Lecture 3
Coordinate Systems and Trigonometry

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Computer Science and Mathematics
Preparatory Course

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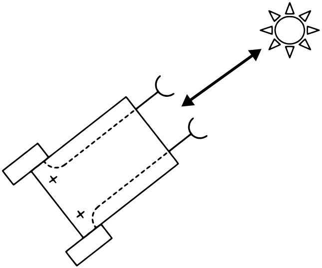
Motivation - Coordinate Systems

How far is the source away?



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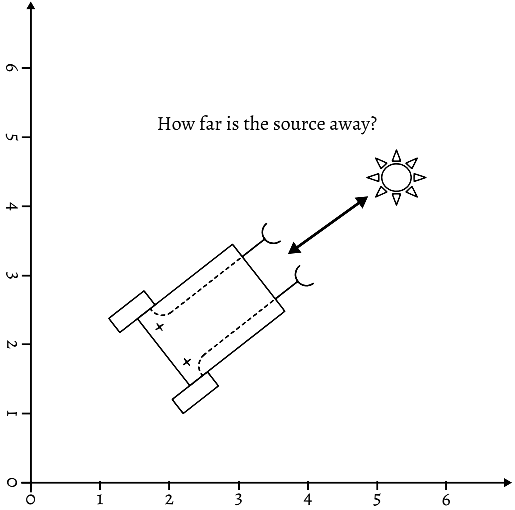

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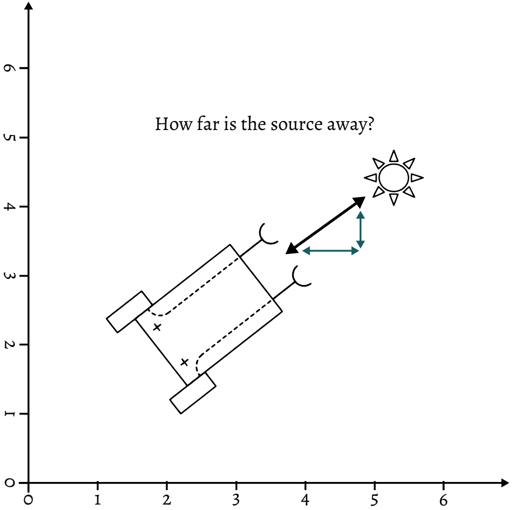

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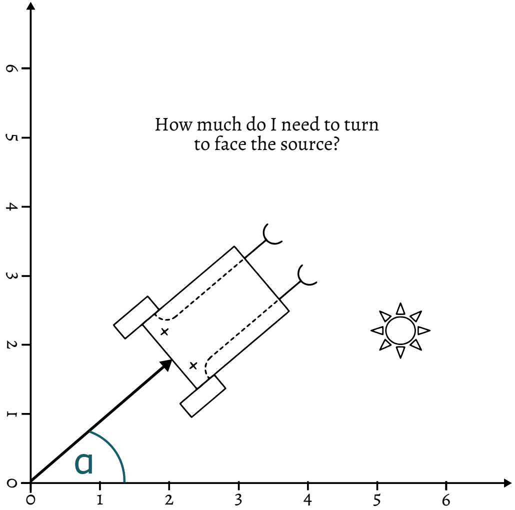

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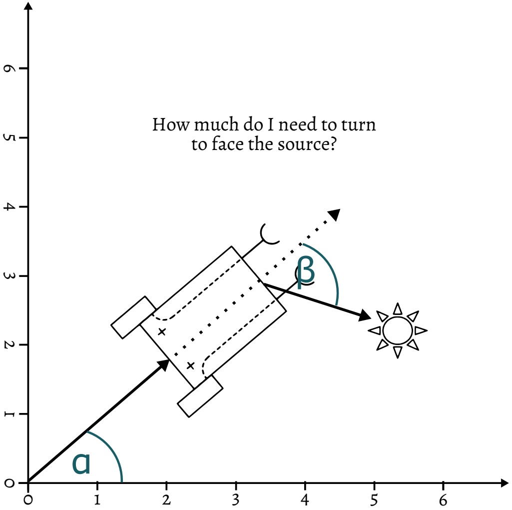

