Lecture 7
Object Oriented Programming

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Computer Science and Mathematics
Preparatory Course

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Overview

Matrix Definition

A Matrix $\boldsymbol{A}_{m,n}$ is a rectangular array arranged in m rows and n columns.

- Example:

$$\boldsymbol{A}_{3,4} = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \end{pmatrix}$$

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- A single element in a matrix is usually denoted by $a_{i,j}$, where i is the row and j the column index. For example $a_{2,3} = 7$.
- A matrix A_m, n, where $m = n$ is called a **square matrix**
- A matrix that has only entries on the diagonal is called a **diagonal matrix**

$$\boldsymbol{D}_{3,3} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 4 \end{pmatrix} \text{ Special case } \textbf{identity matrix } \boldsymbol{I}_{3,3} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

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$$\boldsymbol{A}_{3,2} + \boldsymbol{B}_{3,2} = \begin{pmatrix} 1 & 2 \\ 5 & 6 \\ 9 & 10 \end{pmatrix} + \begin{pmatrix} 4 & 2 \\ 3 & 1 \\ 8 & 2 \end{pmatrix} = \begin{pmatrix} 1+4 & 2+2 \\ 5+3 & 6+1 \\ 9+8 & 10+2 \end{pmatrix} = \begin{pmatrix} 5 & 4 \\ 8 & 7 \\ 17 & 12 \end{pmatrix}$$

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- Subtraction works analogously:

$$\boldsymbol{A}_{3,2} - \boldsymbol{B}_{3,2} = \begin{pmatrix} 1 & 2 \\ 5 & 6 \\ 9 & 10 \end{pmatrix} - \begin{pmatrix} 4 & 2 \\ 3 & 1 \\ 8 & 2 \end{pmatrix} = \begin{pmatrix} 1-4 & 2-2 \\ 5-3 & 6-1 \\ 9-8 & 10-2 \end{pmatrix} = \begin{pmatrix} -3 & 0 \\ 2 & 5 \\ 1 & 8 \end{pmatrix}$$

Scalar Multiplication and Transposition

- Multiplication with scalar values is also applied element-wise:

$$\boldsymbol{A}_{3,2} \cdot 3 = \begin{pmatrix} 1 & 2 \\ 5 & 6 \\ 9 & 10 \end{pmatrix} \cdot 3 = \begin{pmatrix} 1 \cdot 3 & 2 \cdot 3 \\ 5 \cdot 3 & 6 \cdot 3 \\ 9 \cdot 3 & 10 \cdot 3 \end{pmatrix} = \begin{pmatrix} 3 & 6 \\ 15 & 18 \\ 27 & 30 \end{pmatrix}$$

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- The transposition $\boldsymbol{A}^T$ of a matrix switches the roles of row and columns Example:

$$\boldsymbol{A}^T_{3,2} = \begin{pmatrix} 1 & 2 \\ 5 & 6 \\ 9 & 10 \end{pmatrix}^T = \begin{pmatrix} 1 & 5 & 9 \\ 2 & 6 & 10 \end{pmatrix}$$

The transposition turns a $m \times n$ matrix into a $n \times m$ matrix.

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- Matrices $\boldsymbol{A}$ and $\boldsymbol{B}$ can be multiplied with each other, if the number of columns of $\boldsymbol{A}_{m,\boldsymbol{n}}$ matches the number of rows in $\boldsymbol{B}_{\boldsymbol{n},o}$.
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$$\boldsymbol{A}_{2,3} \cdot \boldsymbol{B}_{3,3} = \begin{pmatrix} 3 & 6 & 5 \\ 4 & 2 & 1 \end{pmatrix} \cdot \begin{pmatrix} 4 & 3 & 8 \\ 1 & 2 & 10 \\ 7 & 3 & 2 \end{pmatrix} = \begin{pmatrix} - & - & - \\ - & - & - \end{pmatrix}$$

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The Numpy Module

- Numpy is part of SciPy **the** module for scientific programming
- It should have been installed with matplotlib
- It is usually imported like this:

```python
import numpy as np
```

The Numpy Array

- Numpy brings its own data structure the numpy array

```
import numpy as np
#Arrays can be created from lists
array_example = np.array([1,6,7,9])
#Arrays can be created with arange
#An array with numbers from 4 to 5 and step size 0.2
array2 = np.arange(4,5,0.2) #5 is not in the array
print(array2) # [4.0 4.2 4.4 4.6 4.8]
```

- Elements of an array can be manipulated simultaneously

```
array3 = array2*array2 #For example with multiplication
print(array3)# [16.0 16.64 19.36 21.16 23.04]
```

Matplotlib and Numpy

- Plotting $sin(x)$ from 0 to π with lists

```
listX=[]
listY=[]
step_size = 0.5
for i in range(0,int(math.pi/step_size)):
        xValue = i*step_size
        listX.append(xValue)
        listY.append(math.sin(xValue))
plt.plot(listX,listY)
```

- Plotting $sin(x)$ from 0 to π with numpy

```
xValues = np.arange(0,math.pi,0.5)
yValues = np.sin(xValues)
plt.plot(xValues,yValues)
```

Numpy Arrays as Matrices

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- In numpy a matrix can be created from a multi-dimensional list

```
# This creates a 3x4 Matrix
A = np.array([[1,2,3,4], [5,6,7,8], [9,10,11,12]])
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```

- Numpy treats such an array as a matrix

```
arr_dim = A.shape #Gives you the shape of your matrix
print(arr_dim) #Prints (3,4)
# Access elements with indexing
single_number = A[1,3] #8, 2nd list,4th element
num2 = A[0,1] #2, 1st list, 2nd element
```

Matrix Operations in Numpy

- Matrix Addition: $\begin{pmatrix} 1 & 2 & 3 \\ 5 & 6 & 7 \end{pmatrix} + \begin{pmatrix} 3 & 5 & 1 \\ 5 & -3 & 1 \end{pmatrix} = \begin{pmatrix} 4 & 7 & 4 \\ 10 & 3 & 8 \end{pmatrix}$
- In numpy code:

```
A = np.array([[1,2,3], [5,6,7]])
B = np.array([[3,5,1], [5,-3,1]])
C = A + B
D = A - B #Subtraction works analogously
print(D) #[[-2 -3 2],[0 9 6]]
```

Matrix Operations in Numpy

- Matrix Multiplication: $\begin{pmatrix} 1 & 2 & 3 \\ 5 & 6 & 7 \end{pmatrix} * \begin{pmatrix} 3 & 5 \\ 5 & -3 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 16 & 2 \\ 52 & 14 \end{pmatrix}$
- In numpy code:

```
A = np.array([[1,2,3], [5,6,7]])
E  = np.array([[3,5], [5,-3],[1,1]])
F = np.matmul(A,E)
print(F) # [[16,2],[52,14]]
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```

- Do not confuse with element-wise multiplication

```
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B = np.array([[3,5,1], [5,-3,1]])
G = A*B # [[3,10,3],[25,-18,7]]
```

Matrix Operations in Numpy

- It also works for vectors:

$$< v_1, v_2 >= {v_1}^T v_2 = \begin{pmatrix} 1 & 2 & 3 \end{pmatrix} * \begin{pmatrix} 3 \\ 5 \\ 1 \end{pmatrix} = 16$$

- In numpy code:

```
V1 = np.array([1,2,3])
V2  = np.array([3,5,1])
R = np.matmul(V1,V2)
print(R) # 16
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- Or vectors and matrices if you want to

Other helpful Operations

- Transpose Matrices: $\boldsymbol{A} = \begin{pmatrix} 1 & 2 & 3 \\ 5 & 6 & 7 \end{pmatrix} \quad \boldsymbol{A}^T = \begin{pmatrix} 1 & 5 \\ 2 & 6 \\ 3 & 7 \end{pmatrix}$
- In numpy:

```
A = np.array([[1,2,3], [5,6,7]])
H = A.T # [[1,5],[2,6],[3,7]]
```

- Element-wise summing across arrays:

```
sum = np.sum(H) #24,
V1 = np.array([1,2,3]) #works also for 1D-arrays
sum_v = np.sum(V1) # 6
```

Images as Matrices

$$\begin{pmatrix} x_{0,0} & x_{0,1} & x_{0,2} & x_{0,3} & x_{0,4} \\ x_{1,0} & x_{1,1} & x_{1,2} & x_{1,3} & x_{1,4} \\ x_{2,0} & x_{2,1} & x_{2,2} & x_{2,3} & x_{2,4} \\ x_{3,0} & x_{3,1} & x_{3,2} & x_{3,3} & x_{3,4} \\ x_{4,0} & x_{4,1} & x_{4,2} & x_{4,3} & x_{4,4} \end{pmatrix}$$

Images as Matrices

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Images as Matrices

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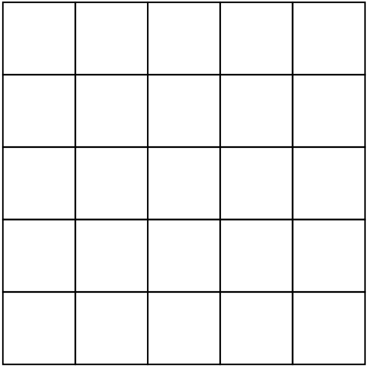
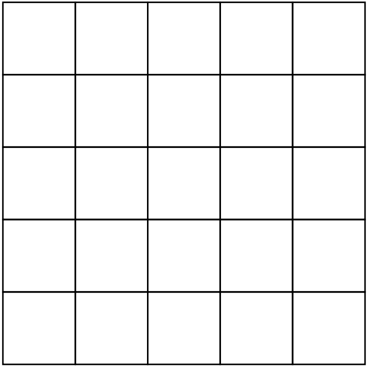


Images as Matrices

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$



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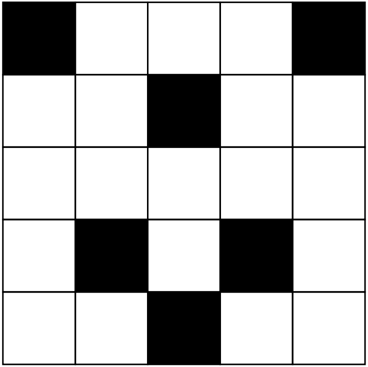

Convolution of Functions

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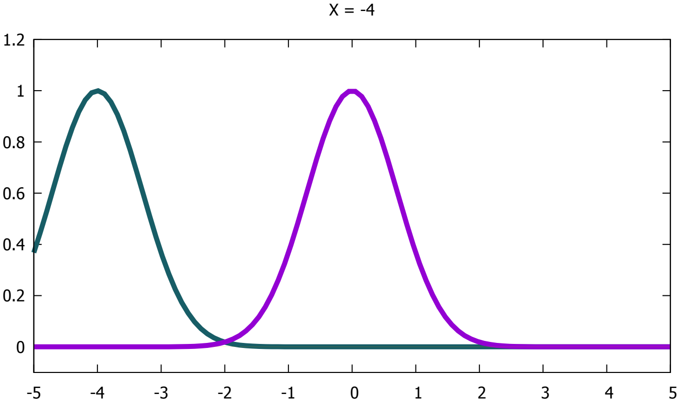
Convolution of the Gaussian function with itself



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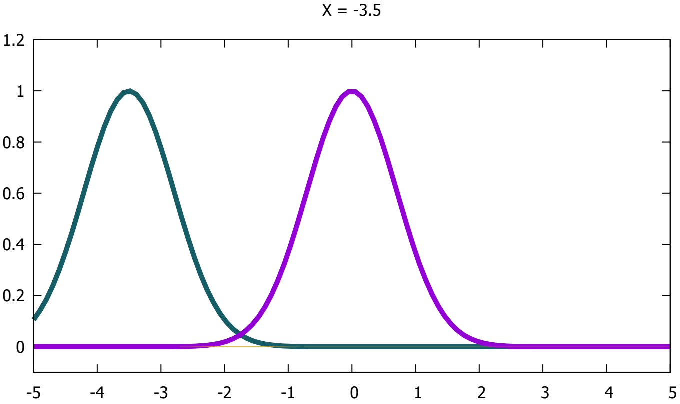

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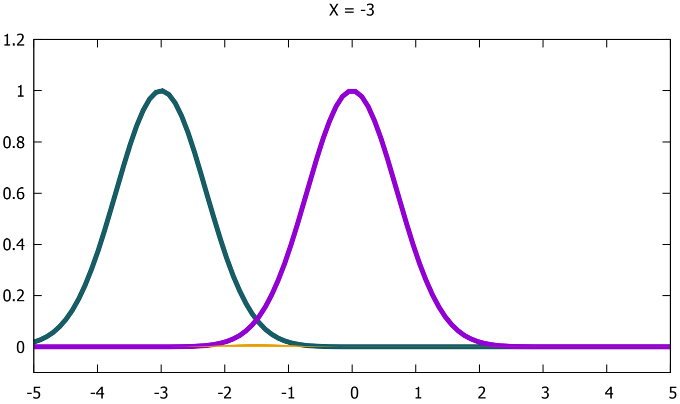

