

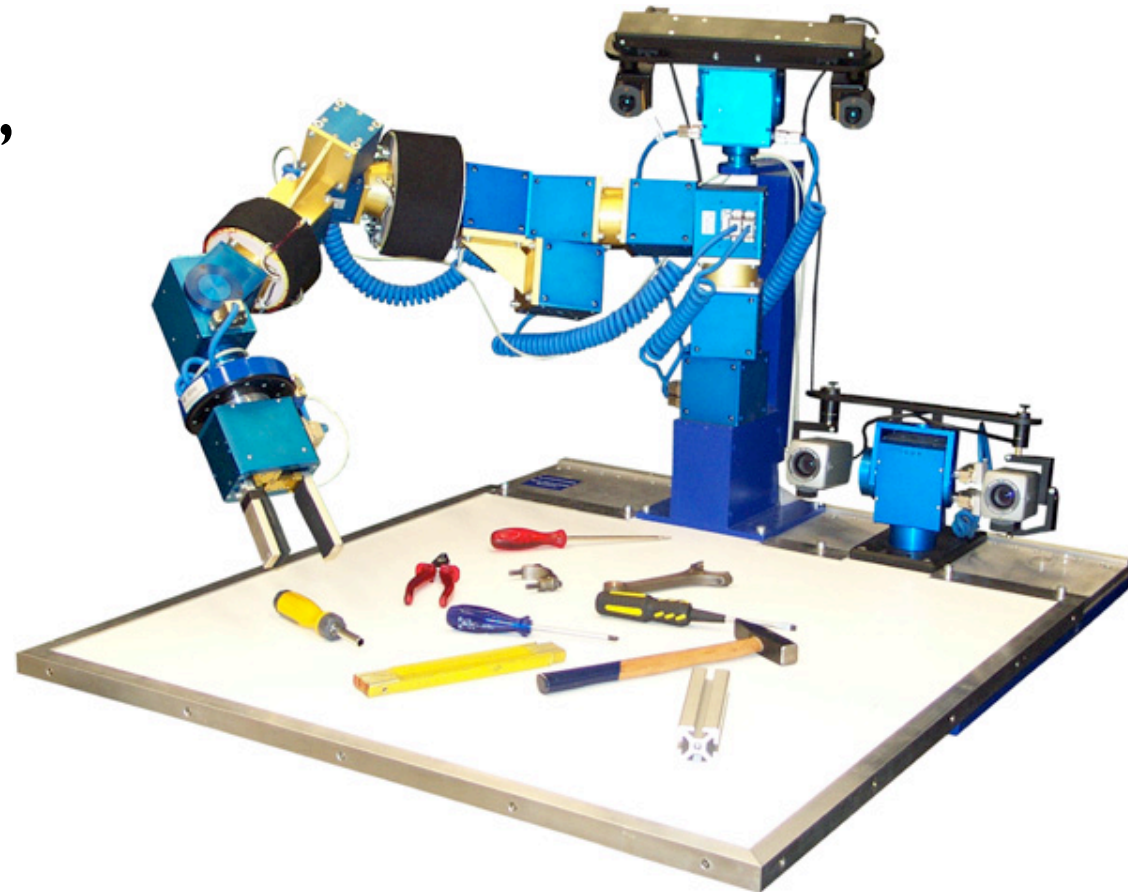
Robotic/human manipulation and the degree of freedom problem

Gregor Schöner

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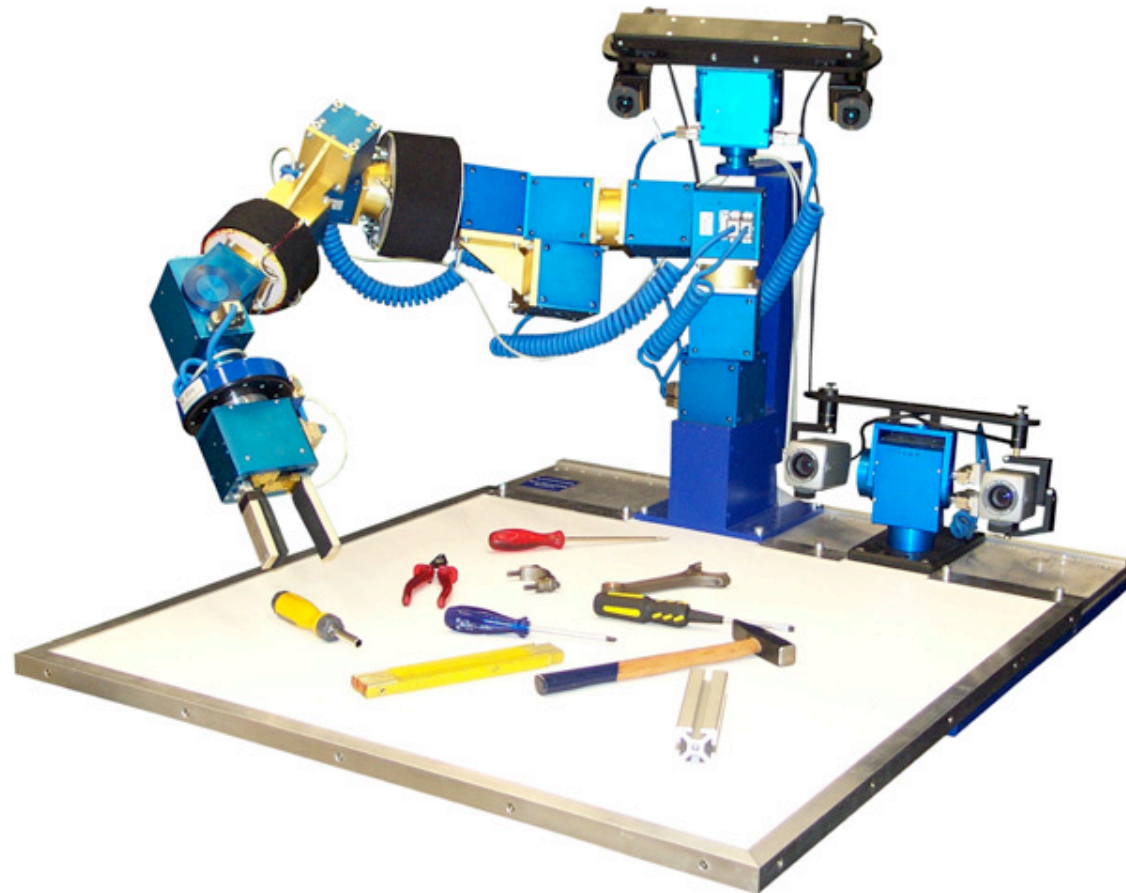
Movement to reach and grasp, lift, transport, manipulate

