# Lecture 2

# Functions in Math and Programming

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Computer Science and Mathematics
Preparatory Course

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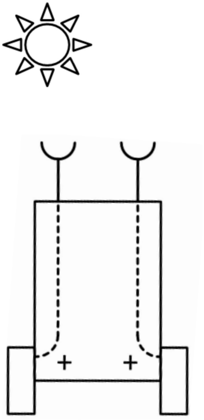
# Overview

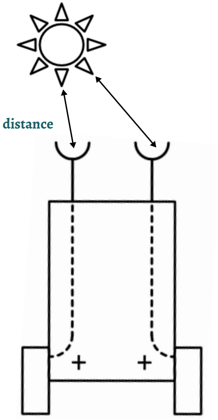
1. **Motivation**
2. **Functions in Math**
   - Definition
   - Parametrization
   - Function Types
   - Properties
3. **Programming**
   - Functions
   - Lists
   - Writing Files
4. **Tasks**

# Functions in Braitenberg Vehicles




2a

# Functions in Braitenberg Vehicles

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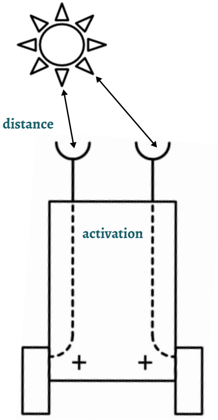


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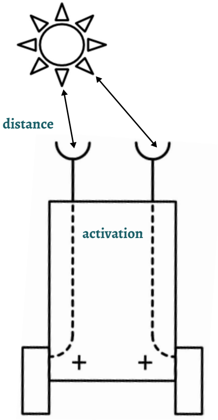


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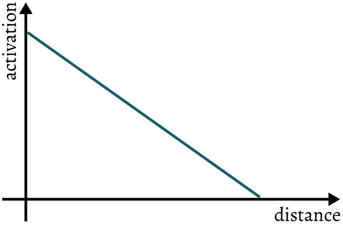

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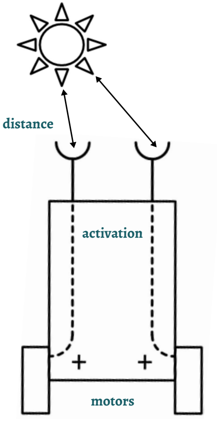


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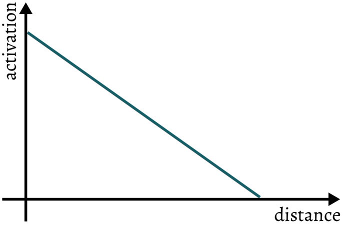

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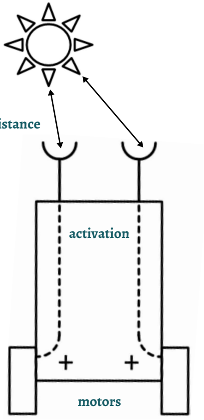


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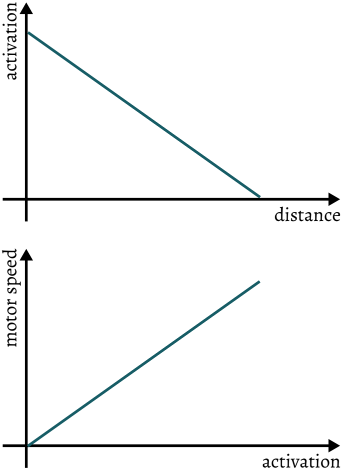

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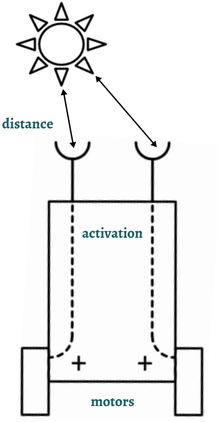


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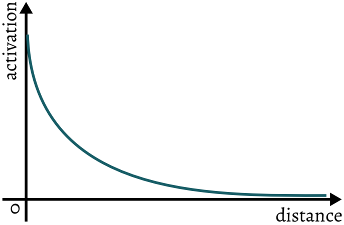
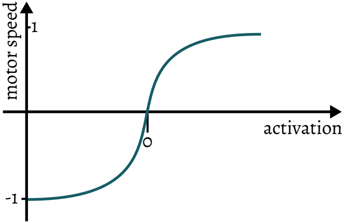


# Function Definition

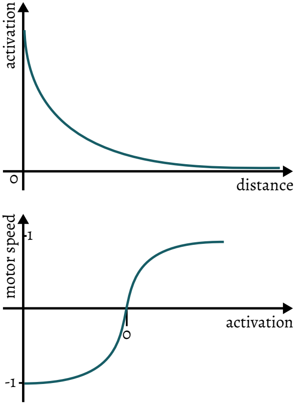
## Function

$X$ and $Y$ are two non-empty sets.
A **function** $f : X \to Y$, assigns each element $x \in X$ exactly one element $y \in Y$.

