Timing and coordination

Gregor Schöner

In vehicle motion planning

- movement is generated through a "behavioral dynamics" that is in closed loop with the environment
- taking into account (possibly time varying) constraints from the perceived environment
- time to reach the target was not a constraint.. and not controlled/stabilized

Reaching movements of an arm

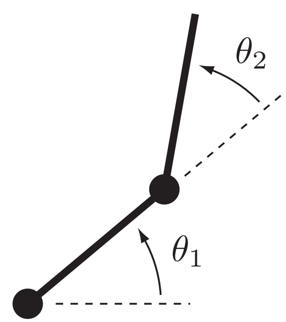
- reaching movements may be generated in open loop.. by an internal "neural" dynamics
- generate movements that are "timed", that is,
  - they arrive "on time"
  - the are coordinated across different effectors
  - the are coordinated with moving objects (e.g., catching)
- timing implies some form of anticipation…

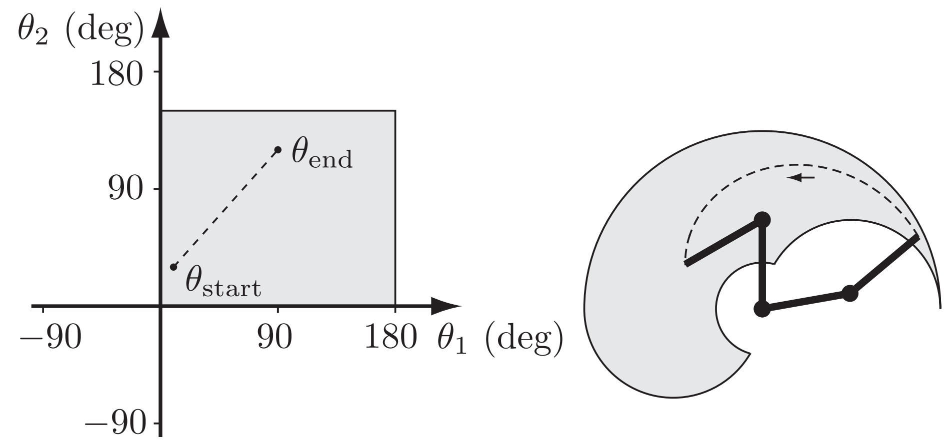
How is timing done in conventional robotics?

- conventional motion planning:
  - compute/design the movement plan, parameterized by a path variable
  - then rescale that path variable to generate a desired timing profile
  - which the robotic controller must track

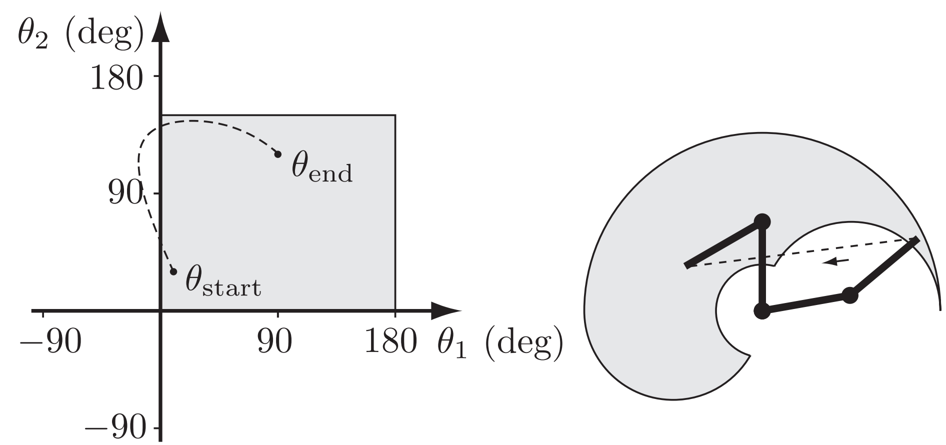
Conventional robotic timing

- paths may be planned in joint or end-effector space





$$\theta(s) = \theta_{\text{start}} + s(\theta_{\text{end}} - \theta_{\text{start}}),$$

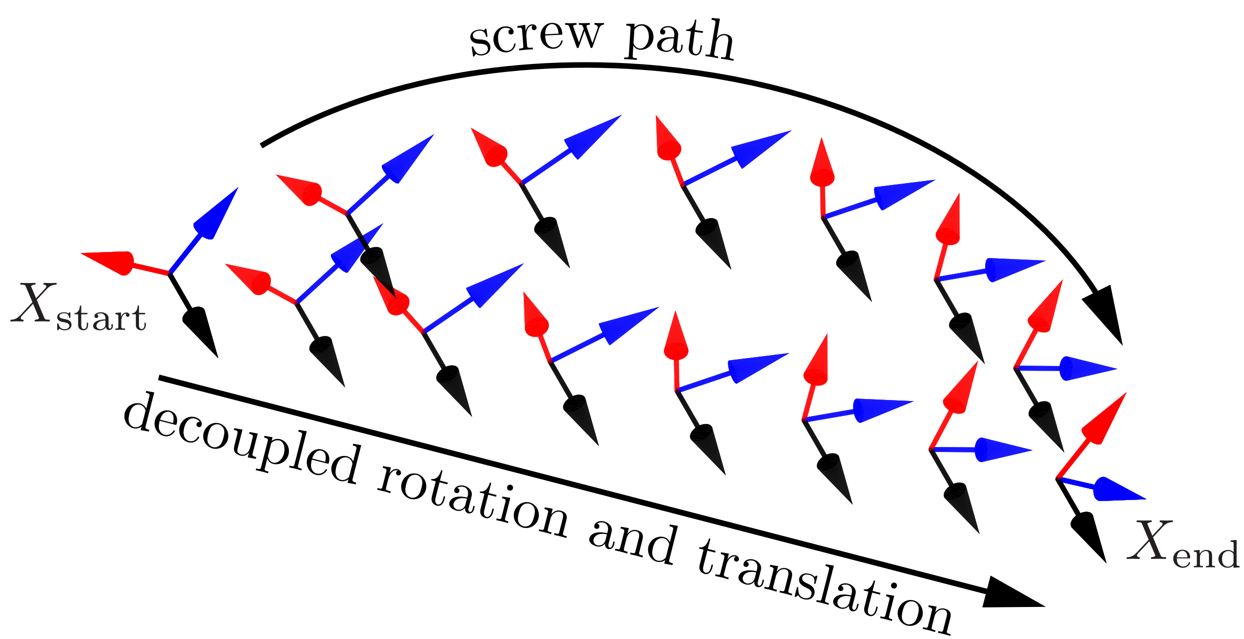


$$X(s) = X_{\text{start}} + s(X_{\text{end}} - X_{\text{start}}), s \in [0,1].$$

[Lynch, Park, 2017 (Chapter 9)]

Conventional robotic timing

- paths are more generally planned in the space of robot arm reconfigurations "screws"



[Lynch, Park, 2017 (Chapter 9)]

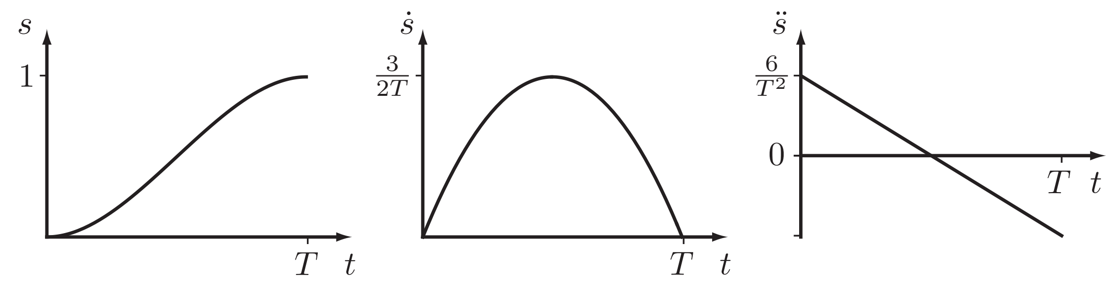
Conventional robotic timing

- time scaling

$$s(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3.$$

$$X(s) = X_{\text{start}} + s(X_{\text{end}} - X_{\text{start}}), s \in [0, 1].$$

$$\theta(s) = \theta_{\text{start}} + s(\theta_{\text{end}} - \theta_{\text{start}}),$$

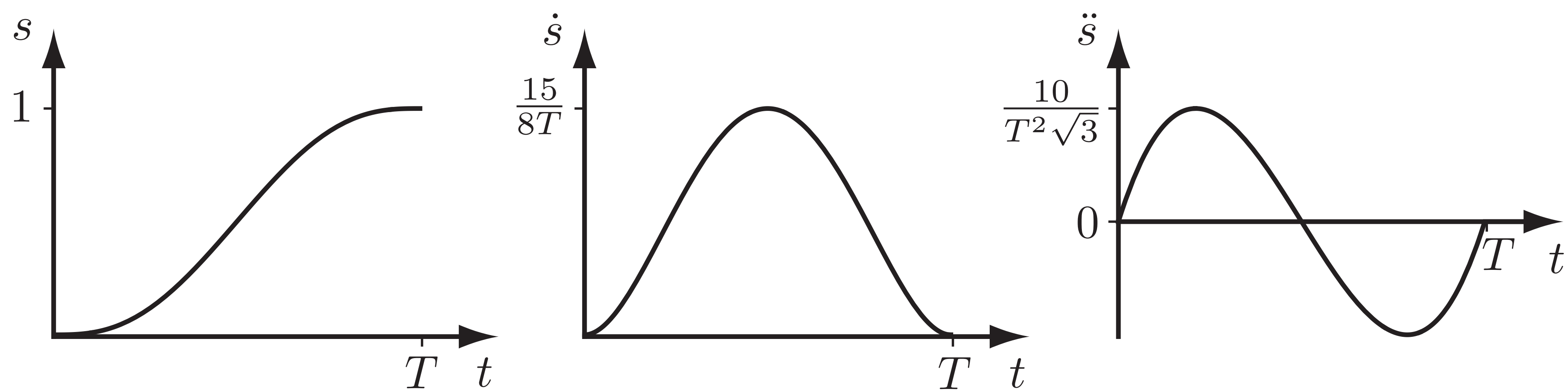


- compute parameters to achieve a particular movement time T, with zero velocity at target

[Lynch, Park, 2017 (Chapter 9)]

Conventional robotic timing

- time scaling: 5th order polynomial



- compute parameters to achieve a particular movement time T, with zero velocity and zero acceleration at target

[Lynch, Park, 2017 (Chapter 9)]

Conventional robotic timing

- time scaling: ramps

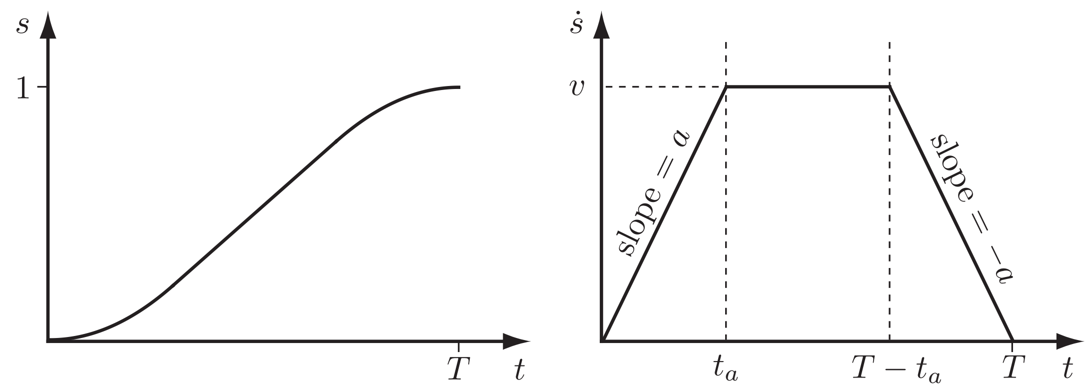
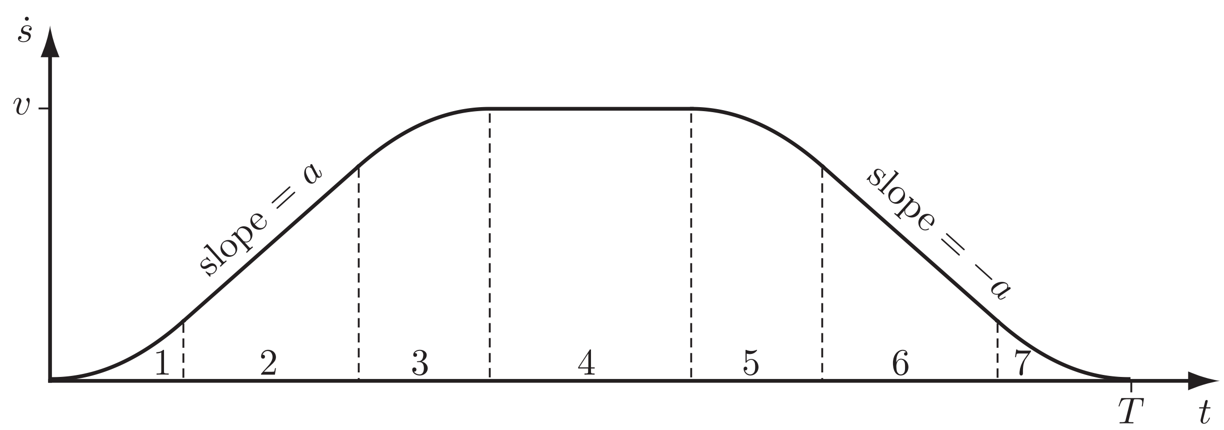


Figure 9.5: Plots of $s(t)$ and $\dot{s}(t)$ for a trapezoidal motion profile.

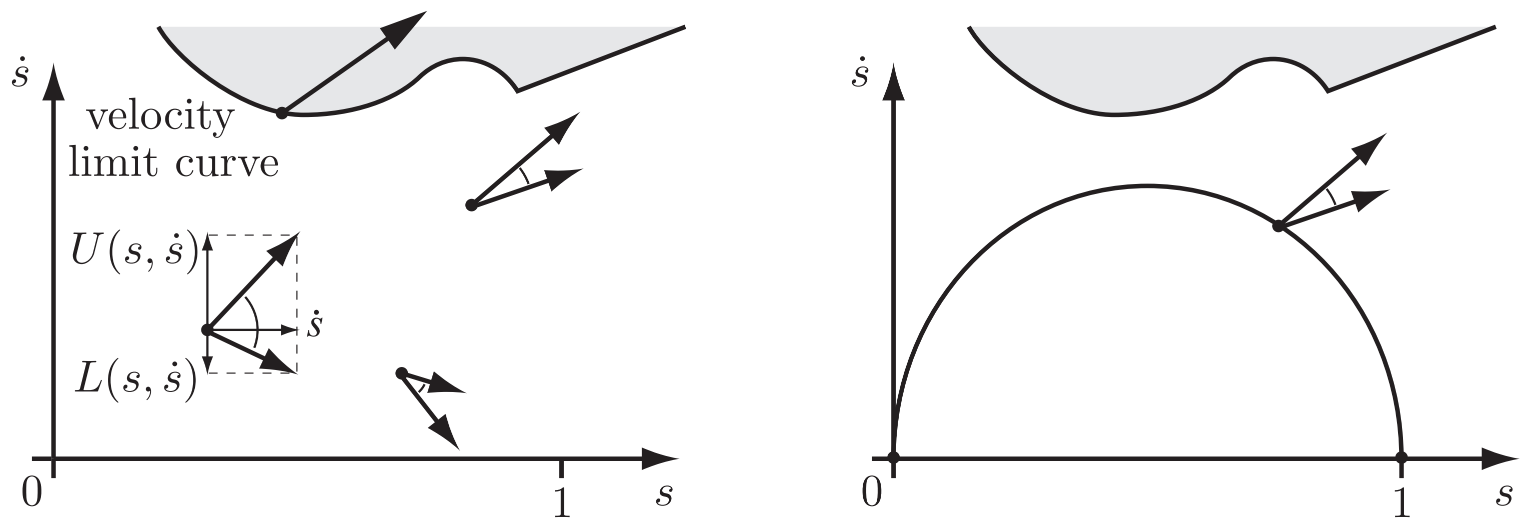
- time scaling: smoothed ramps



[Lynch, Park, 2017 (Chapter 9)]

Conventional robotic timing

- time scaling: taking limits on acceleration into account



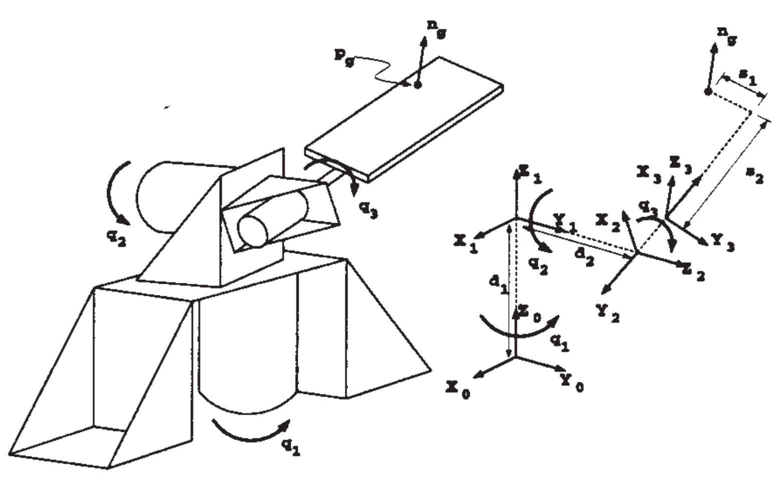
[Lynch, Park, 2017 (Chapter 9)]

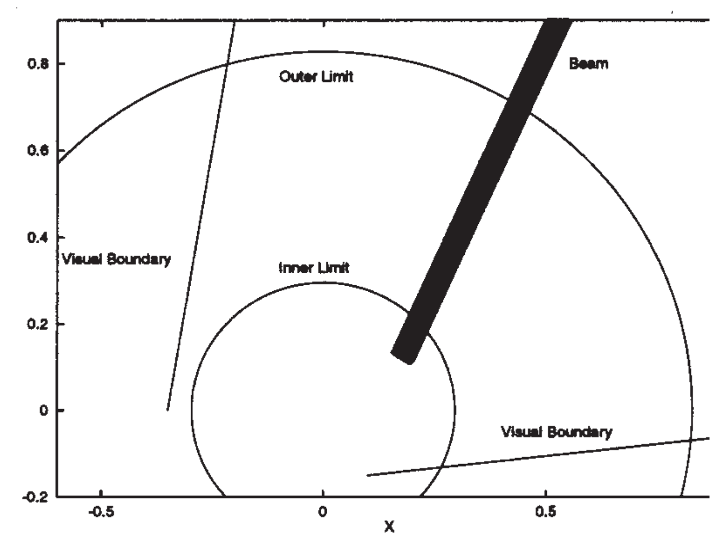
How is timing done in autonomous robotics?

