \alpha r - r^3$$

$$y = r\sin(\phi) \qquad \dot{\phi} = \omega$$







$$x(t) = \sqrt{\alpha}\sin(\omega t)$$

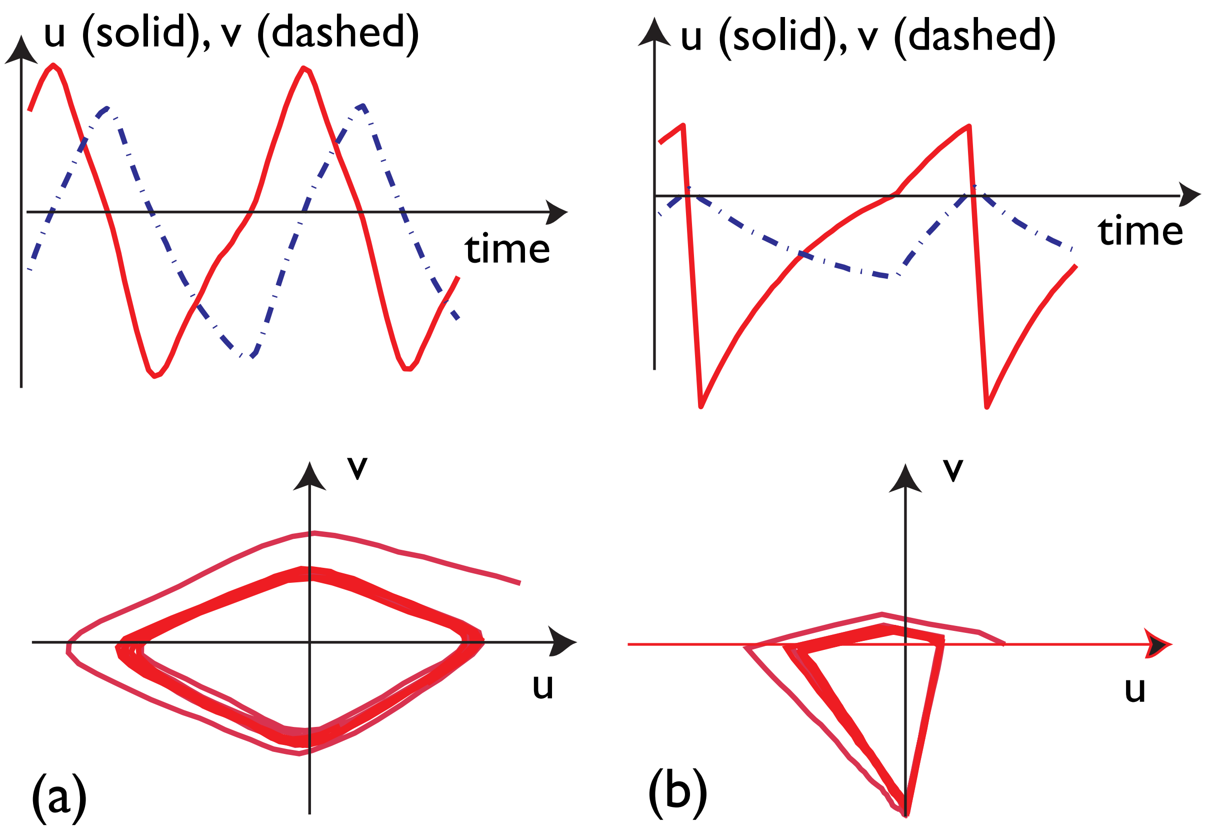
$$\text{amplitude } A = \sqrt{\alpha}$$

$$\text{cycle time } T = 2\pi/\omega,$$

Neural oscillator

- relaxation oscillator

$$\tau\dot{u} = -u + h_u + w_{uu}f(u) - w_{uv}f(v)$$
$$\tau\dot{v} = -v + h_v + w_{vu}f(u),$$



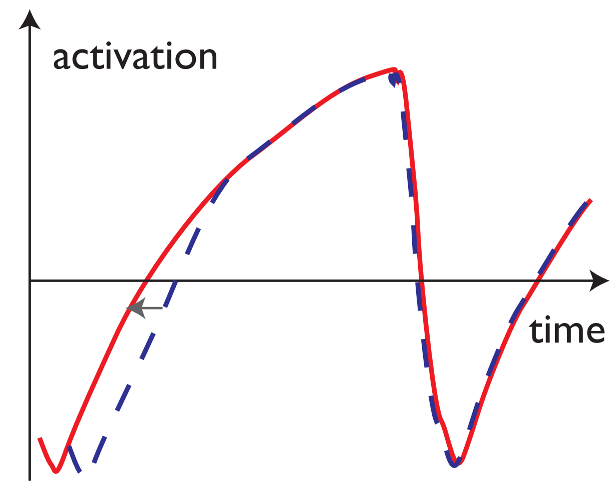
[Amari 77]

Relative timing

- Coordination is the maintenance of stable timing relationships between components of voluntary movement.
- => recovery relative timing after perturbations
- Example: coordination of limbs, of articulators in speech production..
- Example: action-perception patterns

Coordination from coupling

- coordination=stable relative timing emerges from coupling of neural oscillators



$$\tau \dot{u}_1 = -u_1 + h_u + w_{uu} f(u_1) - w_{uv} f(v_1)$$
$$\tau \dot{v}_1 = -v_1 + h_v + w_{vu} f(u_1) + c f(u_2)$$
$$\tau \dot{u}_2 = -u_2 + h_u + w_{uu} f(u_2) - w_{uv} f(v_2)$$
$$\tau \dot{v}_2 = -v_2 + h_v + w_{vu} f(u_2) + c f(u_1)$$

[Schöner: Timing, Clocks, and Dynamical Systems. *Brain and Cognition* 48:31-51 (2002)]

Learn from these ideas for robotics?

- timed reaching that stabilizes timing in response to perturbations

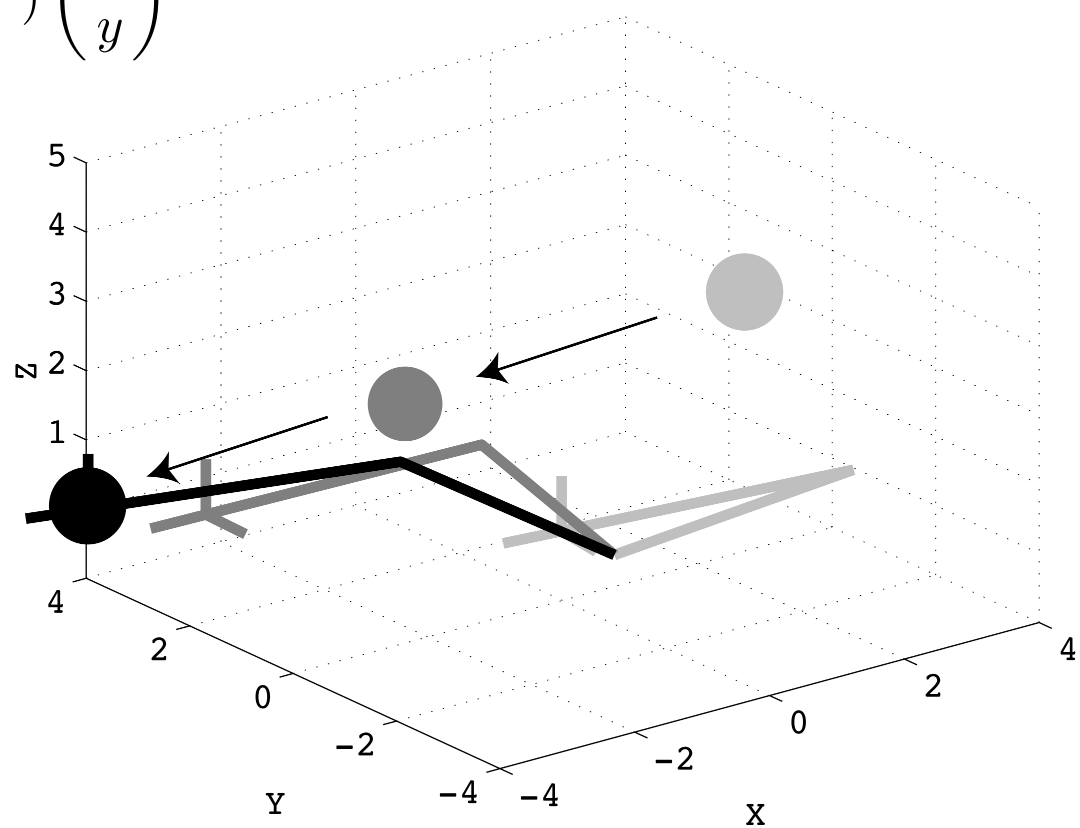
Timed movement to intercept ball

- timing from an oscillator

$$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = -5\ |u_{\text{init}}| \begin{pmatrix} x - x_{\text{init}} \\ y \end{pmatrix} + |u_{\text{hopf}}|\ \mathbf{f}_{\text{hopf}} - 5\ |u_{\text{final}}| \begin{pmatrix} x - x_{\text{final}} \\ y \end{pmatrix} + \text{gwn}$$

$$\mathbf{f}_{\text{hopf}} = \begin{pmatrix} 2.5 & -\omega \\ \omega & 2.5 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} - 2.5\ (x^2 + y^2) \begin{pmatrix} x \\ y \end{pmatrix}$$