- involves a “manipulator”, a robotic/human arm with a grasping mechanism/hand



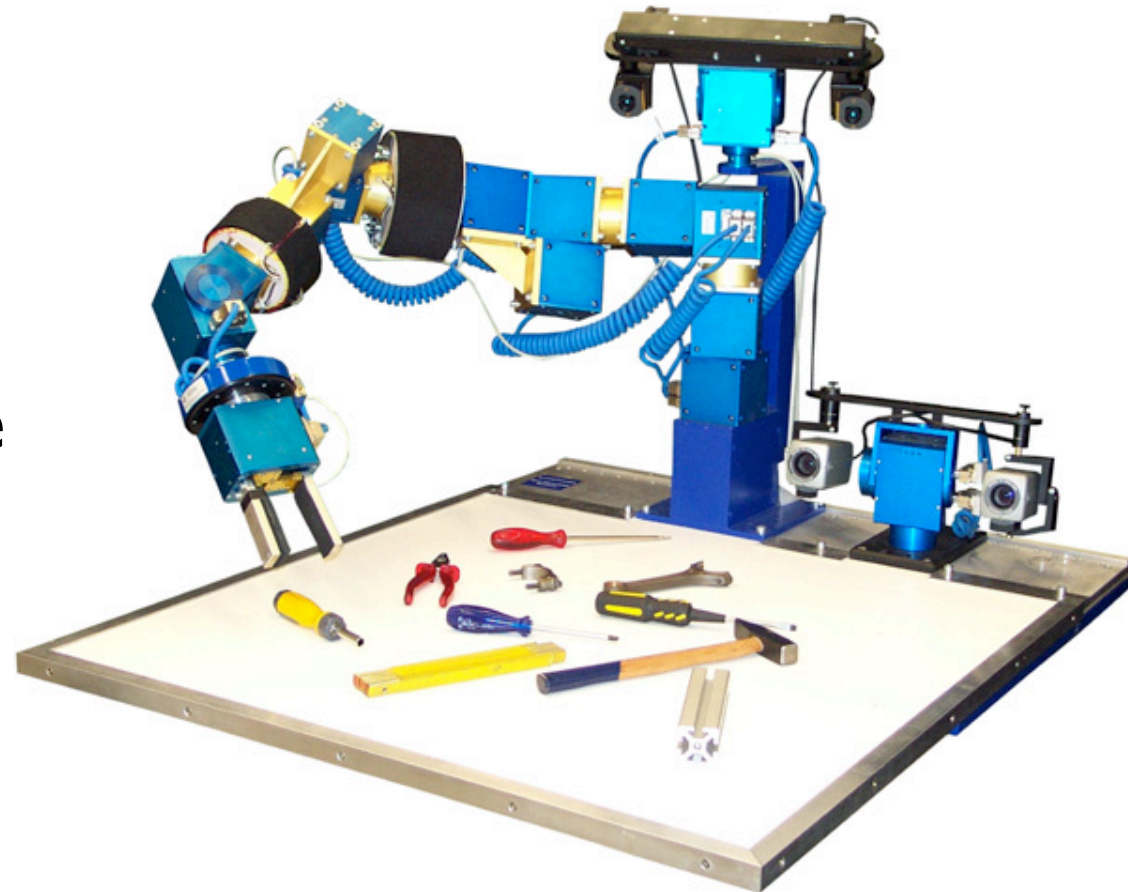
What is entailed in autonomously reaching for objects?

- Perception: recognizing and segmenting objects, estimating their pose
- Scene representation: registering the spatial array in the arms workspace for possible target objects, free space, and obstacles



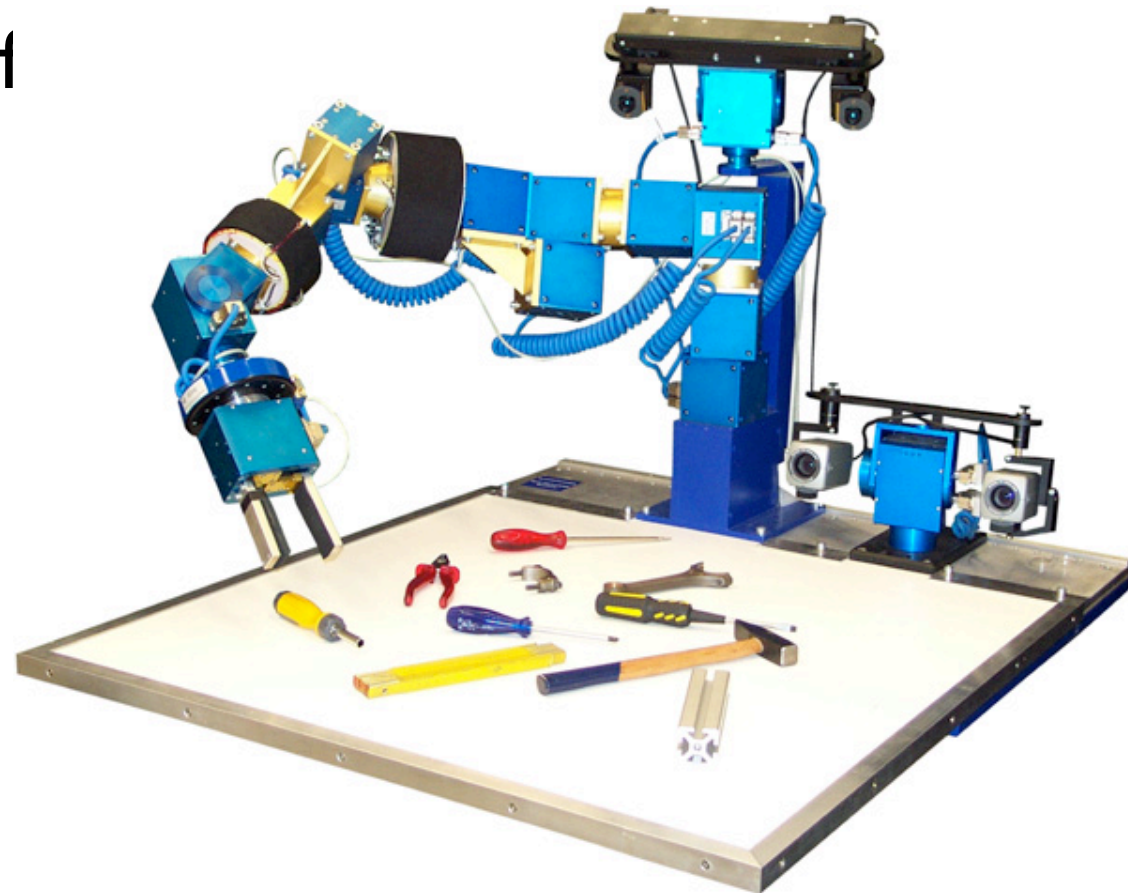
What is entailed in autonomously reaching for objects?