$$x \to y = f(x)$$

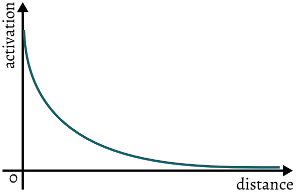
- $x$ is called the **function argument**
- $y$ is called the **function value**
- $X$ is called the **domain of a function**
- $Y$ is called the **codomain of a function**
- The **image** $W$ of $f(x)$ are all values in $Y$ that can be assumed

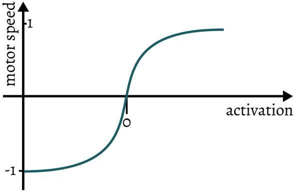
# Braitenberg Examples



# Braitenberg Examples

- Assumptions:
  - Distance $d$ may assume positive values
  - Activation $a$ may assume any value
  - Motor Speed $s$ may assume any value between -1 and 1



# Braitenberg Examples

- Assumptions:
  - Distance $d$ may assume positive values
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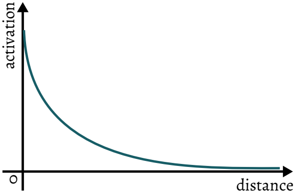
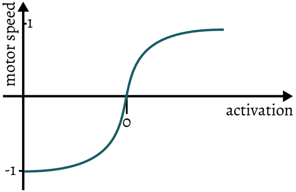
- $f : \mathrm{d} \to \mathrm{a}$
  - $f(d) = e^{-d}$
  - $X = \mathbb{R}^+,\ Y = \mathbb{R}$
  - $W = \mathbb{R}^+$



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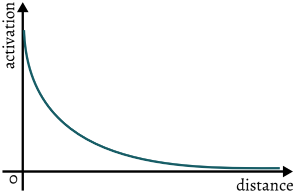
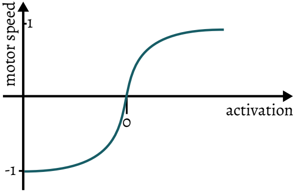
  - $f(d) = e^{-d}$
  - $X = \mathbb{R}^+,\ Y = \mathbb{R}$
  - $W = \mathbb{R}^+$
- $g : \mathrm{a} \rightarrow \mathrm{s}$
  - $g(a) = \tanh(a)$
  - $X = \mathbb{R},\ Y = \mathbb{R}$
  - $W = [-1, 1]$



# Plotting Functions

Tabular Interpretation of: $f(x) = 2x + 3$

| **x** | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| **y** | | | | | | |

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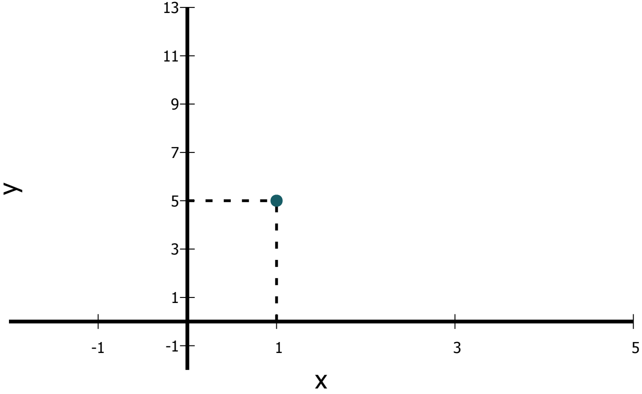

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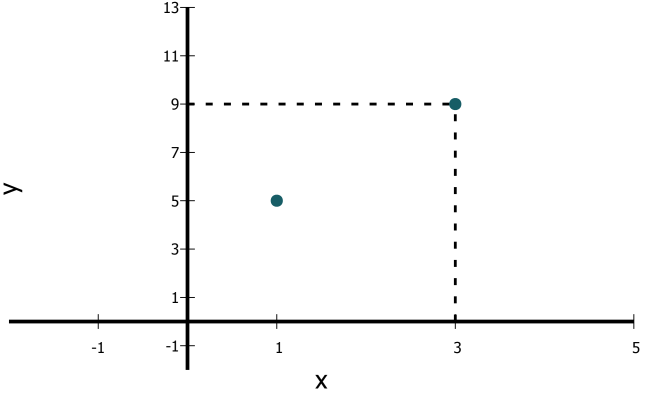
Tabular Interpretation of: $f(x) = 2x + 3$

| **x** | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| **y** | | 5 | | 9 | | |