- all of these methods require detailed models of the task and make demands on the control system… to guarantee soft arrival….
- in autonomous robotics: use more robust heuristics

Timing in autonomous robotics

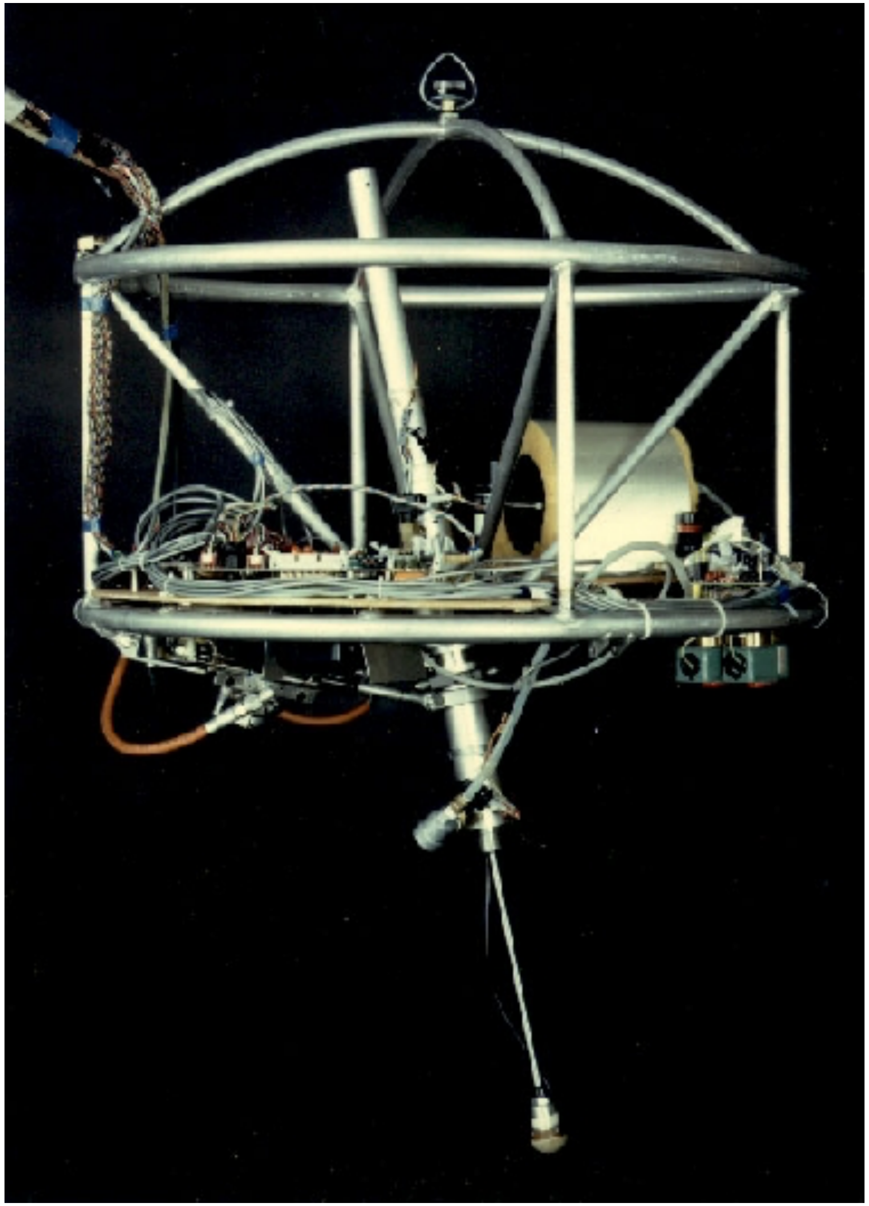
- Koditschek's juggling robot:
  - physical dynamics of bouncing ball modeled… state estimated based on vision, actuator inserts a perturbation so that a periodic solution (limit cycle) results
  - ball is kept within reach by conventional P control from contact to contact





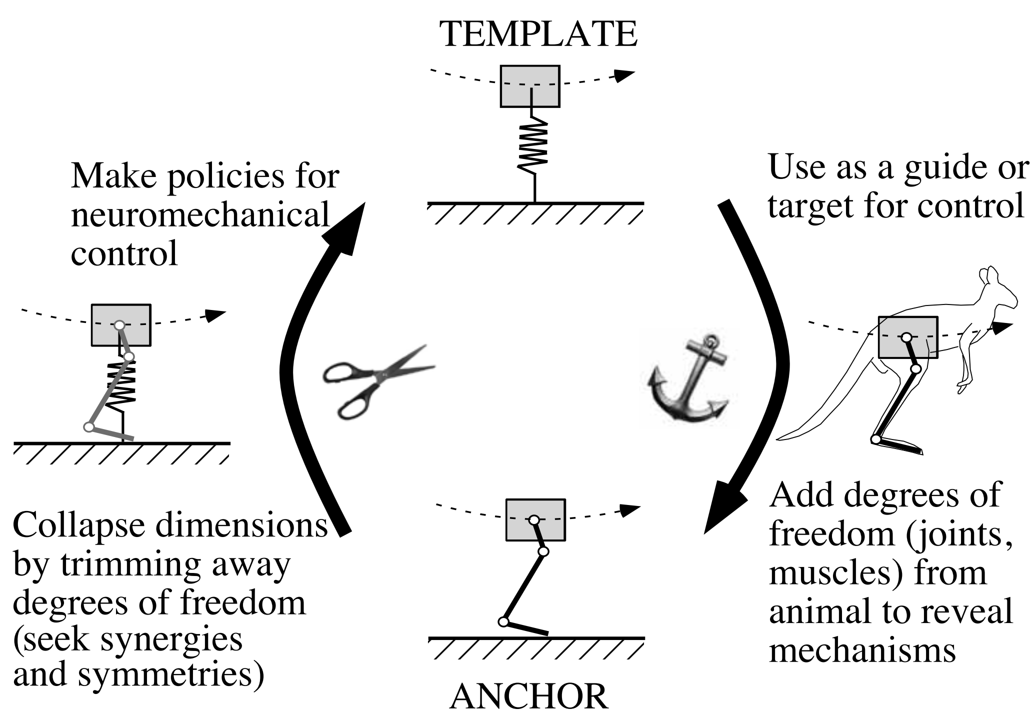
Timing in autonomous robotics

- Raibert's hopping robots
  - dynamics bouncing robot modeled… actuator inserts a term into that dynamics so that a periodic solution (limit cycle) results
  - robot is kept upright by controlling leg angle to achieve particular horizontal position for Center of Mass



Generalization to bipedal/ quadrupedal locomotion

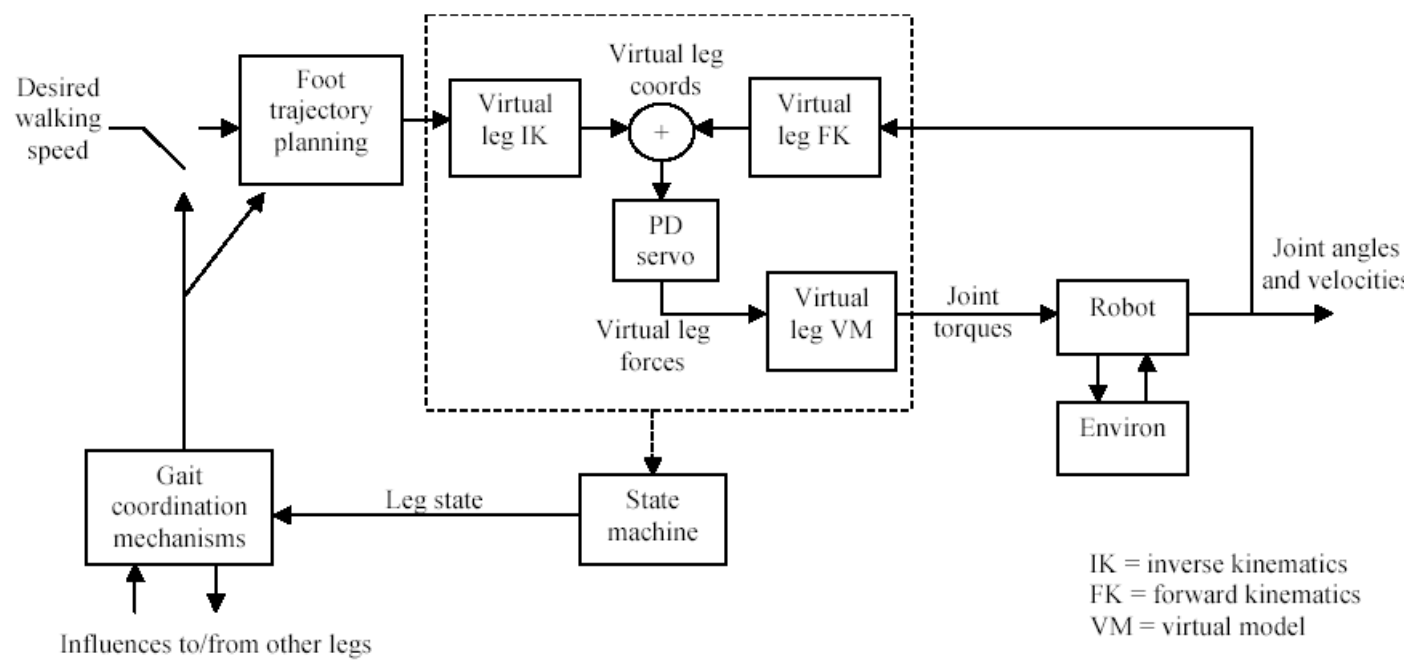
- template…oscillator at macro-level..
- anchor… kinematics at joint/actuator level



[Full Koditschek 99]

Timing in autonomous robotics

- Raibert's bio-dog
  - expand that idea to coordination among limbs
- => technical variant



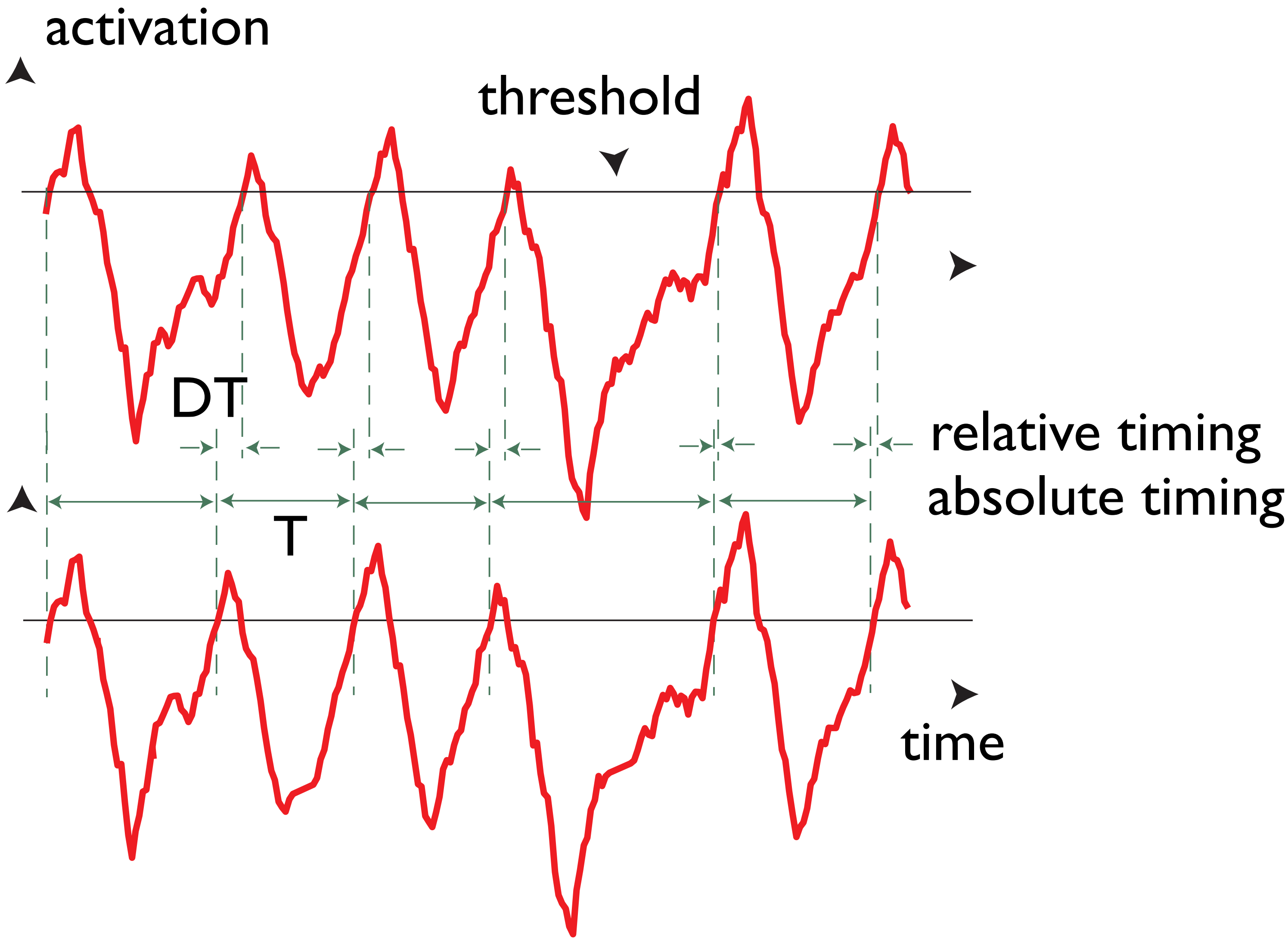
Timing in autonomous robotics

https://www.youtube.com/watch?v=M8YjvHYbZ9w

Some ideas from human movement

- timing
- absolute vs relative timing
- coordination
- coupled oscillators

Relative vs. absolute timing



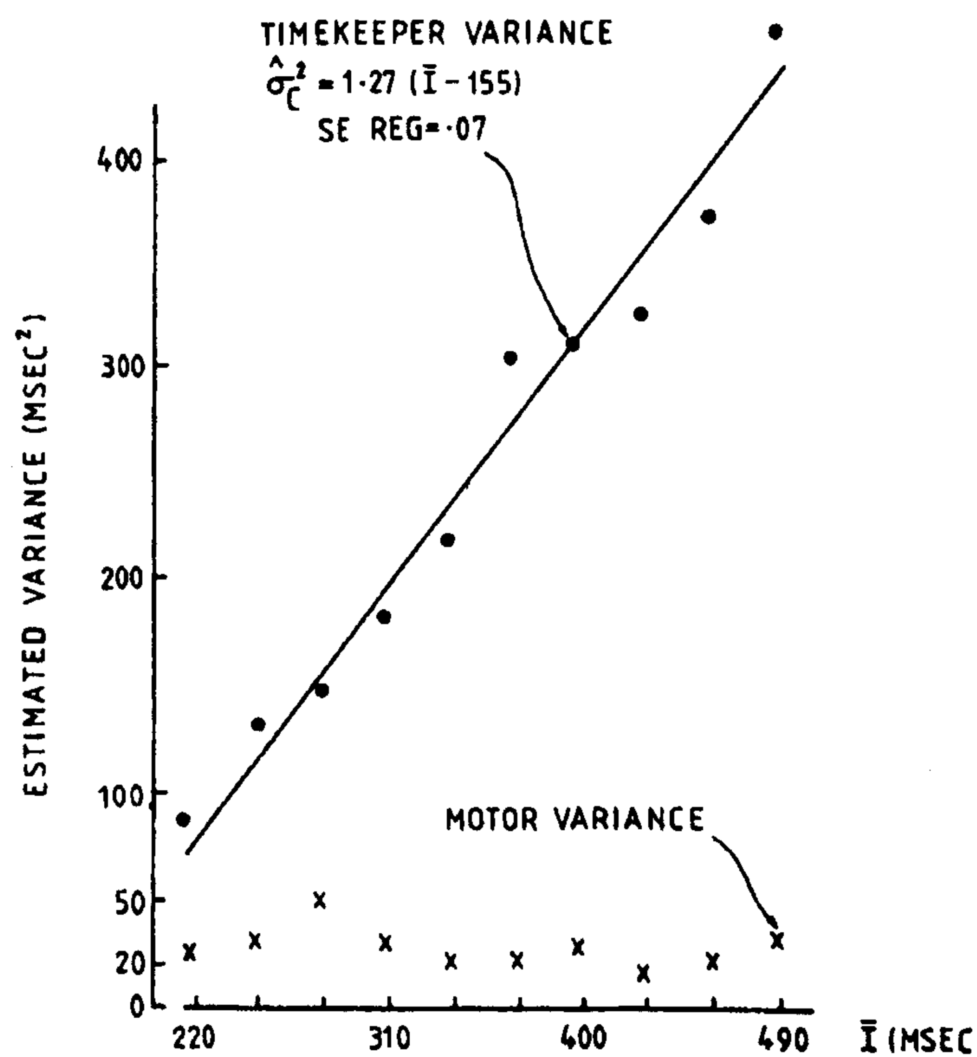
relative phase=DT/T

Absolute timing

- examples: music, prediction, estimating time
- typical task: tapping
- self-paced vs. externally paced

Human performance

- on absolute timing is impressive
- smaller variance than 5% of cycle time in continuation paradigm



[Wing, 1980]

Theoretical account for absolute timing

- (neural) oscillator autonomously generates timing signal, from which timing events emerge
- => limit cycle oscillators
- = clocks