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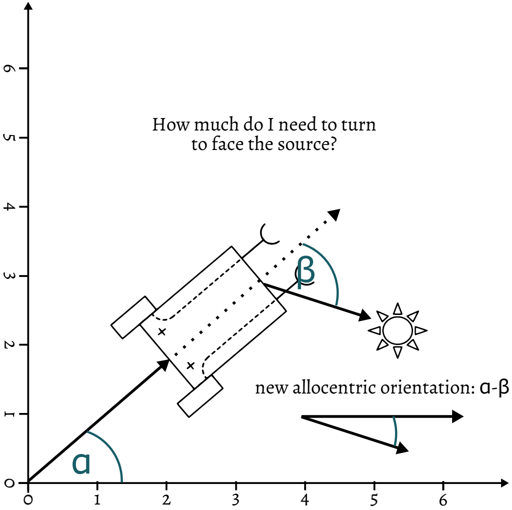

The number π

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$$\frac{75.39}{24} = 3.14159\ldots = \pi$$

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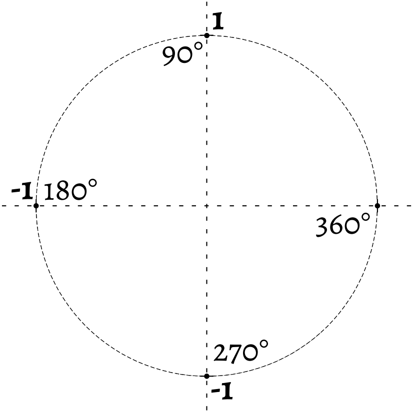
Circumference of a circle: $2\pi r$

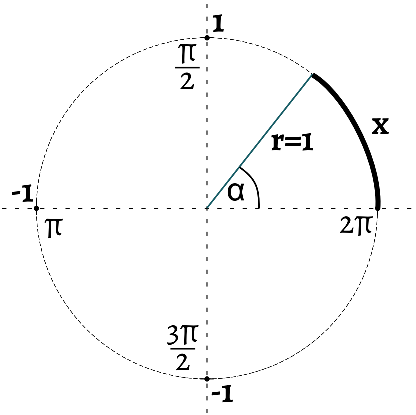
Measuring Angles

- Defining a full angle as 360° is common but actually arbitrary



Measuring Angles

- Defining a full angle as 360° is common but actually arbitrary
- Less arbitrary is the use of the actual length of the enclosed arc-segment called the **Radian**



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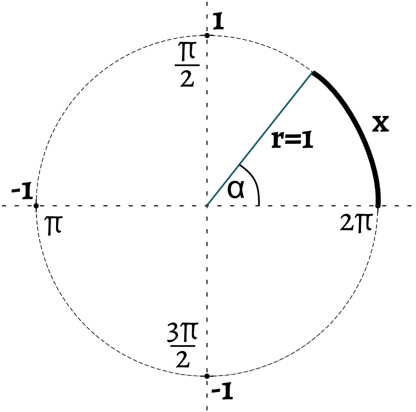
- Thus $360^\circ = 2\pi$, $90^\circ = \frac{\pi}{2}$, $180^\circ = \pi \ldots$



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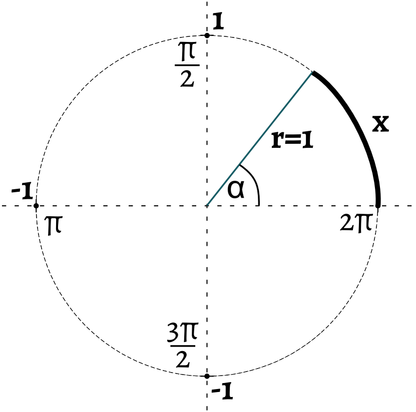
- Rad x to Degree: $x \cdot \frac{180^\circ}{\pi}$



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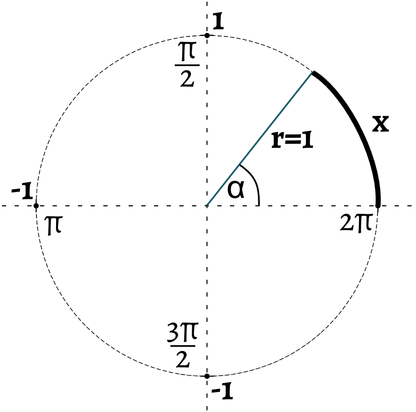
- Degree d to Rad: $d \cdot \frac{\pi}{180^\circ}$



