Dynamical systems tutorial: 2. Numerics

Gregor Schöner, INI, RUB
Dynamical system

\[ \dot{x} = \frac{dx}{dt} = f(x) \]
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the present determines the future

\[ \frac{dx}{dt} = f(x) \]
x spans the state space (can be vector-valued or even function valued)

\( f(x) \) is the "dynamics" of \( x \) (or vector-field)

\( x(t) \) is a solution of the dynamical systems with initial condition \( x_0 \) \( \iff \) the rate of change of \( x(t) \) obeys \( \dot{x}(t) = f(x) \) and \( x(0) = x_0 \)
Numerical solutions

- Sample time discretely, \( t_i \), with \( i \in \{0, 1, \ldots, N\} \),

- For example: \( t_i = i \Delta t \)

- Compute solution, \( x(t_i) = x_i \), by iterating through time,

- For example: \( x_{i+1} = x_i + \Delta t \ f(x_i) \) (forward Euler)

\[
\begin{bmatrix}
\frac{x_{i+1} - x_i}{\Delta t} & \approx & \frac{dx}{dt} = f(x) & \approx & f(x_i)
\end{bmatrix}
\]
code / simulation