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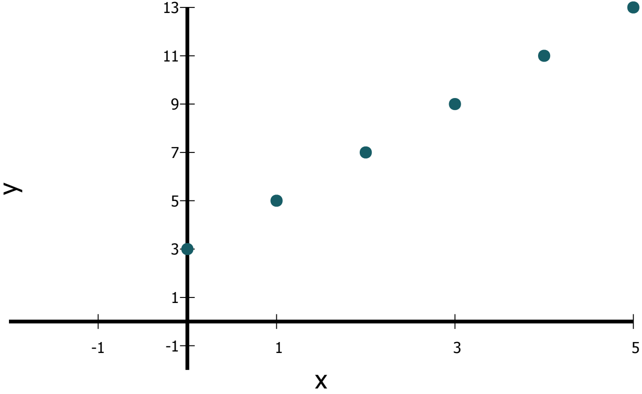
Tabular Interpretation of: $f(x) = 2x + 3$

| | | | | | | |
|---|---|---|---|---|---|---|
| **x** | 0 | 1 | 2 | 3 | 4 | 5 |
| **y** | 3 | 5 | 7 | 9 | 11 | 13 |



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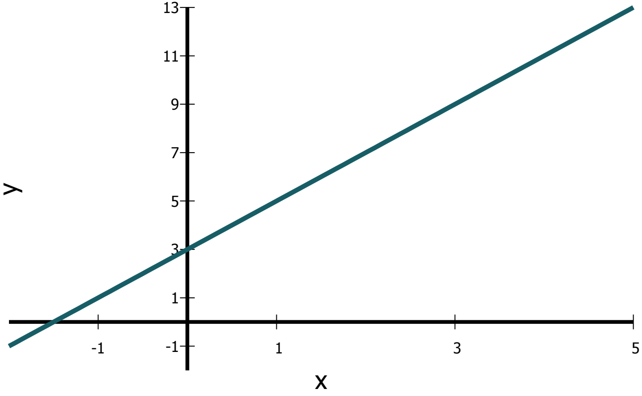
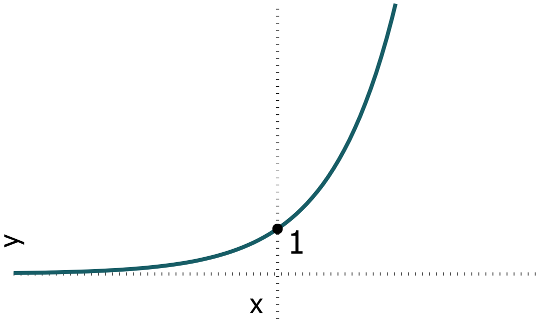


# Function Translation

- Translation in $y$-direction: $\hat{f}(x) = f(x) + b$
- Translation in $x$-direction: $\hat{f}(x) = f(x - a)$




$f(x) = e^x$

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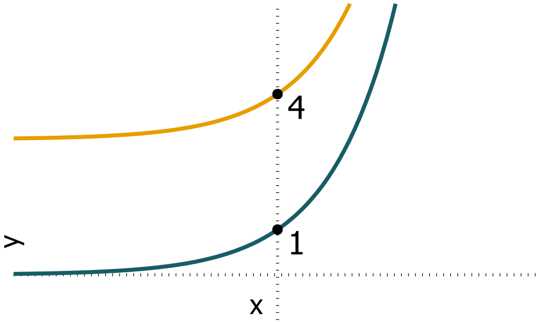


$f(x) = e^x$ $\quad$ $g(x) = e^x + 3$

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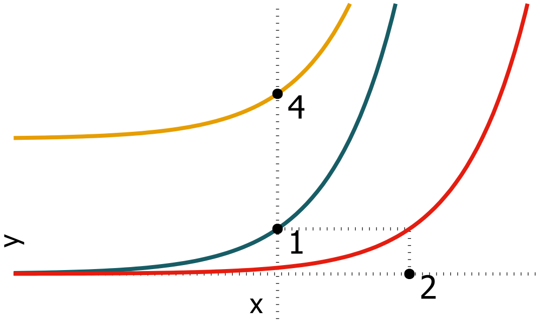
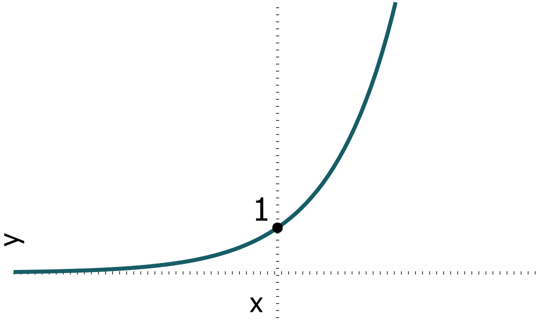