- Sequentially organizing actions (“serial order”) and planning
- Selecting a relevant object or location in the scene



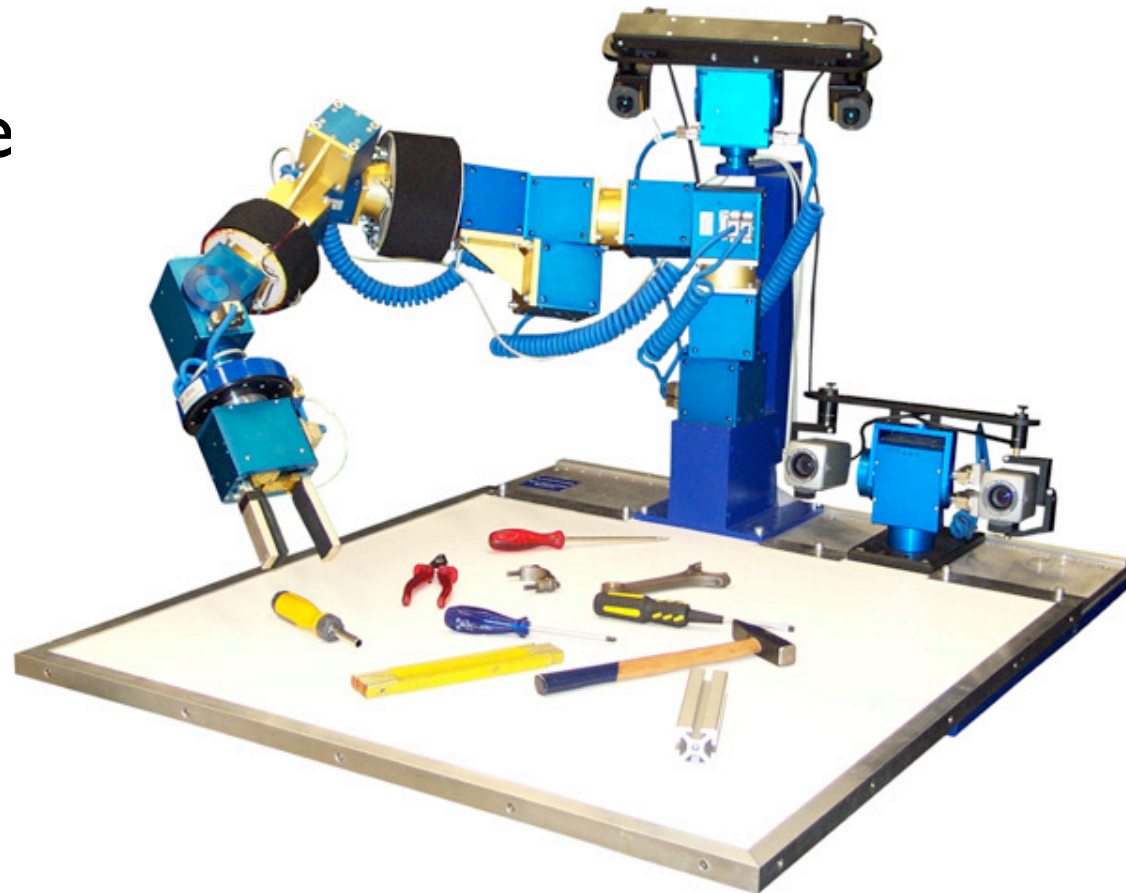
What is entailed in autonomously reaching for objects?

- Extracting parameters of an individual movement segment based on initial posture of arm and target state



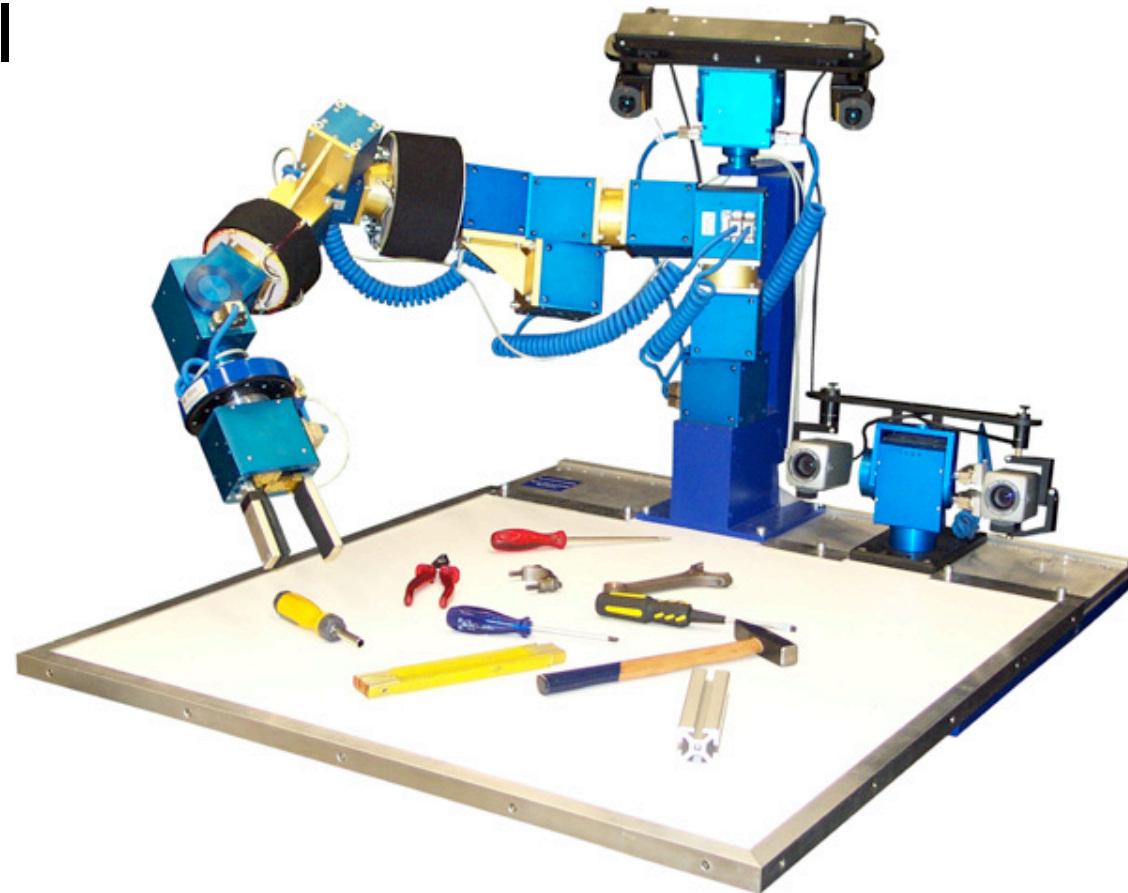
What is entailed in autonomously reaching for objects?

