Dynamical systems tutorial: 2. Numerics

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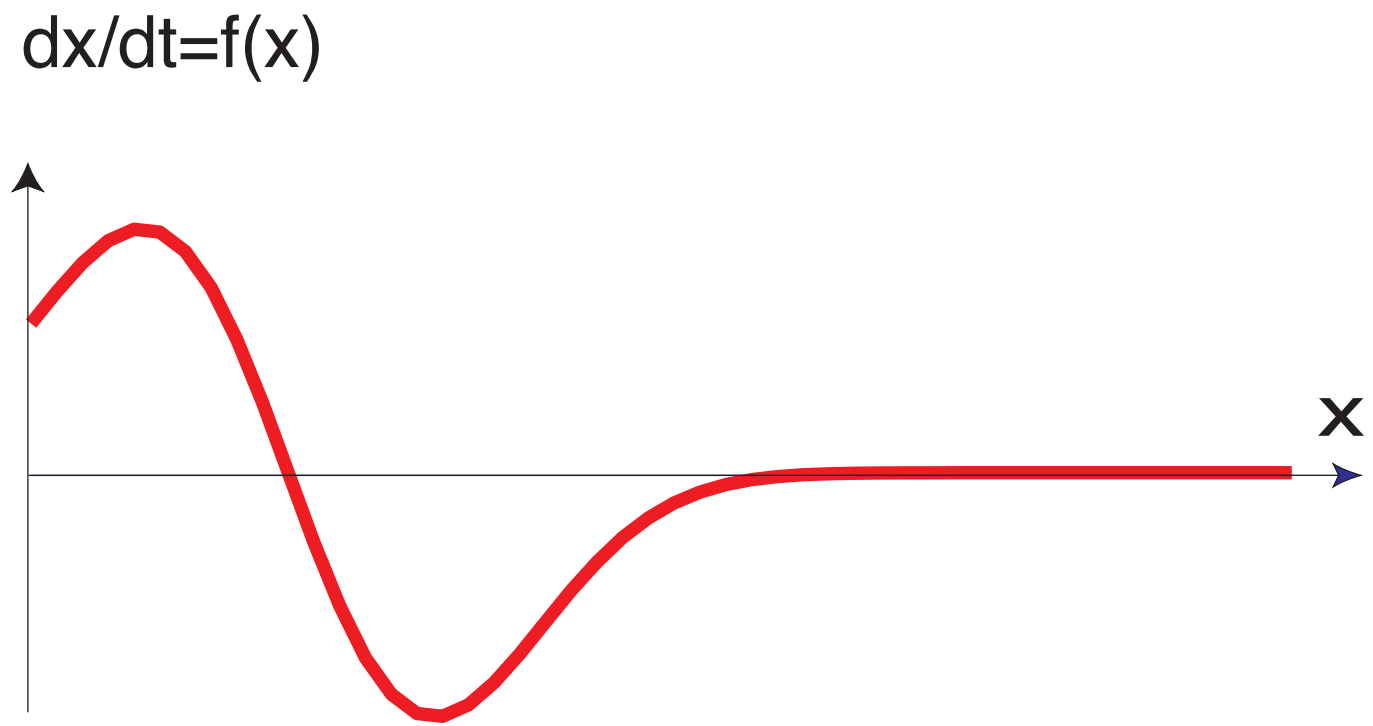
Dynamical system