$f(x) = e^x$ $\quad$ $g(x) = e^x + 3$ $\quad$ $h(x) = e^{x-2}$

# Function Stretching and Compression

- Stretching/Compression in ***y*-direction**: $\hat{f}(x) = \boldsymbol{d}f(x), d > 0$
- Stretching/Compression in ***x*-direction**: $\hat{f}(x) = f(\boldsymbol{c}x), c > 0$

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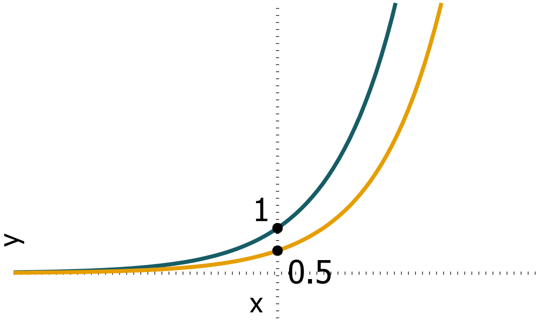


$f(x) = e^x$ $\quad$ $g(x) = \frac{1}{2}e^x$

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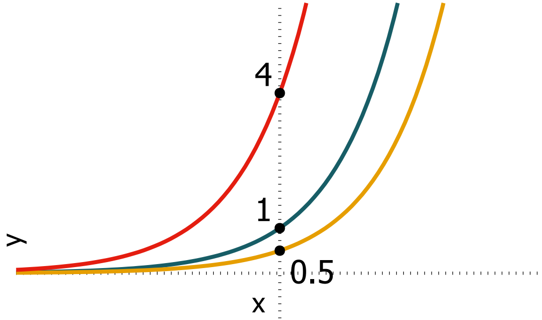


$f(x) = e^x$ $\quad g(x) = \frac{1}{2}e^x$ $\quad h(x) = 4e^x$

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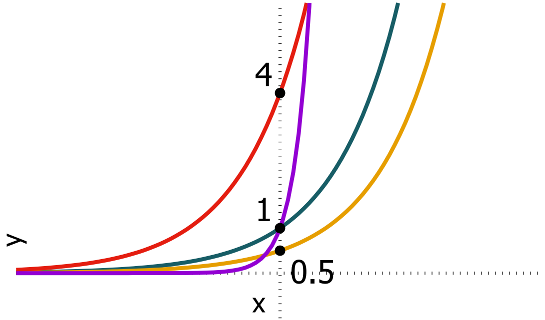
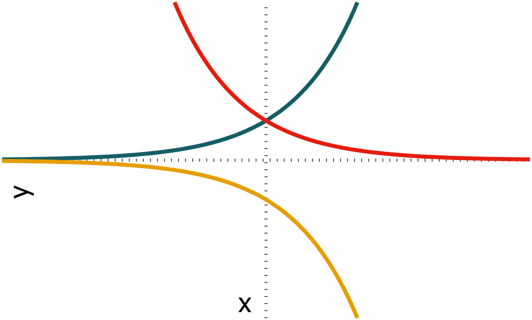



$f(x) = e^x$ $\quad g(x) = \frac{1}{2}e^x$ $\quad h(x) = 4e^x$ $\quad j(x) = e^{4x}$

# Function Reflection

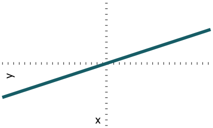
- Reflection across the ***y*-axis**: $\hat{f}(x) = f(-x)$
- Reflection across the ***x*-axis**: $\hat{f}(x) = -f(x)$




$f(x) = e^x$ $g(x) = -e^x$ $\mathrm{h(x)} = e^{-x}$

# Function Types

- **Linear Functions**
  $y = mx + b$



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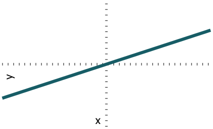
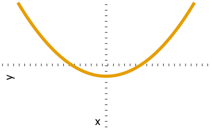



- **Power Functions**
  $y = ax^n$



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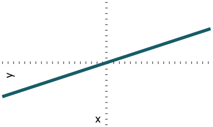


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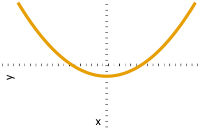
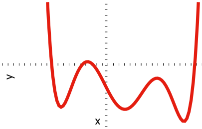