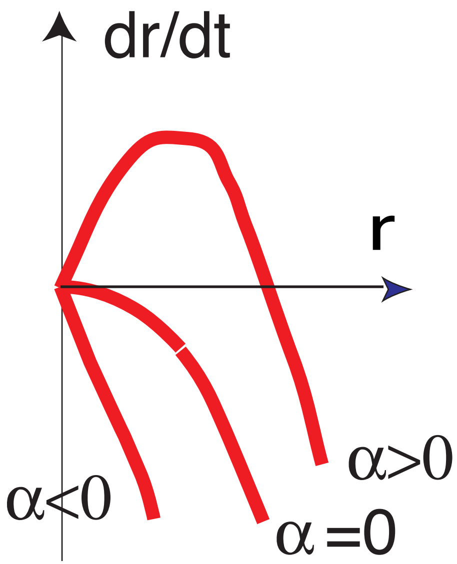
Limit cycle oscillator: Hopf

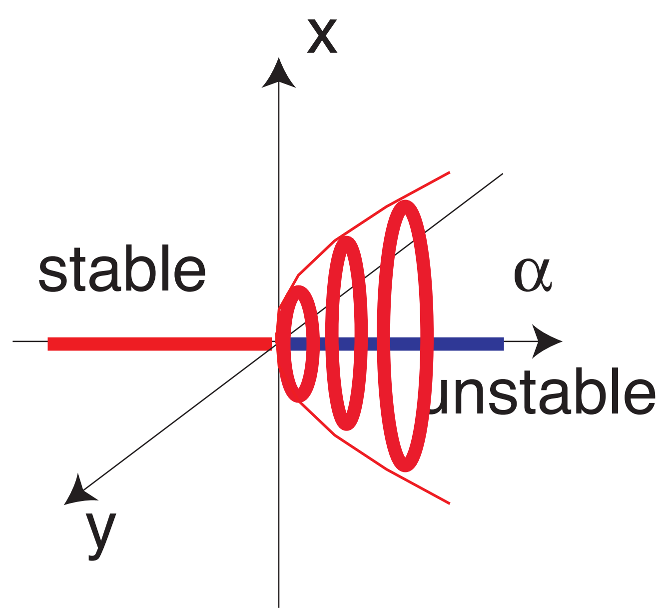
- normal form

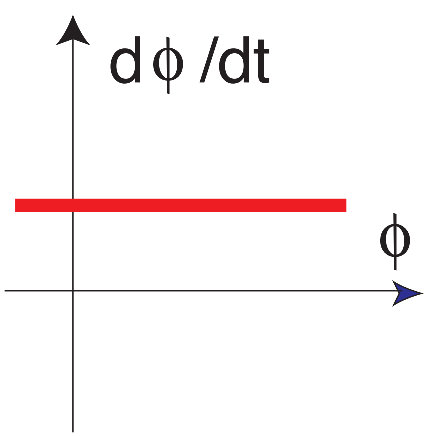
$$\begin{pmatrix}\dot{x}\\ \dot{y}\end{pmatrix} = \begin{pmatrix}\alpha & -\omega\\ \omega & \alpha\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix} - (x^2+y^2)\begin{pmatrix}x\\ y\end{pmatrix}$$

$$x = r\cos(\phi) \qquad \dot{r} = \alpha r - r^3$$

$$y = r\sin(\phi) \qquad \dot{\phi} = \omega$$







$$x(t) = \sqrt{\alpha}\sin(\omega t)$$

$$\text{amplitude } A = \sqrt{\alpha}$$

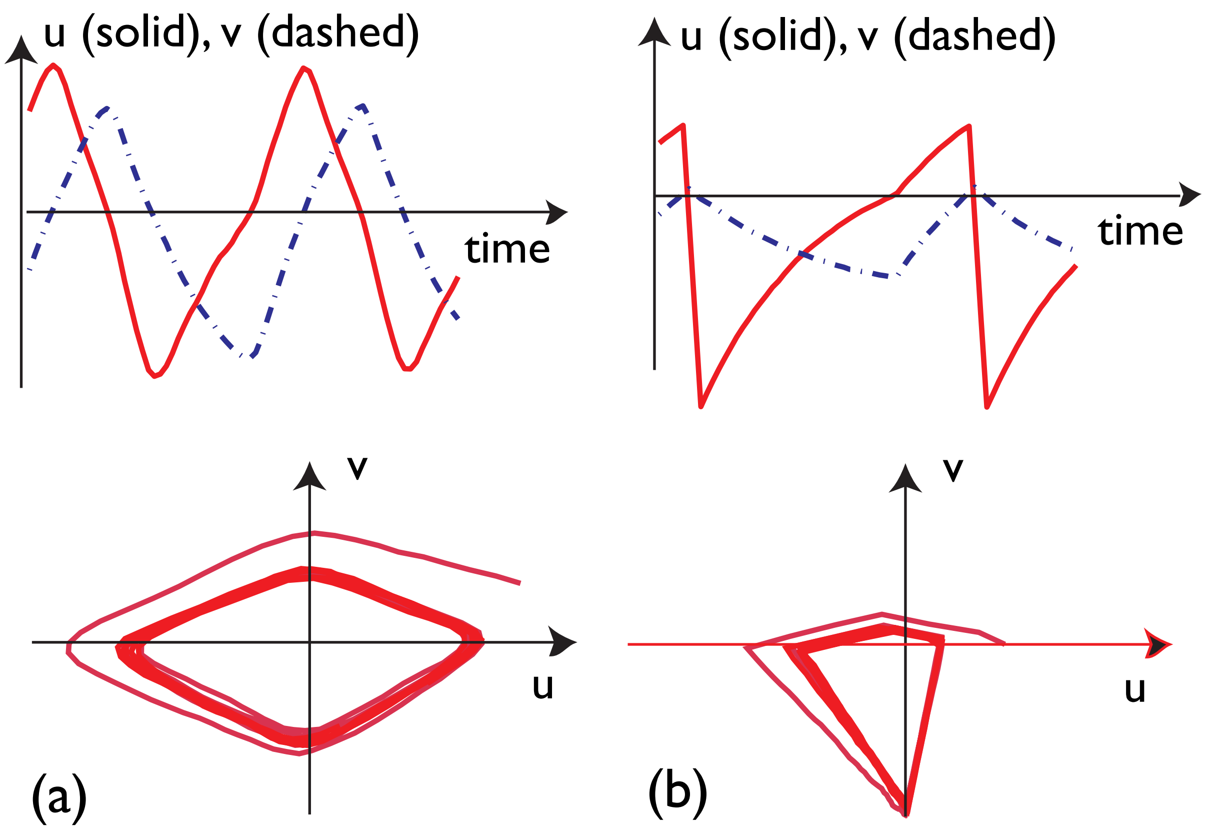
$$\text{cycle time } T = 2\pi/\omega,$$

Neural oscillator

- relaxation oscillator

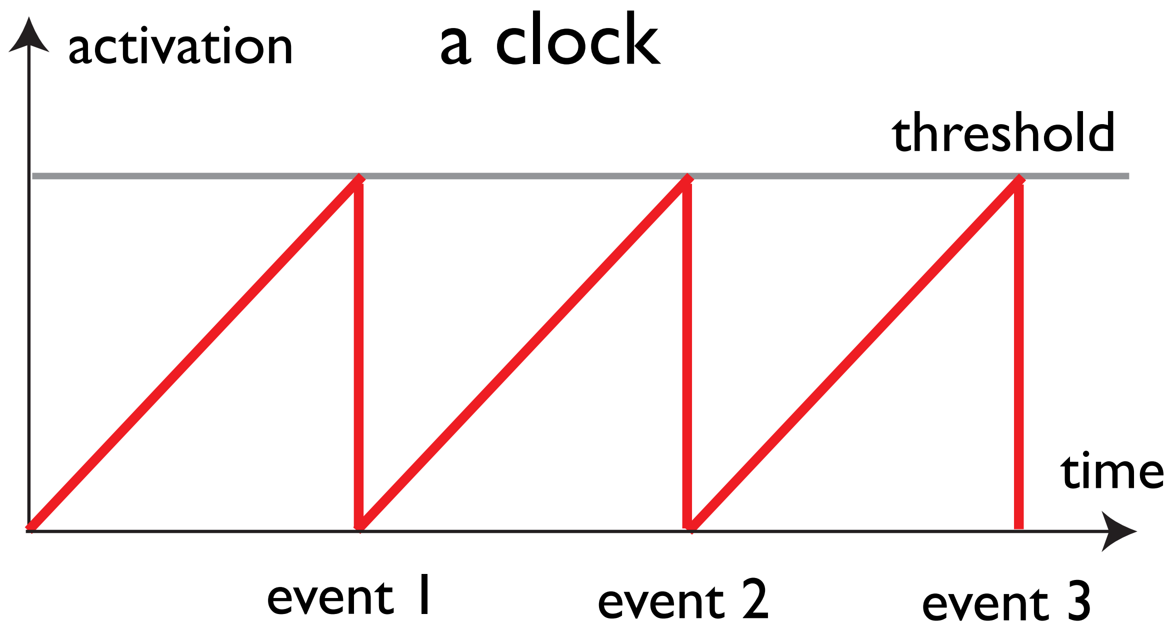
$$\tau \dot{u} = -u + h_u + w_{uu} f(u) - w_{uv} f(v)$$
$$\tau \dot{v} = -v + h_v + w_{vu} f(u),$$

[Amari 77]



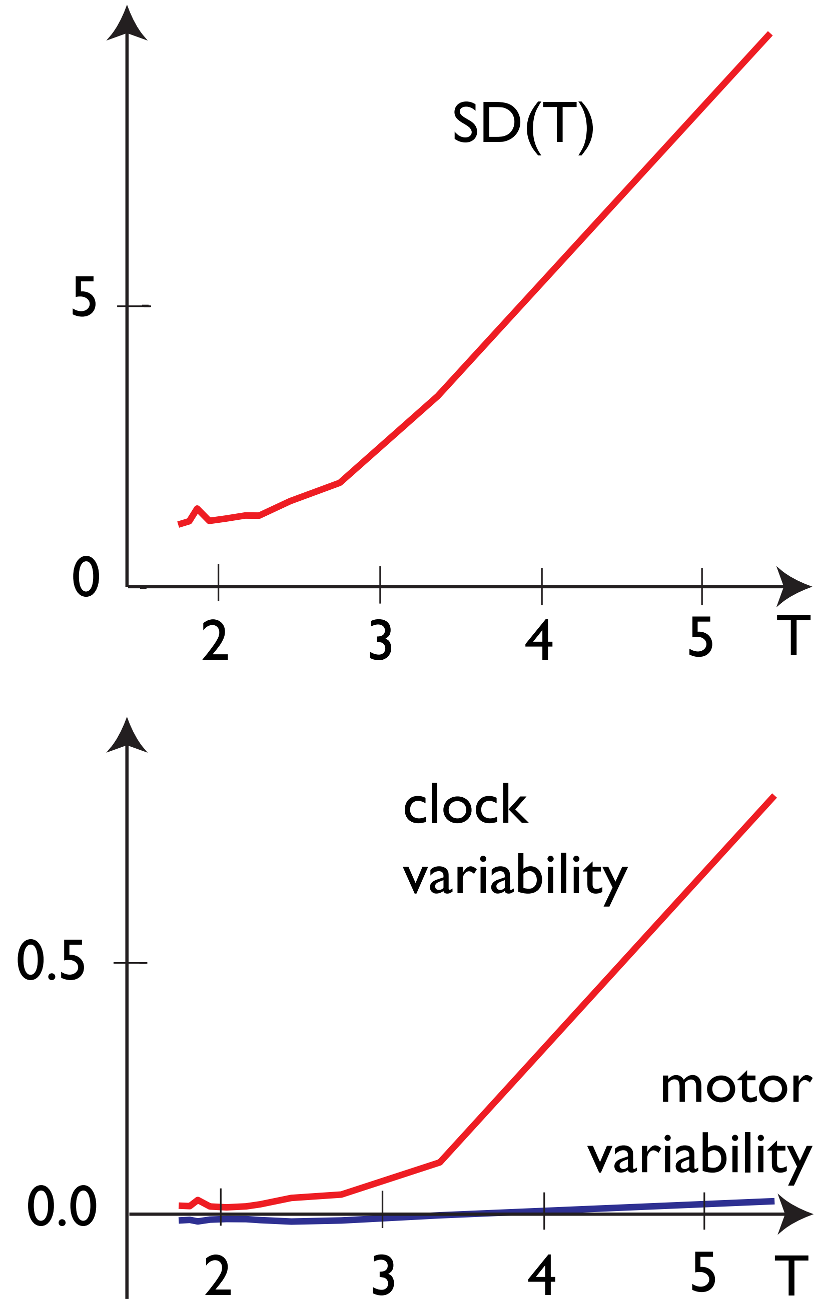
Clocks

- hour glasses are also oscillators
- but: it is critical to include the “resetting”



[from: Schöner, Brain & Cogn 48:31 (2002)]

Neural oscillator accounts for variance of absolute timing



[Schöner 2002]

Relative timing: movement coordination

- locomotion, interlimb and intralimb
- speaking
- mastication
- music production
- ... approximately rhythmic

Examples of coordination of temporally discrete acts:

- reaching and grasping
- bimanual manipulation
- coordination among fingers during grasp
- catching, intercepting

Definition of coordination

- Coordination is the maintenance of stable timing relationships between components of voluntary movement.
- Operationalization: recovery of coordination after perturbations
- Example: speech articulatory work (Gracco, Abbs, 84; Kelso et al, 84)
- Example: action-perception patterns

Is movement always timed/ coordinated?

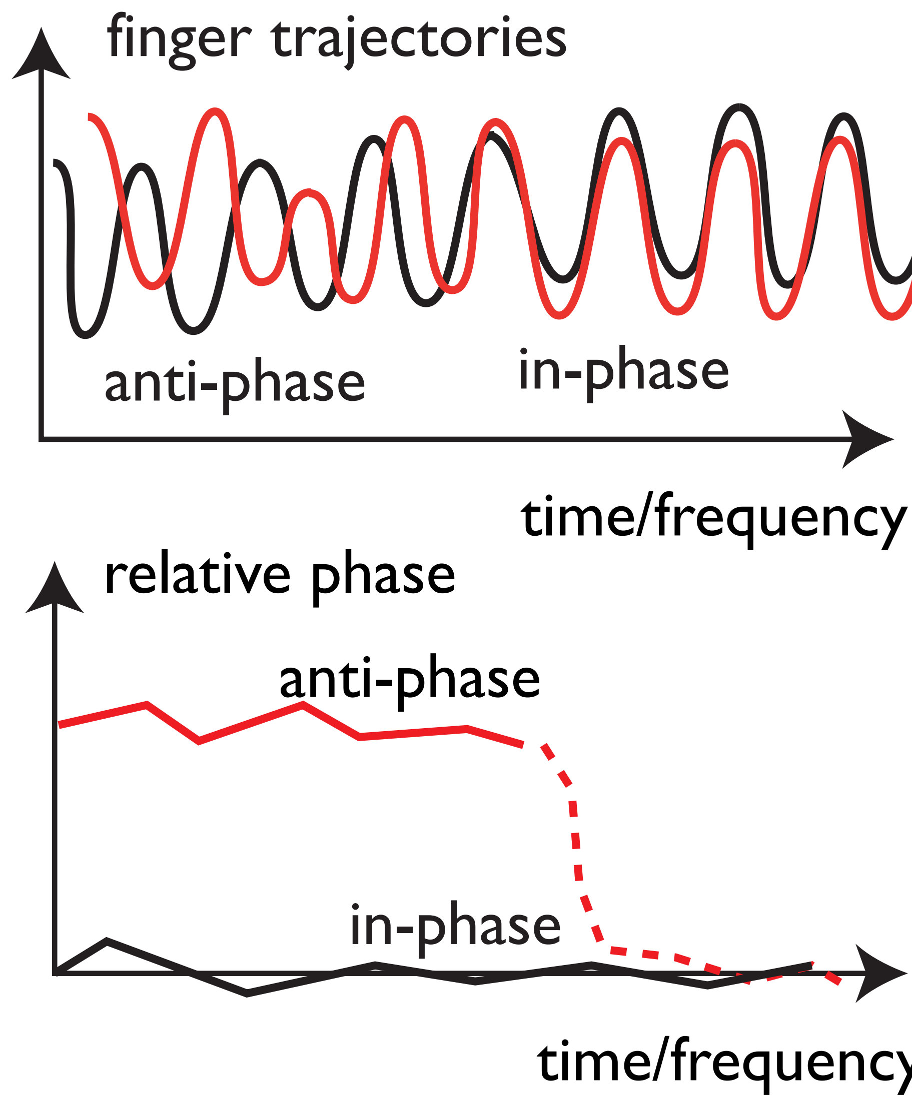
- No, for example:
- locomotion: whole body displacement in the plane
  - in the presence of obstacles takes longer
  - delay does not lead to compensatory acceleration
- but coordination is pervasive...
  - e.g., coordinating grasp with reach

Two basic patterns of coordination

- in-phase
  - synchronization, moving through like phases simultaneously
  - e.g., gallop (approximately)
- anti-phase or phase alternation
  - syncopation
  - e.g., trott

An instability in rhythmic movement coordination

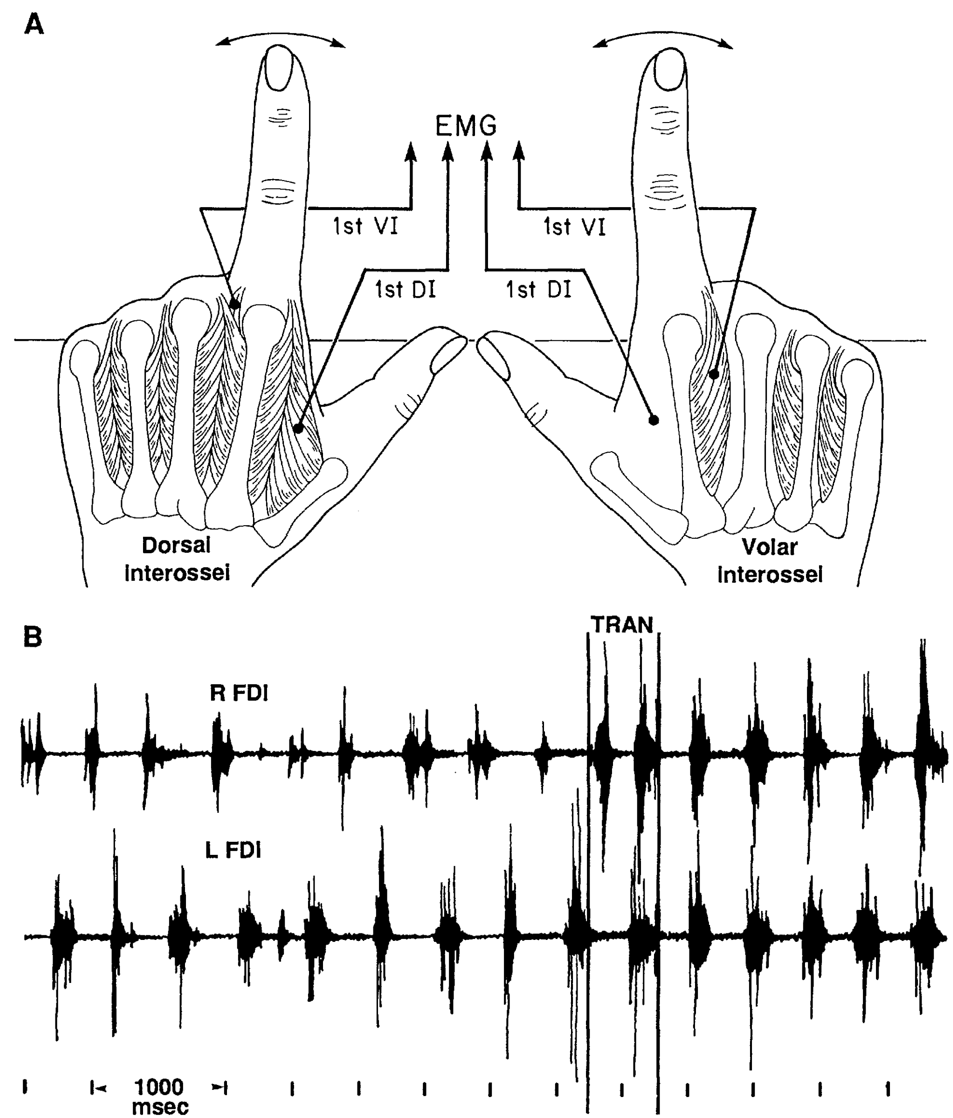
- switch from anti-phase to in-phase as rhythm gets faster