$$x(t) = \sin(\omega t)$$



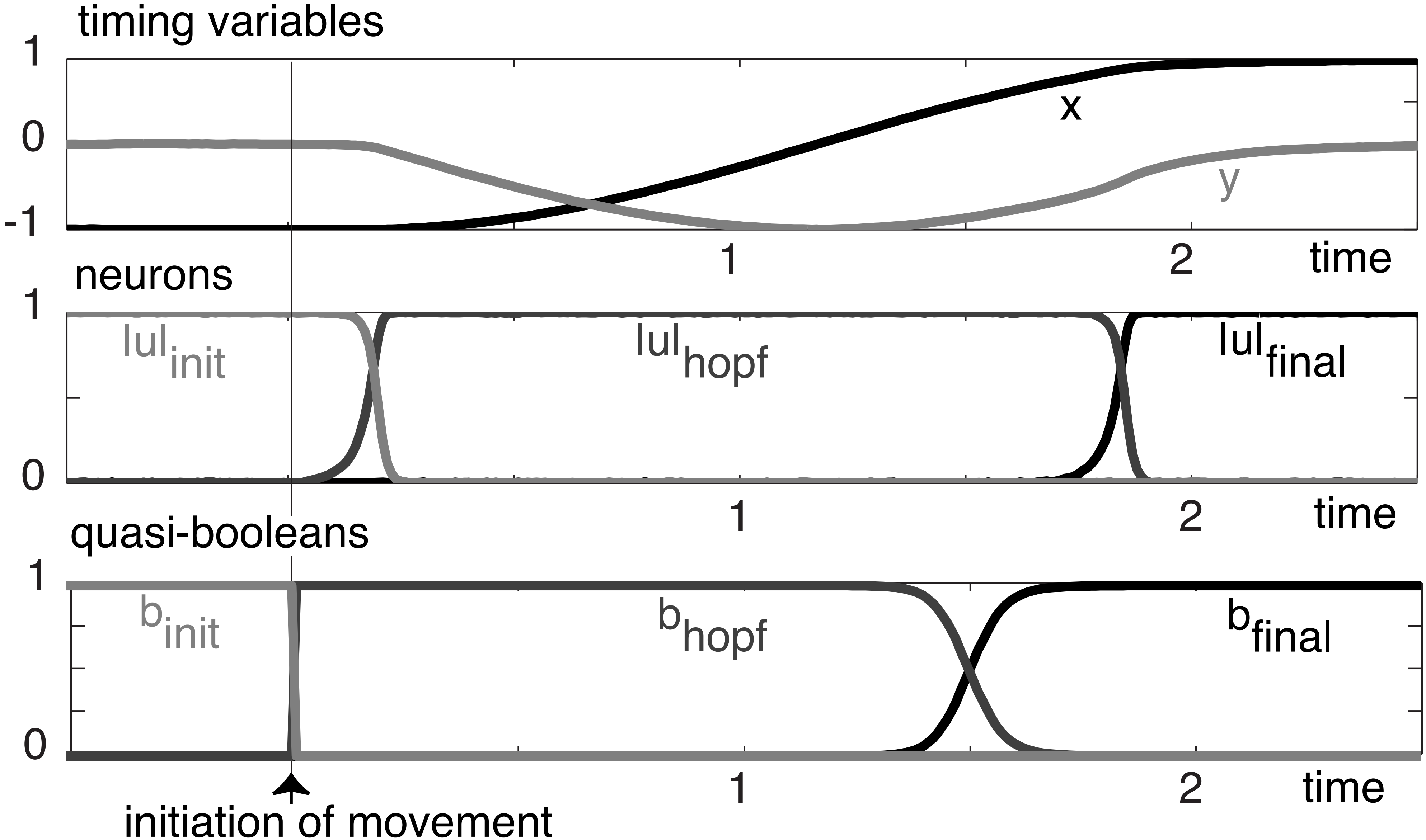
[Schöner, Santos, 2001]

- the oscillator is turned on and off for a single cycle

$$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = -5\ |u_{\text{init}}| \begin{pmatrix} x - x_{\text{init}} \\ y \end{pmatrix} + |u_{\text{hopf}}|\ \mathbf{f}_{\text{hopf}} - 5\ |u_{\text{final}}| \begin{pmatrix} x - x_{\text{final}} \\ y \end{pmatrix} + \text{gwn}$$

$$\begin{aligned}
\alpha\, \dot{u}_{\text{init}} &= \mu_{\text{init}}\ u_{\text{init}} - |\mu_{\text{init}}|\ u_{\text{init}}^3 - 2.1\ (u_{\text{final}}^2 + u_{\text{hopf}}^2)\ u_{\text{init}} + \text{gwn} \\
\alpha\, \dot{u}_{\text{hopf}} &= \mu_{\text{hopf}}\ u_{\text{hopf}} - |\mu_{\text{hopf}}|\ u_{\text{hopf}}^3 - 2.1\ (u_{\text{init}}^2 + u_{\text{final}}^2)\ u_{\text{hopf}} + \text{gwn} \\
\alpha\, \dot{u}_{\text{final}} &= \mu_{\text{final}}\ u_{\text{final}} - |\mu_{\text{final}}|\ u_{\text{final}}^3 - 2.1\ (u_{\text{init}}^2 + u_{\text{hopf}}^2)\ u_{\text{final}} + \text{gwn}
\end{aligned}$$

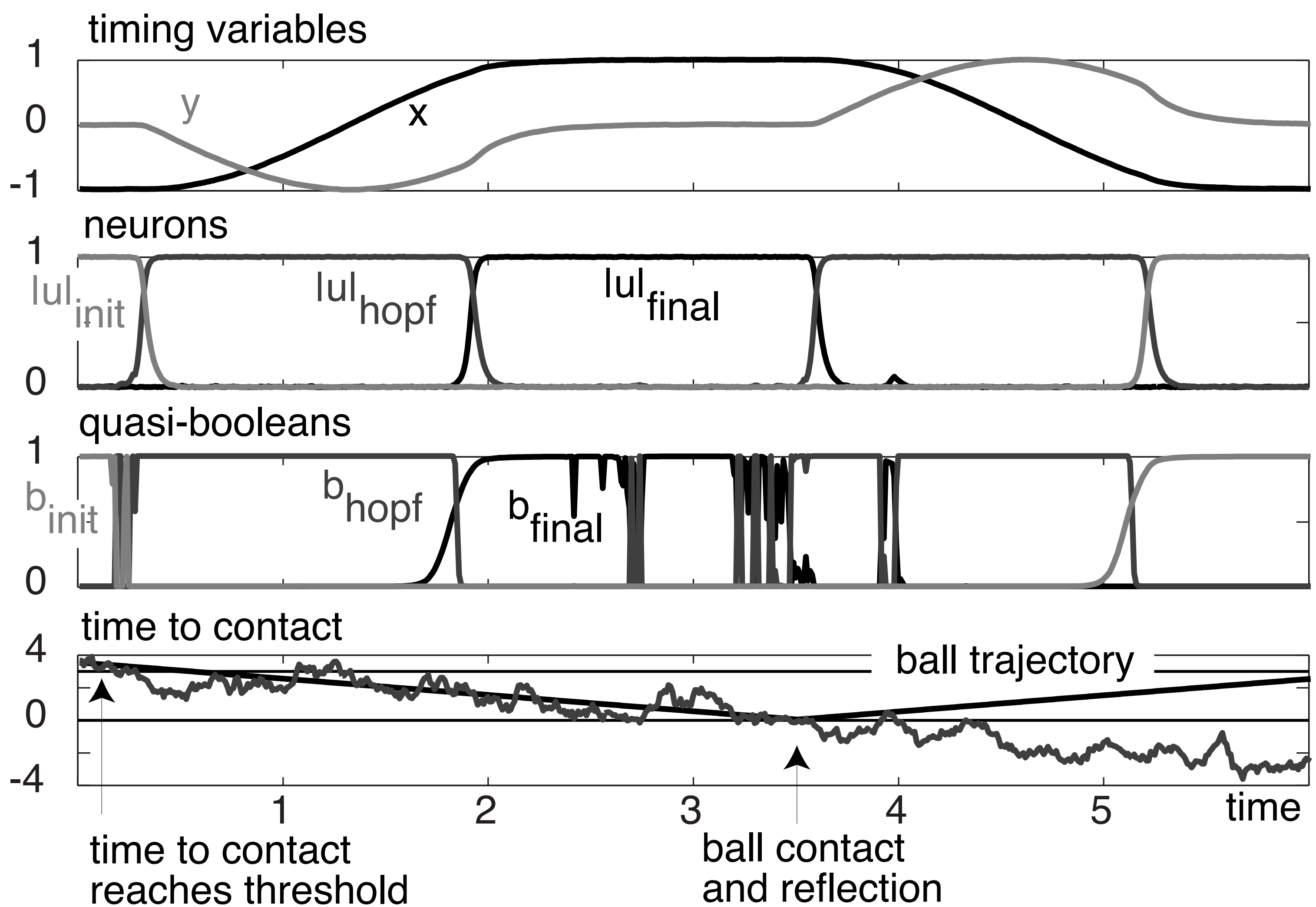
[Schöner, Santos, 2001]



[Schöner, Santos, 2001]