Angle Conversion Examples

- Degree to Radians: $d \cdot \frac{\pi}{180^\circ}$

$$\alpha_{\text{deg}} = 34^\circ$$

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$$\begin{aligned} & \alpha_{\text{deg}} = 34^\circ \\ \iff & 34^\circ \cdot \frac{\pi}{180^\circ} \\ \iff & \frac{34^\circ \cdot \pi}{180^\circ} \iff \frac{106.81^\circ}{180^\circ} \iff 0.593 = \alpha_{\text{rad}} \end{aligned}$$

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$$\alpha_{\text{rad}} = \frac{3}{4}\pi$$

Angle Conversion Examples

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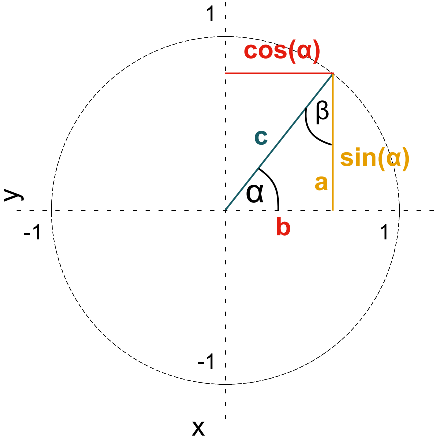
Calculating Angles in a Right triangle



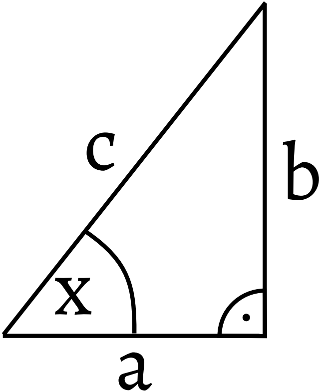
- Sine and cosine are defined in the unit circle.

$$a = cos(\alpha) \iff \alpha = cos^{-1}(a)$$
$$b = sin(\alpha) \iff \alpha = sin^{-1}(b)$$

- Click here for interactive demo.

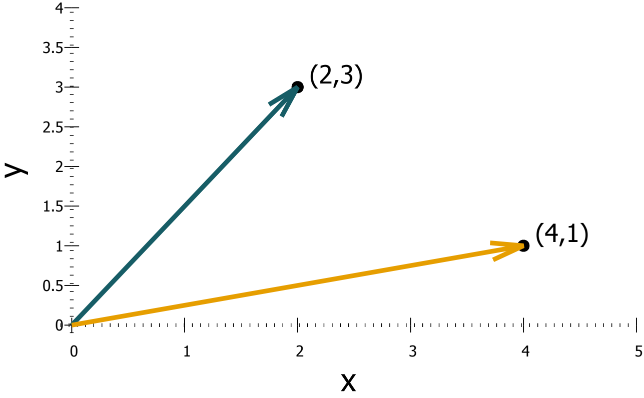
Rules for any Right Triangle

- $a^2 + b^2 = c^2$
- $sin(x) = \frac{b}{c} = \frac{\text{opposite}}{\text{hypothenuse}}$
- $cos(x) = \frac{a}{c} = \frac{\text{adjacent}}{\text{hypothenuse}}$
- $tan(x) = \frac{sin(x)}{cos(x)} = \frac{b}{a} = \frac{\text{opposite}}{\text{adjacent}}$



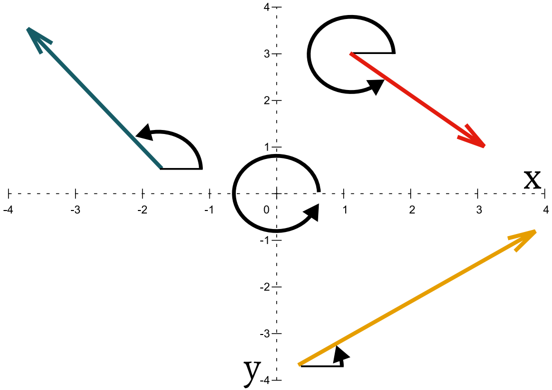
Vectors in the Cartesian Coordinate System