Kelso, 1984

Instability

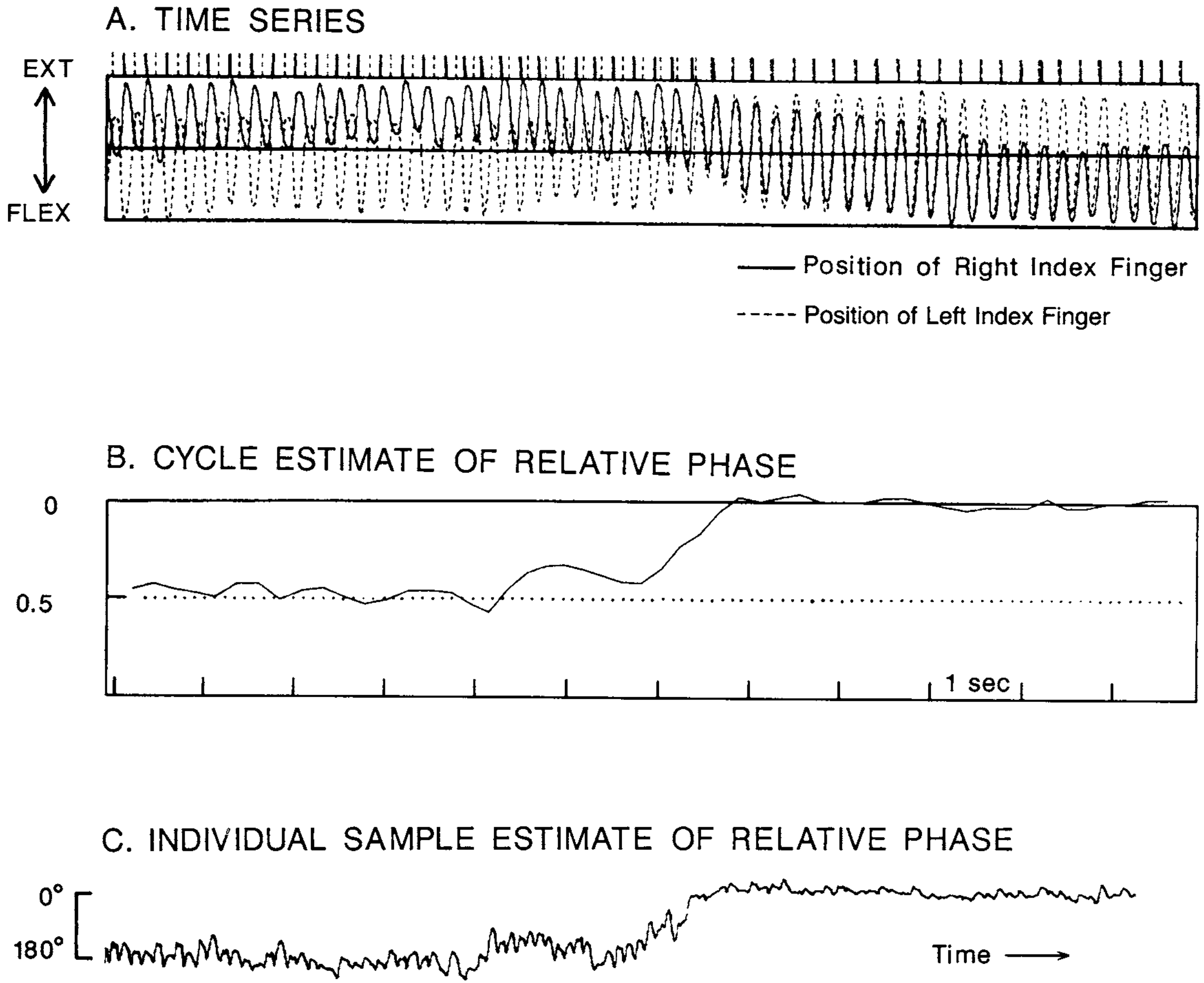
- experiment involves finger movement
  - no mechanical coupling
  - constraint of maximal frequency irrelevant
  - => pure neurallly based coordination



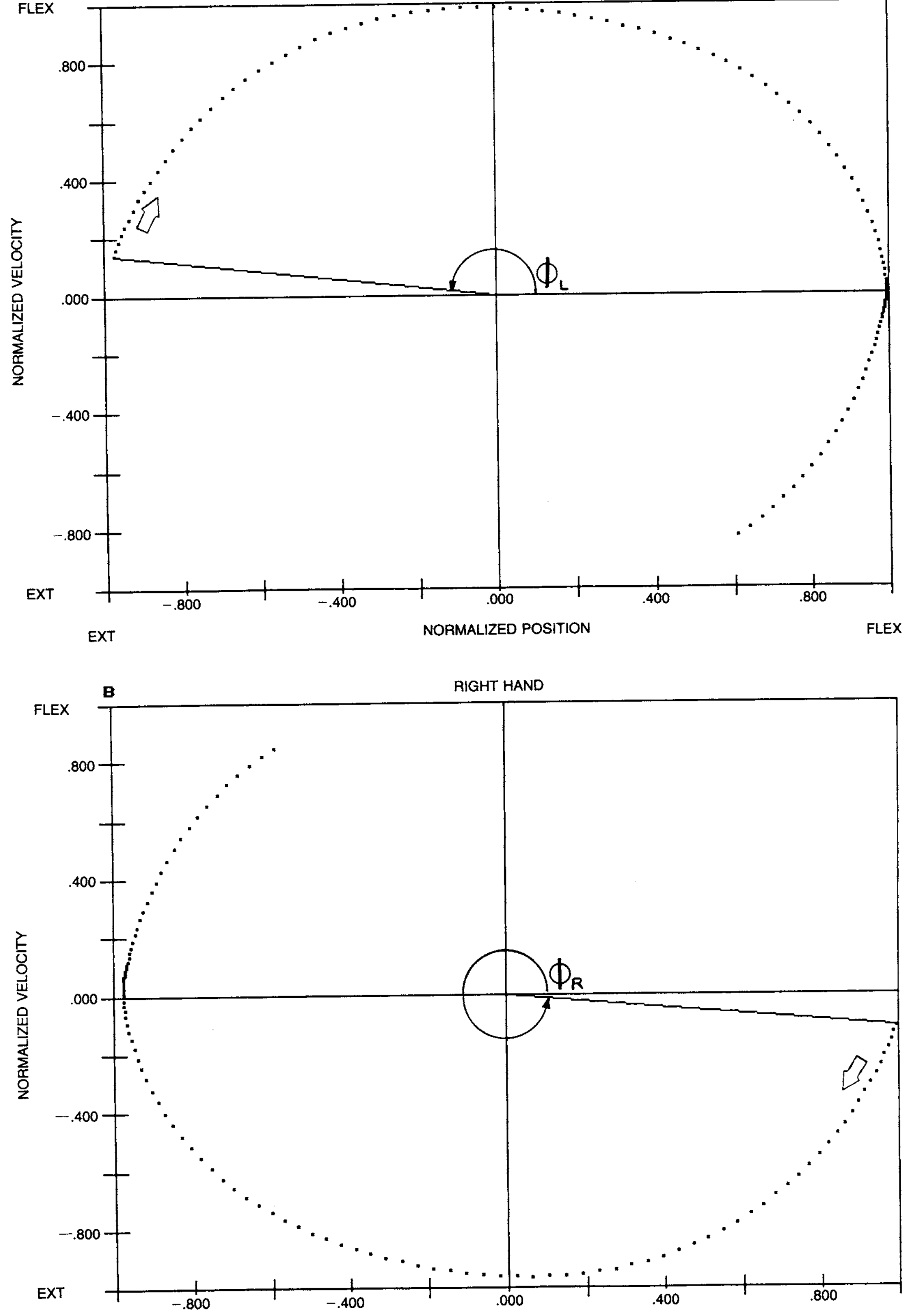
Schöner, Kelso (Science, 1988)

Instability

- frequency imposed by metronomes and varied in steps
- either start out in-phase or anti-phase



data example (Scholz, 1990)



computation of continuous relative phase (Scholz, 1990)

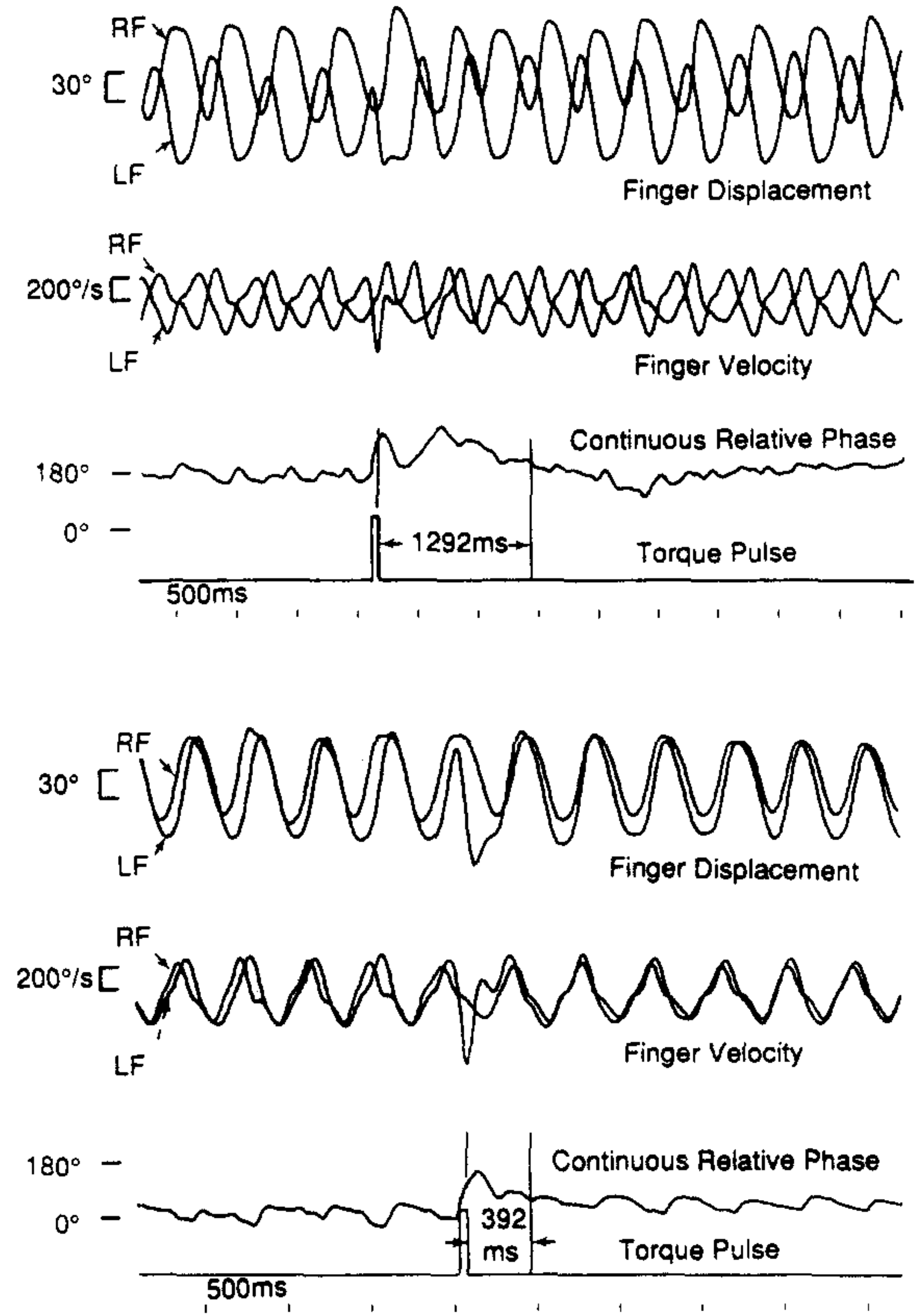
Measures of stability

- variance: fluctuations in time are an index of degree of stability
  - stochastic perturbations drive system away from the coordinated movement
  - the less resistance to such perturbations, the larger the variance

Measures of stability

- relaxation time
  - time need to recover from an outside perturbation
  - e.g., mechanically perturb one of the limbs, so that relative phase moves away from the mean value, then look how long it takes to go back to the mean pattern
  - the less stable, the longer relaxation time

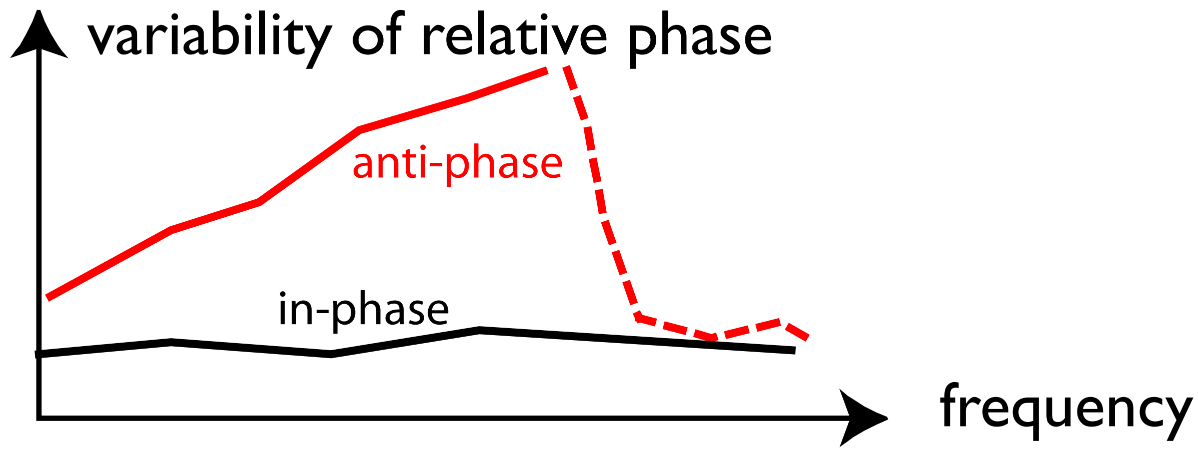
data example
perturbation of
fingers and
relative phase

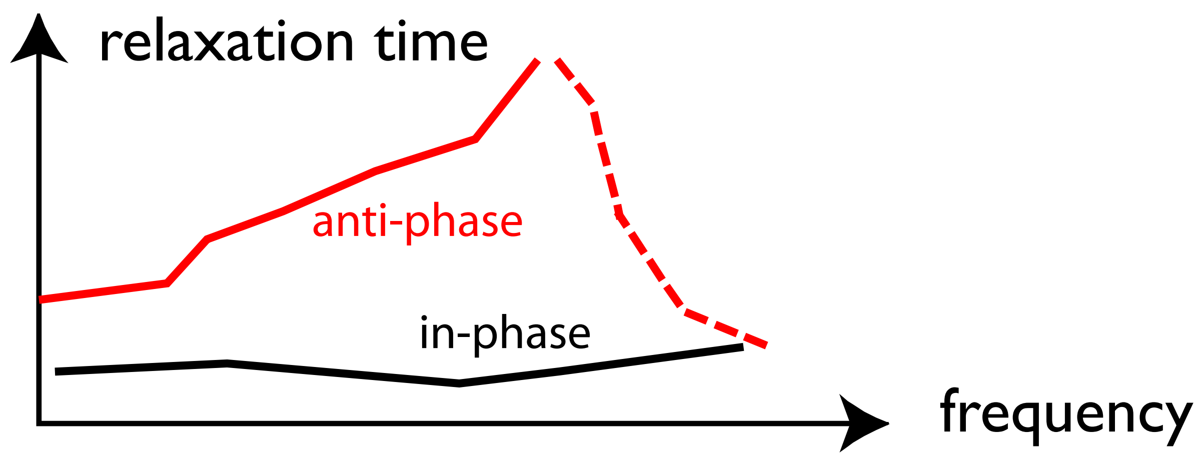


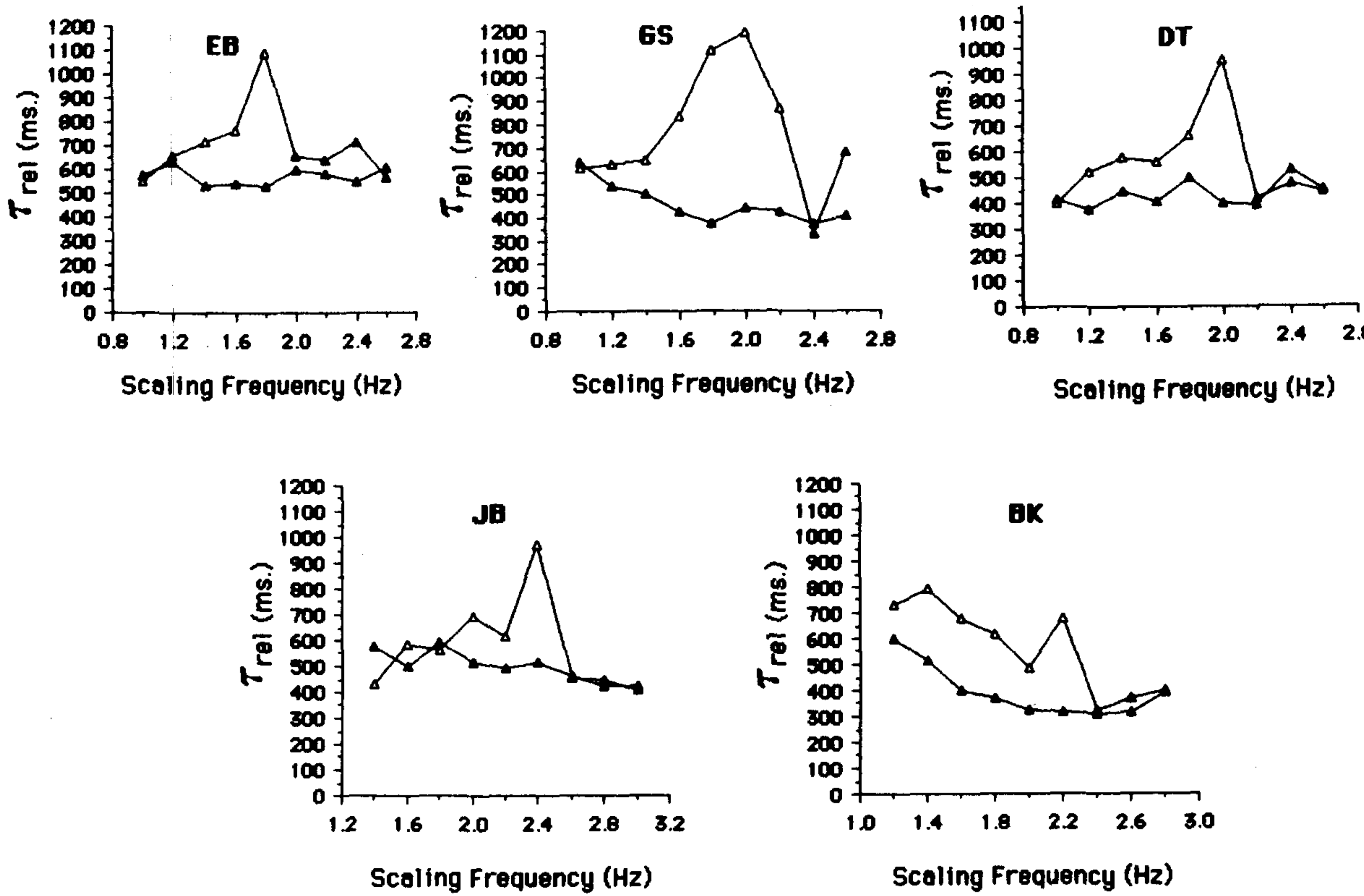
Scholz, Kelso, Schöner, 1987

Signatures of instability

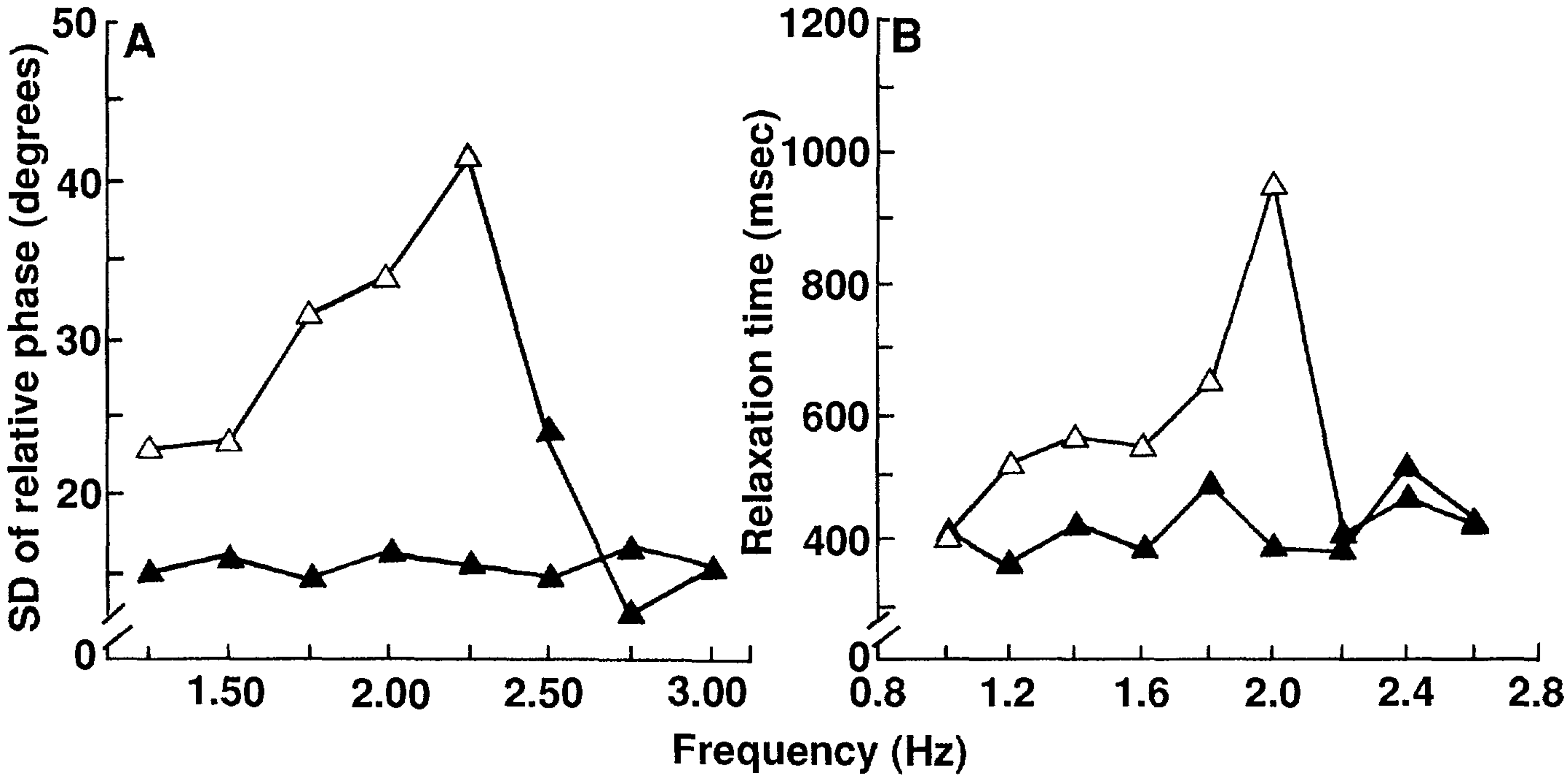
- loss of stability indexed by measures of stability







relaxation times, individual data



data (averaged across subjects)