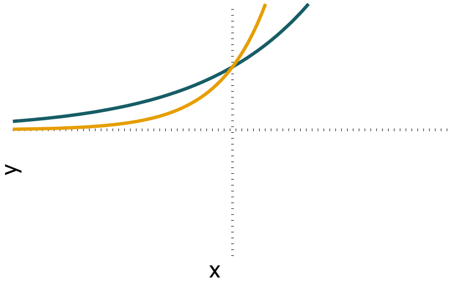

- **Polynomial Functions**
  $y = \sum_{i=0}^{n} a_i x^i$
  $y = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \ldots a_n x^n$
  describe a polynomial of degree $n$, where $a_n \neq 0$



# Exponentials Functions

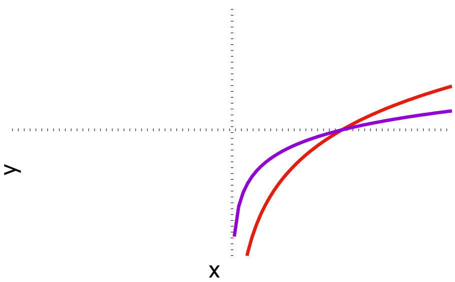
### Exponential Functions

$f(x) = e^x$ $g(x) = 10^x$



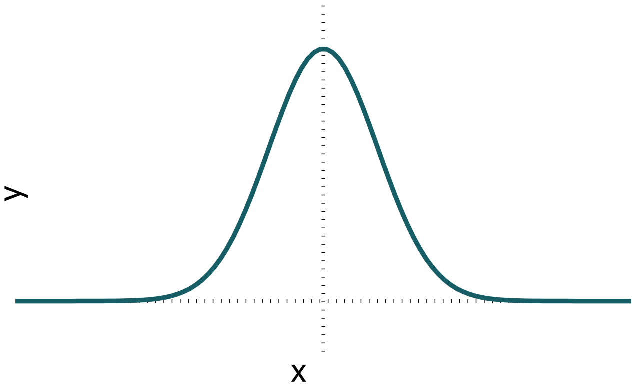

### Logarithmic Functions

$h(x) = ln(x)$ $j(x) = log_{10}(x)$



# The Gaussian Function

$$f(x) = e^{-x^2}$$

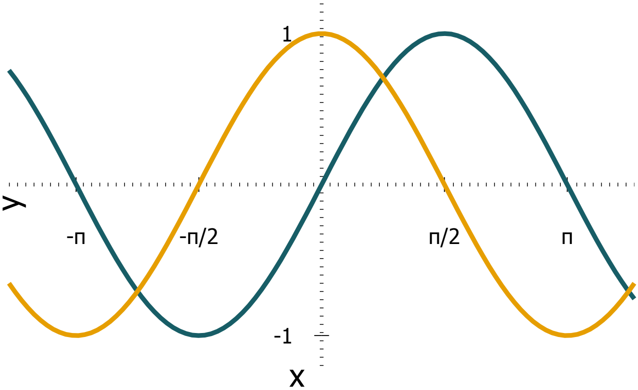


# The Gaussian Function

$f(x) = e^{-x^2}$ $g(x) = e^{-(x-2)^2}$

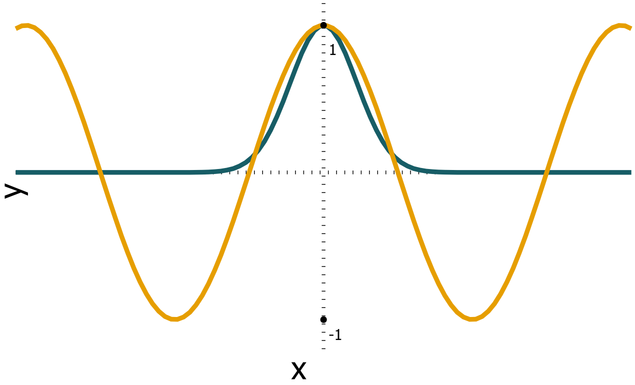
# Trigonometric Functions

$f(x) = sin(x)$ $g(x) = cos(x)$



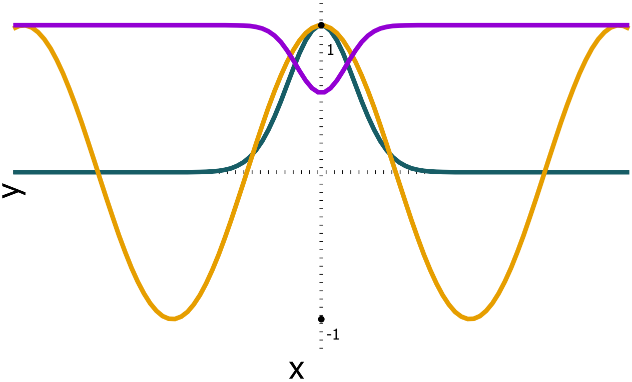
# Chaining Functions

$f(x) = e^{-x^2}$ $\quad$ $g(x) = cos(x)$



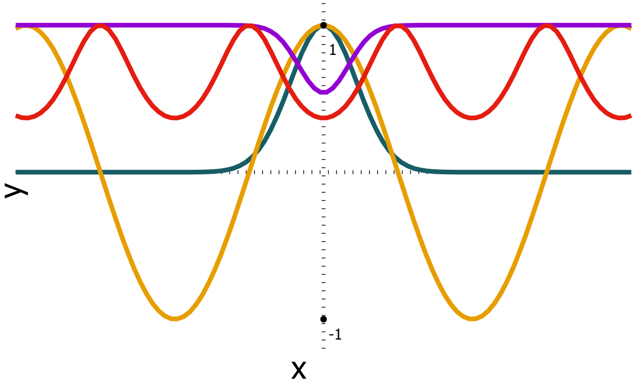
# Chaining Functions

$f(x) = e^{-x^2}$ $\quad g(x) = cos(x)$ $\quad h(x) = g(f(x))$