- Generating a time course for the degrees of freedom of the arm and hand that moves the arm from its initial posture to a state in which the target object is grasped
- timed...



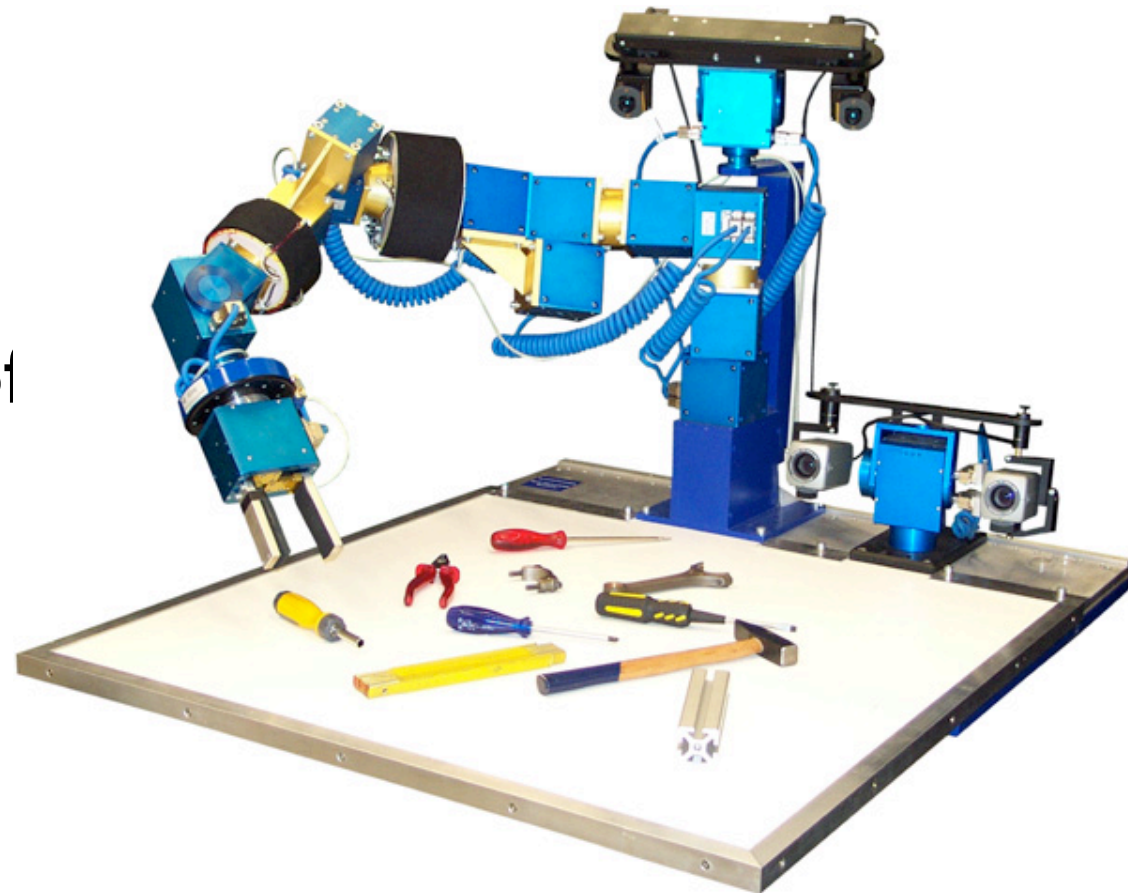
What is entailed in autonomously reaching for objects?

- Controlling the arm: translating the desired time course into control signals to the actuators/muscles that move the arm
- potentially update these signals based on feedback



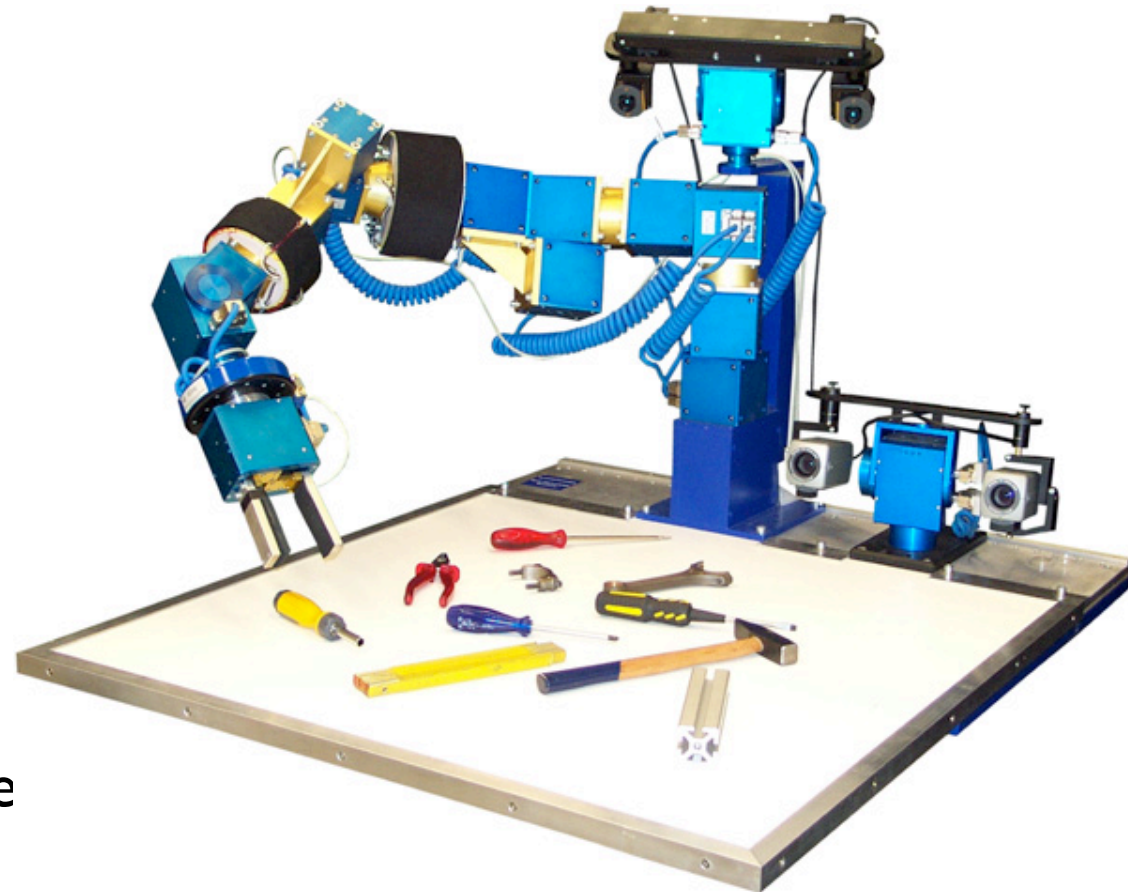
What is entailed in autonomously reaching for objects?