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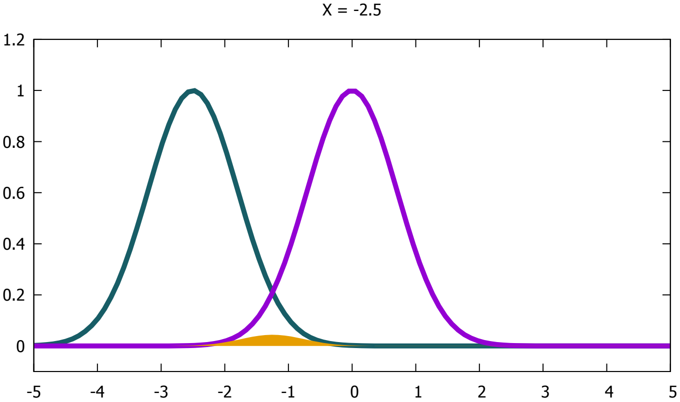

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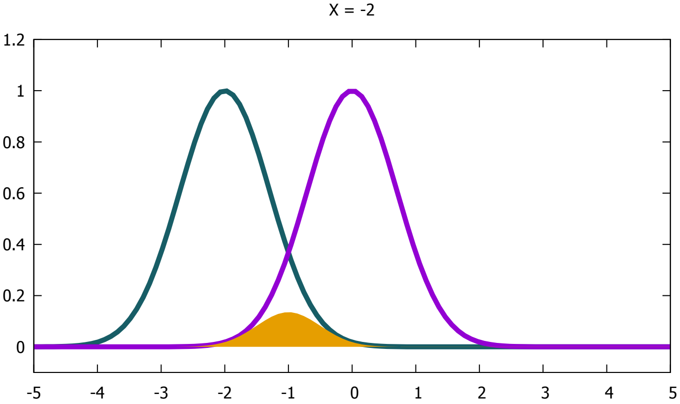

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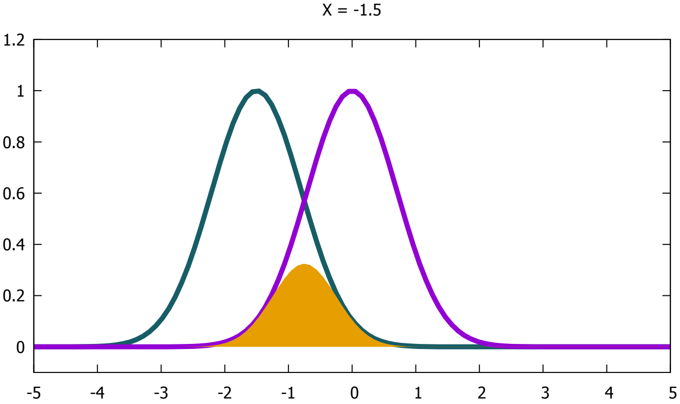

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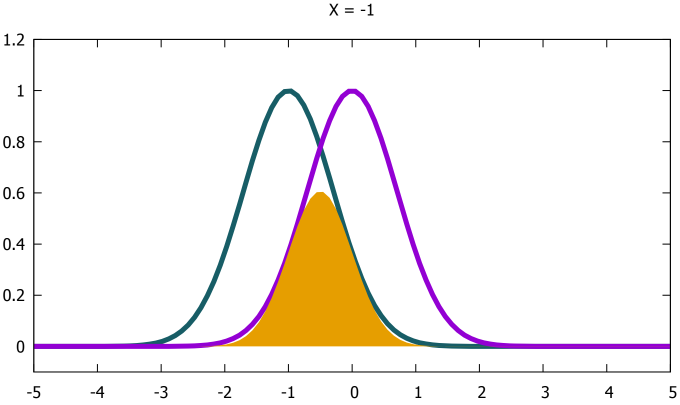

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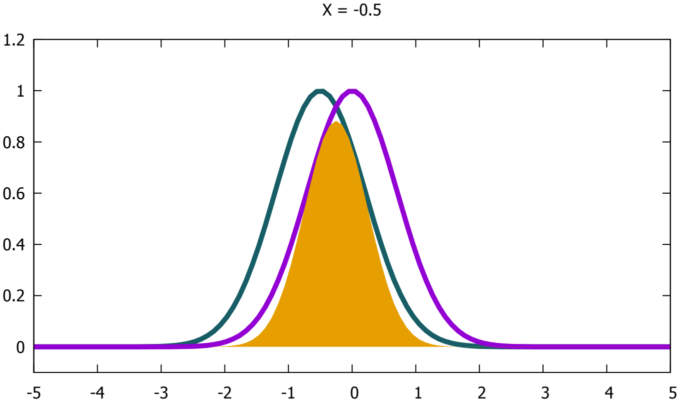

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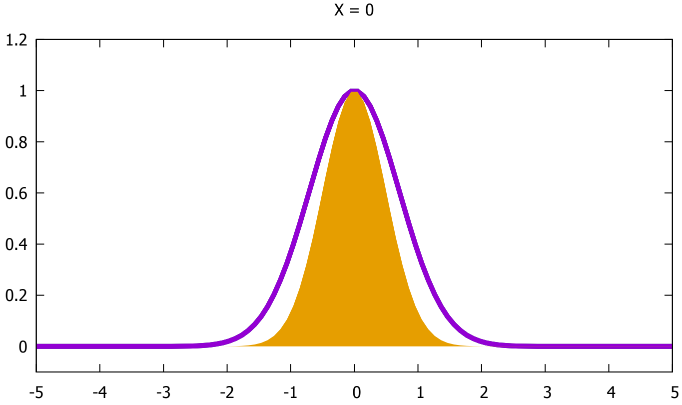

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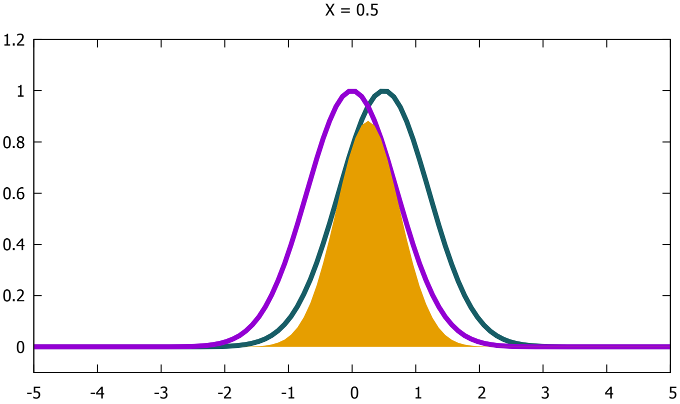

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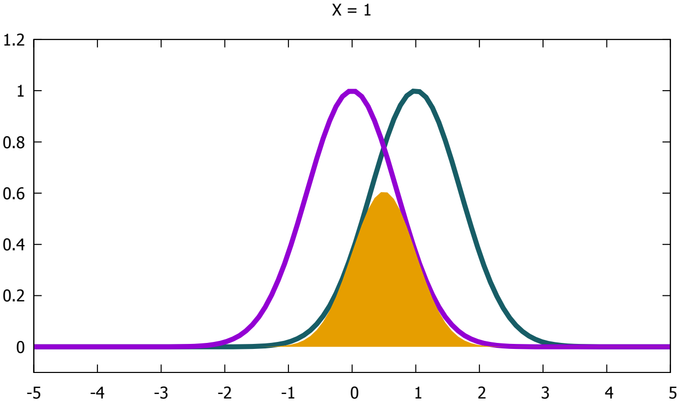

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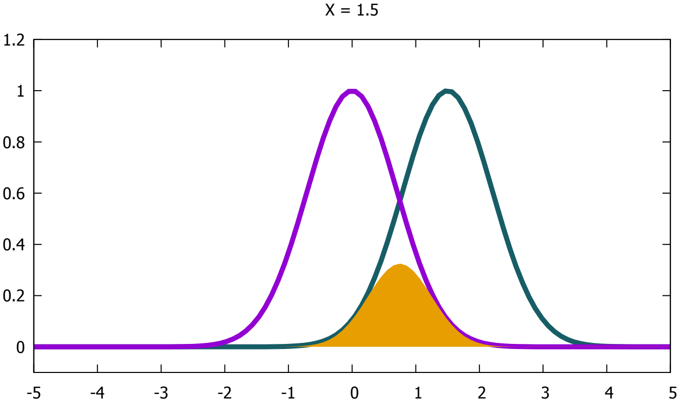

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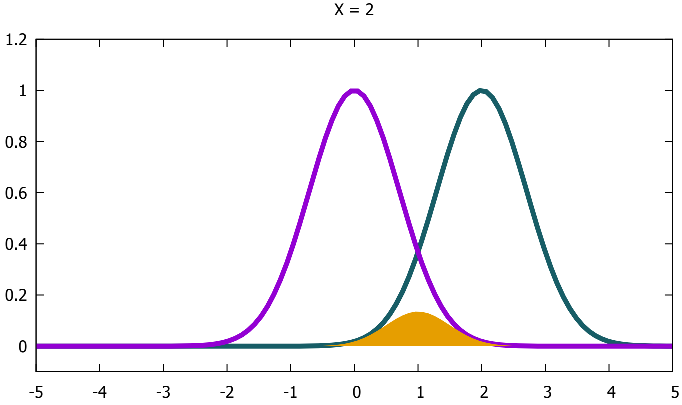

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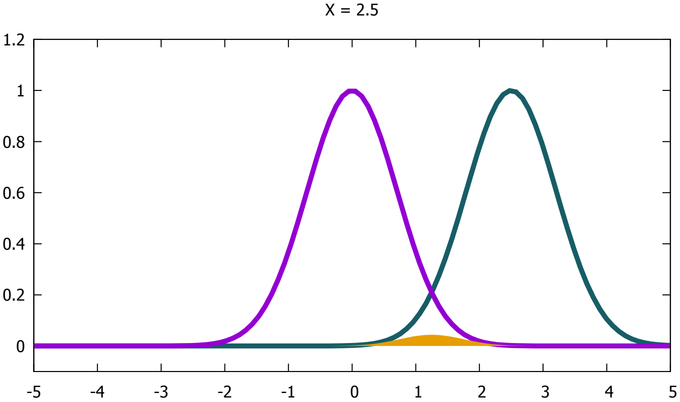

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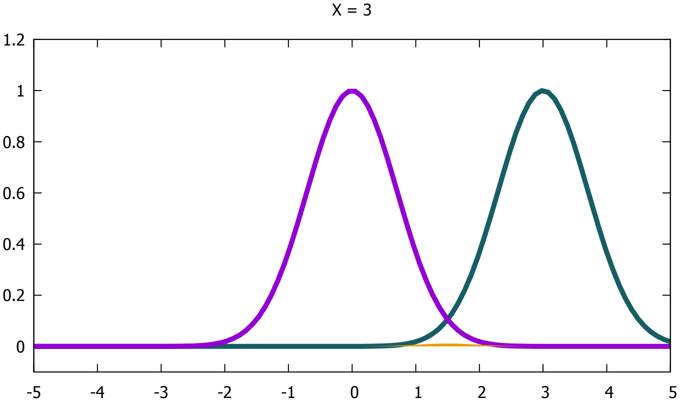

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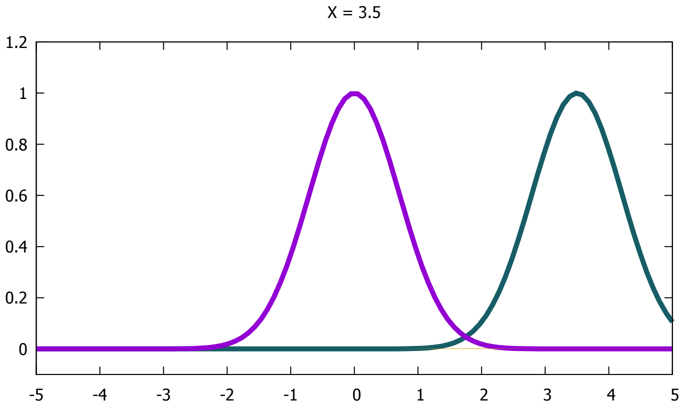