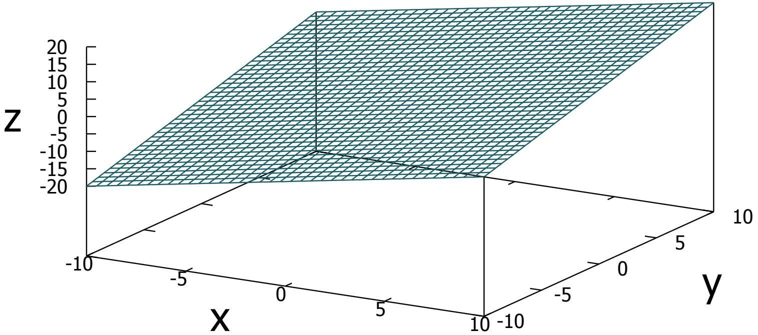
# Chaining Functions

$f(x) = e^{-x^2}$ $g(x) = cos(x)$ $h(x) = g(f(x))$ $j(x) = f(g(x))$



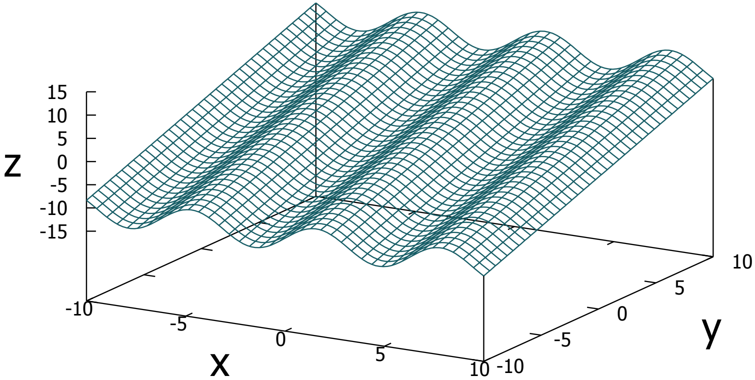
# Multiple Function Arguments

$$f(x,y) = x + y$$



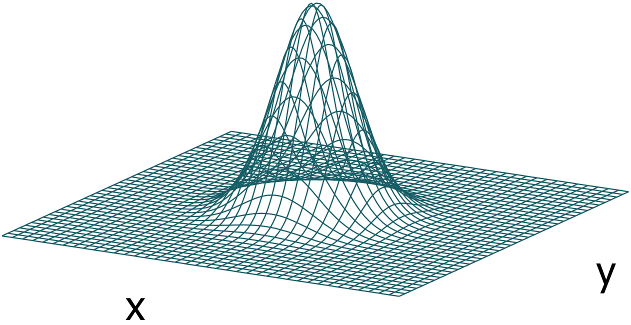
# Multiple Function Arguments

$$f(x, y) = sin(x) + y$$



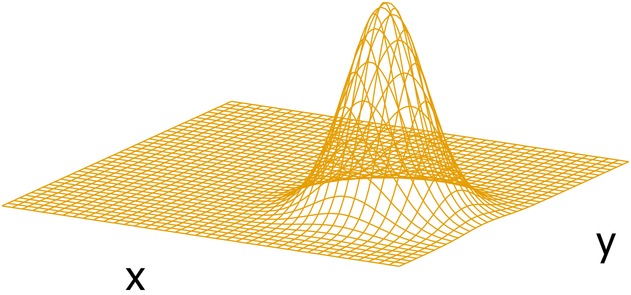
# Multiple Function Arguments

$$f(x, y) = e^{-(x^2+y^2)}$$



# Multiple Function Arguments

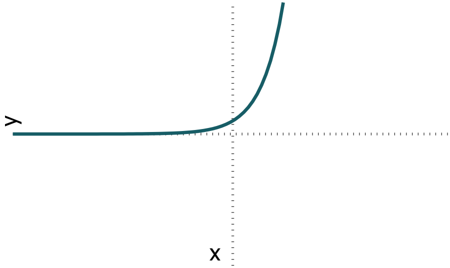
$$f(x,y) = e^{-((x-2)^2+(y+1)^2)}$$



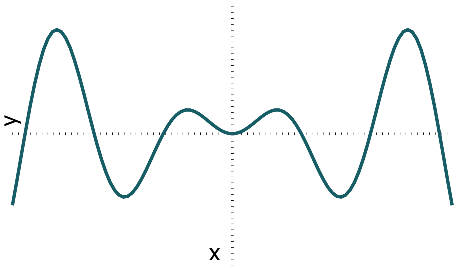
# Injective and Surjective

- An image $f$ is **injective**, if two different elements $x_1 \neq x_2$ are always projected to two different elements $y_1 \neq y_2$
- An image $f$ is **surjective**, if for each element $y \in Y$ one $x \in X$ exists, such that $y = f(x)$

Injective, but not surjective
$f(x) = e^x$



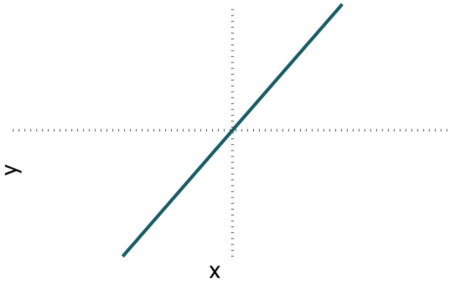