A vector $\boldsymbol{v} = \begin{pmatrix} v_x \\ v_y \end{pmatrix}$ is defined as an arrow from the origin to the point (v_x, v_y)



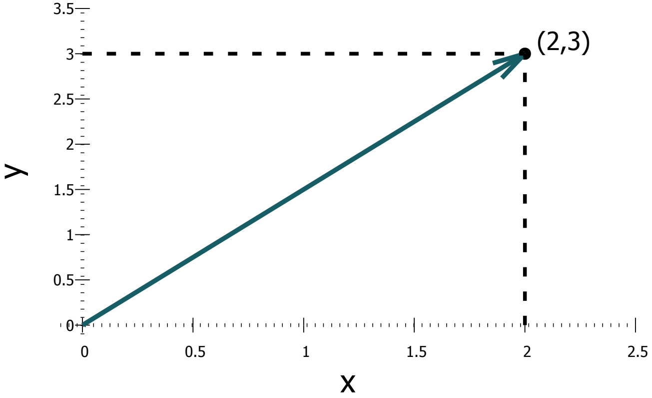
Angles in a Coordinate System

Vector orientation with respect to a coordinate system is defined by translating the origin onto the vectors tail



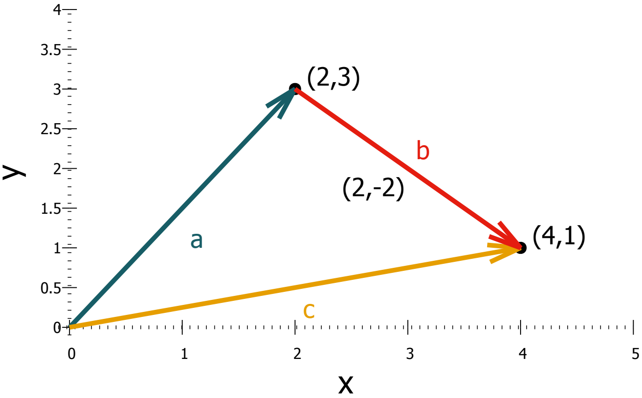
Vector Norm

The norm or length $|\boldsymbol{v}| = \sqrt{{v_x}^2 + {v_y}^2}$ of a vector $\boldsymbol{v} = \begin{pmatrix} v_x \\ v_y \end{pmatrix}$ is calculated using the Pythagorean theorem



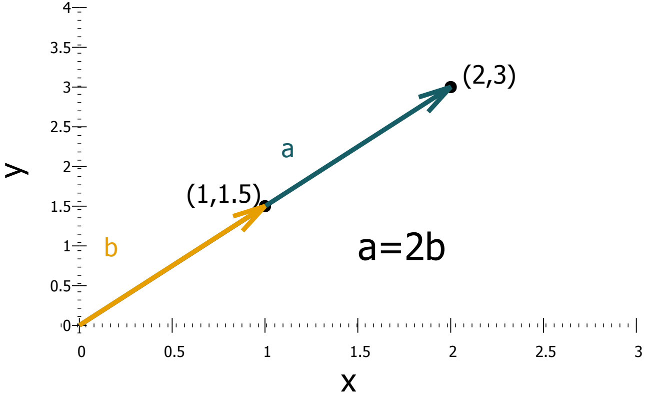
Vector Addition

$$\begin{pmatrix} a_x \\ a_y \end{pmatrix} + \begin{pmatrix} b_x \\ b_y \end{pmatrix} = \begin{pmatrix} a_x + b_x \\ a_y + b_y \end{pmatrix} = \begin{pmatrix} c_x \\ c_y \end{pmatrix}$$



Scalar Multiplication

$$s\boldsymbol{a} = s\begin{pmatrix} a_x \\ a_y \end{pmatrix} = \begin{pmatrix} sa_x \\ sa_y \end{pmatrix}$$



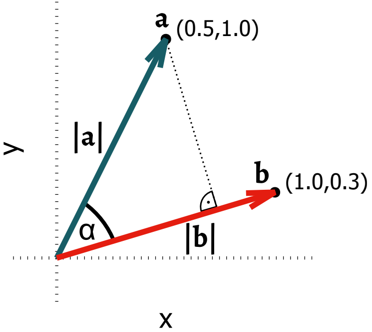
Scalar Product

- The **scalar product** $< \boldsymbol{a}, \boldsymbol{b} >$ or $\boldsymbol{a} \cdot \boldsymbol{b}$ of two vectors is defined as:

$$< \boldsymbol{a}, \boldsymbol{b} >= |\boldsymbol{a}||\boldsymbol{b}|cos(\alpha)$$

  and results in a **scalar** value.