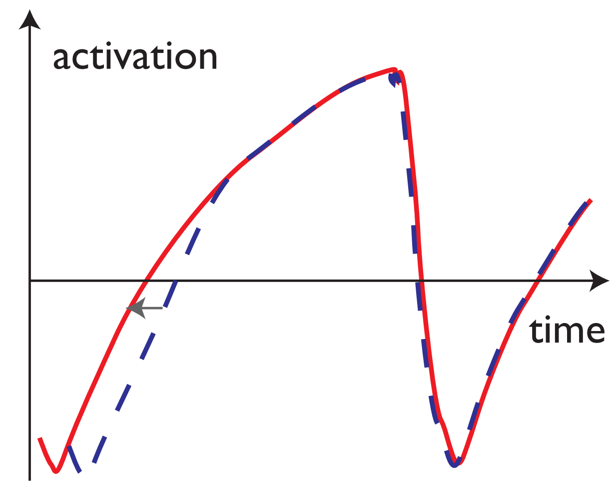
Schöner, Kelso (Science, 1988)

Neuronal process for coordination

- each component is driven by a neuronal oscillator
- their excitatory coupling leads to in-phase
- their inhibitory coupling leads to anti-phase

Coordination from coupling

- coordination=stable relative timing emerges from coupling of neural oscillators

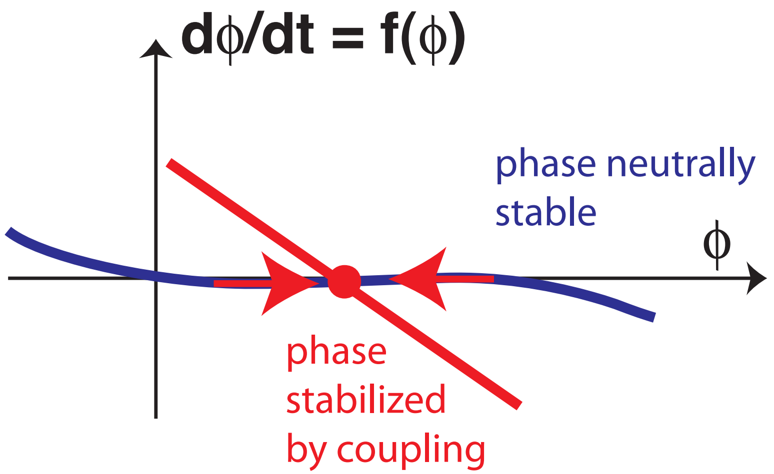


$$\tau \dot{u}_1 = -u_1 + h_u + w_{uu} f(u_1) - w_{uv} f(v_1)$$
$$\tau \dot{v}_1 = -v_1 + h_v + w_{vu} f(u_1) + c f(u_2)$$
$$\tau \dot{u}_2 = -u_2 + h_u + w_{uu} f(u_2) - w_{uv} f(v_2)$$
$$\tau \dot{v}_2 = -v_2 + h_v + w_{vu} f(u_2) + c f(u_1)$$

[Schöner: Timing, Clocks, and Dynamical Systems. *Brain and Cognition* 48:31-51 (2002)]

Movement timing

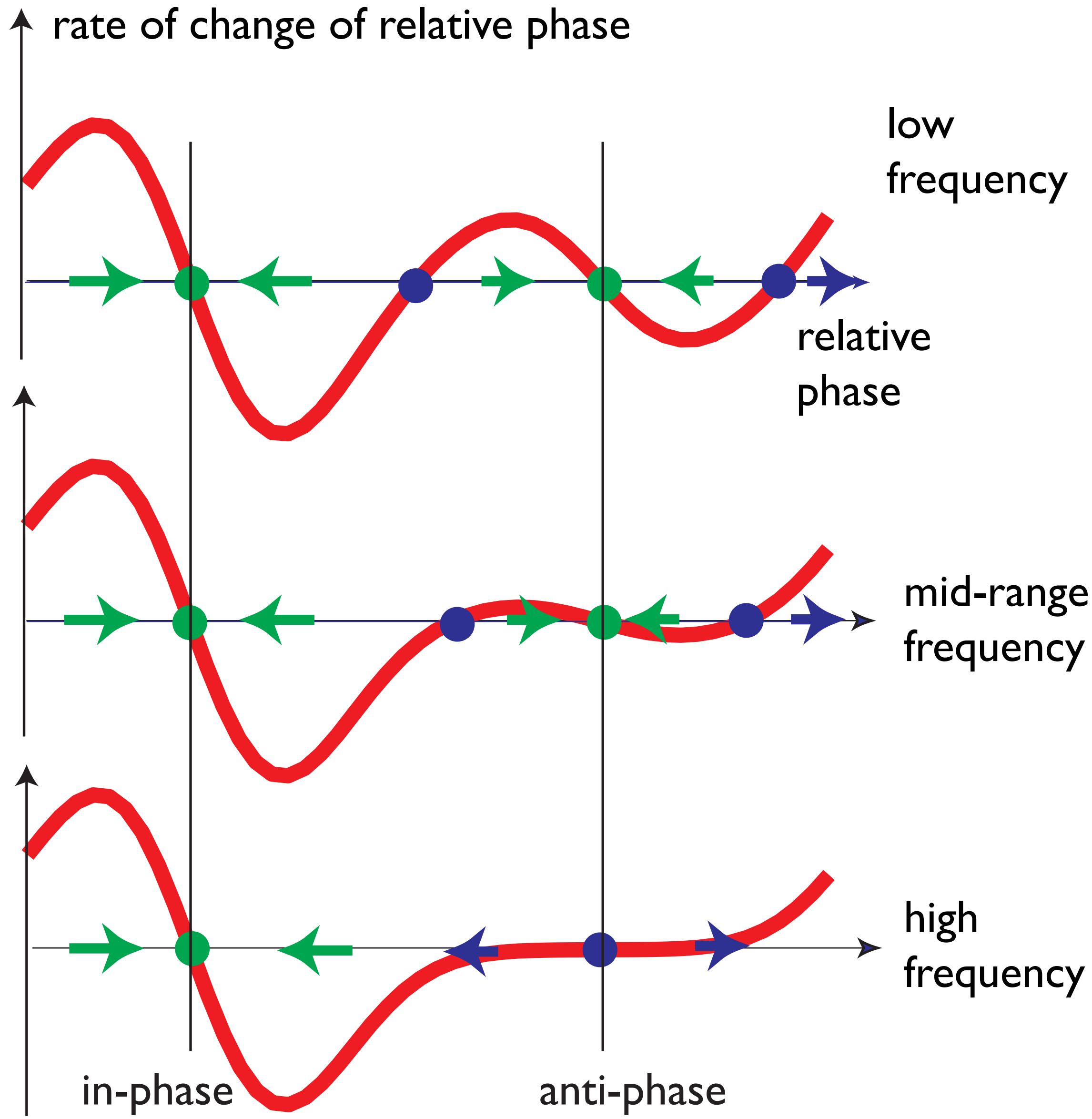
- marginal stability of phase enables stabilizing relative timing while keeping trajectory unaffected



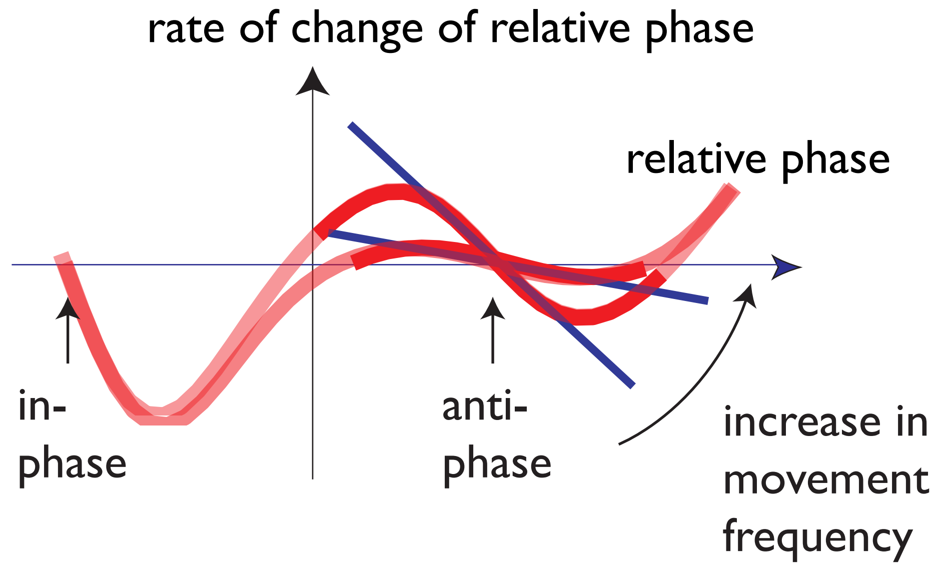
[Schöner: Timing, Clocks, and Dynamical Systems. *Brain and Cognition* 48:31-51 (2002)]

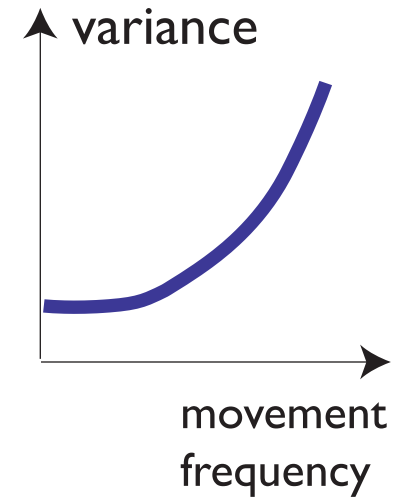
Dynamical systems account of instability

- at increasing frequency stability of anti-phase is lost

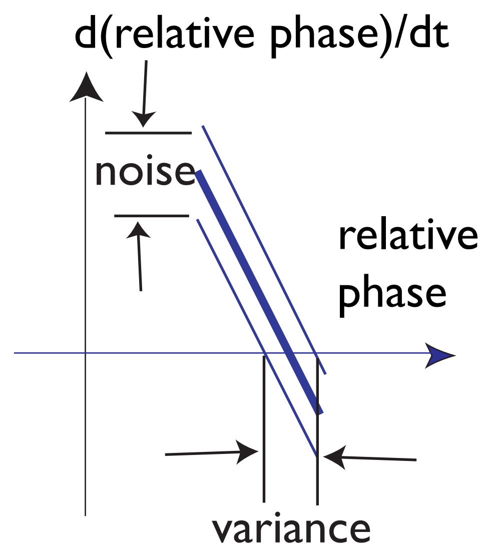


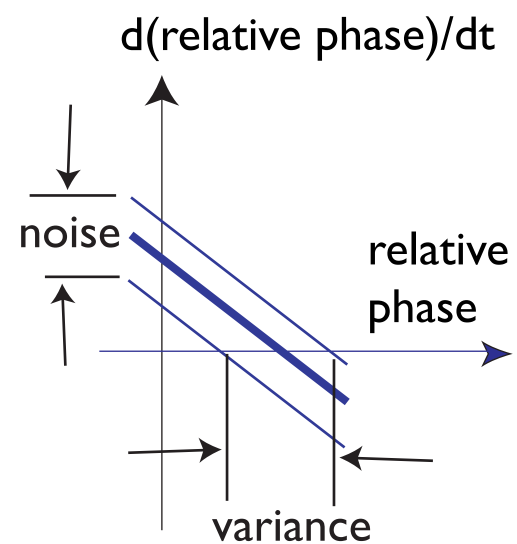
Predicts increase in variance





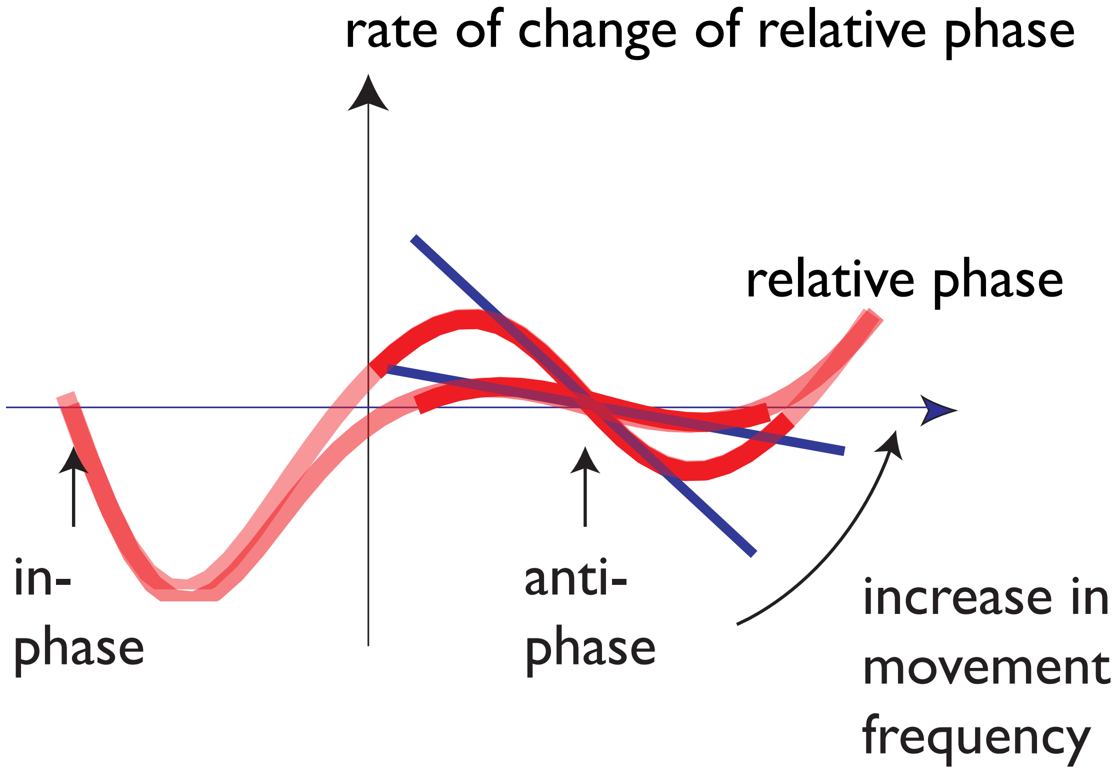
- "critical fluctuations"

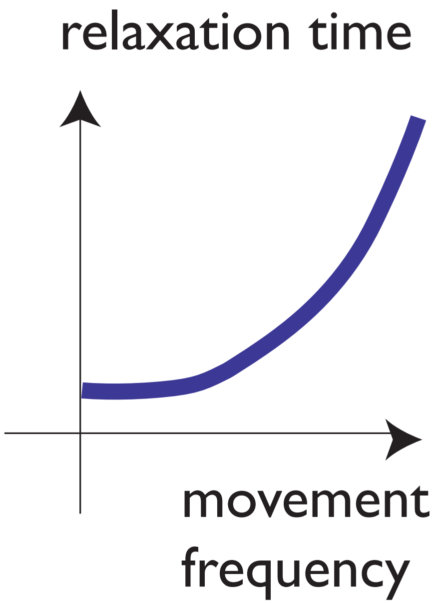




Predicts increase in relaxation time

- "critical slowing down"





=> coordination from coupled oscillators

Learn from these ideas for robotics?