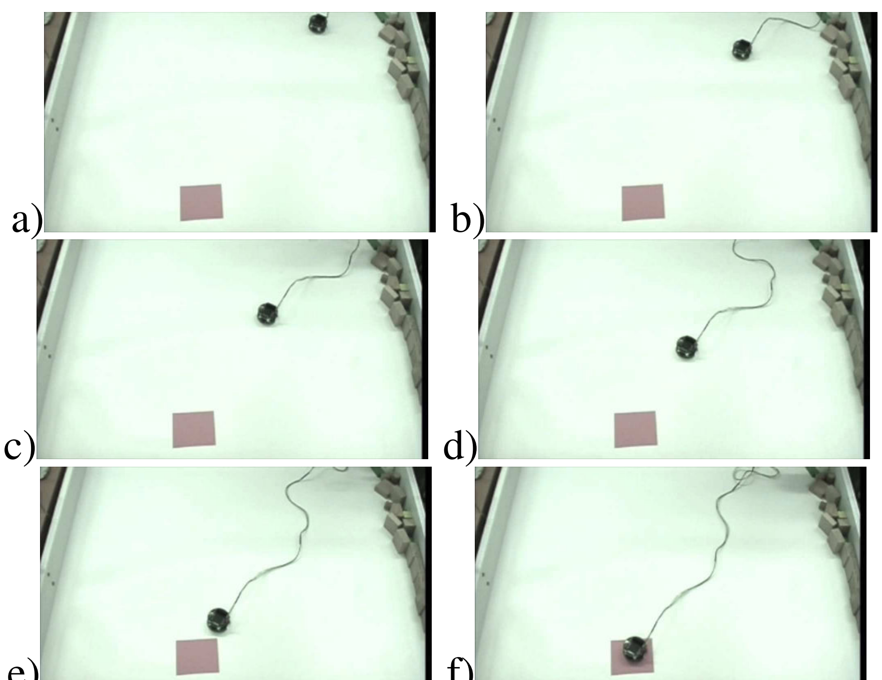
Timed movement to intercept ball

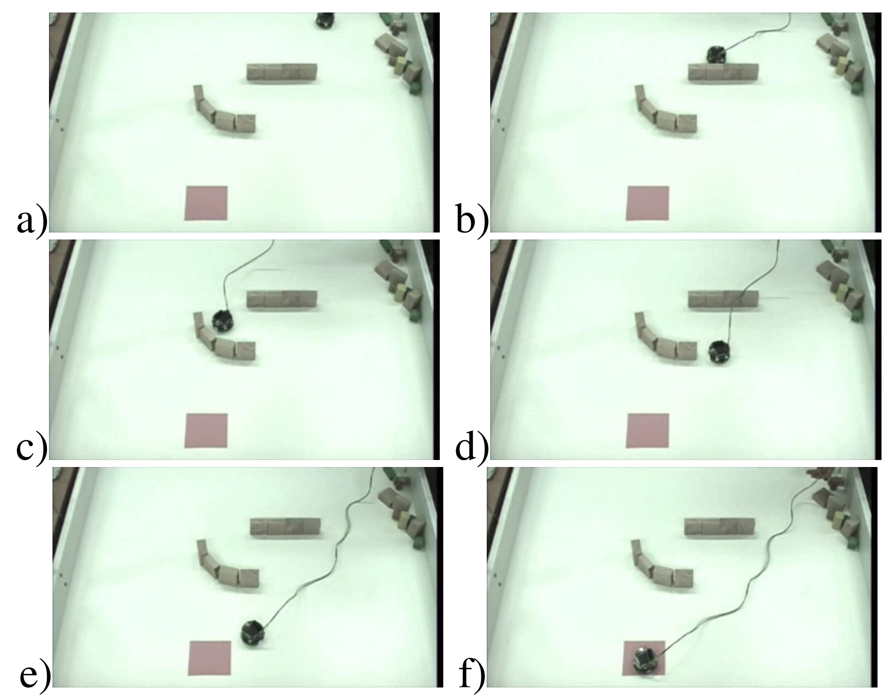
- turn oscillator on in response to detected ball at right time to contact



Compensating for lost time

- plan to reach target at fixed time
- recover time as obstacle forces longer path





[Tuma, Iossifidis, Schöner, ICRA 2009]

behavioral dynamics

heading direction
$$\tau\dot{\phi} = \sum_{i=1}^{N} f_{Obs,i}(\phi, \psi_{Obs,i}) + f_{Tar}(\phi, \psi_{Tar})$$

velocity a
$$\begin{aligned}
\tau \begin{pmatrix} \dot{a} \\ \dot{b} \end{pmatrix} = & - & c_1 \cdot u_{Init}^2 \begin{pmatrix} a \\ b \end{pmatrix} \\
& + & u_{Hopf}^2 \cdot f_{Hopf}(a - R_h, b) \\
& - & c_2 \cdot u_{Final}^2 \begin{pmatrix} a^2 - a \cdot \alpha_{tc} \\ b \end{pmatrix}
\end{aligned}$$

$$\begin{aligned}
f_{Hopf}(a - R_h, b) = & \begin{pmatrix} \lambda & -\omega \\ \omega & \lambda \end{pmatrix} \begin{pmatrix} a - R_h \\ b \end{pmatrix} \\
& -\gamma \left[(a - R_h)^2 + b^2 \right] \begin{pmatrix} a - R_h \\ b \end{pmatrix}
\end{aligned} \quad (7)$$

peak velocity required to arrive at time T
$$R_h = \frac{\omega D(t=0)}{2\pi} \qquad T = 2\pi/\omega$$

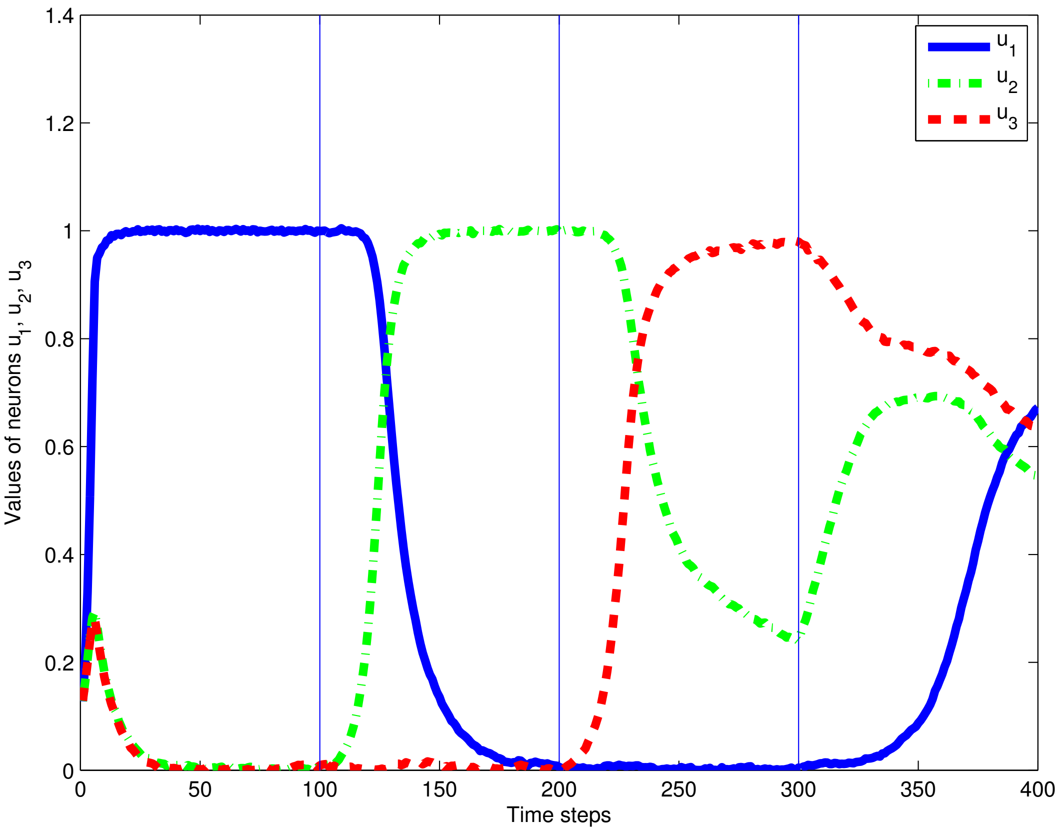
behavioral organization

velocity a

$$\tau \begin{pmatrix} \dot{a} \\ \dot{b} \end{pmatrix} = \begin{array}{ll} - & c_1 \cdot u_{Init}^2 \begin{pmatrix} a \\ b \end{pmatrix} \\ + & u_{Hopf}^2 \cdot f_{Hopf}(a - R_h, b) \\ - & c_2 \cdot u_{Final}^2 \begin{pmatrix} a^2 - a \cdot \alpha_{tc} \\ b \end{pmatrix} \end{array}$$