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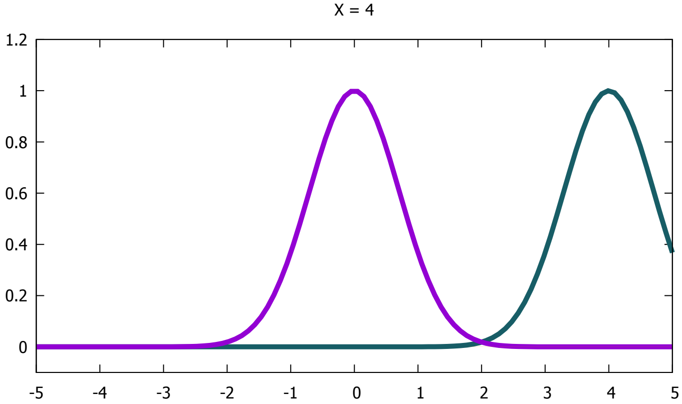

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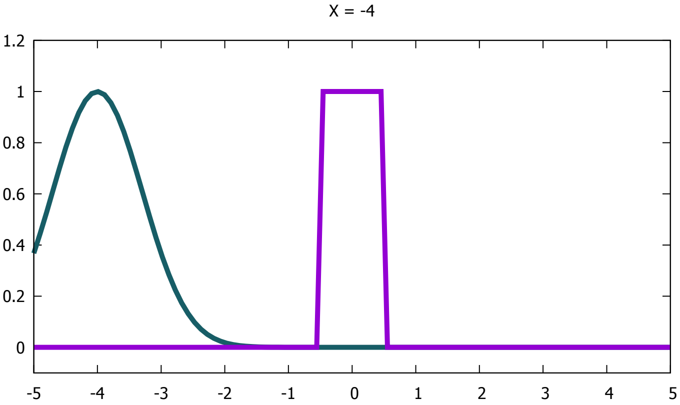

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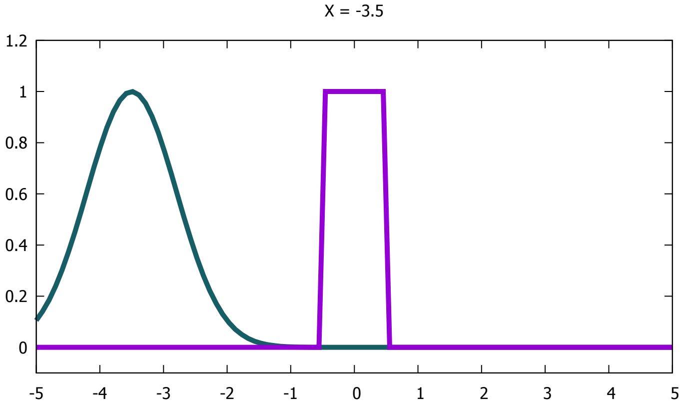

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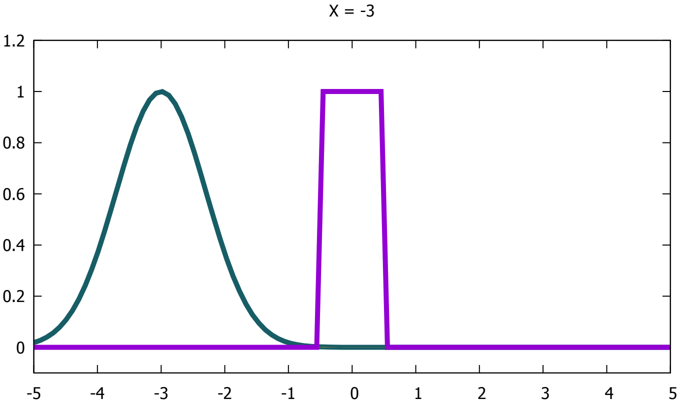

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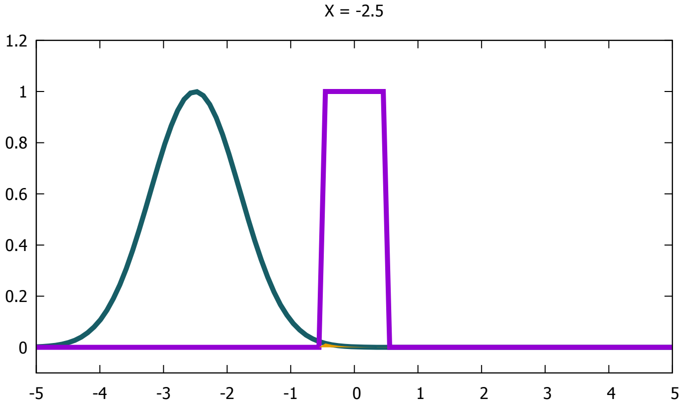

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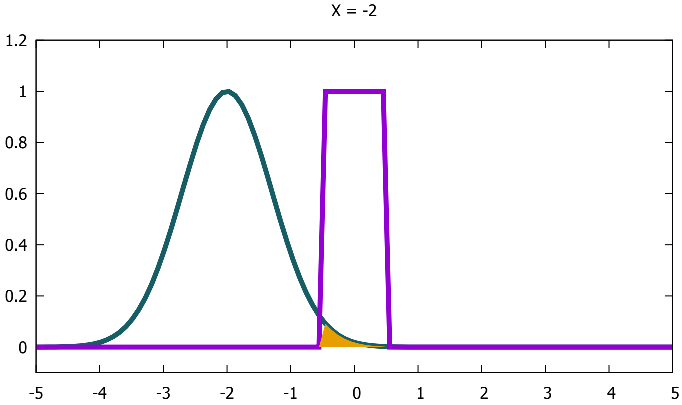

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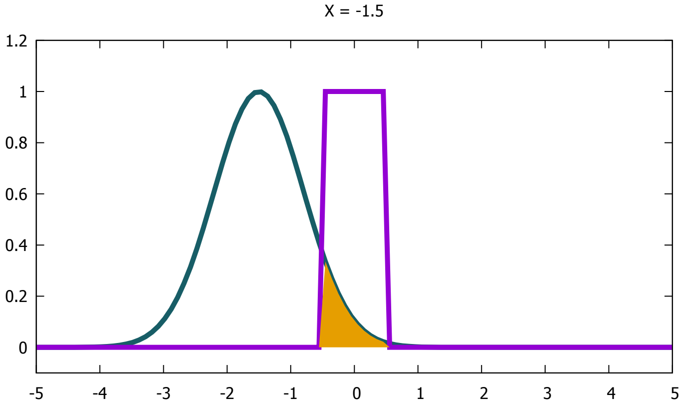

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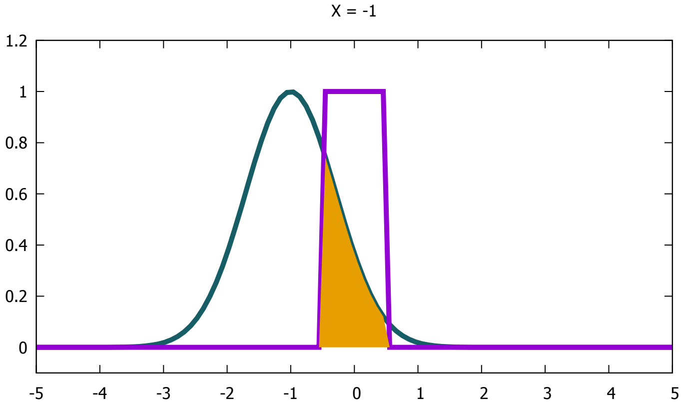

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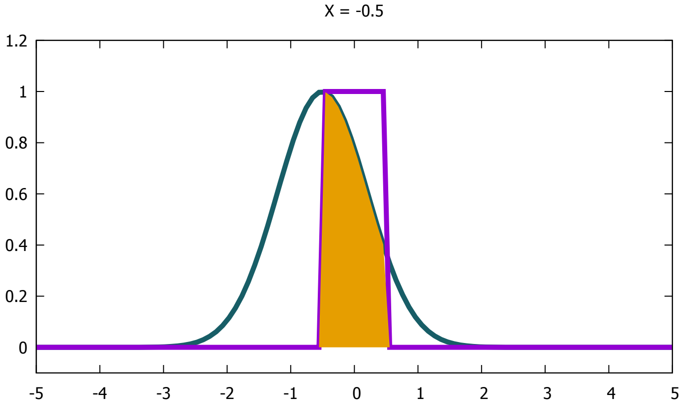

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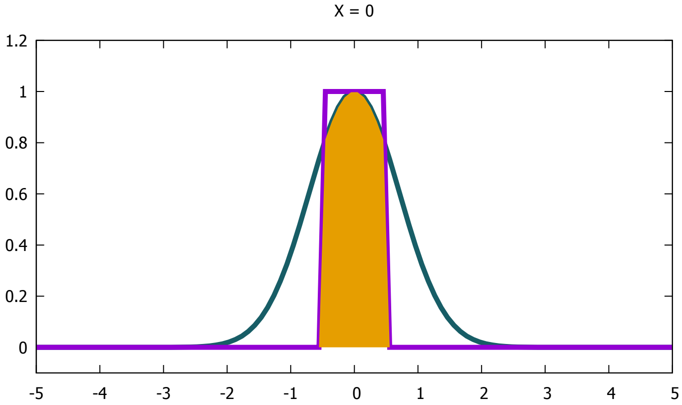

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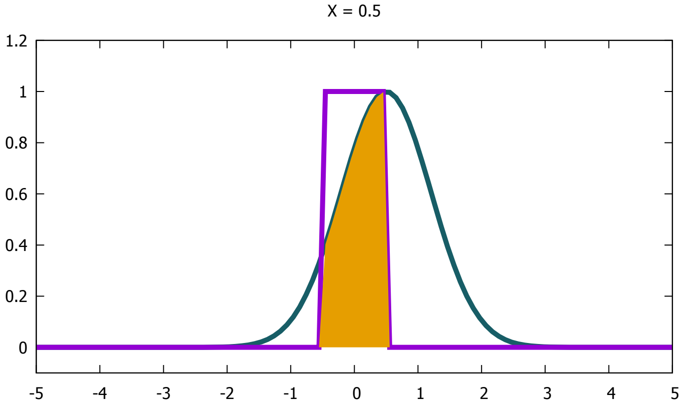

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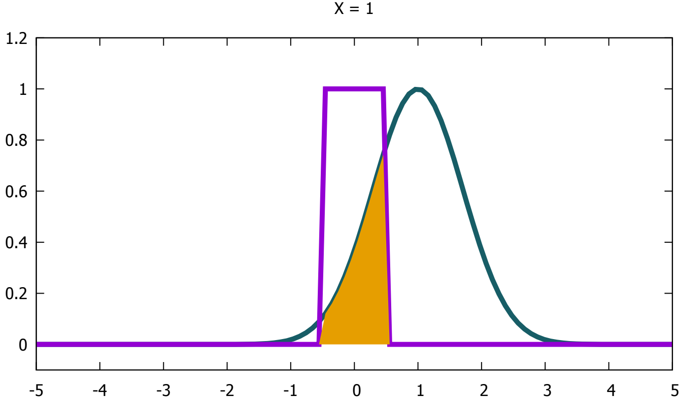

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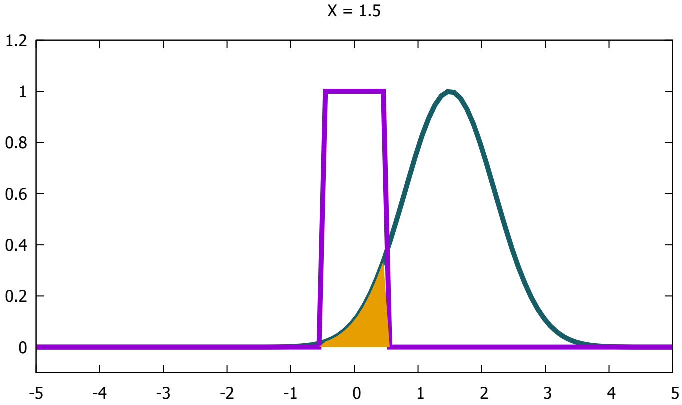