- timed reaching that stabilizes timing in response to perturbations

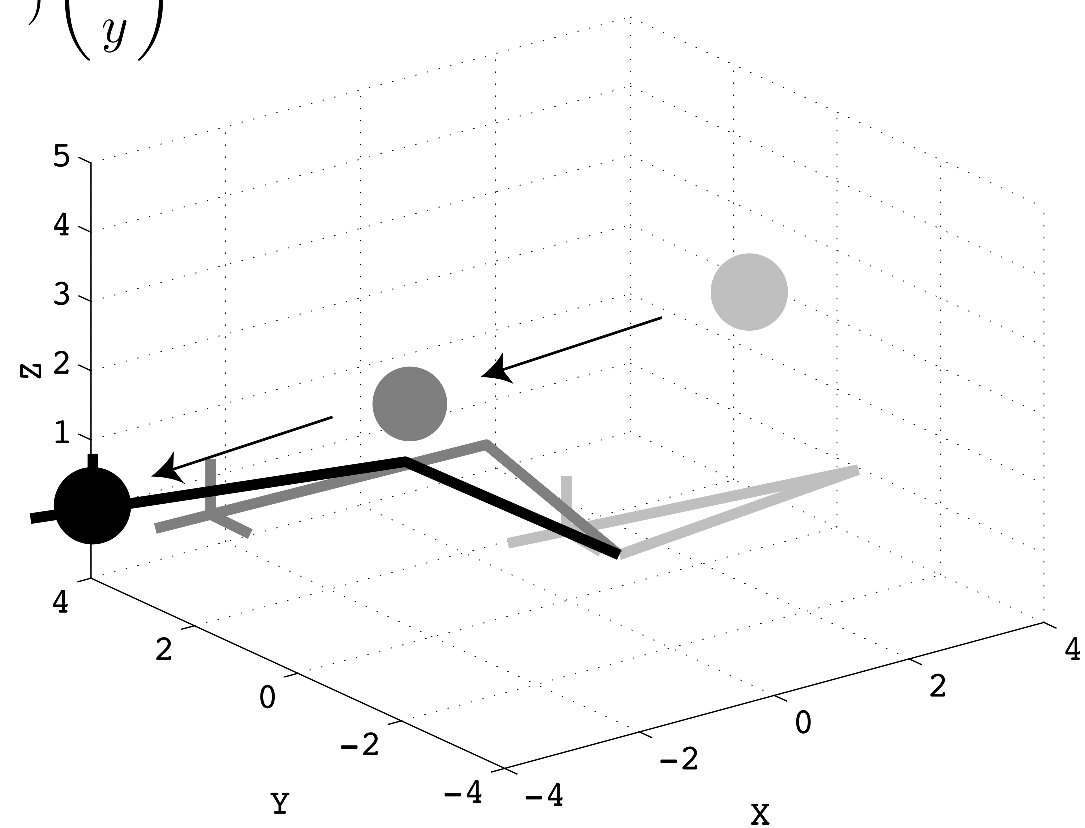
Timed movement to intercept ball

- timing from an oscillator

$$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = -5\ |u_{\text{init}}| \begin{pmatrix} x - x_{\text{init}} \\ y \end{pmatrix} + |u_{\text{hopf}}|\ \mathbf{f}_{\text{hopf}} - 5\ |u_{\text{final}}| \begin{pmatrix} x - x_{\text{final}} \\ y \end{pmatrix} + \text{gwn}$$

$$\mathbf{f}_{\text{hopf}} = \begin{pmatrix} 2.5 & -\omega \\ \omega & 2.5 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} - 2.5\ (x^2 + y^2) \begin{pmatrix} x \\ y \end{pmatrix}$$

$$x(t) = \sin(\omega t)$$



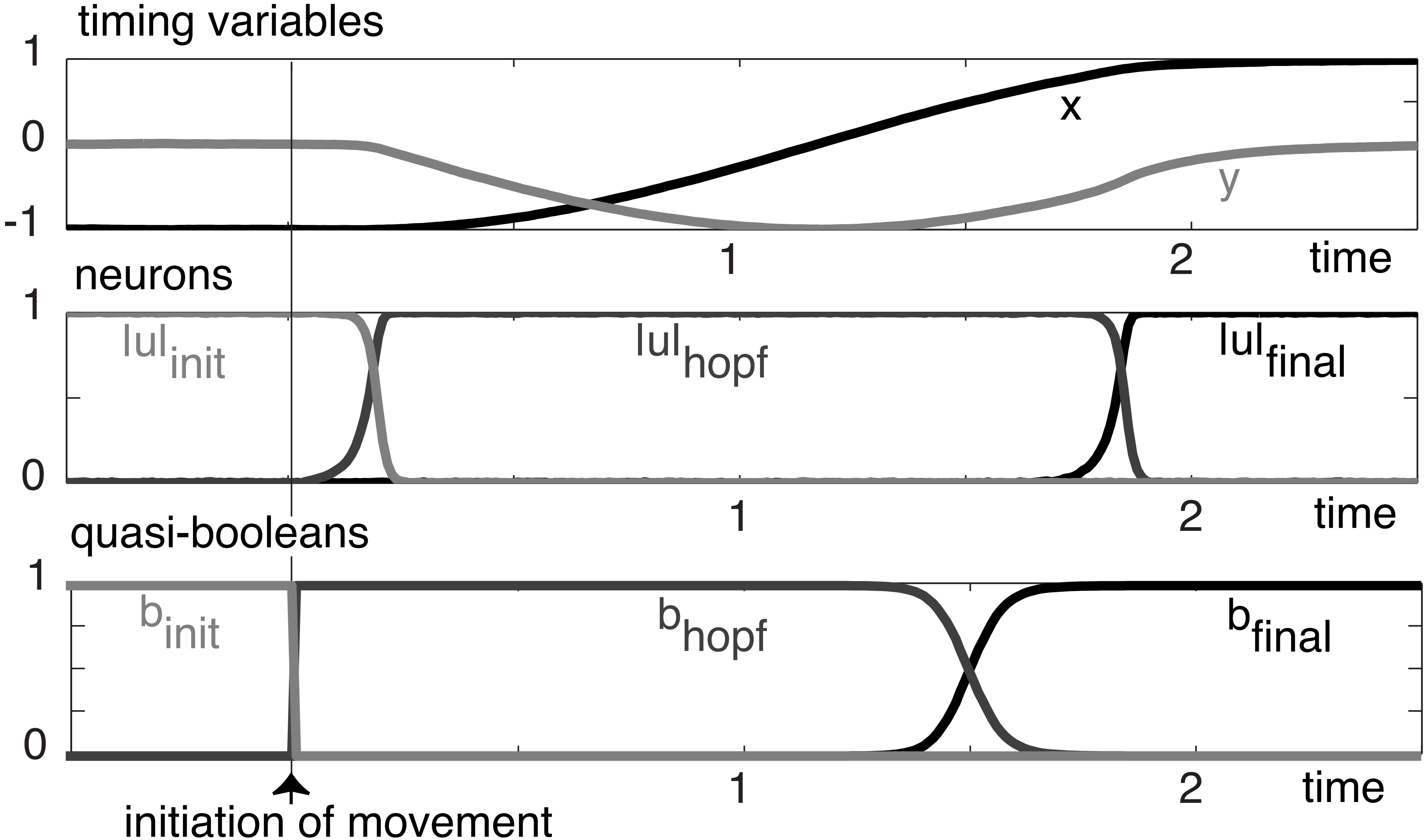
[Schöner, Santos, 2001]

- the oscillator is turned on and off for a single cycle

$$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = -5\ |u_{\text{init}}| \begin{pmatrix} x - x_{\text{init}} \\ y \end{pmatrix} + |u_{\text{hopf}}|\ \mathbf{f}_{\text{hopf}} - 5\ |u_{\text{final}}| \begin{pmatrix} x - x_{\text{final}} \\ y \end{pmatrix} + \text{gwn}$$

$$\begin{aligned}
\alpha\, \dot{u}_{\text{init}} &= \mu_{\text{init}}\ u_{\text{init}} - |\mu_{\text{init}}|\ u_{\text{init}}^3 - 2.1\ (u_{\text{final}}^2 + u_{\text{hopf}}^2)\ u_{\text{init}} + \text{gwn} \\
\alpha\, \dot{u}_{\text{hopf}} &= \mu_{\text{hopf}}\ u_{\text{hopf}} - |\mu_{\text{hopf}}|\ u_{\text{hopf}}^3 - 2.1\ (u_{\text{init}}^2 + u_{\text{final}}^2)\ u_{\text{hopf}} + \text{gwn} \\
\alpha\, \dot{u}_{\text{final}} &= \mu_{\text{final}}\ u_{\text{final}} - |\mu_{\text{final}}|\ u_{\text{final}}^3 - 2.1\ (u_{\text{init}}^2 + u_{\text{hopf}}^2)\ u_{\text{final}} + \text{gwn}
\end{aligned}$$

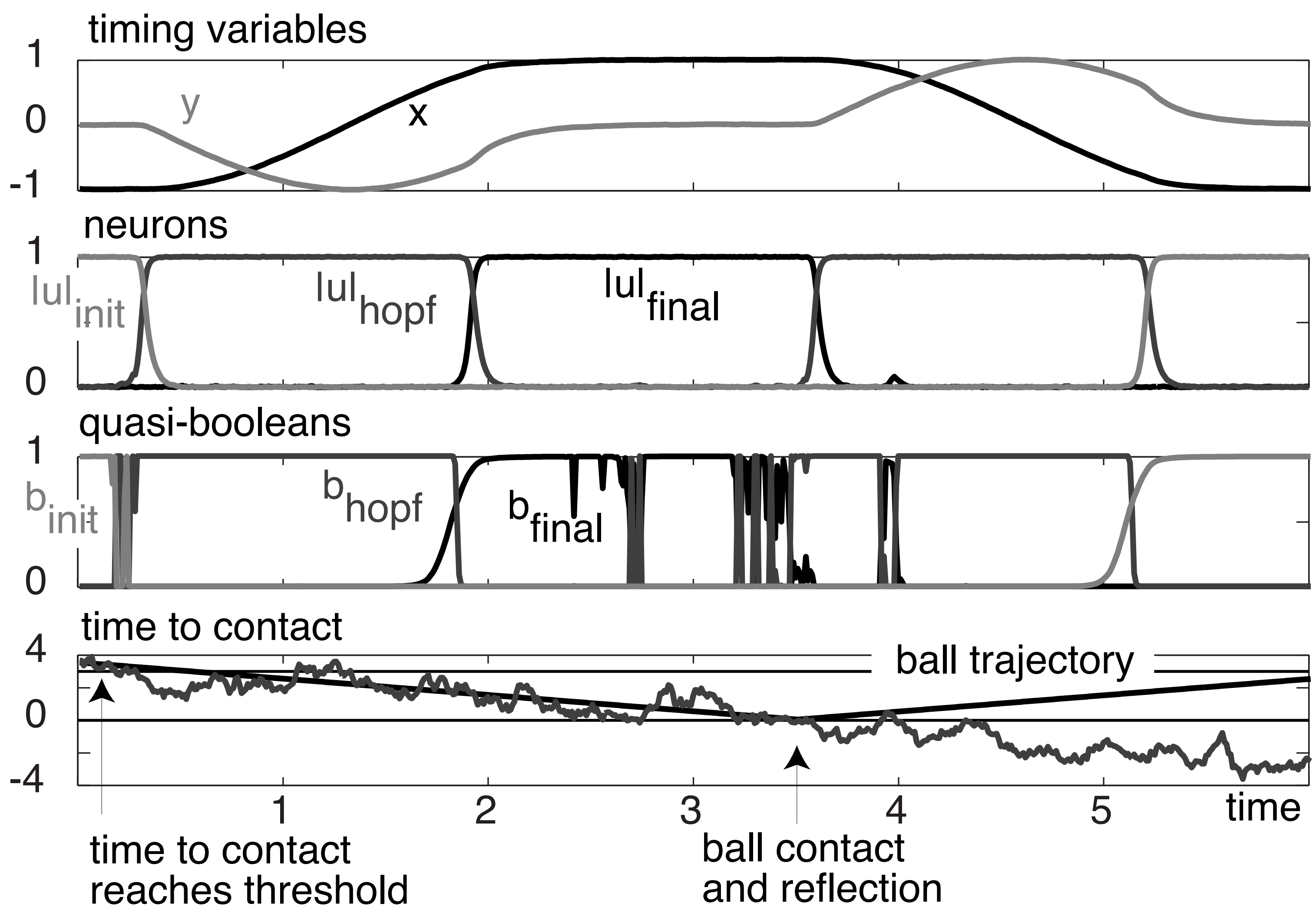
[Schöner, Santos, 2001]



[Schöner, Santos, 2001]

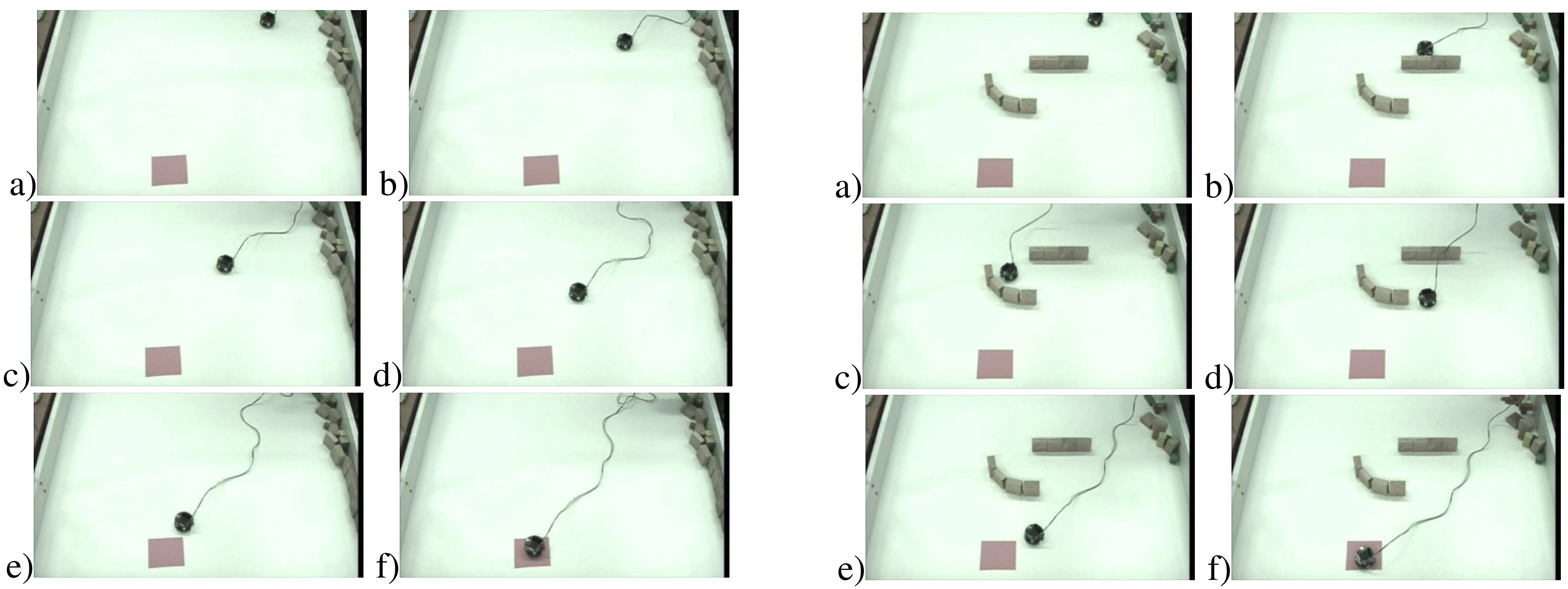
Timed movement to intercept ball

- turn oscillator on in response to detected ball at right time to contact



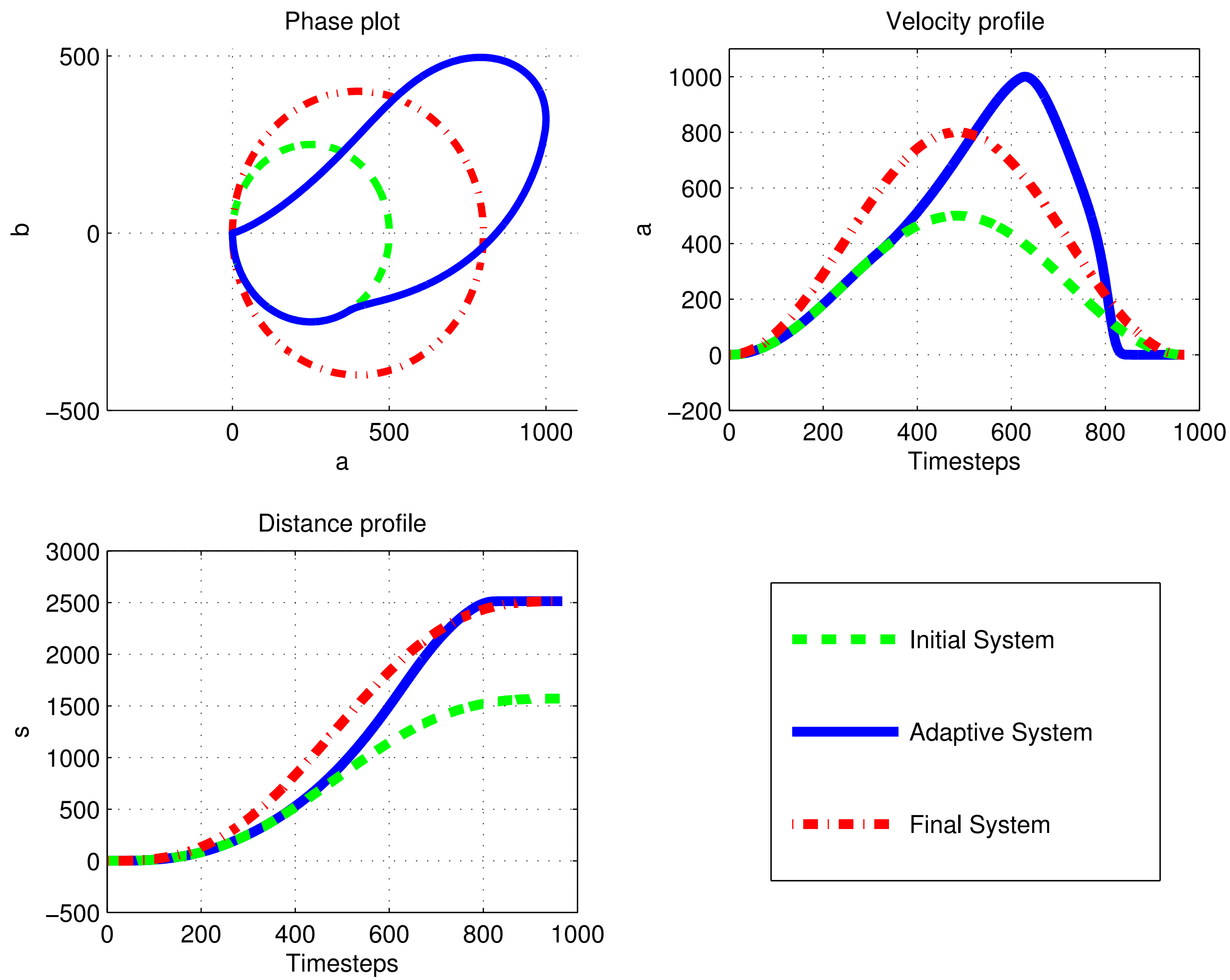
Compensating for lost time

- plan to reach target at fixed time
- recover time as obstacle forces longer path