"neural" dynamics

$$\tau \dot{u}_i = \mu_i u_i - |\mu_i| \, u_i^3 - \nu \sum_{a \neq i} u_a^2 u_i$$



perturbed movement

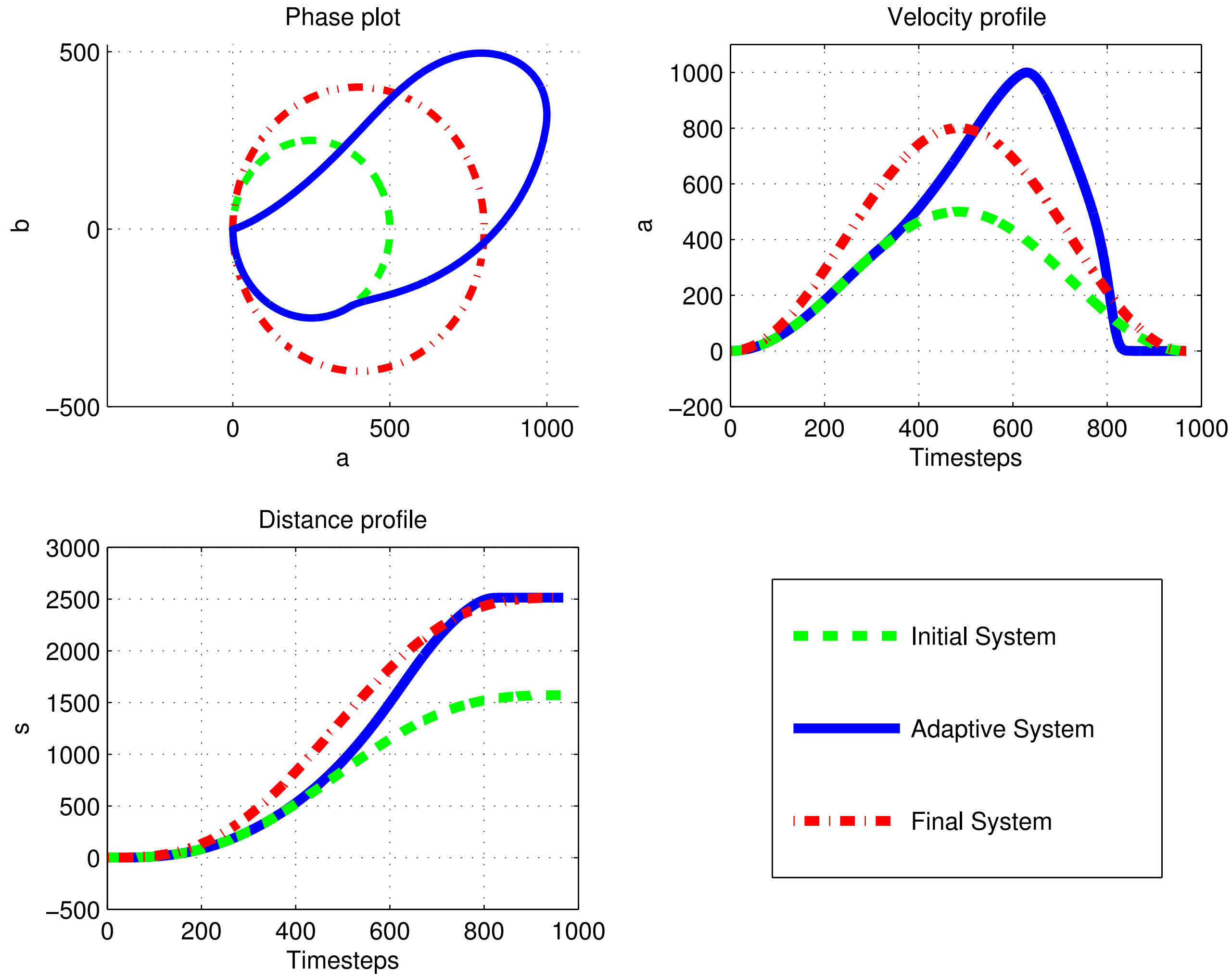
initial distance that system would have had based on remaining distance at time of perturbation

$$D(t=0) = D(t) + \int_0^t v(\tau)d\tau$$
$$= \frac{D(t)}{\left(1 - \frac{t}{T} + \frac{\sin(2\pi \cdot t/T)}{2\pi}\right)}$$

peak velocity needed to reach target in remaining time

$$R_h(t) = \frac{\omega}{2\pi} \frac{D(t)}{\left(1 - \frac{t}{T} + \frac{\sin(2\pi \cdot t/T)}{2\pi}\right)}$$

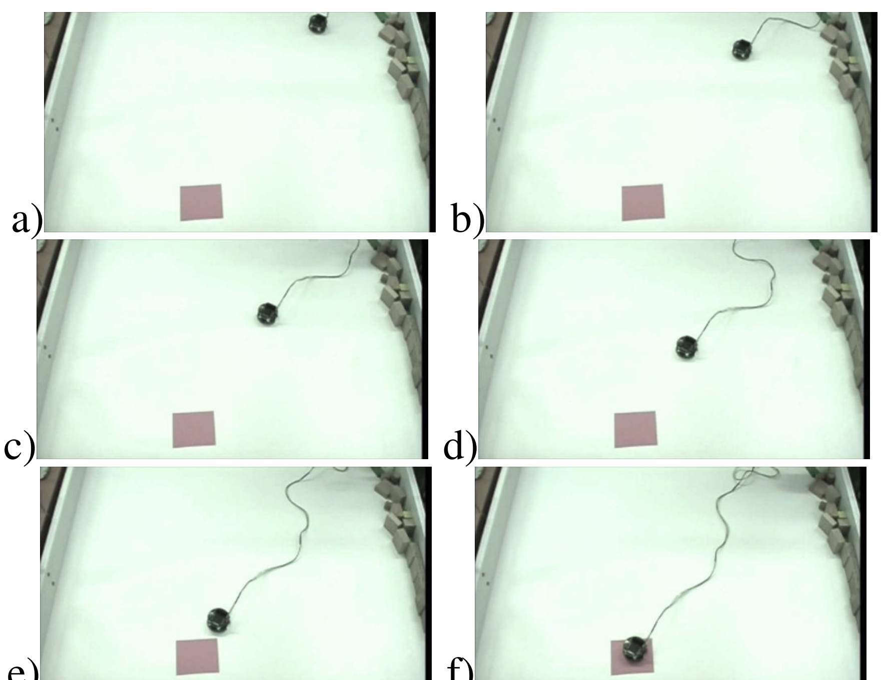
Compensating for lost time

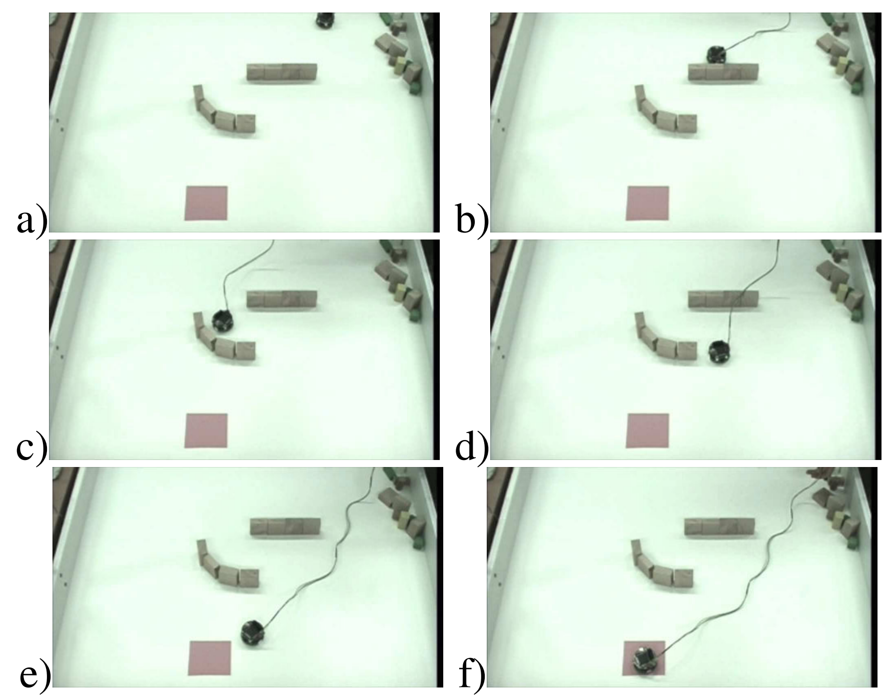


[Tuma, Iossifidis, Schöner, ICRA 2009]

Compensating for lost time

- plan to reach target at fixed time
- recover time as obstacle forces longer path





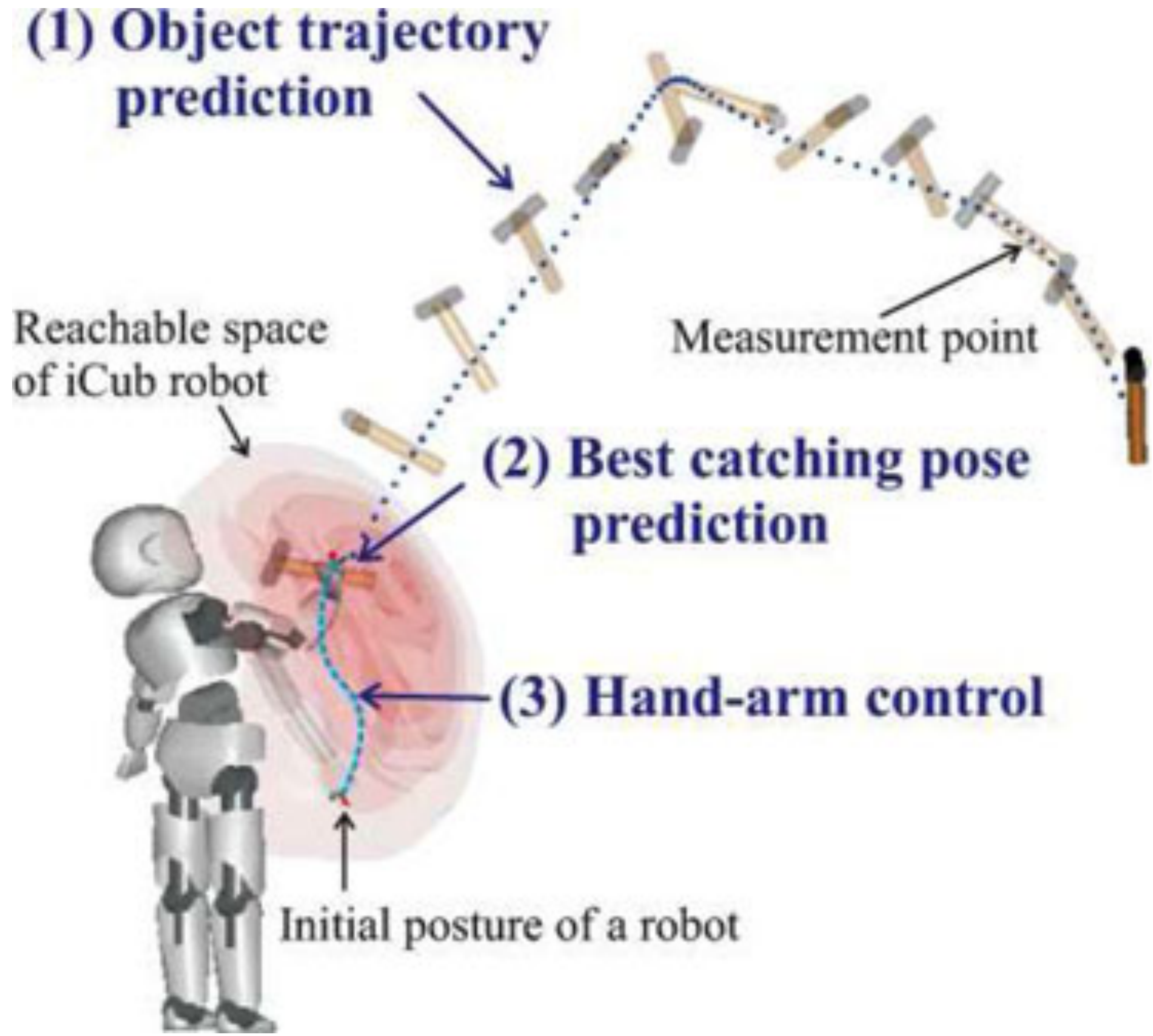
[Tuma, Iossifidis, Schöner, ICRA 2009]

