Dynamical systems tutorial: 1. Basic concepts

Gregor Schöner, INI, RUB

Dynamical systems: Tutorial

- the word "dynamics"
  - time-varying measures
  - range of a quantity
  - forces causing/accounting for movement => dynamical systems
- dynamical systems are the universal language of science
  - physics, engineering, chemistry, theoretical biology, economics, quantitative sociology, ...

time-variation and rate of change

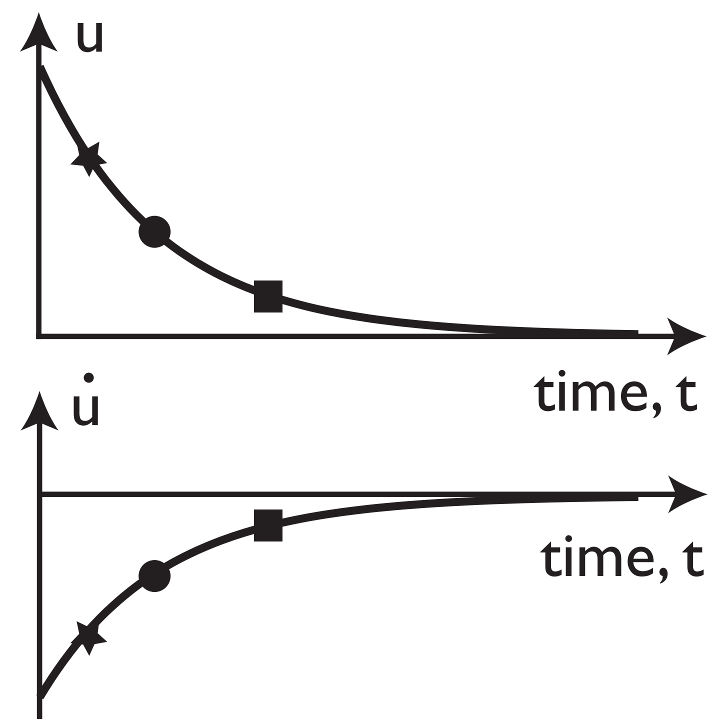
- variable $x(t)$;
- rate of change dx/dt

time-variation and rate of change

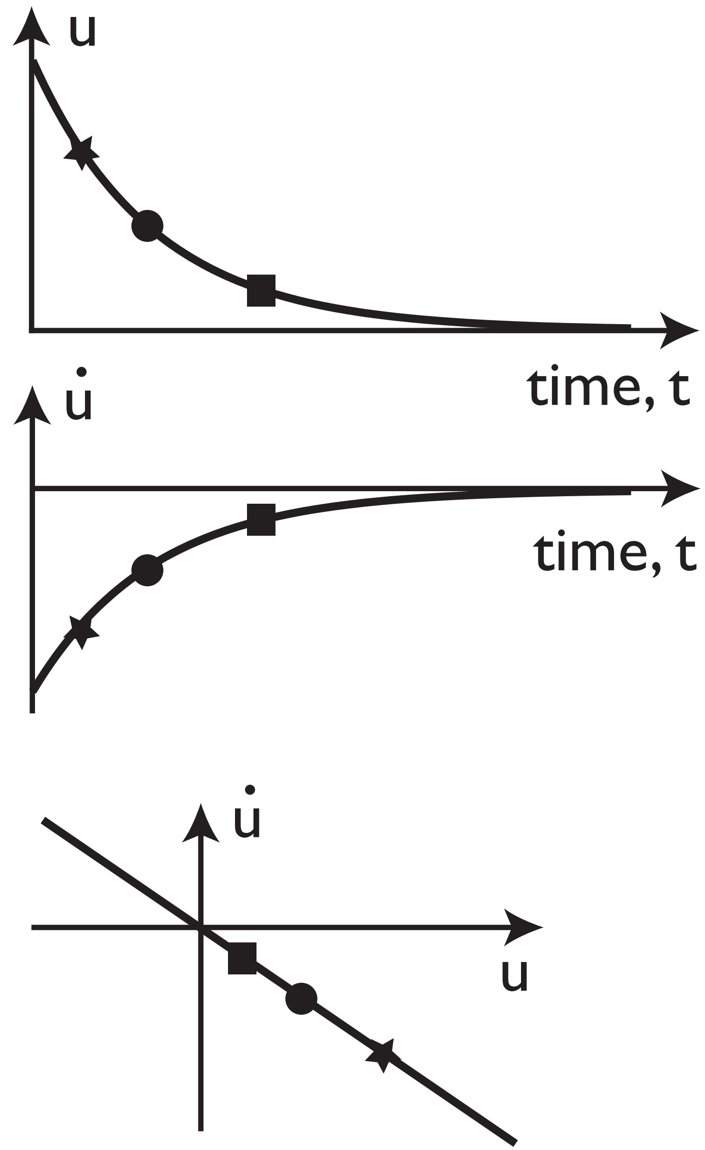
- example:
  - variable $x(t)$ = position
  - rate of change dx/dt = velocity
- example
  - variable $v(t)$ = velocity
  - rate of change ?