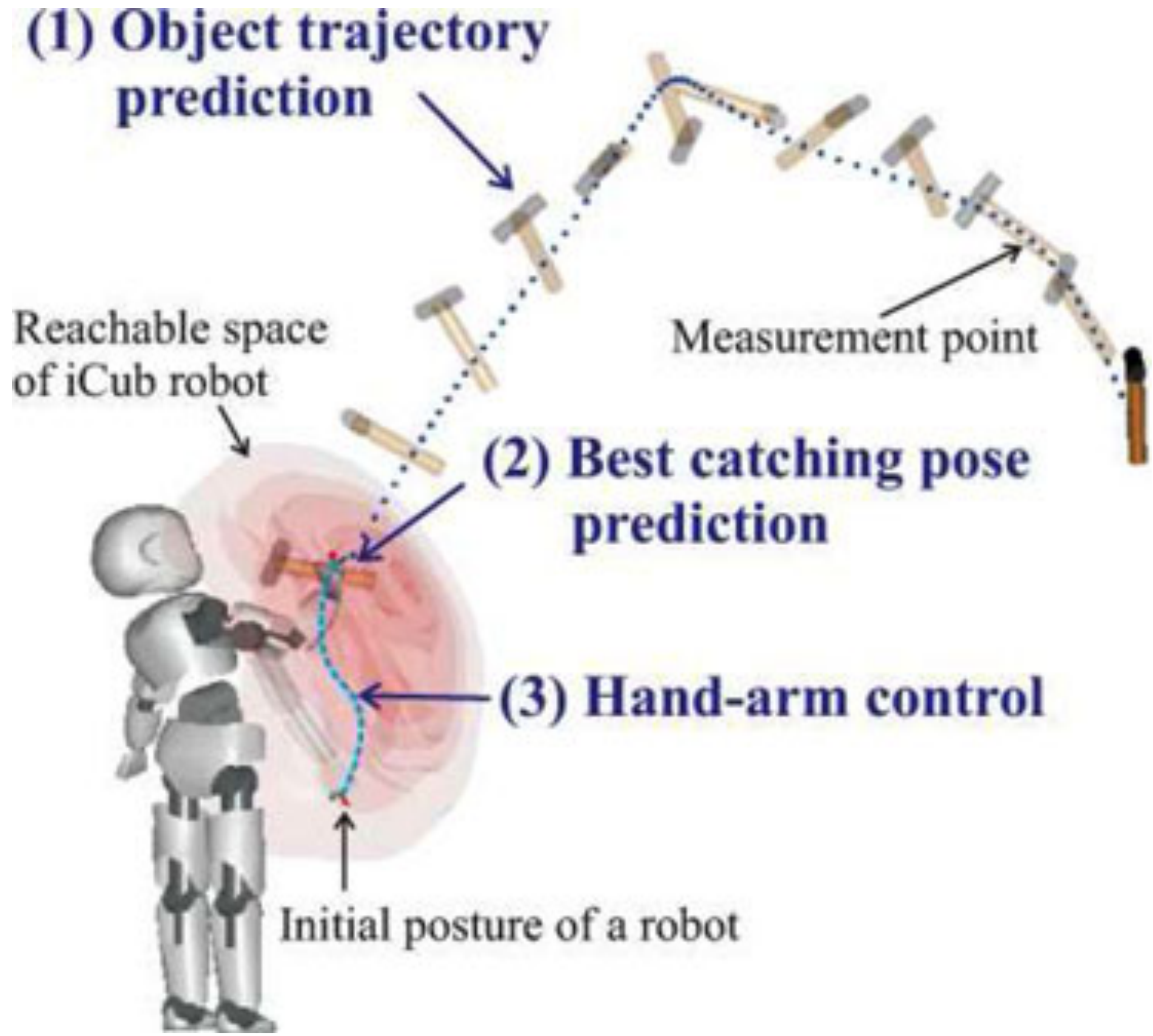
[Tuma, Iossifidis, Schöner, ICRA 2009]

Compensating for lost time



[Tuma, Iossifidis, Schöner, ICRA 2009]

Catching



[Kim, Shukla, Billard, 2014]

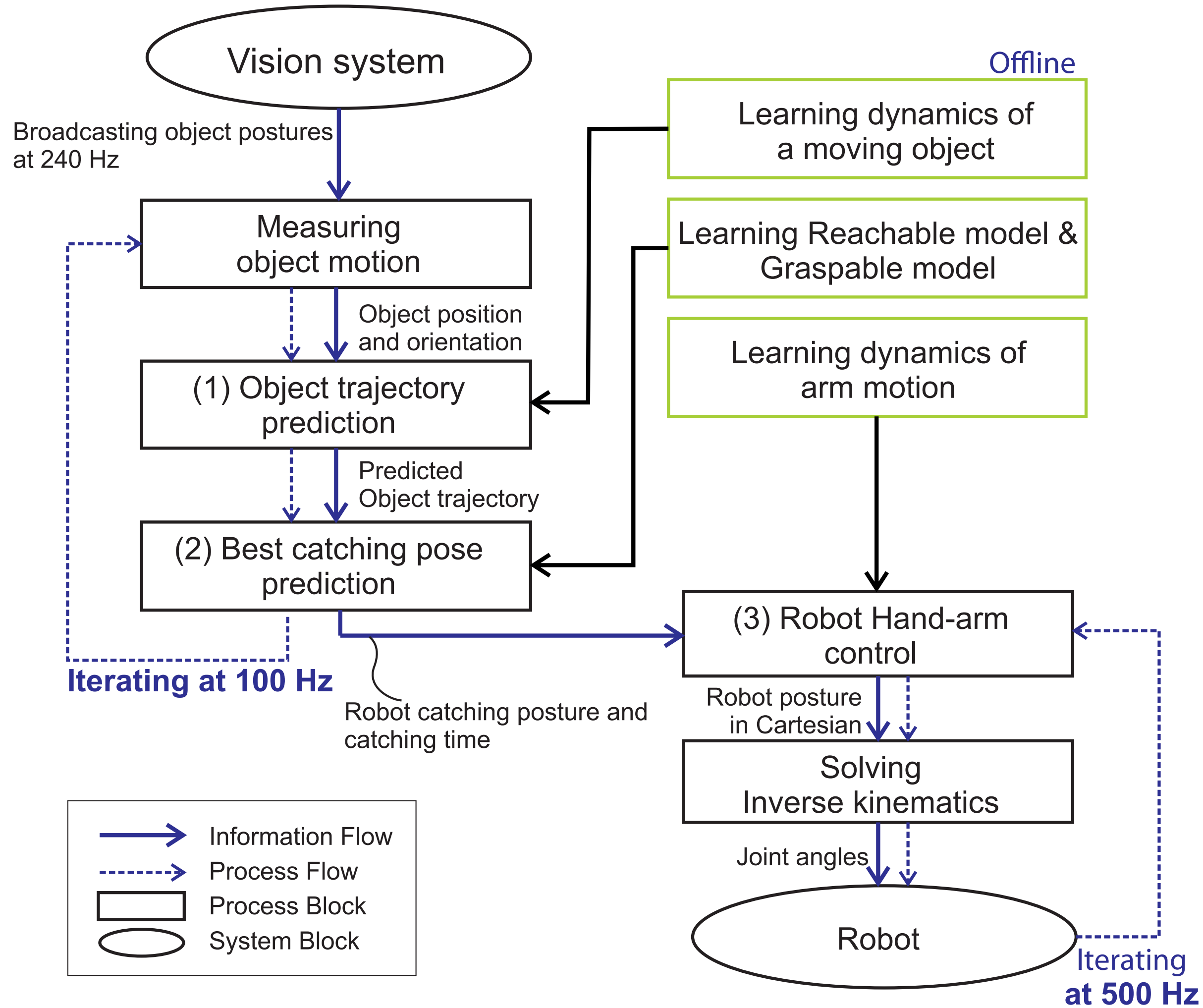
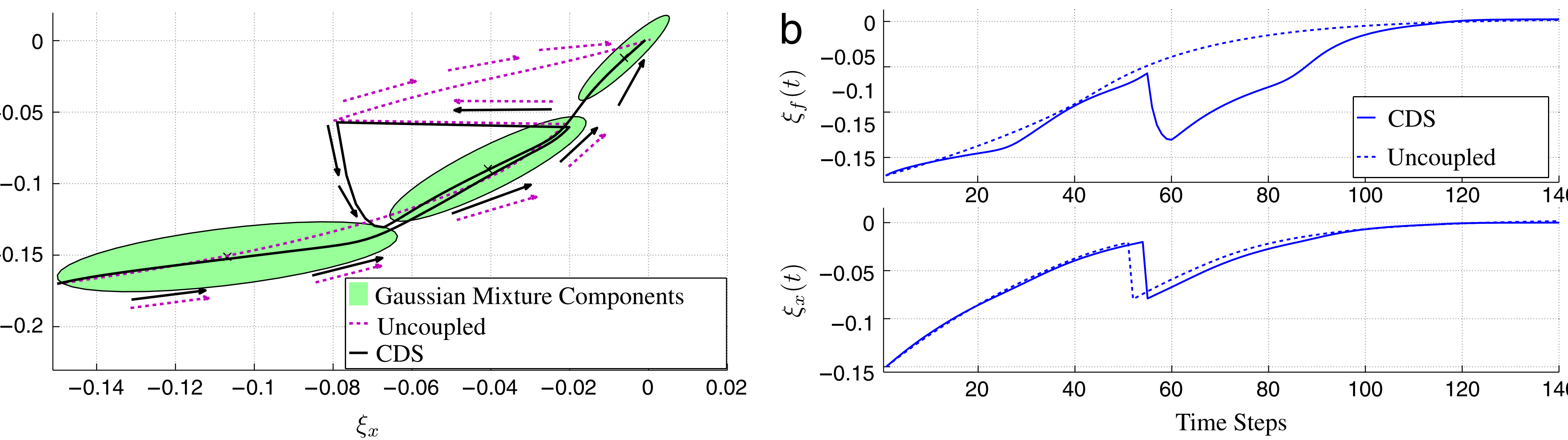


Fig. 2. Block diagram for robotic catching.

[Kim, Shukla, Billard, 2014]

coupled dynamical systems approach



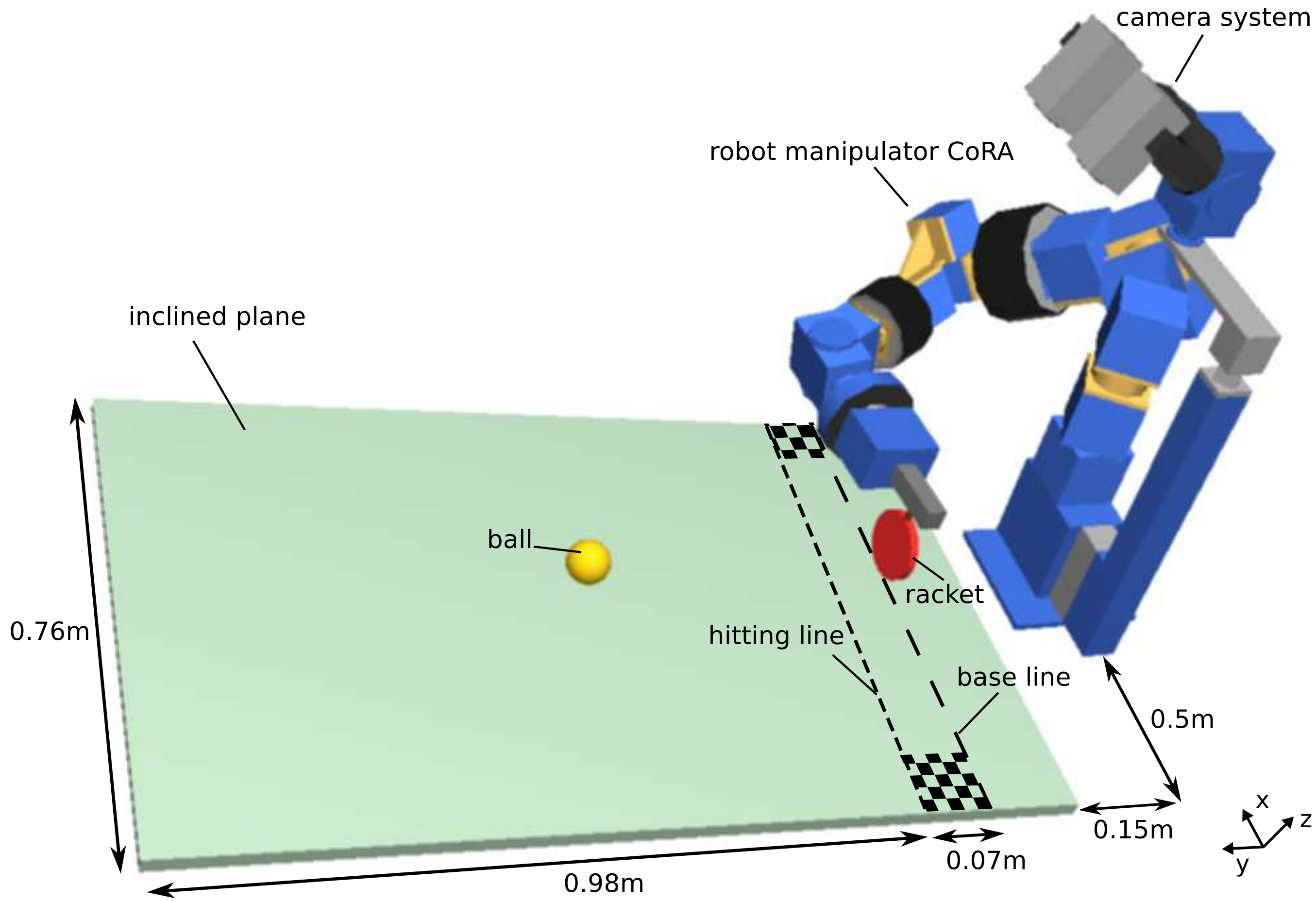
[Shukla, Billard, 2012]

video

- https://youtu.be/M413ILWvrbI?t=3

Timing and behavioral organization

- sequences of timed actions to intercept ball



[Oubatti, Richter, Schöner, 2013]

Timing and behavioral organization

- timing from oscillator, whose cycle time is adjusted to perceived time to contact

$$\tau \begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = -c_{\text{post}}\, a \begin{pmatrix} x - x_{\text{post}} \\ y \end{pmatrix} + c_{\text{hopf}}\, H(x, y) + \eta,$$

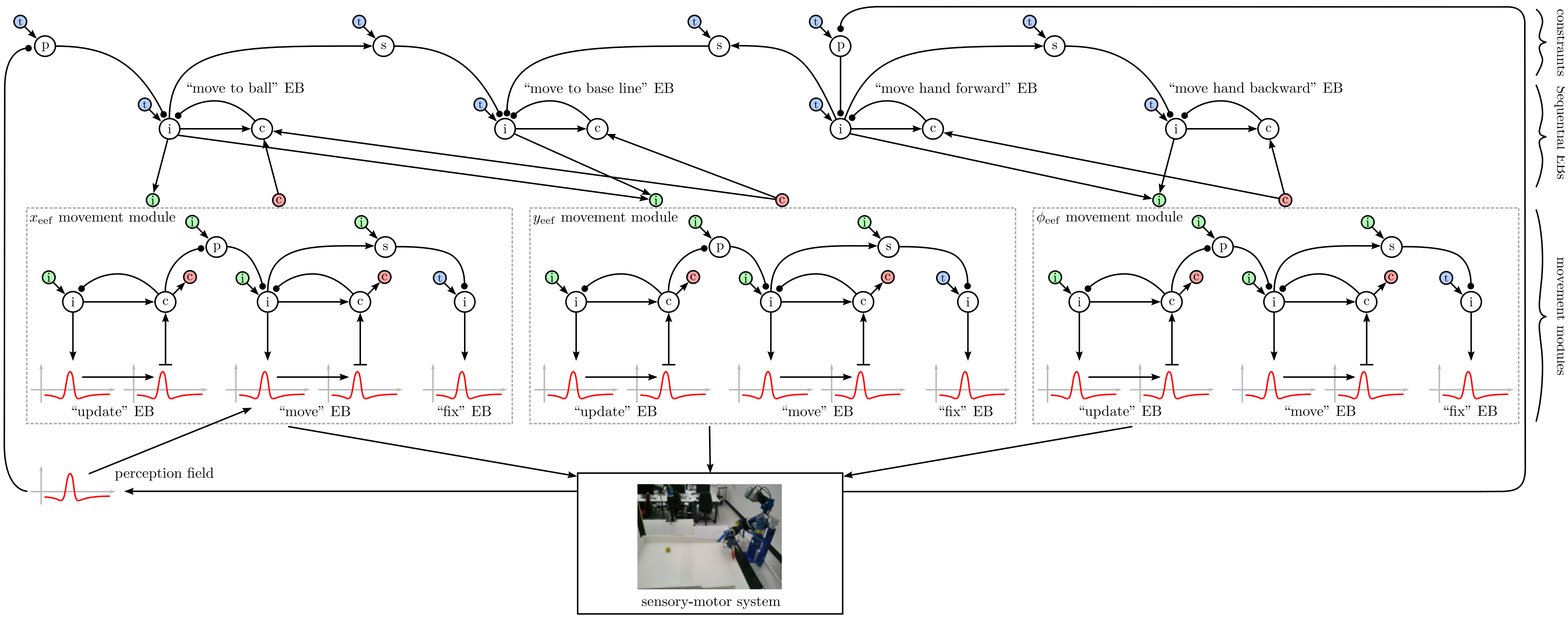
$$H(x, y) = \begin{pmatrix} \lambda & -\omega \\ \omega & \lambda \end{pmatrix} \begin{pmatrix} x - r - x_{\text{init}} \\ y \end{pmatrix} - \left((x - r - x_{\text{init}})^2 + y^2\right) \begin{pmatrix} x - r - x_{\text{init}} \\ y \end{pmatrix}$$

$$\frac{T}{2d_{\text{init}}} = \frac{t_{\text{tim}}}{d(t)}.$$

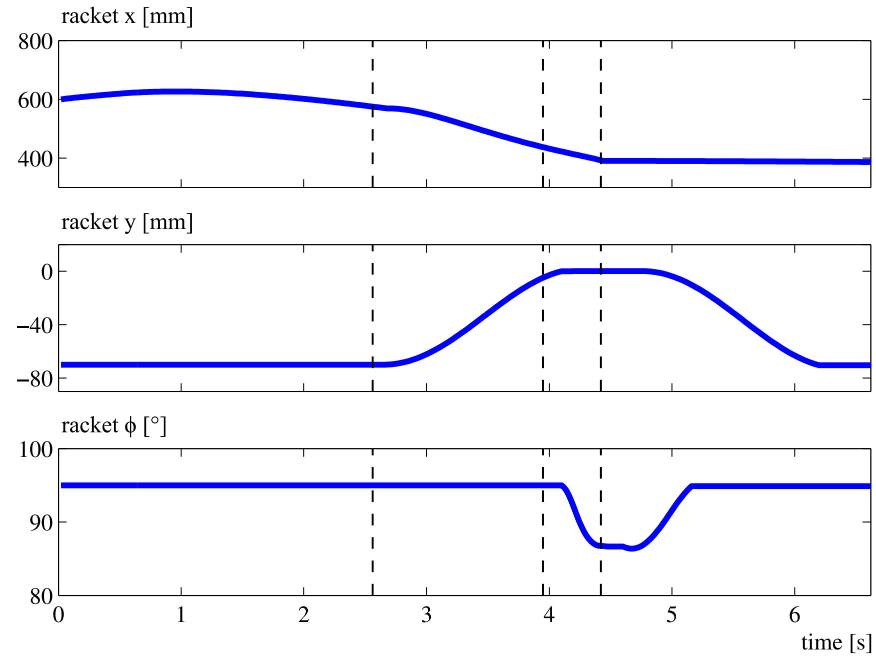
[Oubatti, Richter, Schöner, 2013]