Compensating for lost time

- plan to reach target at fixed time
- recover time as obstacle forces longer path

Setup	Total Distance driven (cm)	MT (s)	Increase in Distance (Factor)	Increase in Time (Factor)
Undisturbed	72.4	12.3		
Medium Disturb.	96.0	12.7	1.33	1.03
High Disturb.	109.7	12.9	1.52	1.05

Catching



[Kim, Shukla, Billard, 2014]

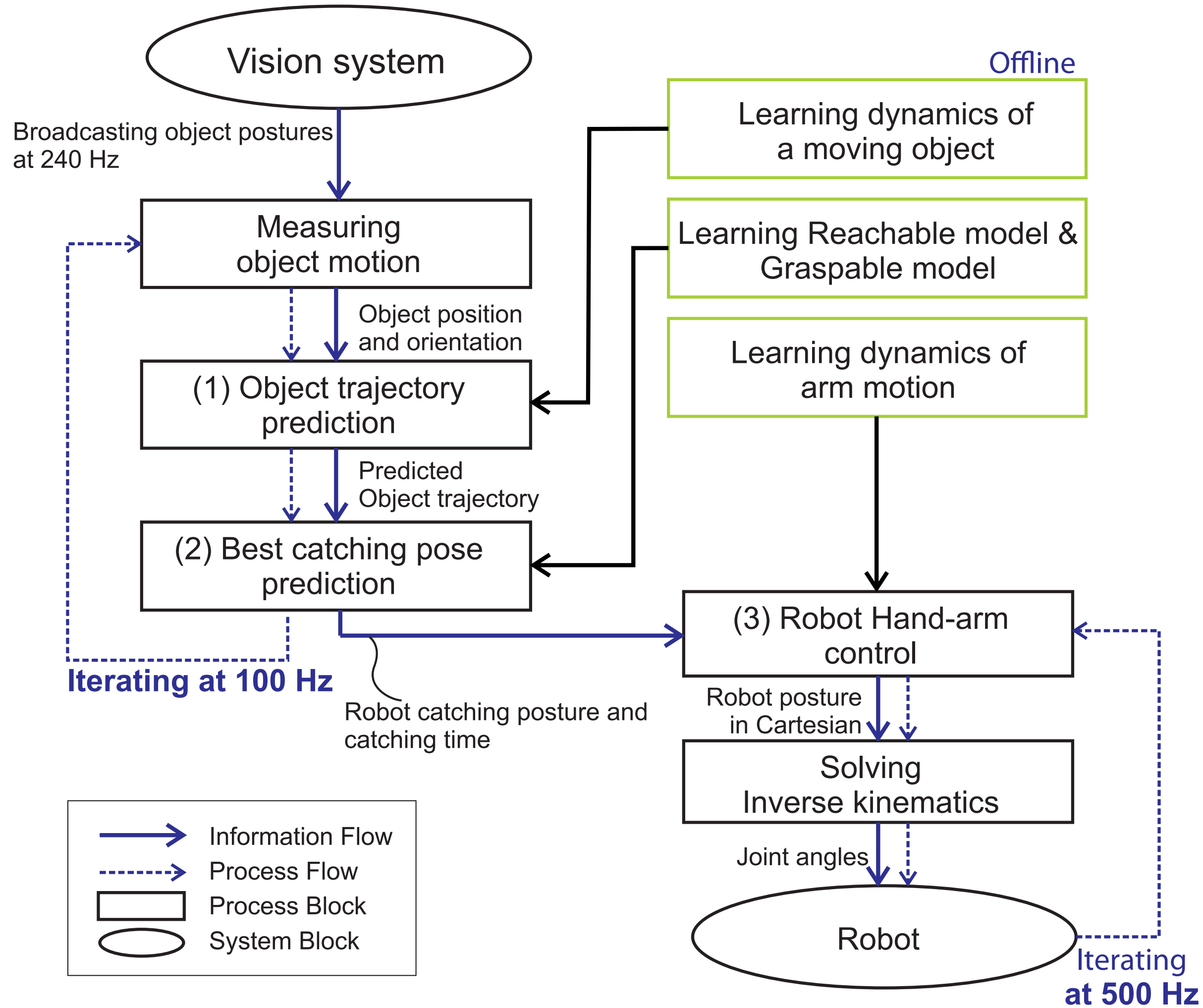
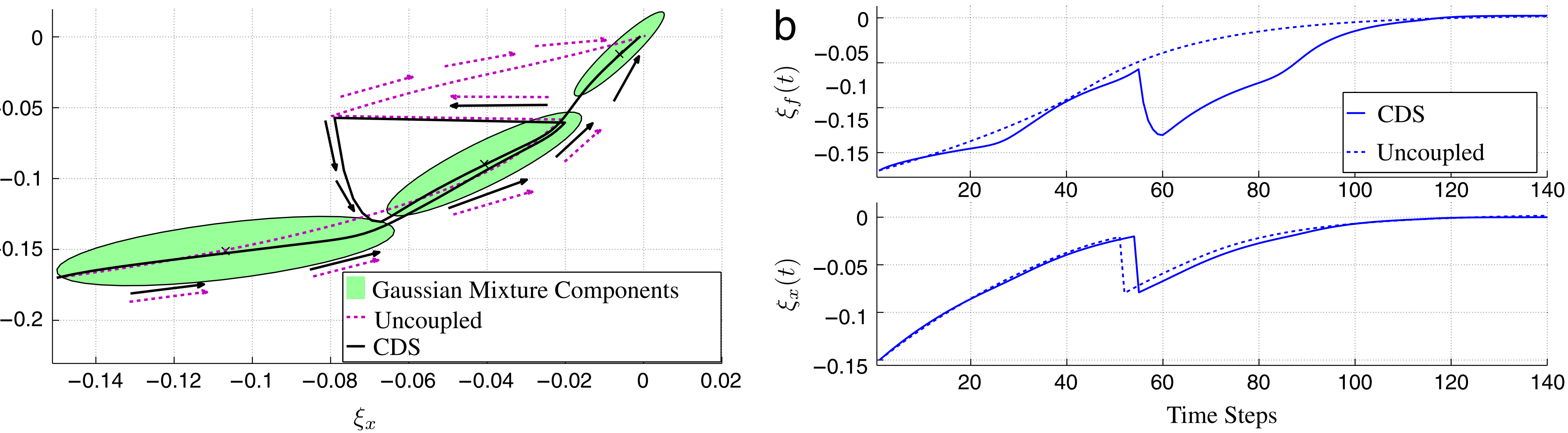


Fig. 2. Block diagram for robotic catching.