time-variation and rate of change

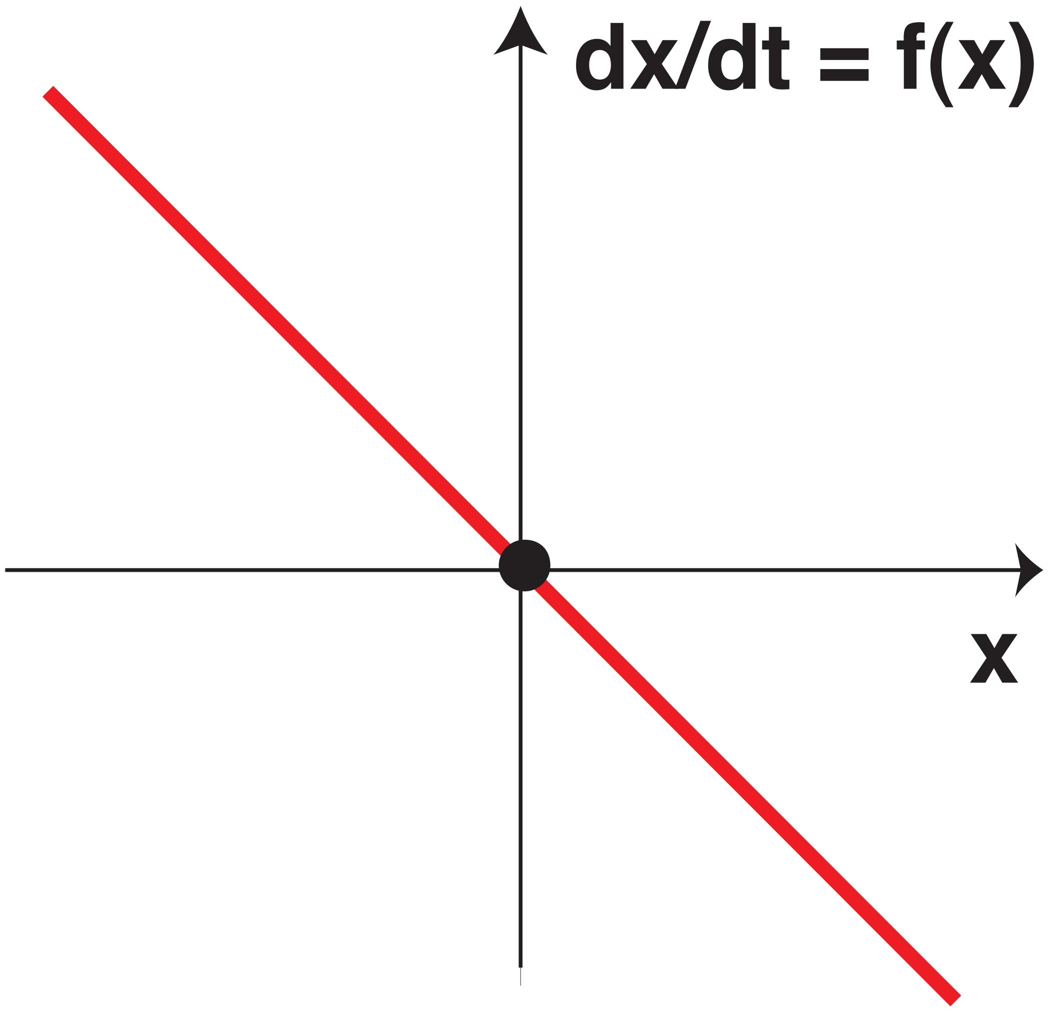
- trajectory: time course of a dynamical variable
- rate of change: slope of the trajectory