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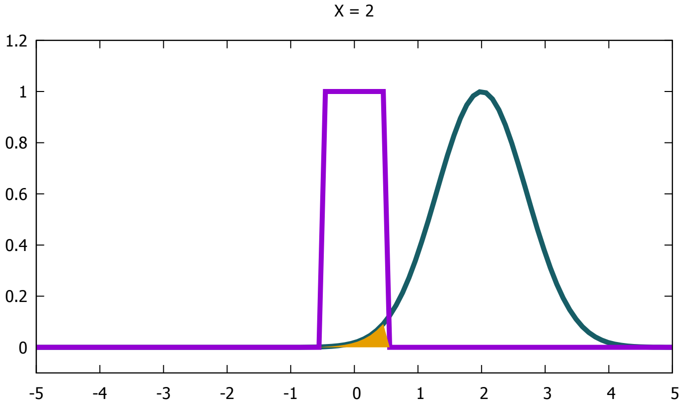

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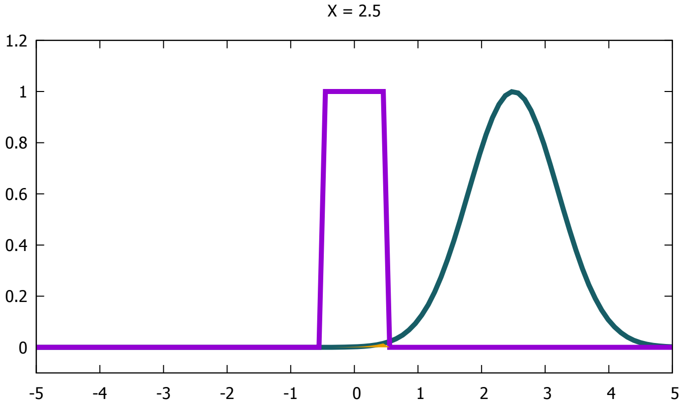

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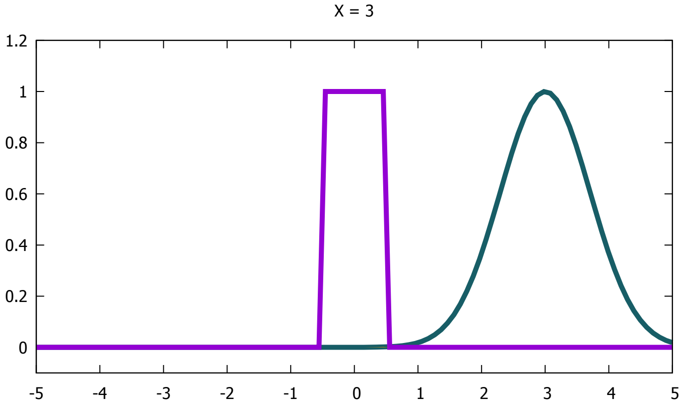

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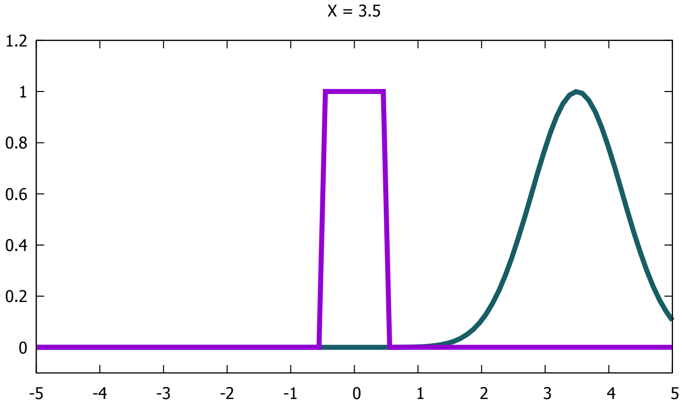

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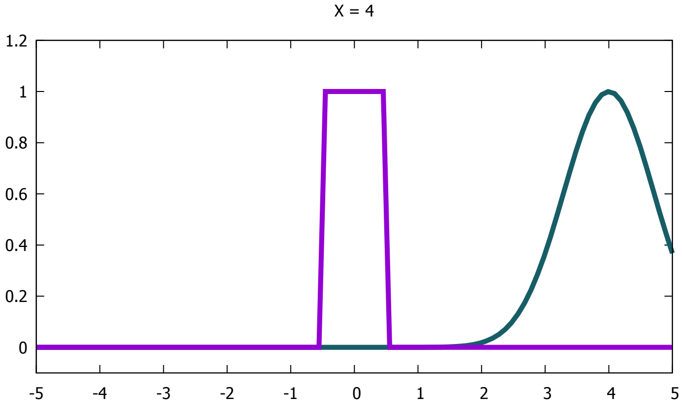
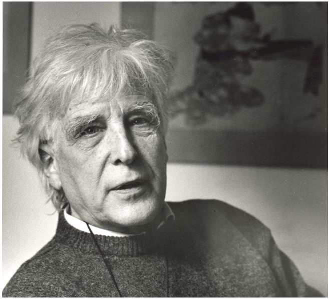


Applying Filters to Images

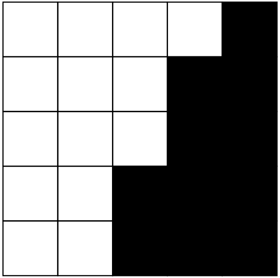


Convolution with Matrices

Edge Filter

-1	0	1

Image

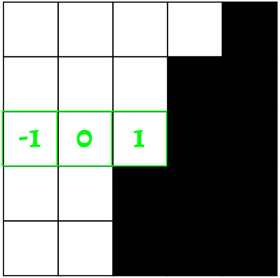


Convolution with Matrices

Edge Filter

-1	0	1

Image

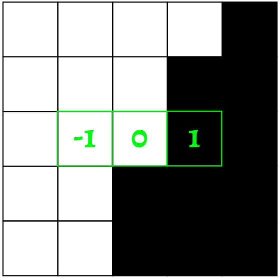


Convolution with Matrices

Edge Filter

-1	0	1

Image

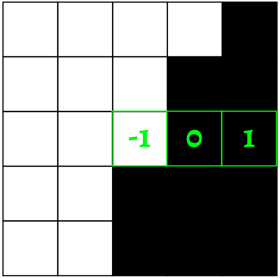


Convolution with Matrices

Edge Filter

-1	0	1

Image



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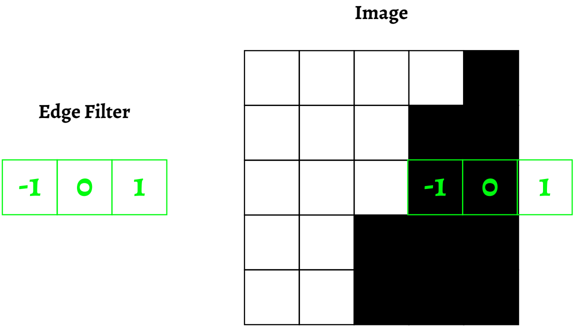

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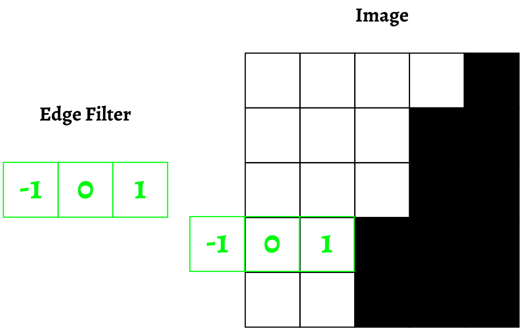

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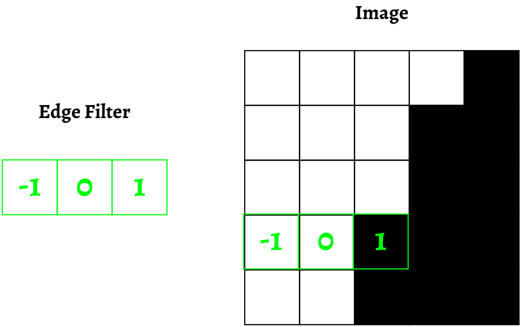