[Kim, Shukla, Billard, 2014]

coupled dynamical systems approach



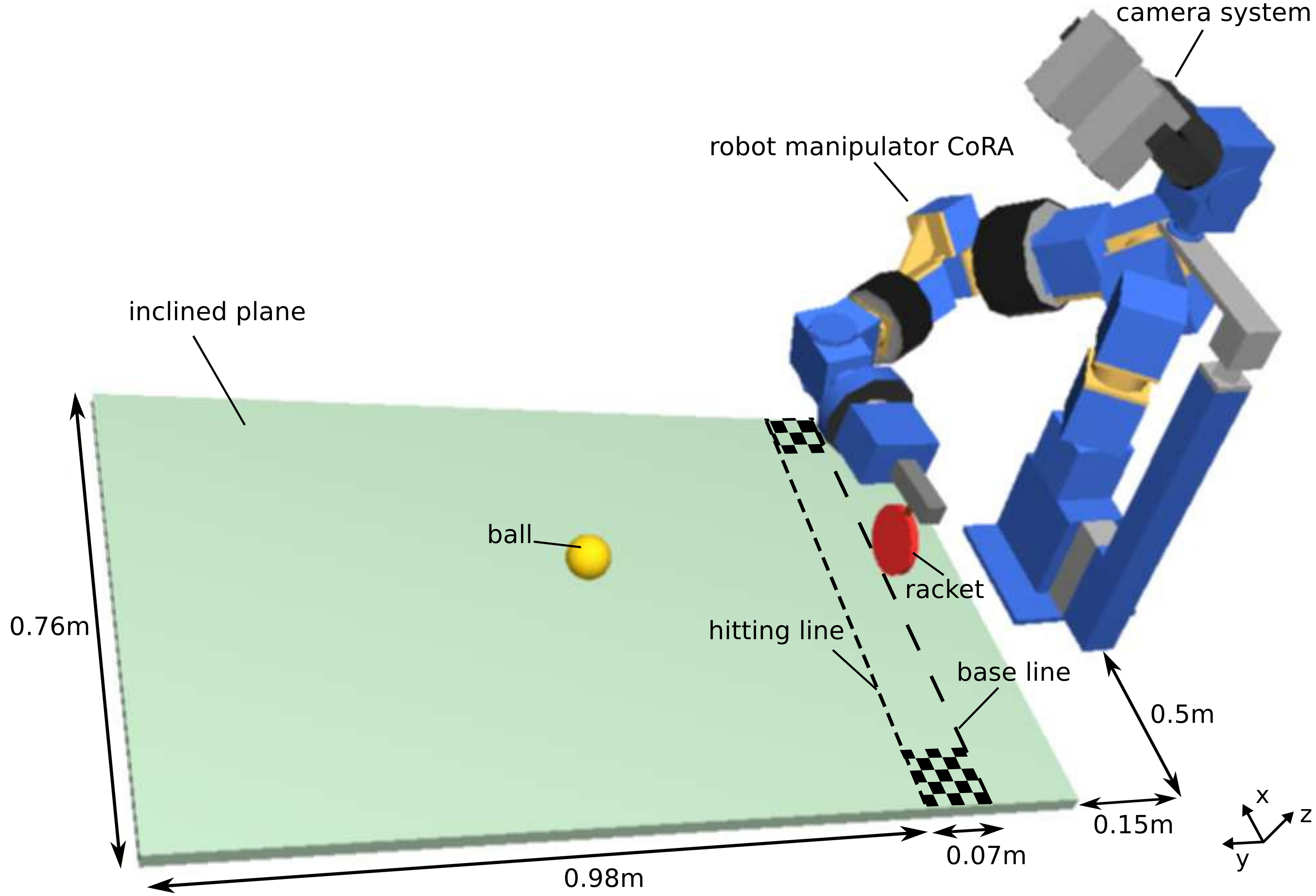
[Shukla, Billard, 2012]

video

- https://youtu.be/M413ILWvrbI?t=3

Timing and behavioral organization

- sequences of timed actions to intercept ball



[Oubatti, Richter, Schöner, 2013]

Timing and behavioral organization

- timing from oscillator, whose cycle time is adjusted to perceived time to contact

$$\tau \begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = -c_{\text{post}}\, a \begin{pmatrix} x - x_{\text{post}} \\ y \end{pmatrix} + c_{\text{hopf}}\, H(x,y) + \eta,$$

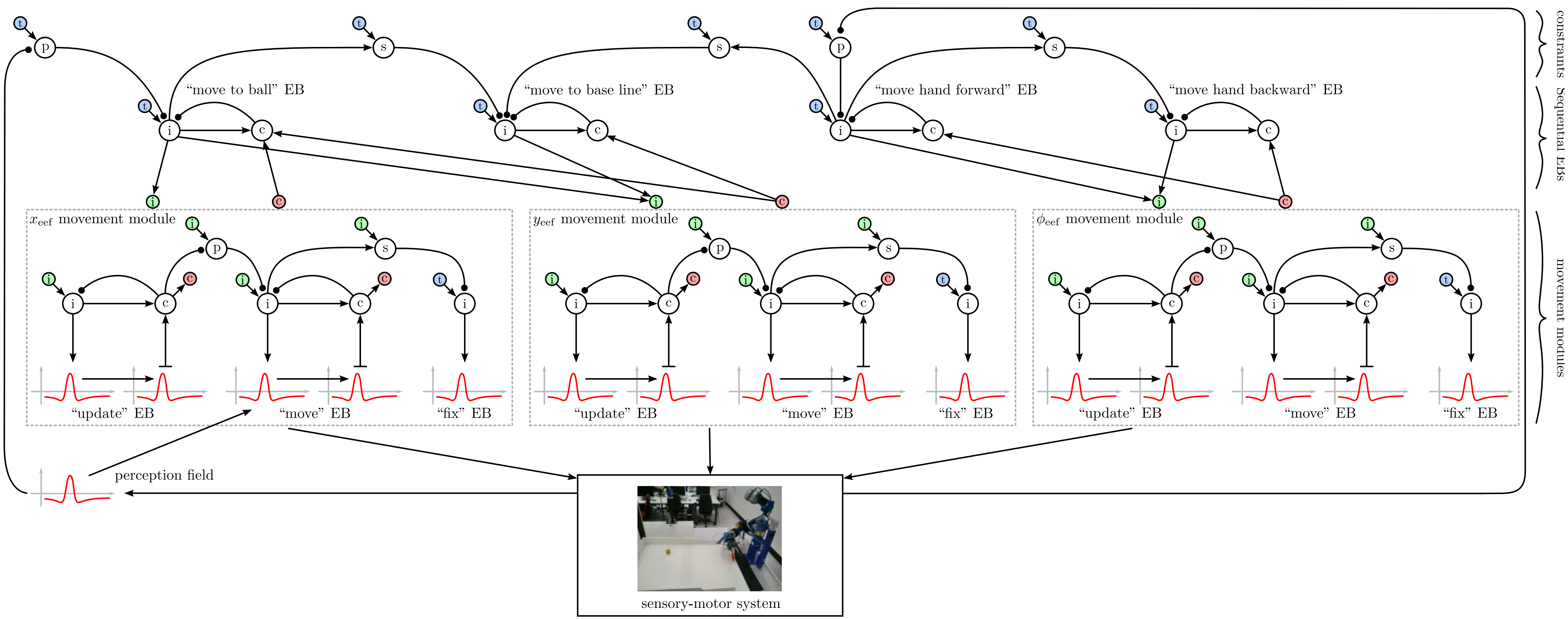
$$\begin{aligned} H(x,y) = & \begin{pmatrix} \lambda & -\omega \\ \omega & \lambda \end{pmatrix} \begin{pmatrix} x - r - x_{\text{init}} \\ y \end{pmatrix} \\ & - \left((x - r - x_{\text{init}})^2 + y^2\right) \begin{pmatrix} x - r - x_{\text{init}} \\ y \end{pmatrix} \end{aligned}$$

$$\frac{T}{2d_{\text{init}}} = \frac{t_{\text{tim}}}{d(t)}.$$

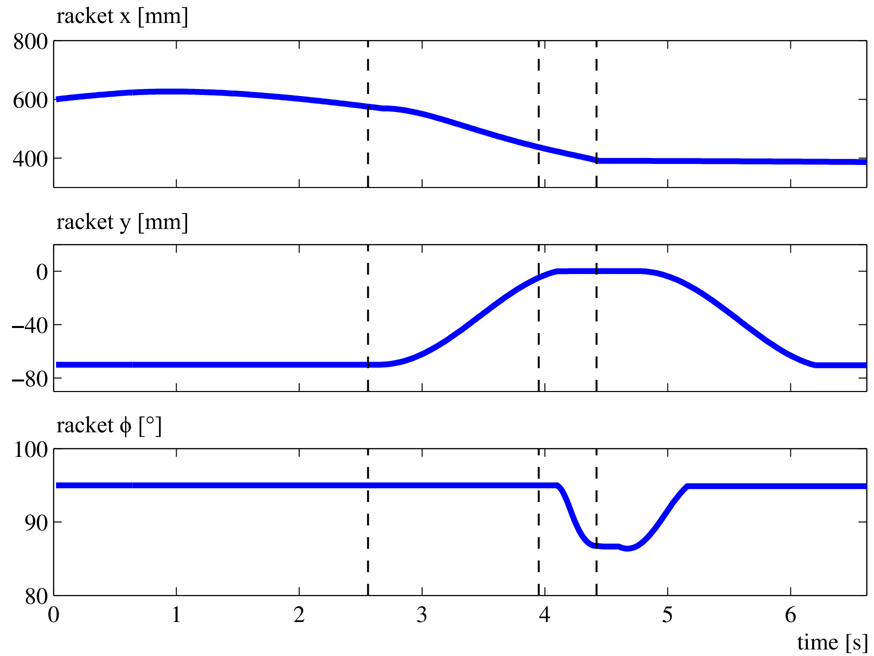
[Oubatti, Richter, Schöner, 2013]