- Detect termination of the movement
- Transition to the next element in a sequence of movements...



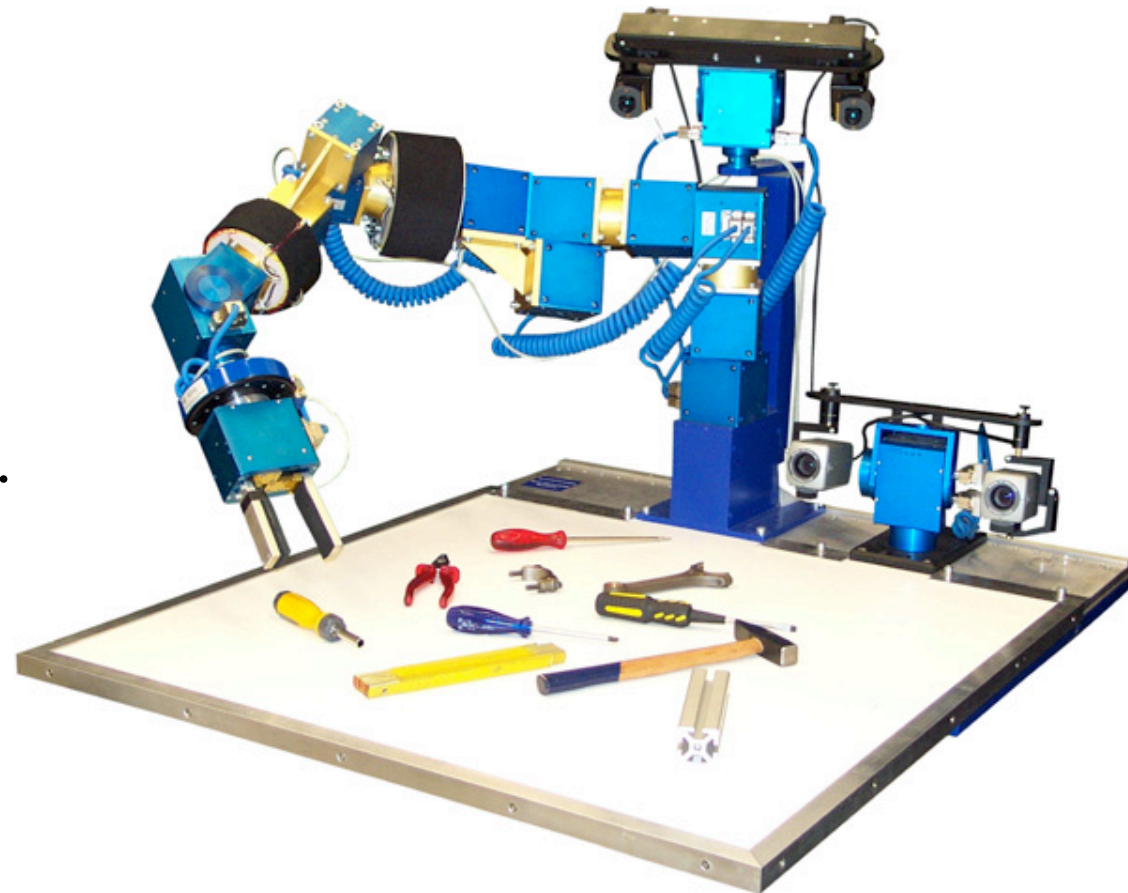
The kinematic gap... the kinematic degree of freedom problem

- The target state is defined by task variables
 - e.g. 3D position of gripper/hand
 - e.g. 2D or 3D orientation of gripper/hand
- Other task constraints may invoke other task variables
 - e.g. 3D position of arm surface for obstacle avoidance



The kinematic gap... the kinematic degree of freedom problem

- The time course/motor plan may also be about such task variables
- e.g. hand at the right position at the right time for catching..