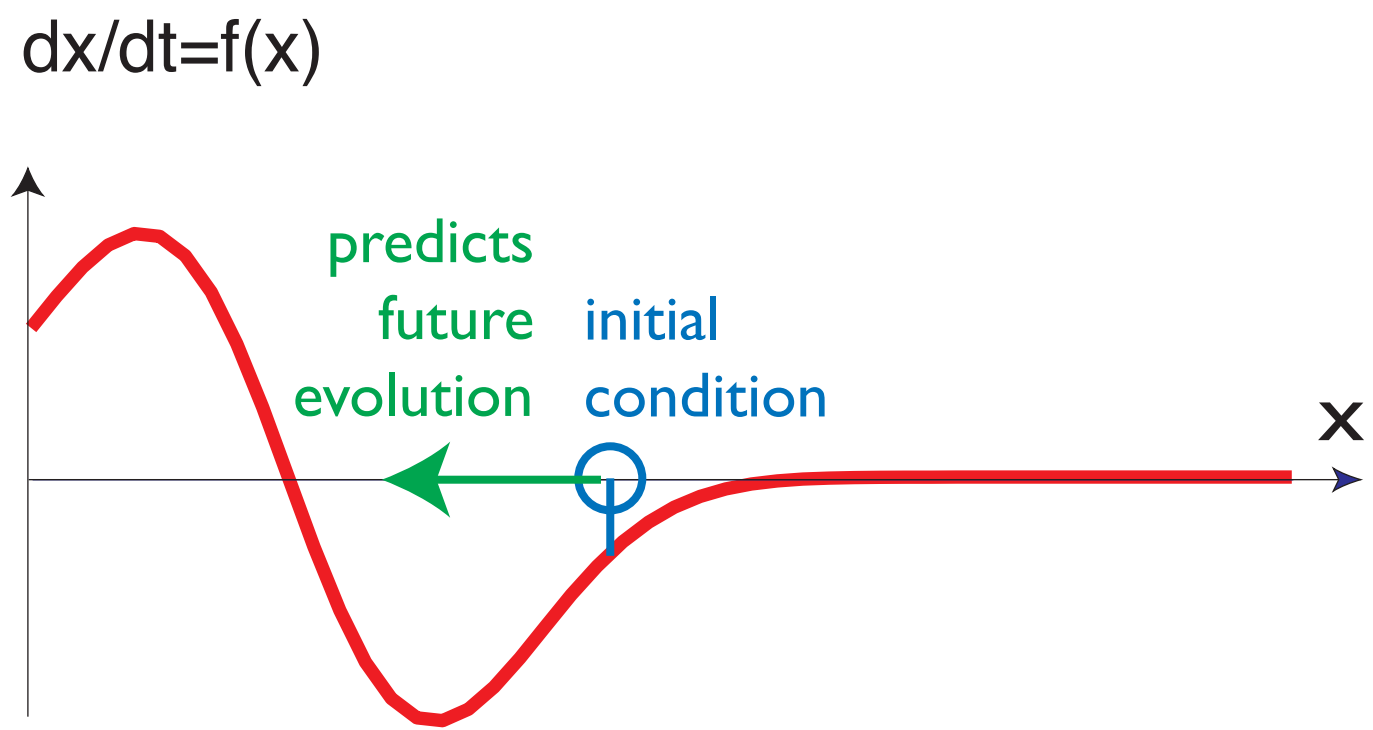
$$\dot{x} = \frac{dx}{dt} = f(x)$$



Dynamical system

$$\dot{x} = \frac{dx}{dt} = f(x)$$

- the present determines the future



Dynamical system

$$\dot{x} = \frac{dx}{dt} = f(x)$$

- x spans the state space (can be vector-valued or even function valued)
- $f(x)$ is the "dynamics" of x (or vector-field)
- $x(t)$ is a solution of the dynamical systems with initial condition $x_0 \iff$ the rate of change of $x(t)$ obeys $\dot{x}(t) = f(x)$ and $x(0) = x_0$

Numerical solutions

- sample time discretely, t_i, with $i \in \{0,1,...,N\}$,
- for example: $t_i = i\ \Delta t$
- compute solution, $x(t_i) = x_i$, by iterating through time,
- for example: $x_{i+1} = x_i + \Delta t\ f(x_i)$ (forward Euler)

$$\left[\frac{x_{i+1} - x_i}{\Delta t} \approx \frac{dx}{dt} = f(x) \approx f(x_i)\right]$$

- => code / simulation