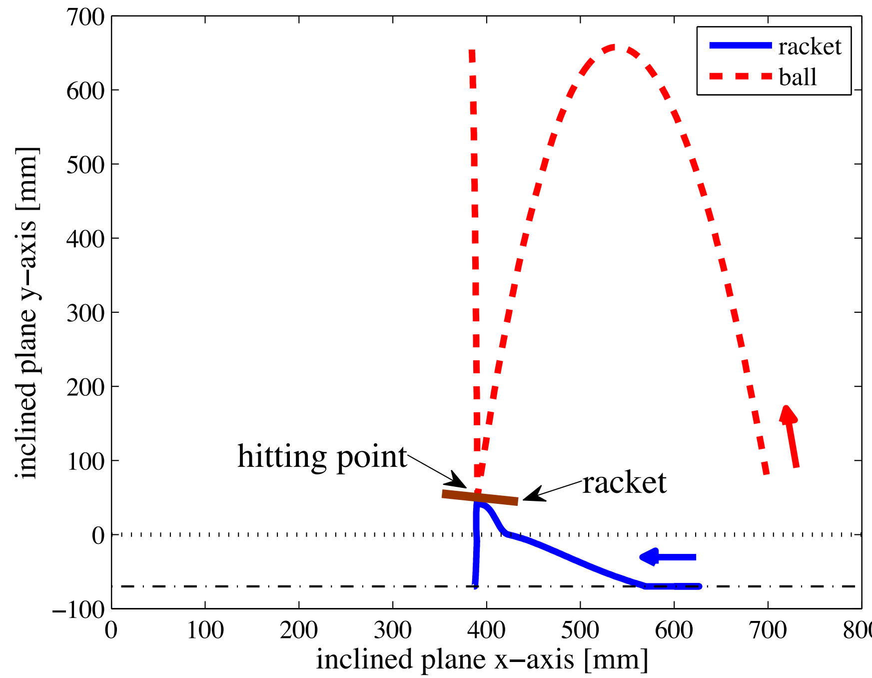
Timing and behavioral organization

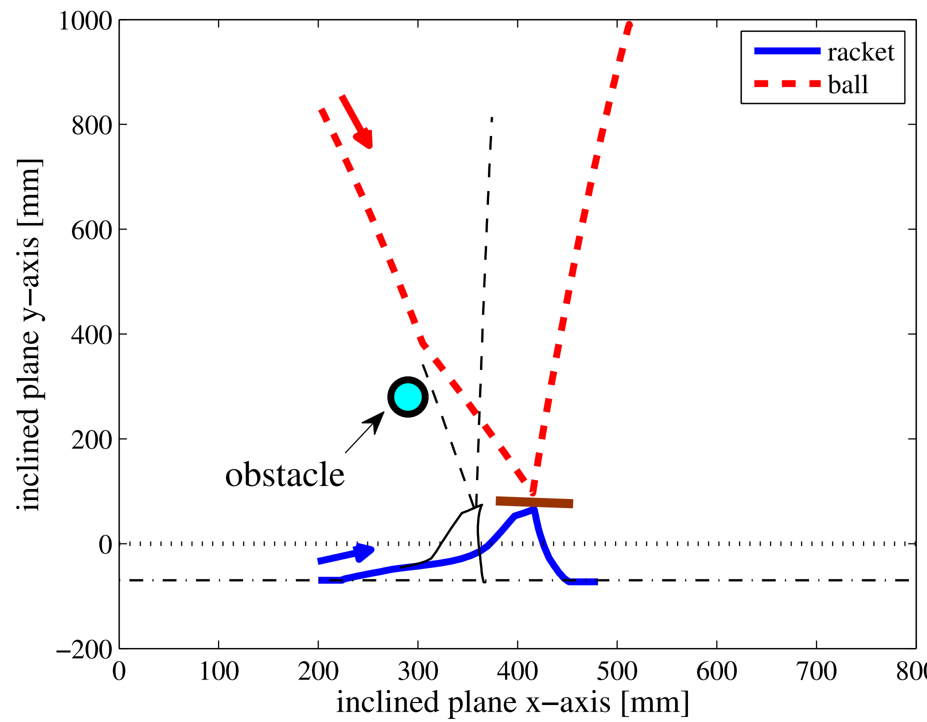
- coupled neural dynamics to organize the sequence



[Oubatti, Richter, Schöner, 2013]