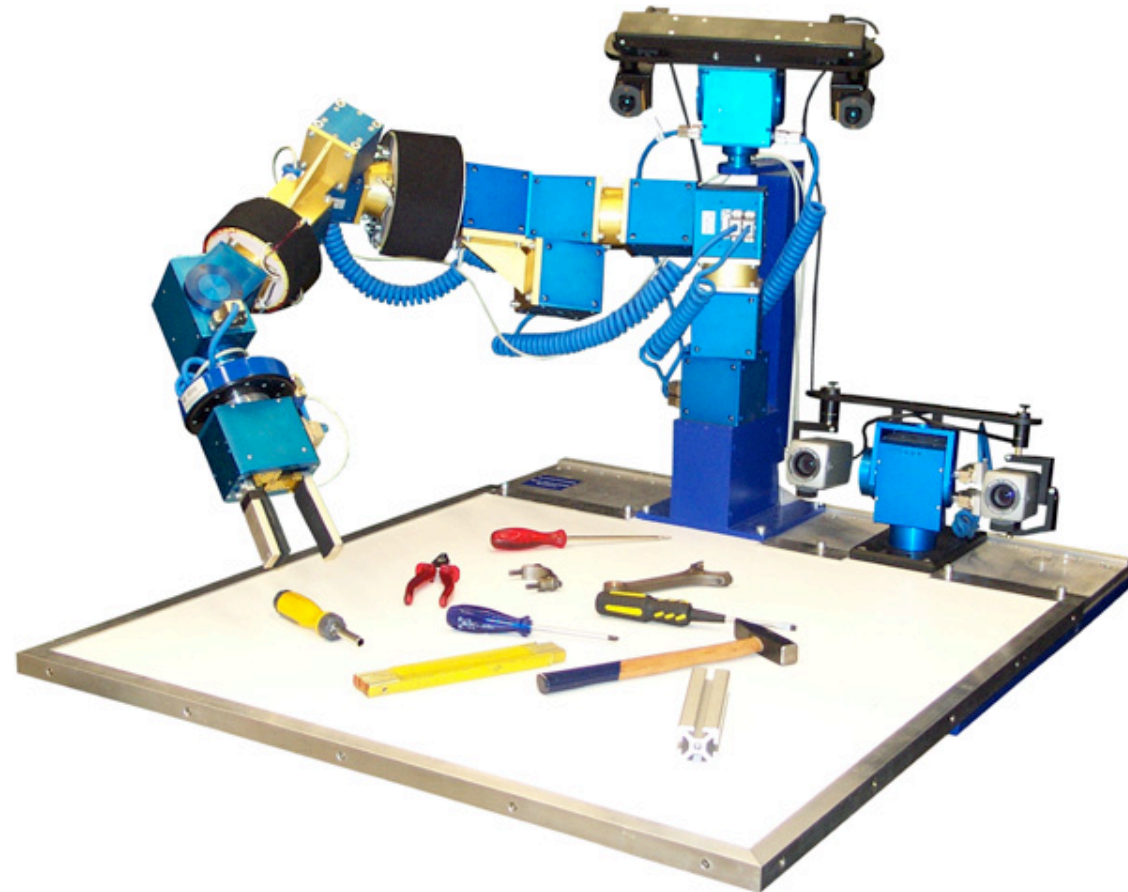


The kinematic gap... the kinematic degree of freedom problem

■ The control signals are at the level of the actuated degrees of freedom...