Surjective, but not Injective
$f(x) = x \sin(x)$



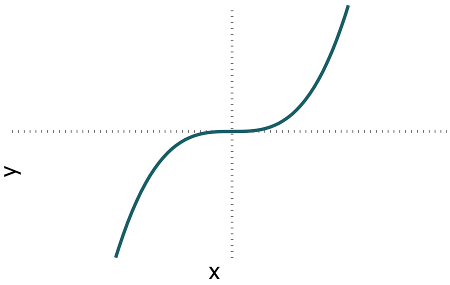
# Bijective Functions

- An image $f$ is **bijective**, if it is injective and surjective

$$f(x) = 4x$$



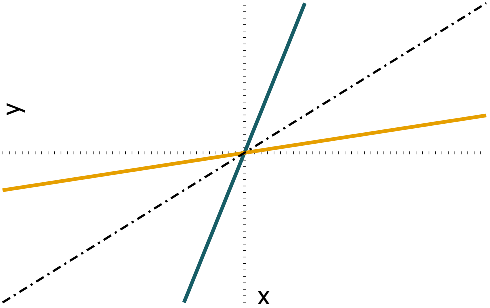

$$f(x) = x^3$$



# Inverse Function

## Definition

If a **function** $f : X \to Y$ is bijective, than $f^{-1} : Y \to X$ describes the **inverse function** of $f$




$f(x) = 4x$ $\quad$ $f^{-1}(x) = \frac{1}{4}x$

# Monotonicity

## Definition

- A function $f : \mathbb{R} \to \mathbb{R}$ is called **monotonically increasing**, if for all $x_1, x_2$ order is preserved by applying $f$:

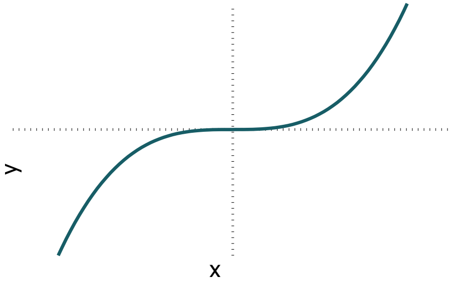
$$x_1 \leq x_2 \Rightarrow f(x_1) \leq f(x_2)$$

- A function $f : \mathbb{R} \to \mathbb{R}$ is called **monotonically decreasing**, if for all $x_1, x_2$ order is reversed by applying $f$:

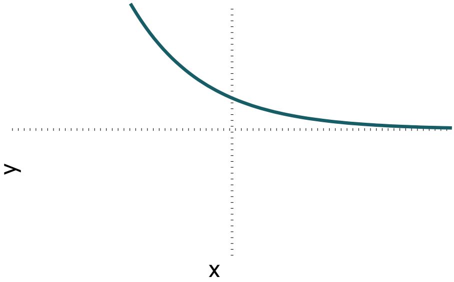
$$x_1 \leq x_2 \Rightarrow f(x_1) \geq f(x_2)$$