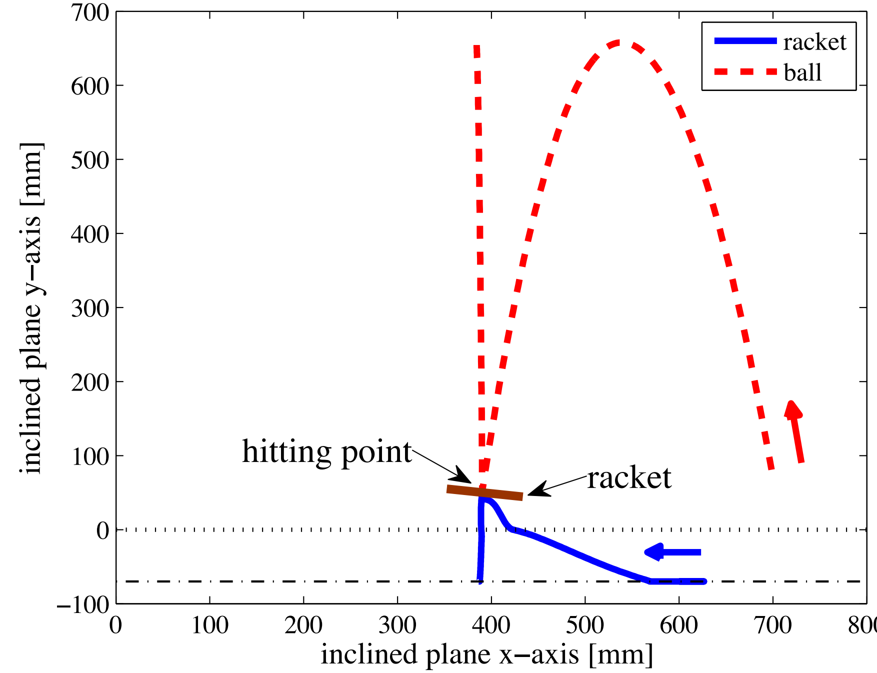
Timing and behavioral organization

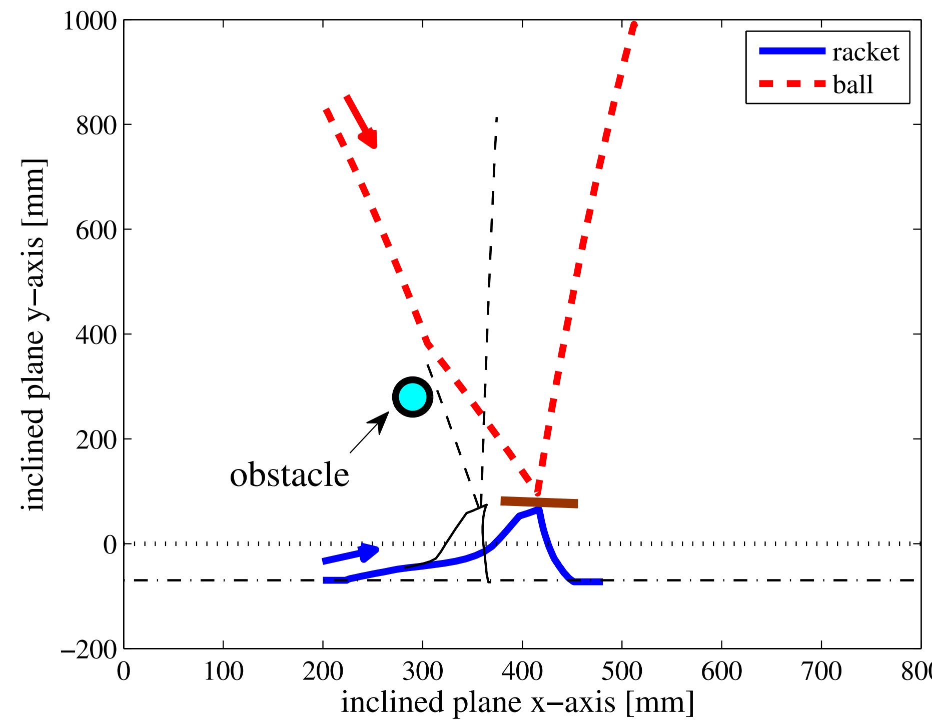
- coupled neural dynamics to organize the sequence



[Oubatti, Richter, Schöner, 2013]

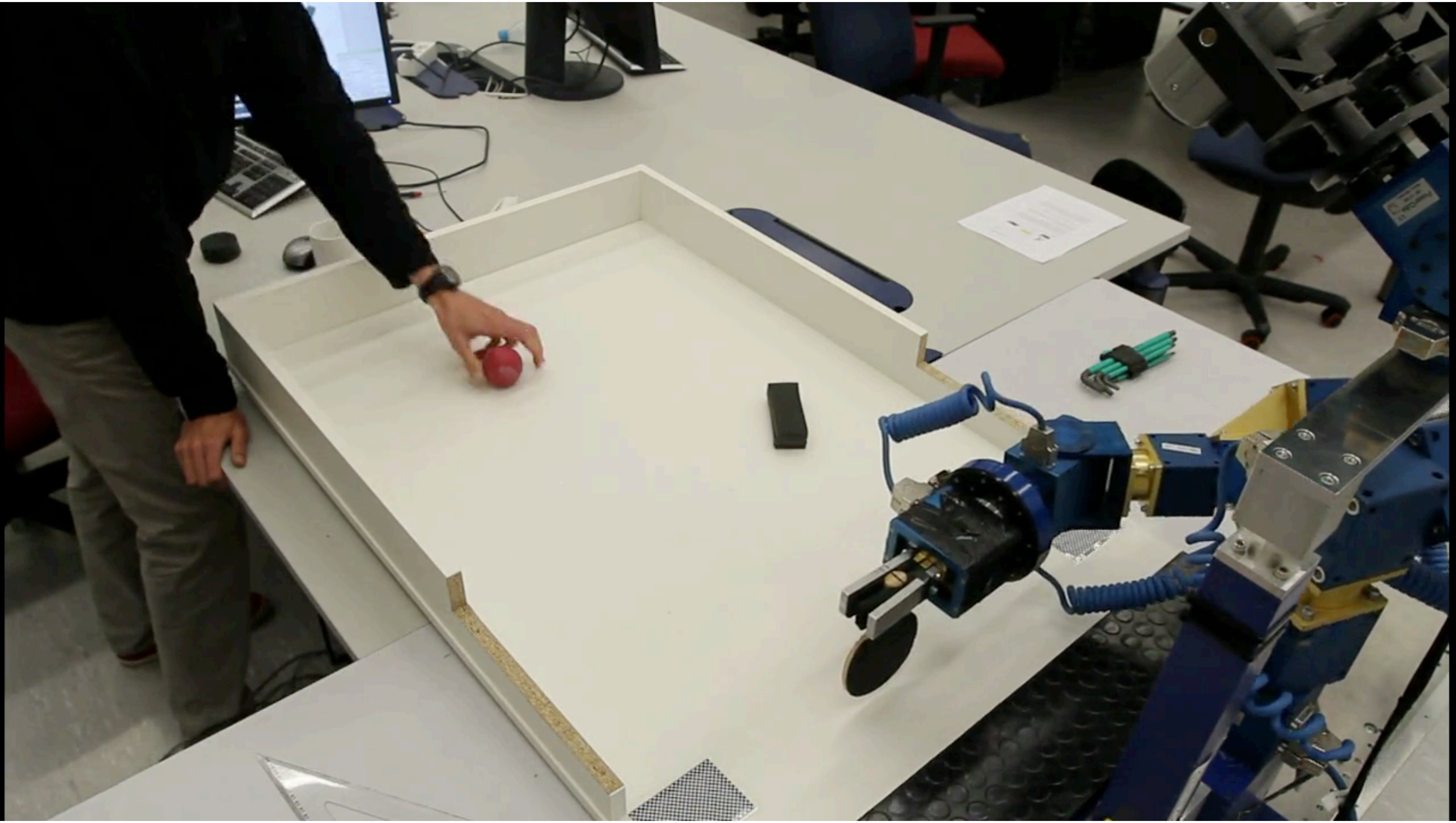


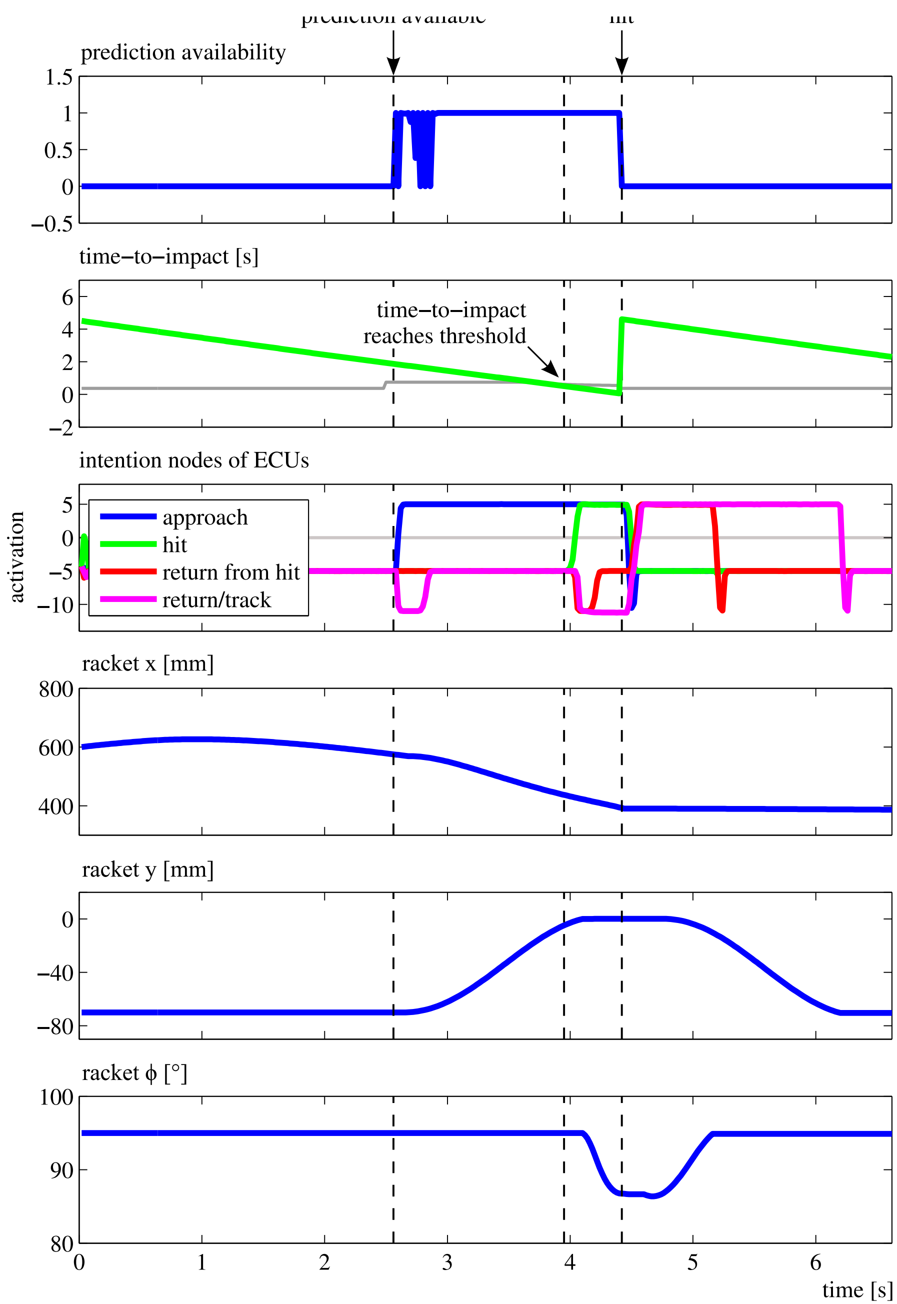


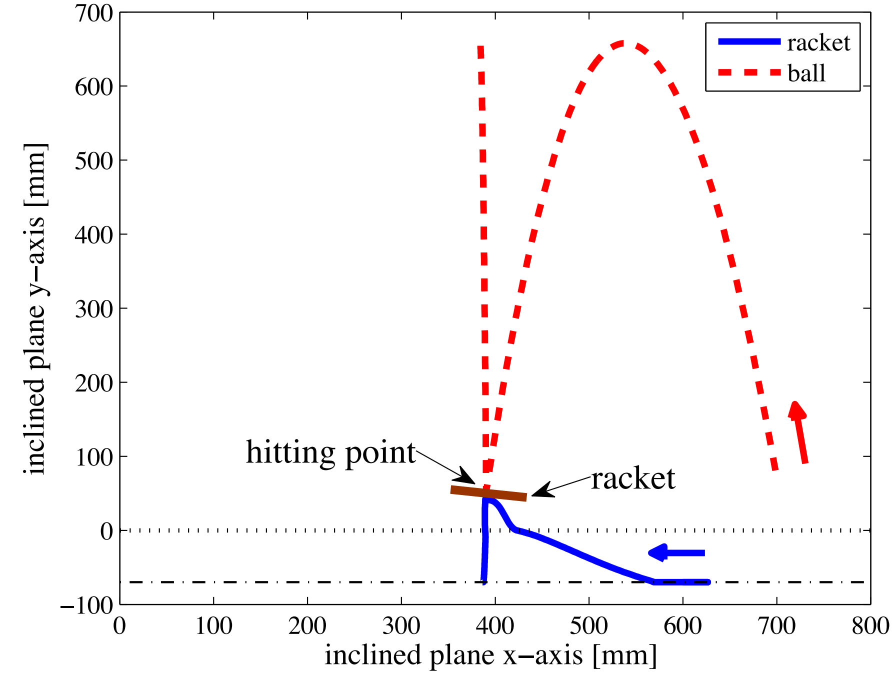


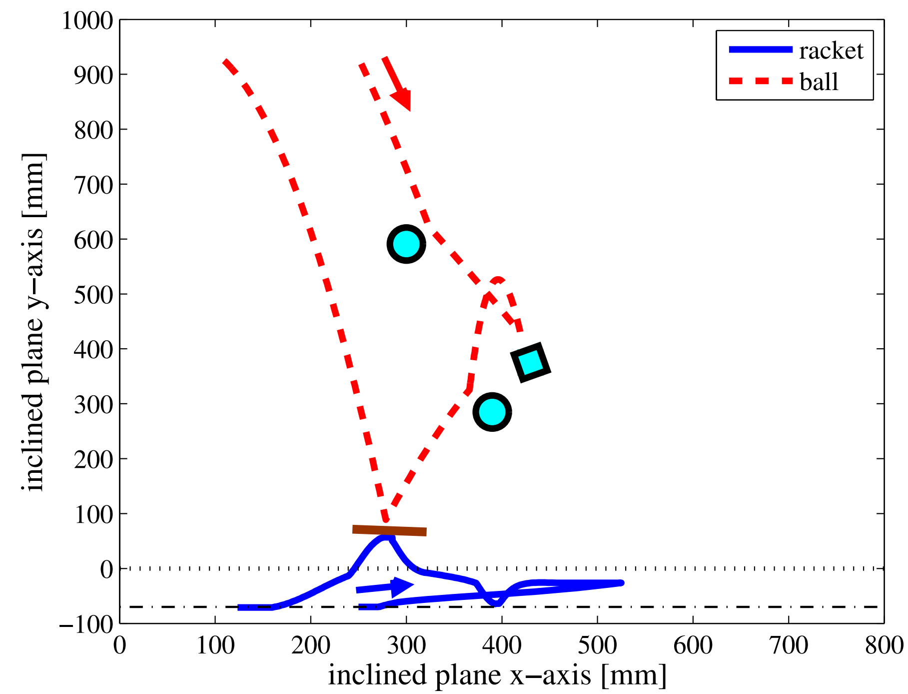
[Oubbati, Richter, Schöner, 2013]

Timed movement with online updating [Faroud Oubatti]



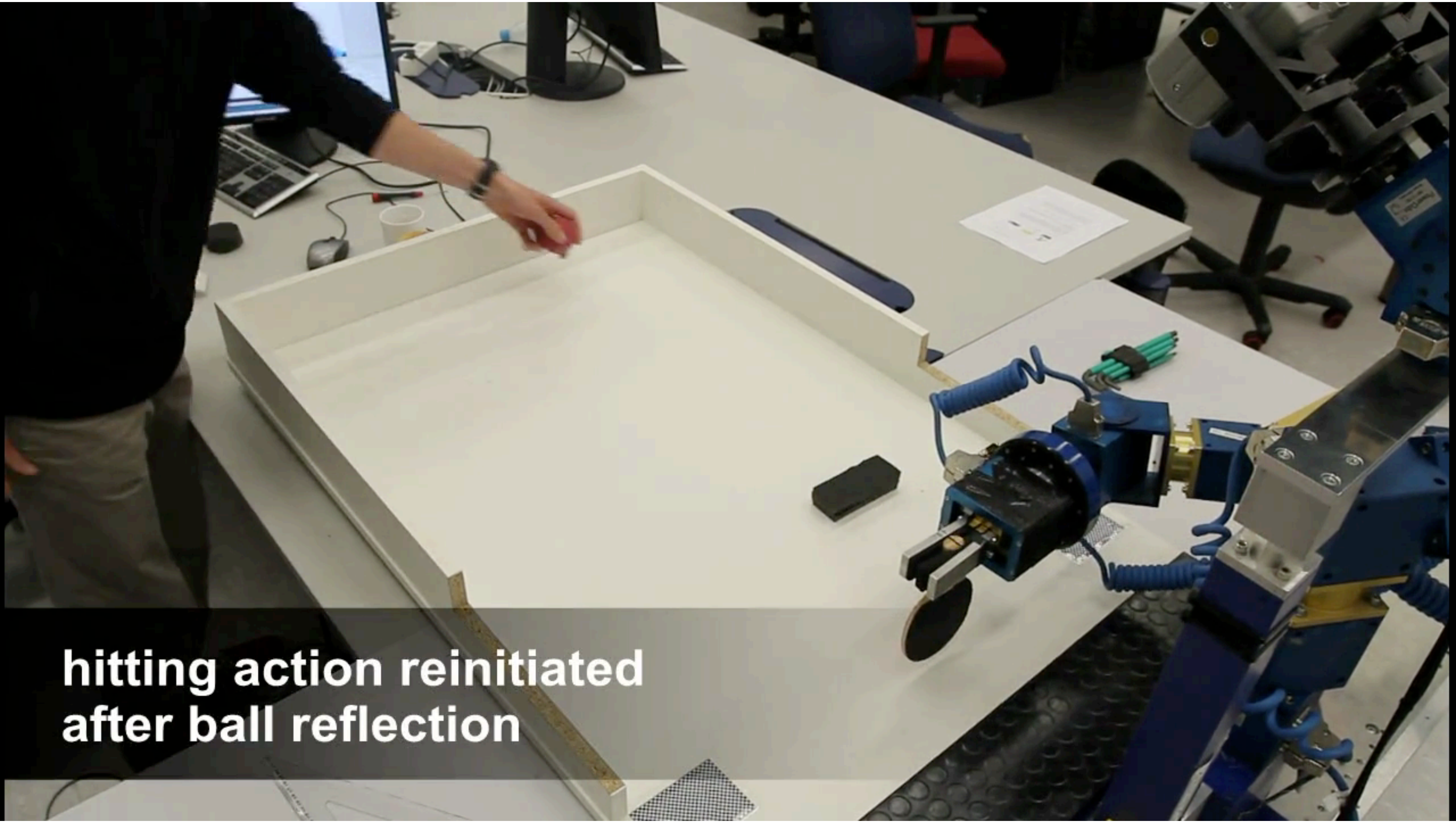






[Oubbati, Richter, Schöner, 2013]

Timing and reorganization of movement



Conclusion

- timing in autonomous robotics is best framed as a problem of stable oscillators and their coupling

Conclusion

- timing is linked to many problems
  - arriving "just in time", estimating time to contact
  - on line updating: planning and timing tightly connected
  - timed movement sequences: behavioral organization
  - coordinating timing across movements, coarticulation
  - timing and control