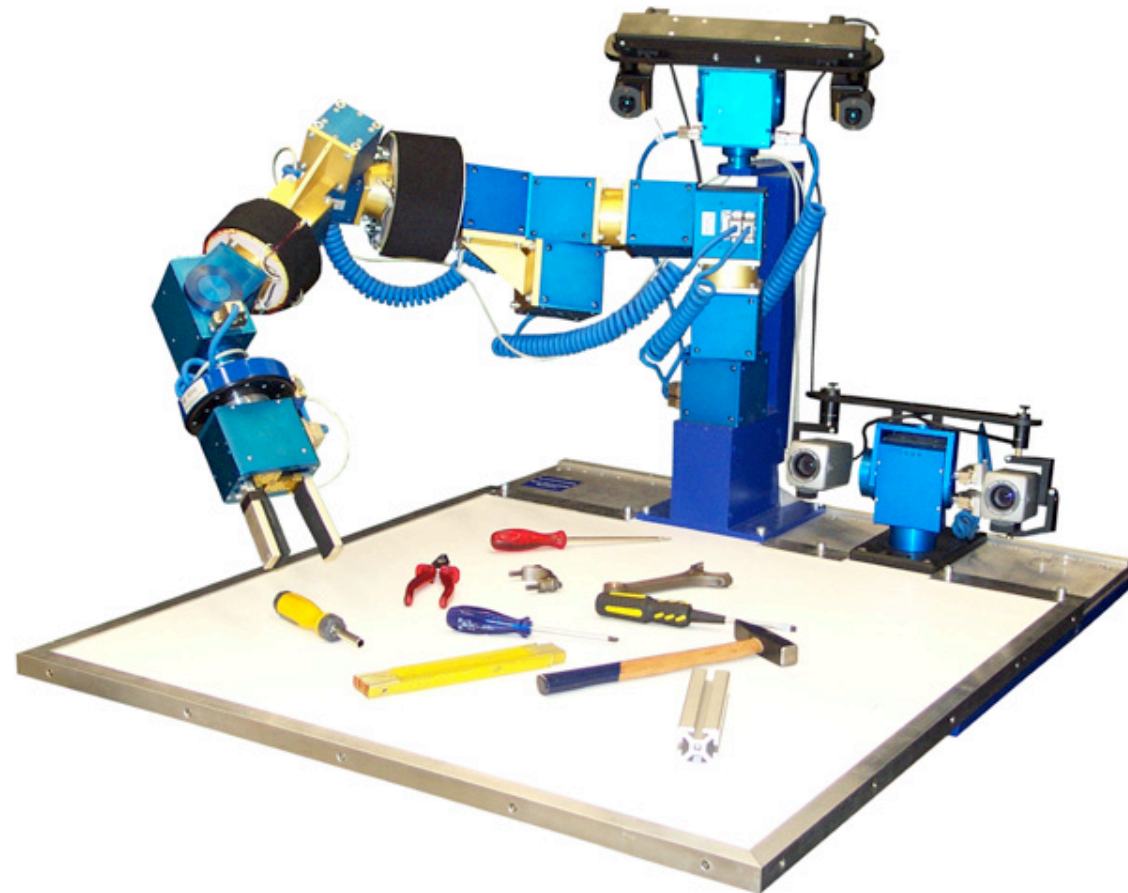
■ e.g. joint angles

■ e.g. muscle activation levels



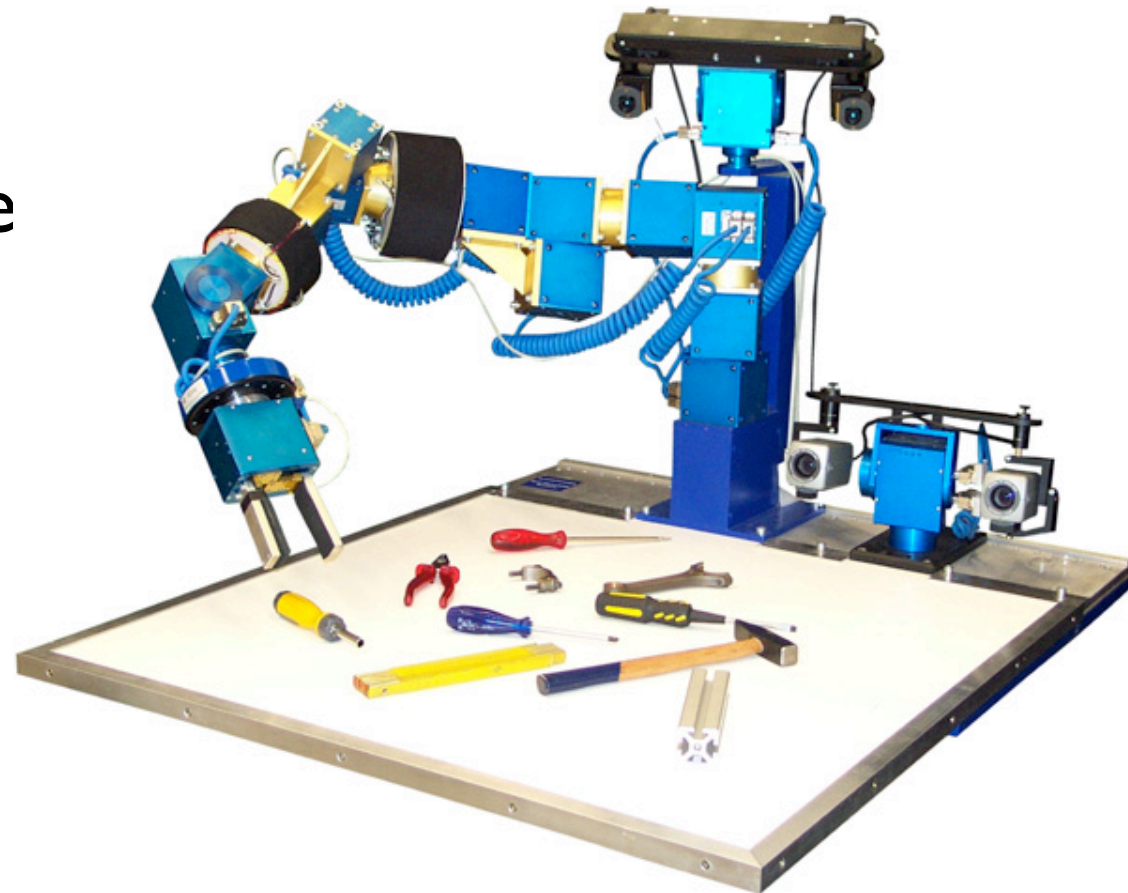
The kinematic gap... the kinematic degree of freedom problem

- For each individual task, there are typically more such control variables than task variables
- e.g. 10 joints for human arm vs 3+3 coordinates for hand position and orientation



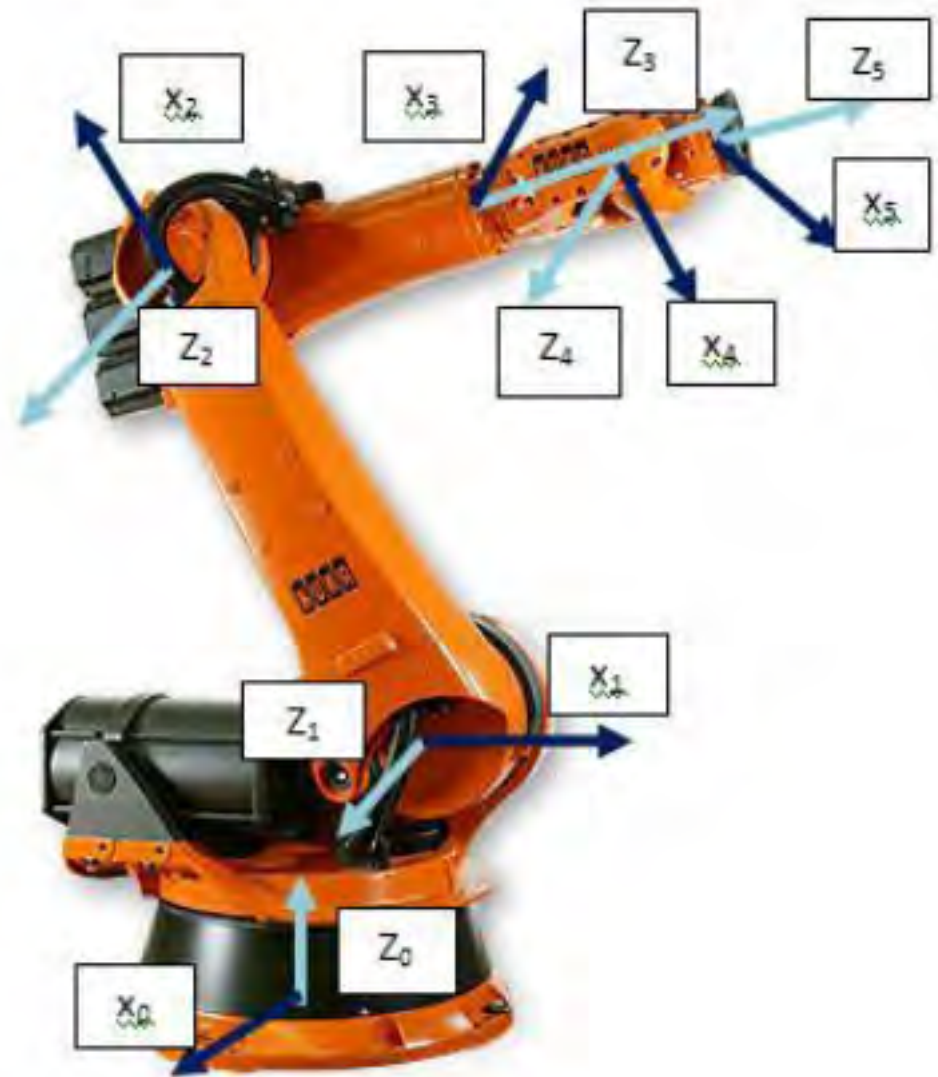
The kinematic gap... the kinematic degree of freedom problem

- That gap between task and actuation level is the “degree of freedom problem”