- Graphical Interpretation:



Scalar Product: Special Cases

- If both vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ point in the same direction:

$$< \boldsymbol{a}, \boldsymbol{b} >= |\boldsymbol{a}||\boldsymbol{b}|cos(0) = |\boldsymbol{a}||\boldsymbol{b}|$$

- If both vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ are orthogonal to each other:

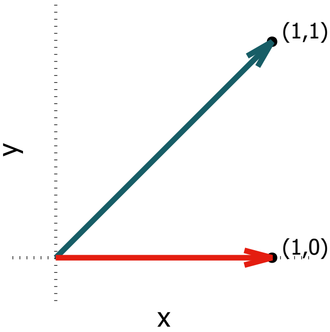
$$< \boldsymbol{a}, \boldsymbol{b} >= |\boldsymbol{a}||\boldsymbol{b}|cos(90^\circ) = 0$$

- Alternatively it can be calculated the following way:

$$< \boldsymbol{a}, \boldsymbol{b} >=< \begin{pmatrix} a_x \\ a_y \end{pmatrix}, \begin{pmatrix} b_x \\ b_y \end{pmatrix} >= a_x b_x + a_y b_y$$

Angle between Vectors

- The scalar product can be used to calculate the angle between two vectors



$$< \boldsymbol{a}, \boldsymbol{b} > = |\boldsymbol{a}||\boldsymbol{b}| cos(\alpha)$$

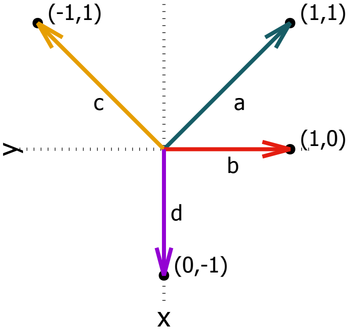
$$\alpha = arccos\left(\frac{< \boldsymbol{a}, \boldsymbol{b} >}{|\boldsymbol{a}||\boldsymbol{b}|}\right)$$

$$\alpha = arccos\left(\frac{1 * 1 + 1 * 0}{\sqrt{2} * 1}\right)$$

$$\alpha = arccos\left(\frac{1}{\sqrt{2}}\right)$$

$$\alpha = \frac{\pi}{4} = 45^\circ$$

Orthogonal Vectors



$$< \boldsymbol{a}, \boldsymbol{c} > = a_x c_x + a_y c_y$$
$$= -1 \cdot 1 + 1 \cdot 1 = 0$$

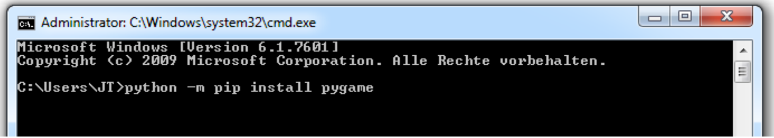
$$< \boldsymbol{b}, \boldsymbol{d} > = b_x d_x + b_y d_y$$
$$= 1 \cdot 0 + 0 \cdot -1 = 0$$

PIP installs Packages

- Pip is a helper tool that downloads and installs additional python modules. You **need an internet connection**.
- Pip can be called from the console with

```
python -m pip install <modulename>
```

- Example:



Modules for the Course

- We will need the *pygame* and *matplotlib* modules
- Matplotlib should already be installed through Anaconda
- Make sure you have a working internet connection
- Execute the following command:

```
python -m pip install pygame
```

- A message like “Sucessfully installed ...” should be displayed after each command terminated.