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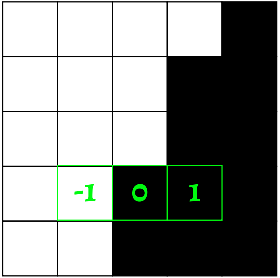
Edge Filter

-1	0	1

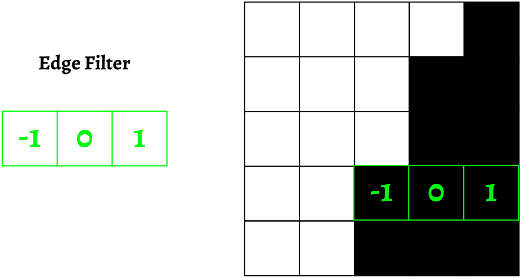
Image



Convolution with Matrices

Image

Edge Filter



Programming Paradigms

Procedural Programming

- A problem is solved by manipulating data structures through procedures
- The key is to write the right logic
- Efficiency is a main focus of procedural programming

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Procedural Programming

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- The key is to write the right logic
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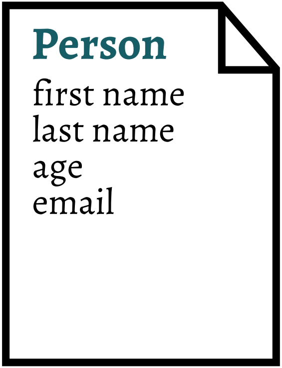
Object oriented Programming

- A problem is solved by modeling it's processes
- The key is to figure out the relevant entities and their relations
- Programming Logic is tightly coupled to entities

Classes vs. Objects

Class



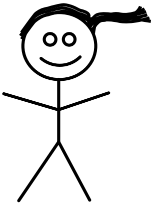
Classes vs. Objects

Class

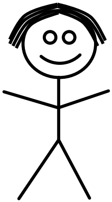
Person

first name
last name
age
email

Objects (Instances)



Alice
Anderson
28
a.anders@gmail.com



Rob
Robertson
17
cool_dude@aol.com

Classes Bind Variables Together

- Instead of writing something like this