dynamical system: relationship between a variable and its rate of change

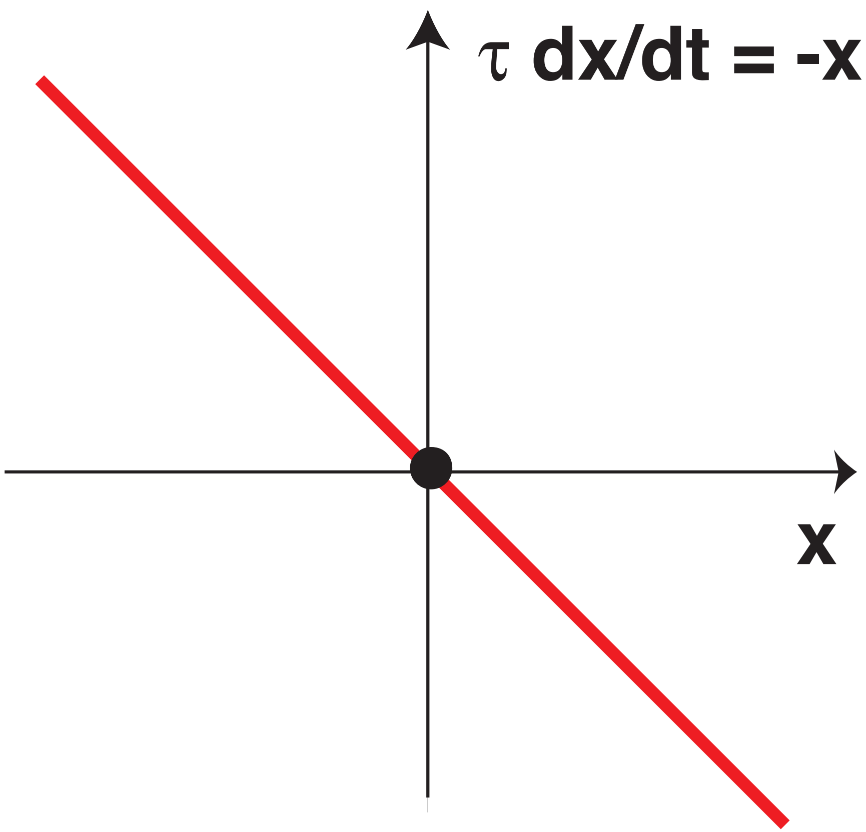


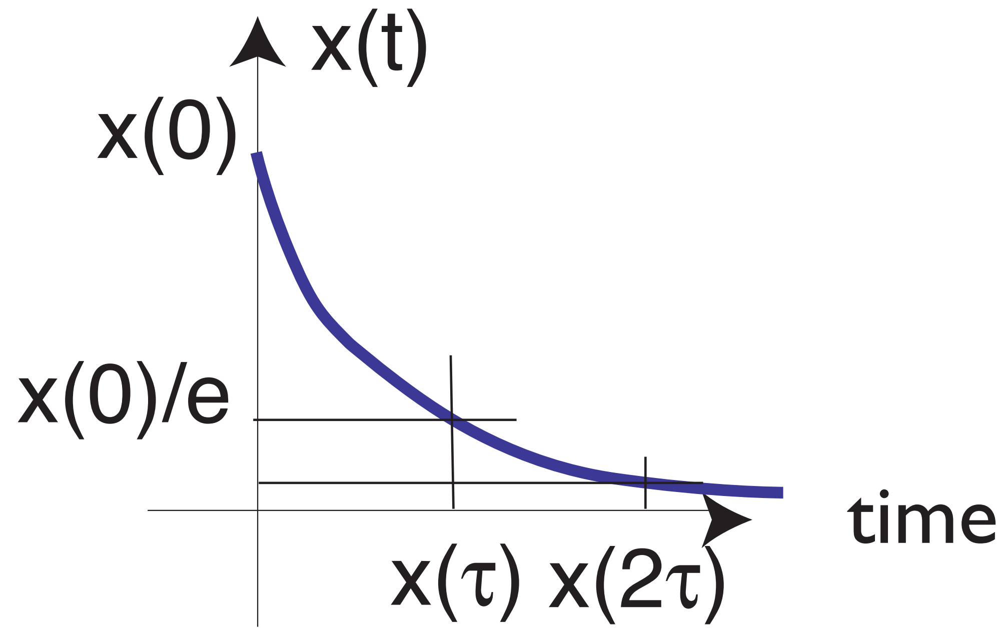
linear dynamical system



solution of linear dynamical systems

$$x(t) = x(0)\exp[-t/\tau]$$





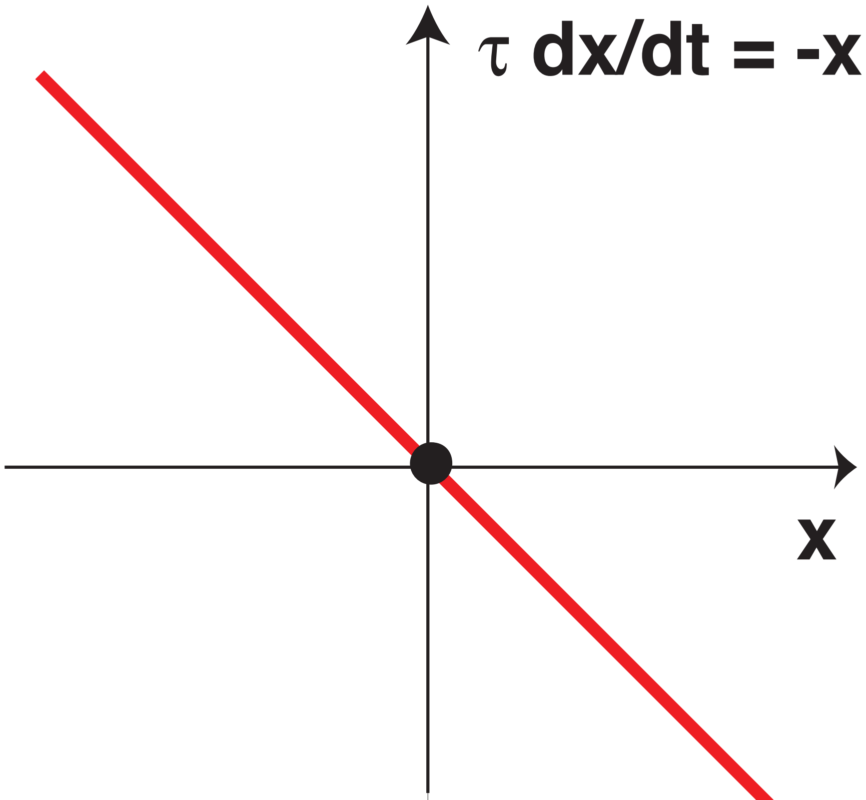
exponential relaxation to attractors

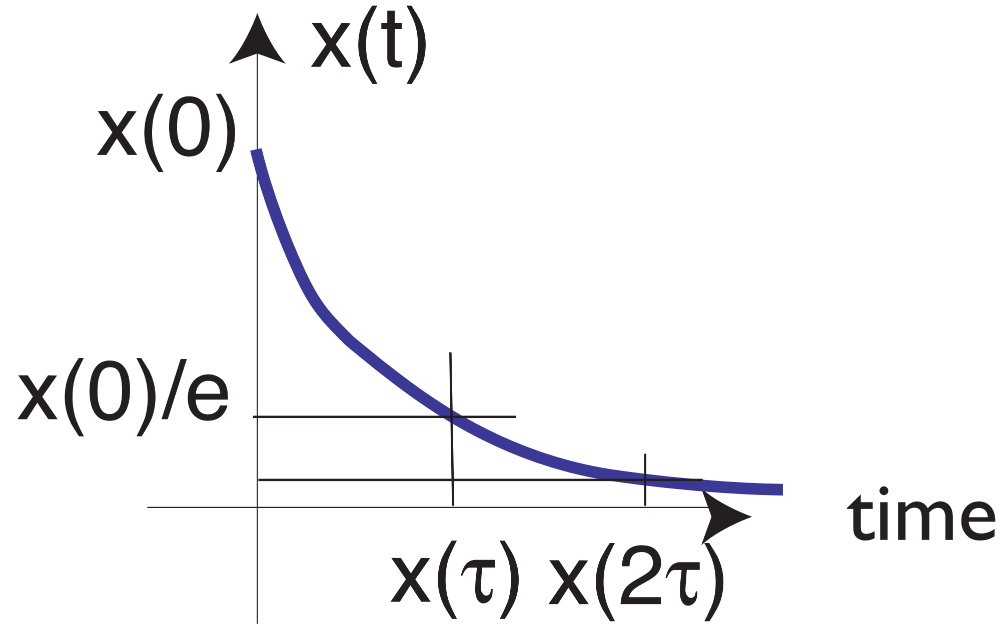
$$x(t) = x(0)\exp[-t/\tau]$$

$$\dot{x}(t) = x(0)\exp[-t/\tau](-1/\tau)$$