The Matplotlib Module

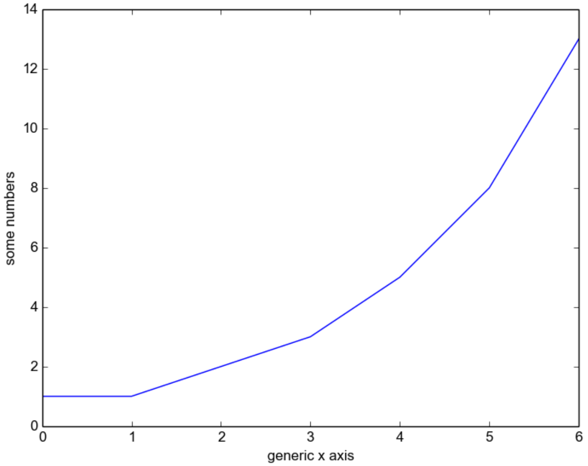
- Matplotlib is the most prominent plotting library for Python
- It was originally developed to create Matlab-like plots for free

Matplotlib.pyplot

- We will use the pyplot submodule

```
# A submodule can be imported with the . operator
import matplotlib.pyplot as plt
# The as operator allows renaming for convenience
numbers = [1,1,2,3,5,8,13]
# It is assumed that the list is a list of y-values
plt.plot(numbers)
# This generates the plot, but does not display
plt.ylabel("some numbers")
plt.xlabel("generic x axis")
plt.show()
# An alternative to showing would be to save the image
```

Result



Pyplot

- Helpful Pyplot Commands

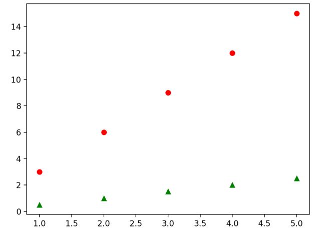
```
#Define the x and y arrays and the line appearance
#'ro' stands for red dots, 'b-' for blue lines
plt.plot([1,2,3,4], [1,4,9,16], "ro",linewidth=2.0)
#Explicitly define the range of the axis
plt.axis([0, 6, 0, 20])
#Save the plot as an image with a desired resolution
plt.savefig("myplot.png",dpi=200)
```

- Find detailed examples here https://matplotlib.org/users/pyplot tutorial.html

Pyplot

- Multiple plots in one figure

```
x = [1,2,3,4,5]
y1 = [3,6,9,12,15]
y2 = [0.5,1,1.5,2,2.5]
plt.plot(x,y1,"ro",x,y2,"g^")
```



The Pygame module

- Pygame contains a set of modules designed for video game writing
- It is an open-source project since 2000, latest update 2017
- Its classes allow high-level game programming

Setting up an environment

- Every pygame script should contain this

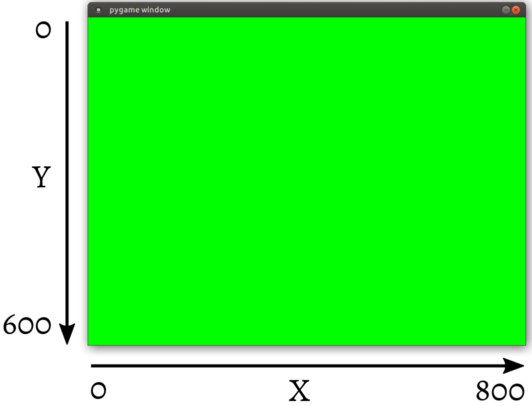
```
import pygame, sys #Import pygame and system functions
from pygame.locals import * #Import all pygame modules
pygame.init() #Initialize all modules
```

- Set up a 800x600 frame

```
#Define the frame size
frame = pygame.display.set_mode((800, 600))
#Fill the frame with a R,G,B color
green = (0,255,0)
frame.fill(green)
pygame.display.flip() #!Important! Update the display
```

Pygame Coordinate System

The Pygame coordinate System has its origin in the top left corner



The Game Loop

- A game should only end through user interaction

```
#This loop runs forever
while True:
    #pygame.event catches user interaction in a list
    for event in pygame.event.get():
            #For example a click on the close-button
            if event.type == QUIT:
                #This exits the game appropriately
                pygame.quit()
                sys.exit()
```