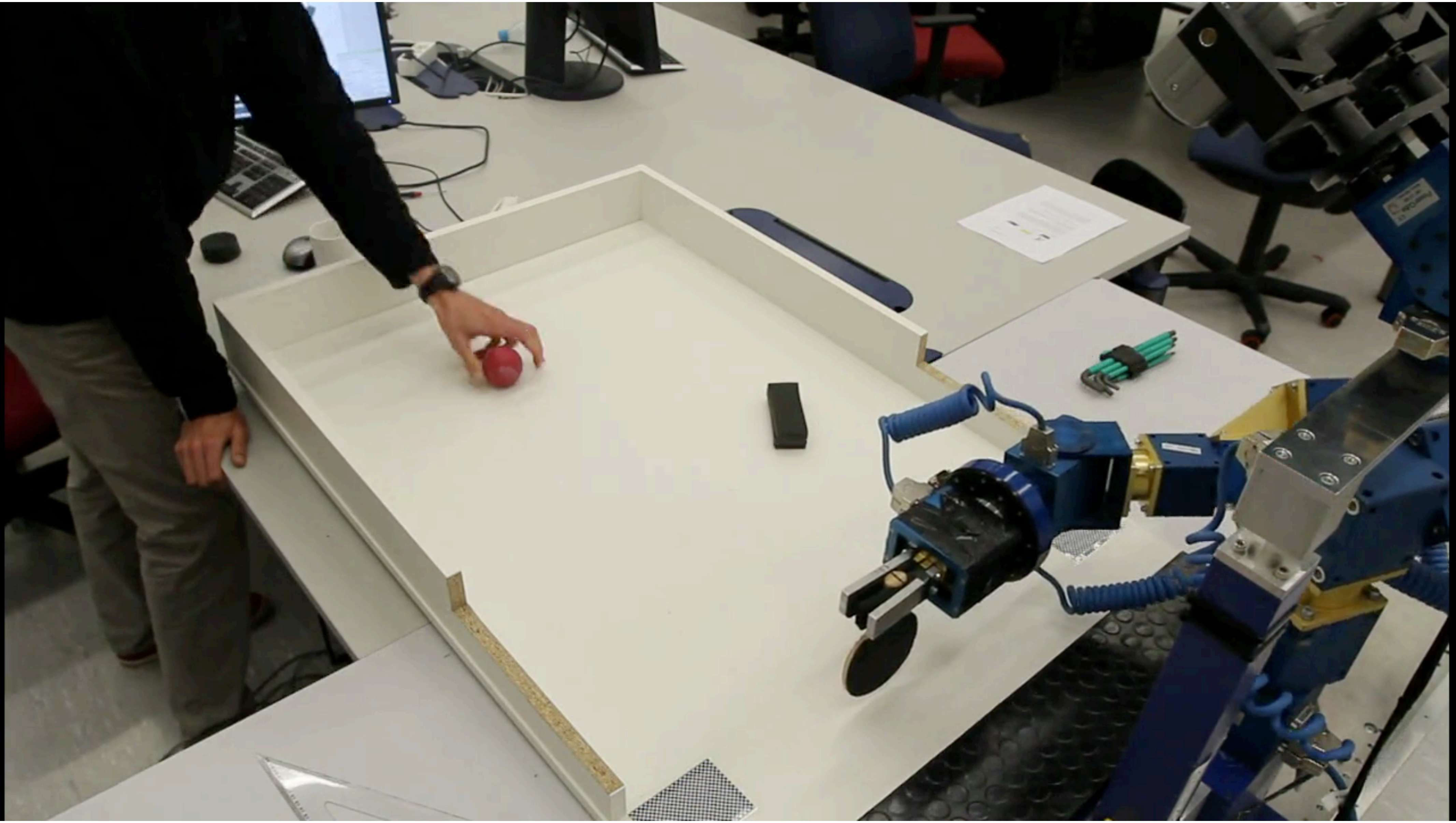


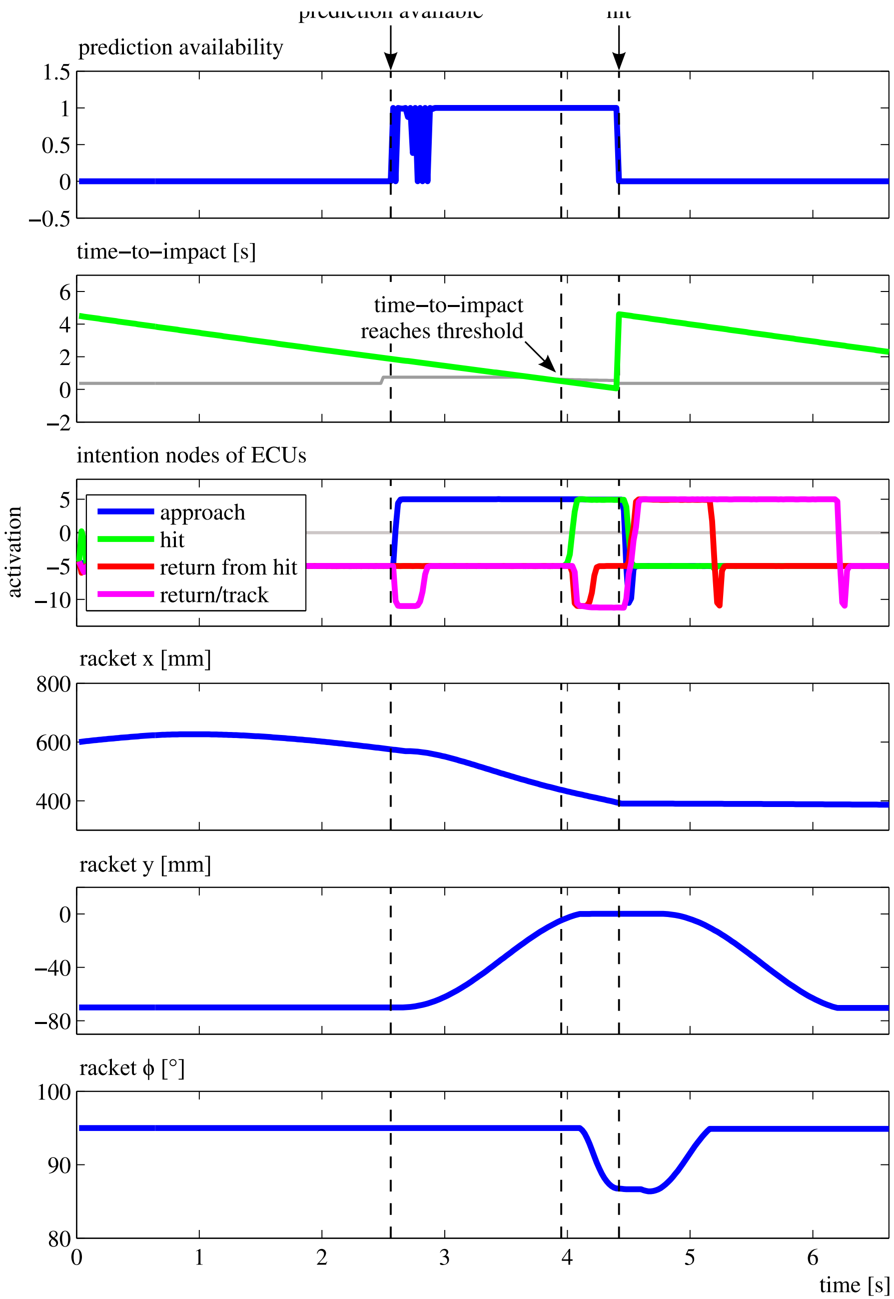


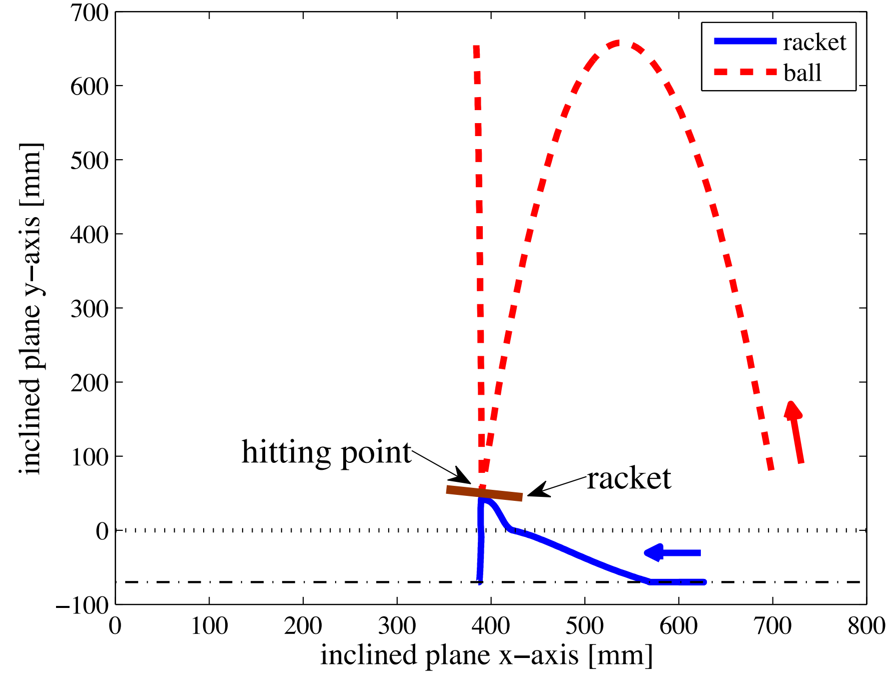


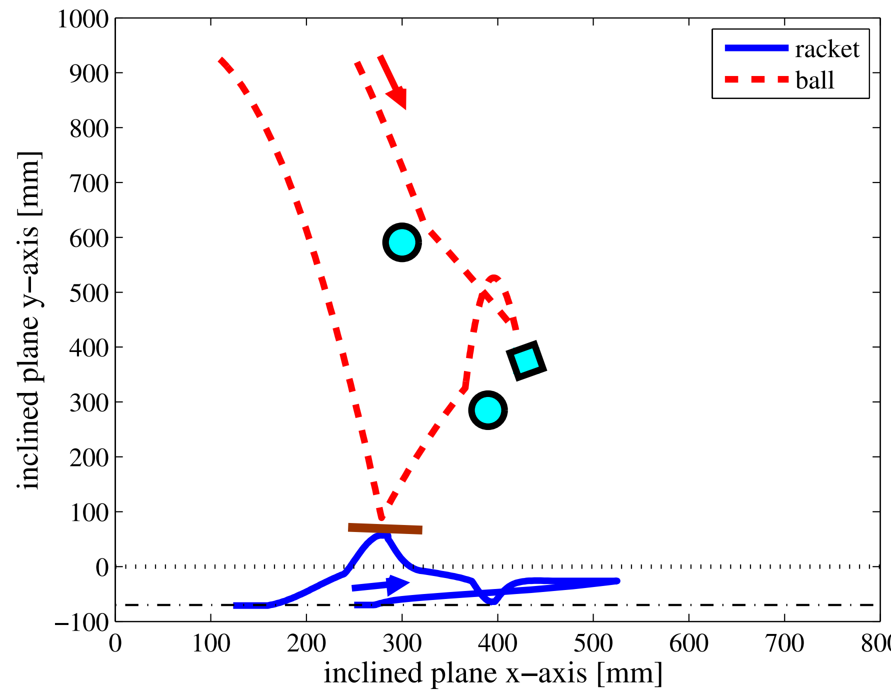
[Oubbati, Richter, Schöner, 2013]

Timed movement with online updating [Faroud Oubatti]



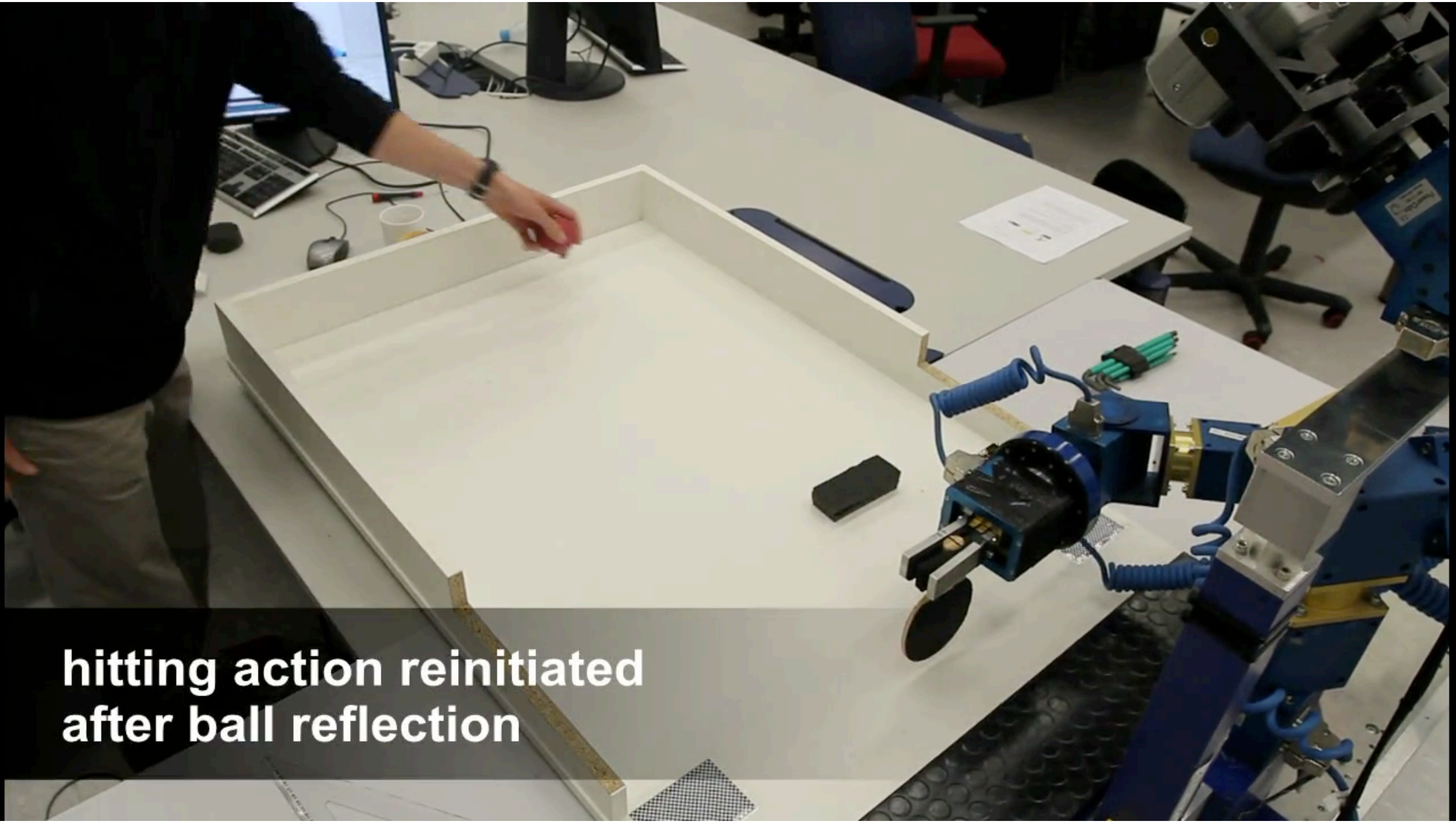






[Oubbati, Richter, Schöner, 2013]

Timing and reorganization of movement



Conclusion

- timing in autonomous robotics is best framed as a problem of stable oscillators and their coupling

Conclusion

- timing is linked to many problems
  - arriving "just in time", estimating time to contact
  - on line updating: planning and timing tightly connected
  - timed movement sequences: behavioral organization
  - coordinating timing across movements, coarticulation
  - timing and control