# Monoticity Examples

monotonically increasing




monotonically decreasing



# Functions in Python

- Functions in programming describe a parameterizable routine
- Functions are defined like this:

```
def greeting(person): #greeting is the function name
  print("Hello " +person) #person is the argument
#print is also a function
```

- Functions are called like this:

```
greeting("Bob") #"Hello Bob"
name = "Alice"
greeting(name) #"Hello Alice"
```

# Returning Values

- Functions may return values to the program

```
def square(value):
  return value*value
```

- The return value may be assigned like any variable

```
x = 3
y = square(x) #y=9, x=3 arguments stay the same
square(square(2)) #8 Functions may be chained
```

- **!** Data types are not explicitly specified

```
x = square("Bob") #This results in an error!
```

# Multiple Function Arguments

- Functions may have multiple arguments

```
def subract(minuend,subtrahend):
  return minuend-subtrahend
```

- Advanced Function calling:

```
result = subract(9,2) #result=7
result = subtract(minuend=9,subtrahend=2) #result=7
result = subtract(subtrahend=2,minuend=9) #result=7
result = subtract(9,subtrahend=2) # error!
```

# Optional Function Arguments

- Some Arguments may be declared optional

```
def subract(minuend,subtrahend,console=False):
  if console:
    print(str(minuend-subtrahend))
  return minuend-subtrahend
```

- Optional arguments may be ignored while calling:

```
result = subract(9,2) #result=7
result = subtract(9,2,True) #result=7 and prints
result = subtract(9,2,False) #is equal to subtract(9,2)
```

# The List Datatype

- Lists allow to manage a collection of variables

```
names = ["Alice","Bob","Carl","Dora"]
numbers = [1,2,3,5,8]
```

- Accessing and modifying elements in a lists

```
print(names) #['Alice','Bob','Carl','Dora']
single_name = names[2] #single_name = 'Carl'
first_element = numbers[0] #first_element = 1
last_name = names[len(names)-1]#last_name = 'Dora'

names[1] = "Bert" #names ['Alice','Bert','Carl','Dora']
```

# Operations on Lists

- Example Operations

```
numbers = [1,2,3,5,8]
names = ["Alice","Bob","Carl"]
count = len(names) #count=3
names.append("Daisy") #['Alice','Bob','Carl','Daisy']
numbers2 = [13,21,34]
numbers3 = numbers + numbers2 #[1,2,3,5,8,13,21,34]
subset = numbers3[2:5] #[3,5,8]
#characters from position 2 (included) to 5 (excluded)
```

# The For Loop

- The For Loop runs for a fixed number of times

```
for x in range(0, 3): This runs a loop 3 times
      print(x) # prints 0,1,2
```

- x is automatically incremented with each iteration
- A while-loop would look like this

```
counter = 0
while counter < 3:
    print(counter)
    counter = counter + 1
```

# Lists and Loops

- The for-loop is especially helpful for lists

```
numbers = [3,7,9]
for number in numbers: #This runs for each list element
    square = number * number
    print(square) #9,49,81
```

- A while-loop would look like this

```
counter = 0
while counter < len(numbers):
    square = numbers[counter] * numbers[counter]
    counter = counter + 1
    print(square)
```

# Writing Files

- Opening a file

```
#This creates the file if it does not exist
fileObject = open("fileOutput.txt", "w")
#Option 'w' will overwrite existing files
#Use the option 'a' to append to a file instead
```

- Writing to the file

```
#Add \n to end a line and \t to create a tab
fileObject.write("Hello you!\n")
```

- Close the file after usage:

```
fileObject.close()
```

# Tasks

1. Write a function that calculates $f(x) = 4x + 3$ for any number $x$
   - Define the function with a single parameter and a return value
   - Call the function with three different inputs and print the output each time
2. Define a second function $g(x, a_0, a_1, a_2, a_3)$ that calculates a polynomial of degree 3 with variable coefficients $a_0$ to $a_3$.
   - To calculate the power function you need to *import math* at the beginning of the script
   - Calculating $x^3$ is then done via $math.pow(x, 3)$
3. Calculate $g(x)$ for all integer values from 0 to 20 with coefficients of your choice and store the result in a list.
   - Define an empty list variable
   - Run a for loop for an appropriate number of times
   - During each loop calculate $g(x)$ and append the result to your list

# Tasks continued

4*. Write a script that writes down the list to a file:

- Start by opening the file
- First write "Coefficients:\n" to the file to create the first line
- Write your coefficients in the second line separated by commas
- Write "Values:" to the next line
- Run a loop through your list and in each loop write down $x$ and the function value g(x) stored in the list

File Content Sketch:

```
Coefficients:
  a_3,        a_2,    a_1,    a_0
 Values:
  0,          g(0)
  1,          g(1)
        ⋮
  19,         g(19)
  20,         g(20)
```