$$\tau\dot{x}(t) = -\,x(0)\exp[-t/\tau]$$

$$\tau\dot{x}(t) = -\,x(t)$$

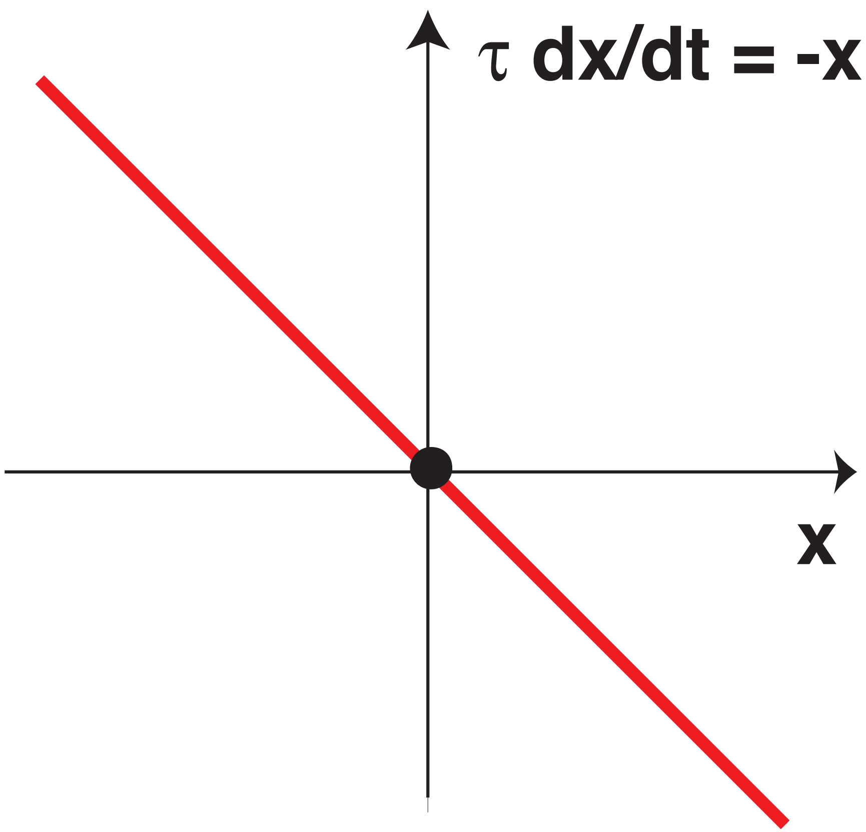


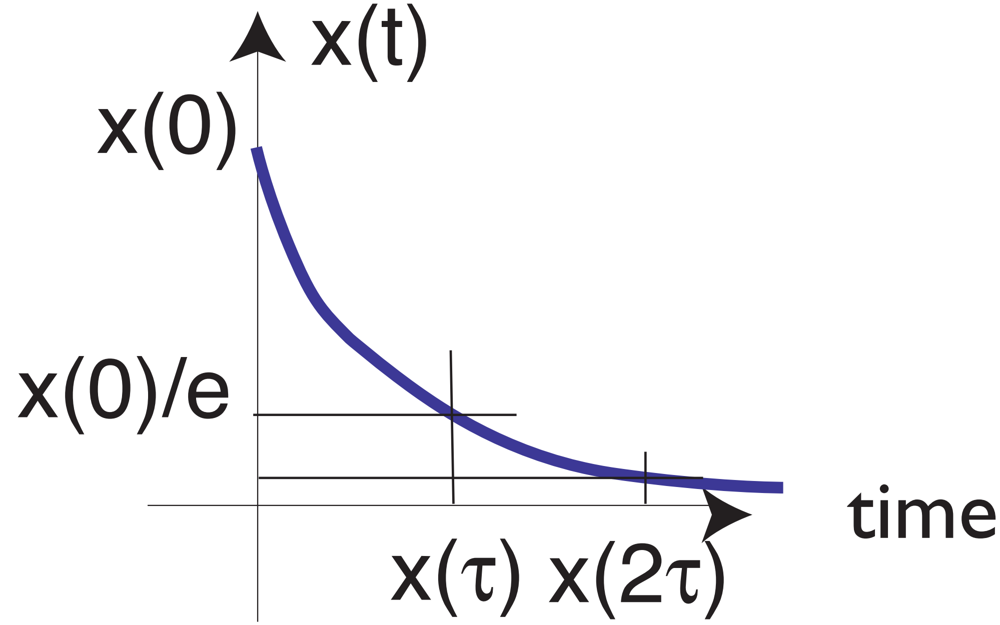


exponential relaxation to attractors

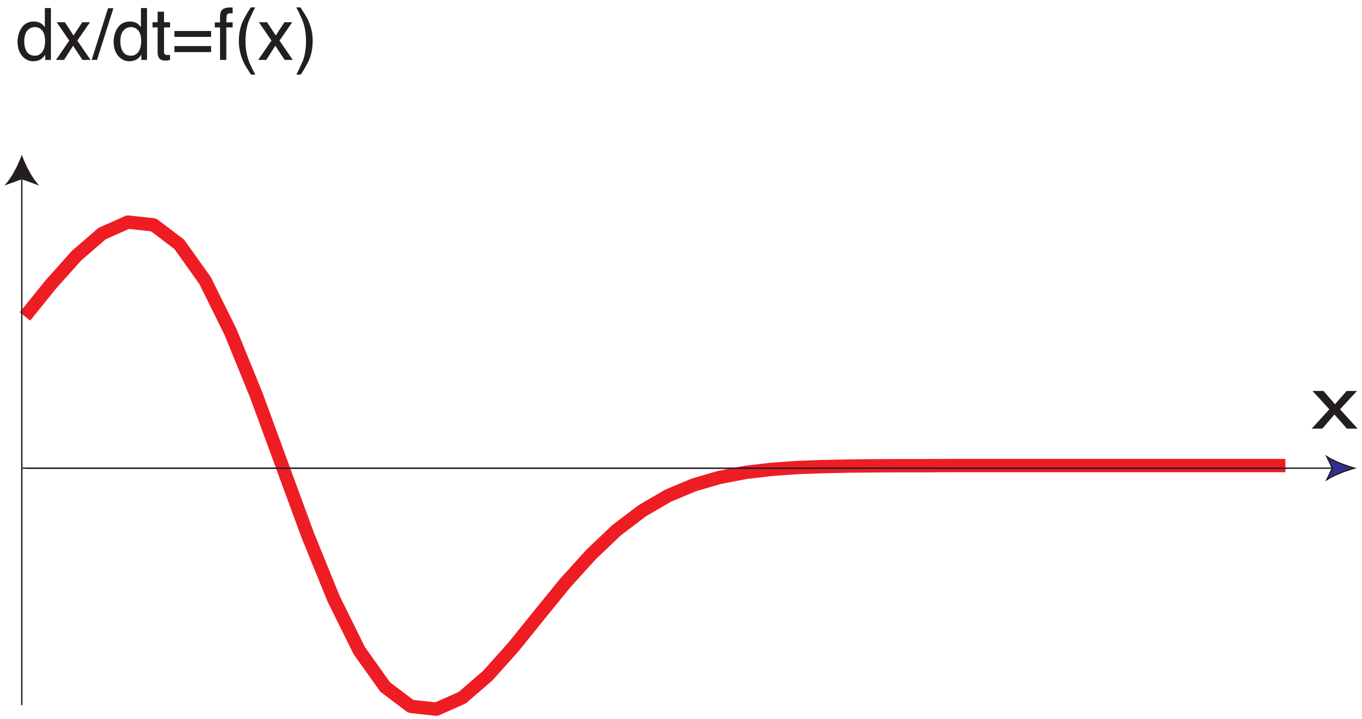
- => time scale

$$x(t) = x(0)\exp[-t/\tau]$$





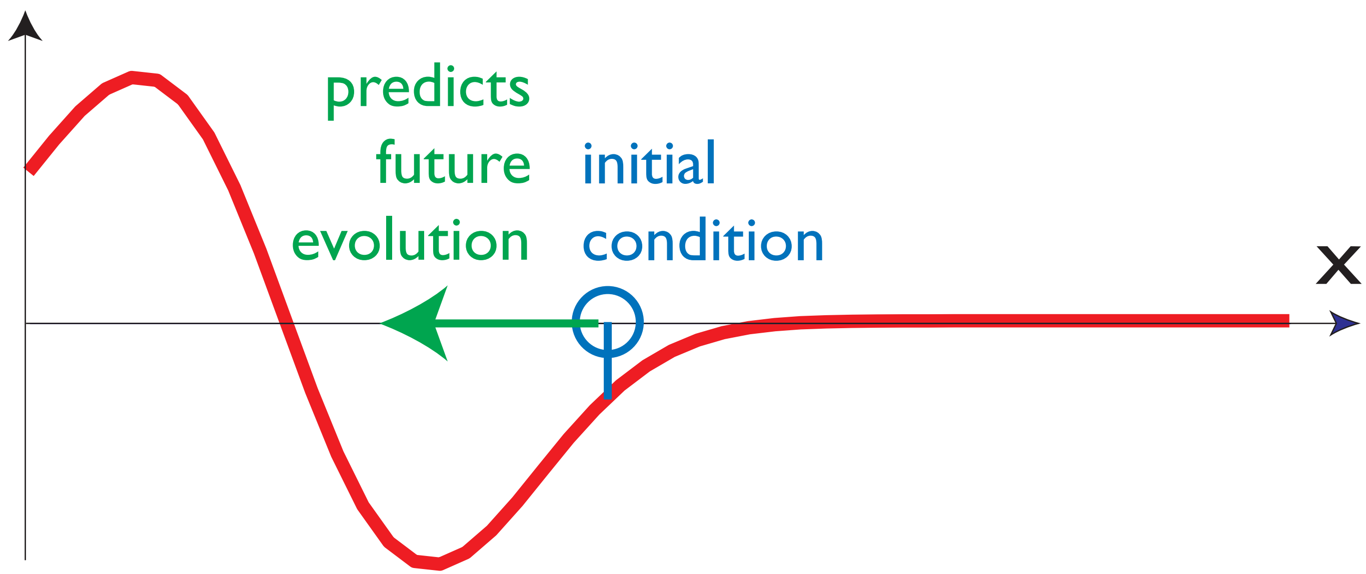
(non-linear) dynamical system



Defining property of dynamical systems

- present determines the future:
  - given an initial condition, predict future time course (or postdict the past)

$dx/dt=f(x)$



Concepts dynamical systems

- x: spans the state space (or phase space)
- f(x): is the "dynamics" of x (or vector-field)
- x(t) is a solution of the dynamical systems to the initial condition x_0
  - if its rate of change = f(x)
  - and x(0)=x_0

Different forms of dynamical systems

- one-dimensional differential equation: initial value determines the future

$$\dot{x} = f(x)$$

Different forms of dynamical systems

- vector-valued differential equation
- a vector of initial states determines the future: systems of differential equations:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) \qquad \text{where} \qquad \mathbf{x} = (x_1, x_2, \ldots, x_n)$$

Different forms of dynamical systems

- continuously many variables x(y) determine the future = an initial function x(y) determines the future
  - partial differential equations
  - integro-differential equations

$$\dot{x}(y,t) = f\left(x(y,t), \frac{\partial x(y,t)}{\partial y}, \ldots\right)$$

$$\dot{x}(y,t) = \int dy' g\left(x(y,t), x(y',t)\right)$$

Different forms of dynamical systems

- a piece of past trajectory determines the future
  - delay differential equations
  - functional differential equations

$$\dot{x}(t) = f(x(t - \tau))$$

$$\dot{x}(t) = \int^{t} dt' f(x(t'))$$