```
#Alice's attributes
alice_name = "Alice"
alice_last_name = "Anderson"
alice_age = 28
```

Classes Bind Variables Together

- Instead of writing something like this

```
#Alice's attributes
alice_name = "Alice"
alice_last_name = "Anderson"
alice_age = 28
```

- Objects encapsulate multiple variables in one place

```
#A Person-object variable
alice = Person("Alice","Anderson",28)
```

Classes are Advanced Data Types

- Object variables can be treated like simple types

```
#Two Person-object variables
alice = Person("Alice","Anderson",28)
rob = Person("Rob","Robertson",17)
#Objects can be stored in lists
myPersonList = [] #I want to manage persons
myPersonList.append(rob)
#Objects can be arguments of self-defined functions
calculate_year_of_birth(alice)
```

Class Definition

- A class needs to be defined

```
class Person: #This defines the class Name
#The __init__ function is responsible for class creation
    ↪  and defines its' attributes
    def __init__(self, first_name,last_name,age):
        #The passed values are stored in the class
        self.first_name = first_name
        self.last_name = last_name
        self.age = age
```

- This is enough to create a class-object

```
robby = Person("Rob","Robertson",17)
```

Accessing Class Attributes

- Class attributes can be accessed via the '.' operator

```
robby = Person("Rob","Robertson",17)

f_name = robby.first_name #"Rob"
l_name = robby.last_name #"Robertson"
age = robby.age #17
```

Accessing Class Attributes

- Class attributes can be accessed via the '.' operator

```
robby = Person("Rob","Robertson",17)

f_name = robby.first_name #"Rob"
l_name = robby.last_name #"Robertson"
age = robby.age #17
```

- They can also be assigned after initialization

```
robby.age = 18 #As he gets older
robby.l_name = "Peterson" #If he marries
```

Objects and Functions

- We can use objects as function arguments

```
#Definition
def print_info(person):
        print(person.first_name +" " +person.last_name +"
            ↪  is " +str(person.age) +" years old.")
```

Objects and Functions

- We can use objects as function arguments

```
#Definition
def print_info(person):
        print(person.first_name +" " +person.last_name +"
        ↪ is " +str(person.age) +" years old.")
```

- Usage:

```
robby = Person("Rob","Robertson",17)
print_info(robby)
#This prints: "Rob Robertson is 17 years old"

alice = Person("Alice","Anderson",28)
print_info(alice)
#This prints: "Alice Anderson is 28 years old"
```

Function Encapsulation

- Functions can even be defined inside classes

```
class Person: #This defines the class Name
    #The __init__ function
    def __init__(self, first_name,last_name,age):
        #The passed values are stored in the class
        self.first_name = first_name
        self.last_name = last_name
        self.age = age

    #Our print_info function
    def print_info(self): #Note how the argument changed
        print(self.first_name +" " +self.last_name +" is
            ↪ " +str(self.age) +" years old.")
```

Function Encapsulation

- A function can be called directly from the object

```
robby = Person("Rob","Robertson",17)
robby.print_info()
#This prints: "Rob Robertson is 17 years old"

alice = Person("Alice","Anderson",28)
alice.print_info()
#This prints: "Alice Anderson is 28 years old"
```

Function Encapsulation

- A function can be called directly from the object

```
robby = Person("Rob","Robertson",17)
robby.print_info()
#This prints: "Rob Robertson is 17 years old"

alice = Person("Alice","Anderson",28)
alice.print_info()
#This prints: "Alice Anderson is 28 years old"
```

- This way a potential programmer/user does not need to know the internal structure of the particular class, e.g. *list.append()*.

Course Management Program

- We want to write a program for the university
- It should give an overview over the different courses
- It should track each course, its lecturer and its students

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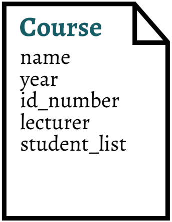
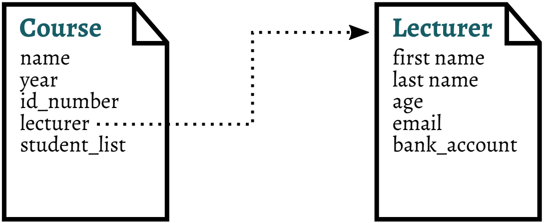
- It should give an overview over the different courses
- It should track each course, its lecturer and its students

How would an OOP model look like?

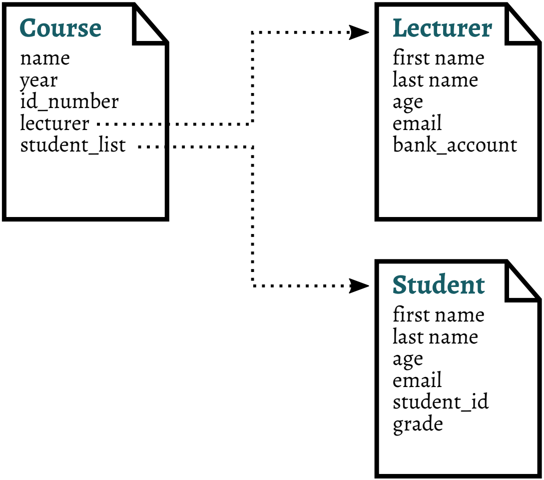
Course Management Program



Course Management Program



Course Management Program



Example Code

- The course class