- For simplicity the pygame.event for-loop will be omitted in future slides

Positioning Objects

- pygame.Rect - object for storing rectangular coordinates

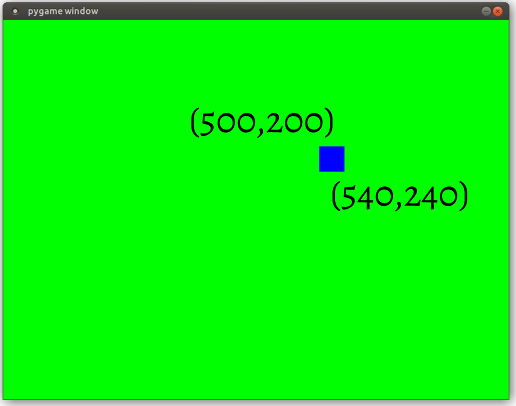
```
#pygame.Rect((left, top), (width, height))
#A square at Pos 500,200 with size 40
square = pygame.Rect((500,200),(40,40))
```

- Rects can be drawn on the screen

```
#pygame.draw.rect(screen, color, pygame.rect)
pygame.draw.rect(frame, (0,0,255), square)
pygame.display.flip() #Always call flip to draw
```

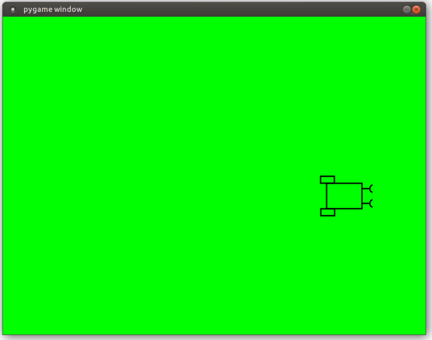
Draw Rectangle Example

```
square = pygame.Rect((500,200),(40,40))
pygame.draw.rect(frame, (0,0,255), square)
```



Loading Images

```
#Loads the Image
vehicle = pygame.image.load("braitenberg.png")
vehicle.convert() #Converts the image to game coordinates
frame.blit(vehicle,(600,300)) #Places it on the screen
```



Using the Game-Loop

- Moving the vehicle across the screen

```
# We loaded the image in vehicle
# and set up a screen in frame
xPos = 100 #Start Position
frame.blit(vehicle,(xPos,300)) #Draw the vehicle

while True:
    xPos = xPos +1 #Increase the xPos

    frame.blit(vehicle,(xPos,300)) #Draw at the new pos
    pygame.display.flip() #Show the Updates
```

Using the Game-Loop

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- **This draws all vehicles on top of another!**

Using the Game-Loop

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# and set up a screen in frame
xPos = 100 #Start Position
frame.blit(vehicle,(xPos,300)) #Draw the vehicle

while True:
    xPos = xPos +1 #Increase the xPos
    frame.fill((0,255,0)) #Paint over the old canvas
    frame.blit(vehicle,(xPos,300)) #Draw at the new pos
    pygame.display.flip() #Show the Updates
```

Helpful Functions

- Pygame

```
# Introduces a pause between each game loop
pygame.time.delay(100)
rot_sprite = pygame.transform.rotozoom(player_image,
    angle,1)
# This rotates an image to angle degrees
```

- Trigonometry:

```
math.pi #The number pi
math.asin(x) # sin^{-1}(x)
math.acos(x) # cos^{-1}(x)
math.degrees(radianValue) # radian to degree
```

Task Template

- For this task download the file *task_3_1.zip* from the course website and extract its contents in a folder of your choice.
- It contains the files *task_3_1_environment.py* and *task_3_1_student_code.py*.
- You will only change code in *task_3_1_student_code.py*.

Explain Task Template!

Angle Calculation

1. Calculate the angle between Vector A and Vector B by implementing *calculate_angles_via_trigonometry*.
   - Implement *calculate_angles_via_trigonometry* using the appropriate formula for a right triangle
   - Use *math.asin* to get sin^{-1} and *math.degrees()* to convert radian in angle
2. Calculate the angle between Vector A and Vector B by implementing the remaining functions.
   - Calculate the scalar product using a for loop that iterates through each element of the vector
   - In each loop multiply the current element of each vector and add the result to a sum variable
   - Similarly implement the norm function by iterating through a single vector
   - Use both functions to implement the scalar product to angle formula

Pyplot Task (optional)

Take your script from the previous lecture that stores function values in a list.

1. Extend the script by also storing the x-values in a second list. Use the x and $f(x)$ list to plot your polynomial function.
2. Generate another $f(x)$ list with the same x-values, but other coefficients a_0 to a_4. Plot both functions in the same plot.
3. Save one of your plots as a '.png' image with 300 dpi. Add labels at your own discretion.