```
class Course: #This defines the class Name
    #The __init__ function
    def __init__(self, name,year,id_number,lecturer):
        #The passed values are stored in the class
        self.name = name
        self.year = year
        self.id_number = id_number
        self.lecturer = lecturer
        self.student_list = [] #empty upon creation
```

Example Code

- The lecturer class

```
class Lecturer: #This defines the class Name
    #The __init__ function
    def __init__(self, first_name,last_name,age,email,
        ↪ bank_account):
        #The passed values are stored in the class
        self.first_name = first_name
        self.last_name = last_name
        self.age = age
        self.email = email
        self.bank_account = bank_account
```

Example Code

- Create the Course

```
lecturer_jan = Lecturer("Jan","Tekuelve",30,"jan.
    ↪ tekuelve@ini.rub.de",1234567)
cscience_course = Course("Computer Science and
    ↪ Mathematics",2019,1234,lecturer_jan)
```

Example Code

- Create the Course

```
lecturer_jan = Lecturer("Jan","Tekuelve",30,"jan.
    ↪ tekuelve@ini.rub.de",1234567)
cscience_course = Course("Computer Science and
    ↪ Mathematics",2019,1234,lecturer_jan)
```

- At the end of the year access the bank account:

```
c_bank_account = cscience_course.lecturer.bank_account
```

Example Code

- Create the Course

```
lecturer_jan = Lecturer("Jan","Tekuelve",30,"jan.
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    ↪ Mathematics",2019,1234,lecturer_jan)
```

- At the end of the year access the bank account:

```
c_bank_account = cscience_course.lecturer.bank_account
```

- This works independent of course and lecturer

The Student Class

- This class looks similar to the lecturer

```
class Student: #This defines the class Name
    #The __init__ function
    def __init__(self, first_name,last_name,age,email,
        ↪ student_id):
        #The passed values are stored in the class
        self.first_name = first_name
        self.last_name = last_name
        self.age = age
        self.email = email
        self.student_id = student_id
        self.grade = -1
```

Code Redundancy

Course
name
year
id_number
lecturer
student_list

Lecturer
first name
last name
age
email
bank_account

Student
first name
last name
age
email
student_id
grade

Code Redundancy

Course
name
year
id_number
lecturer
student_list

Lecturer
first name
last name
age
email
bank_account

Student
first name
last name
age
email
student_id
grade

Code Redundancy

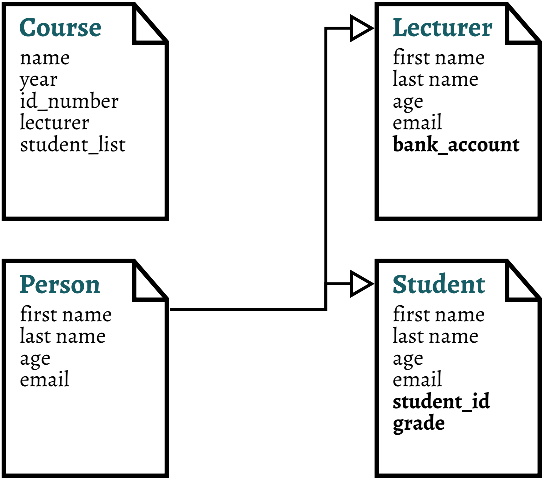
Course
name
year
id_number
lecturer
student_list

Lecturer
first name
last name
age
email
bank_account

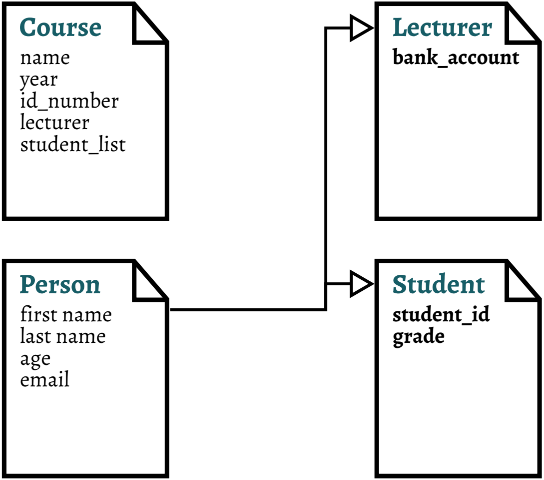
Person
first name
last name
age
email

Student
first name
last name
age
email
student_id
grade

Code Redundancy



Code Redundancy



The Person Class

- We will use the Class Person as *Super*-Class

```
class Person: #This defines the class Name
    #The __init__ function
    def __init__(self, first_name,last_name,age,email):
        #The passed values are stored in the class
        self.first_name = first_name
        self.last_name = last_name
        self.age = age
        self.email = email
```

Inheritance

- Lecturer and Student will inherit from Person

```
class Lecturer(Person): #Brackets declare inheritance
   #The __init__ function is overrriden
   def __init__(self,f_name,l_name,age,email,b_acc):
      #The super() calls the parent function
      super().__init__(f_name,l_name,age,email)
      self.bank_account = b_acc

class Student(Person): #Brackets declare inheritance
   #The __init__ function is overrriden
   def __init__(self,f_name,l_name,age,email,stud_id):
      super().__init__(f_name,l_name,age,email)
      self.student_id = stud_id
      self.grade = -1
```

Modifiying the Parent Class

- Functions of the parent class are available to child classes

```
class Person: #This defines the class Name
    def __init__(self, first_name,last_name,age,email):
        #The passed values are stored in the class
        self.first_name = first_name
        self.last_name = last_name
        self.age = age
        self.email = email

    #Our print_info function
    def print_info(self): #Note how the argument changed
        print(self.first_name +" " +self.last_name +" is
          ↪ " +str(self.age) +" years old.")
```

Using Parent Functions

- Functions of the parent class are available to child classes

```
student_rob = Student("Rob","Robertson",25,"rob.
    ↪ robson@rub.de","108001024")
lecturer_jan = Lecturer("Jan","Tekuelve",30,"jan.
    ↪ tekuelve@ini.rub.de",1234567)

student_rob.print_info()
lecturer_jan.print_info()
#Prints:
#Rob Robertson is 25 years old.
#Jan Tekuelve is 30 years old.
```

Completing the Example

- The course needs to be able to add students

```
#Inside the Course class
def enroll(self,student):
        self.student_list.append(student)
        #Enroll adds them to the course internal list
```

- Minimal example:

```
cscience_course = Course("Computer Science and
    ↪ Mathematics",2019,1234,lecturer_jan)
student_rob = Student("Rob","Robertson",25,"rob.
    ↪ robson@rub.de","108001024")
cscience_course.enroll(student_rob)
```

Creating your own Python Modules

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- You can access the definitions in another script from the same folder:

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student_rob = unimanager.Student("Rob","Robertson",25,"
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```

- This allows for flexible re-usability of code

Advantages/Disadvantages of OOP

Advantages:

- Design Benefit: Real/World processes are easily transferable in code
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Disadvantages:

- Desing Overhead: Modeling requires longer initial development time
- Originally OOP required more “coding”

Tasks

1. Download todays class definitions *unimanager.py* and create a separate script that uses this module to create a course, a lecturer and three sample students.
   - Enroll all students to the course.
   - After enrolling iterate through the student list to print the info of all enrolled students. You can access the student_list via the course object.
   - In the loop use the *print_info()* function.

2.* Add a *print_info()* function to the class definition of Course in *unimanager.py*. This function should print the course name, its lecturer and each student of the course with his/her student ID.
   - The function should be defined in the Course class and its only argument should be self
   - The course name, the lecturer and its student_list can be accessed via the self keyword.

This concludes the Preparatory Course.

Any Questions or Feedback?