The kinematic gap... the kinematic degree of freedom problem

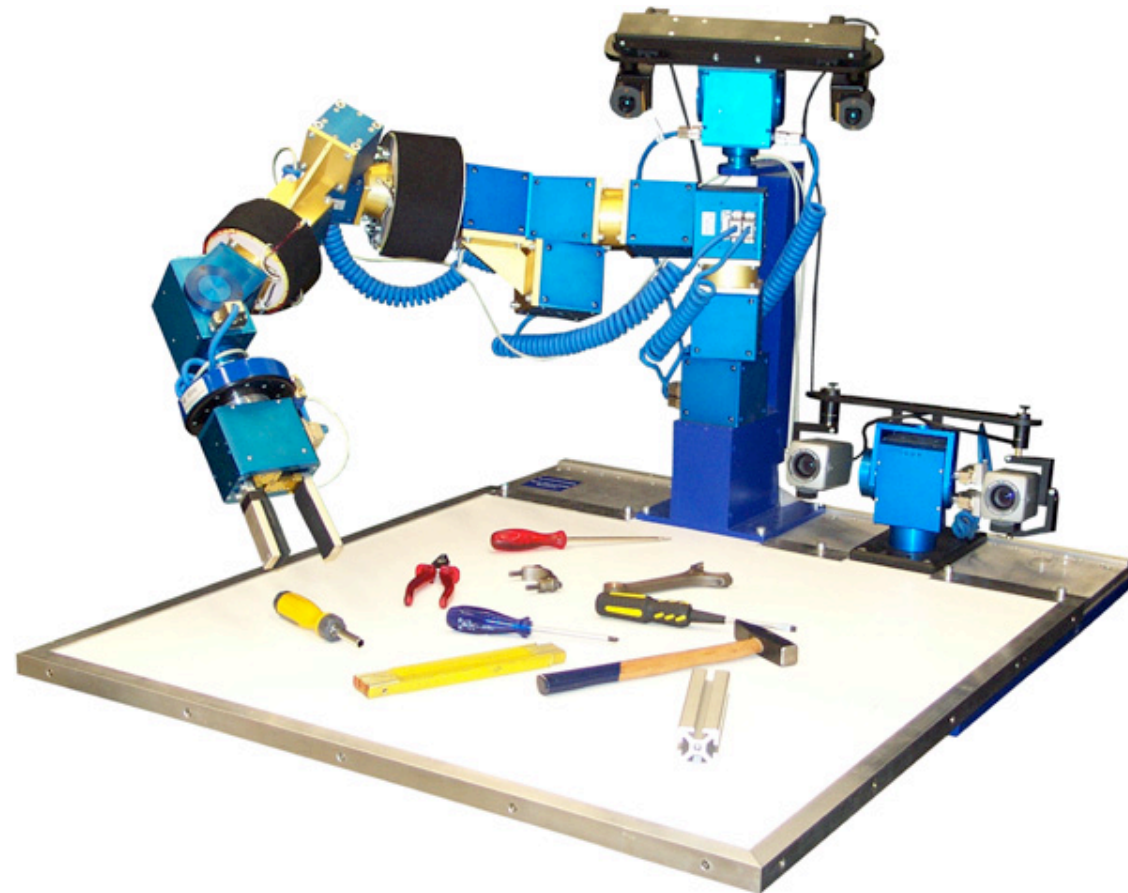
- Many conventional robot arms do not have the DoF problem because they are build for a fixed task for which they have the right number of degrees of freedom
- most commonly: 3+3 hand/ gripper taks variables and 6 joints



[Kuka KR 16KS: Dahari, Tan 2012]]

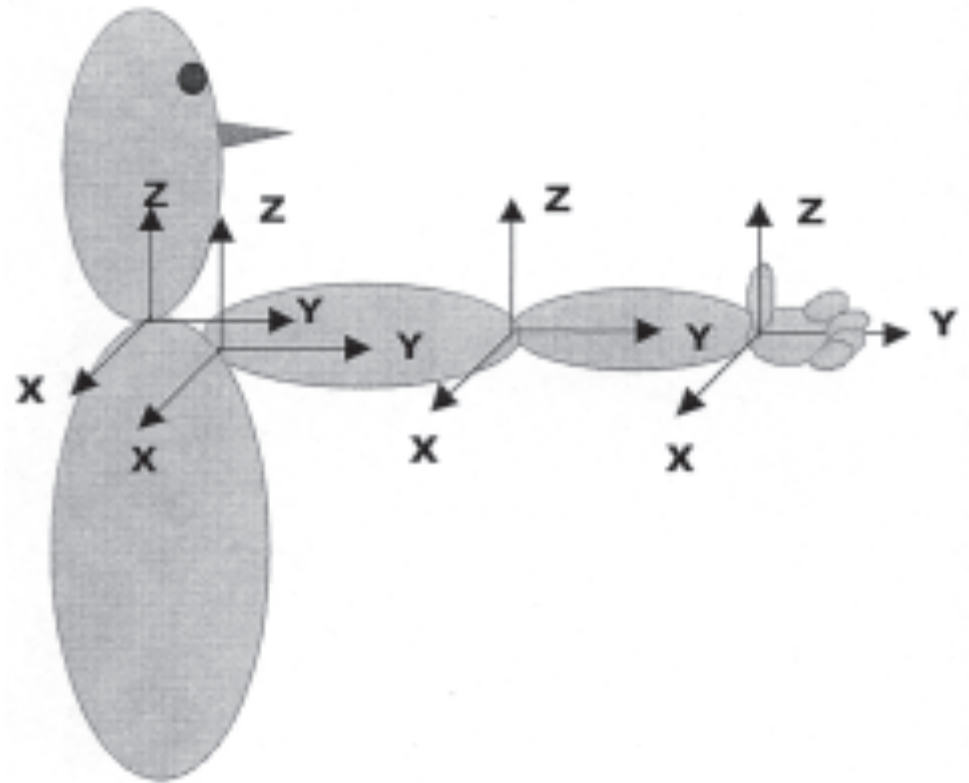
The kinematic gap... the kinematic degree of freedom problem

- But some robot arms are redundant: more DoF than needed for any single task
- so that they can combine multiple tasks...



The kinematic gap... the kinematic degree of freedom problem

- The human motor system is redundant...
- > 10 Dof
- ca. 40 muscles
- 3-6 task variables hand



[Tseng, Scholz, Schöner, 2002]

Rigid body motion

- very good source (from which I will use some illustrations)
- Murray, Li, Sastry: A Mathematical Introduction to Robotic Manipulation, CRC Press, Boca Raton FL USA 1994
- a pdf is made available by the authors
- (quite an advanced text)

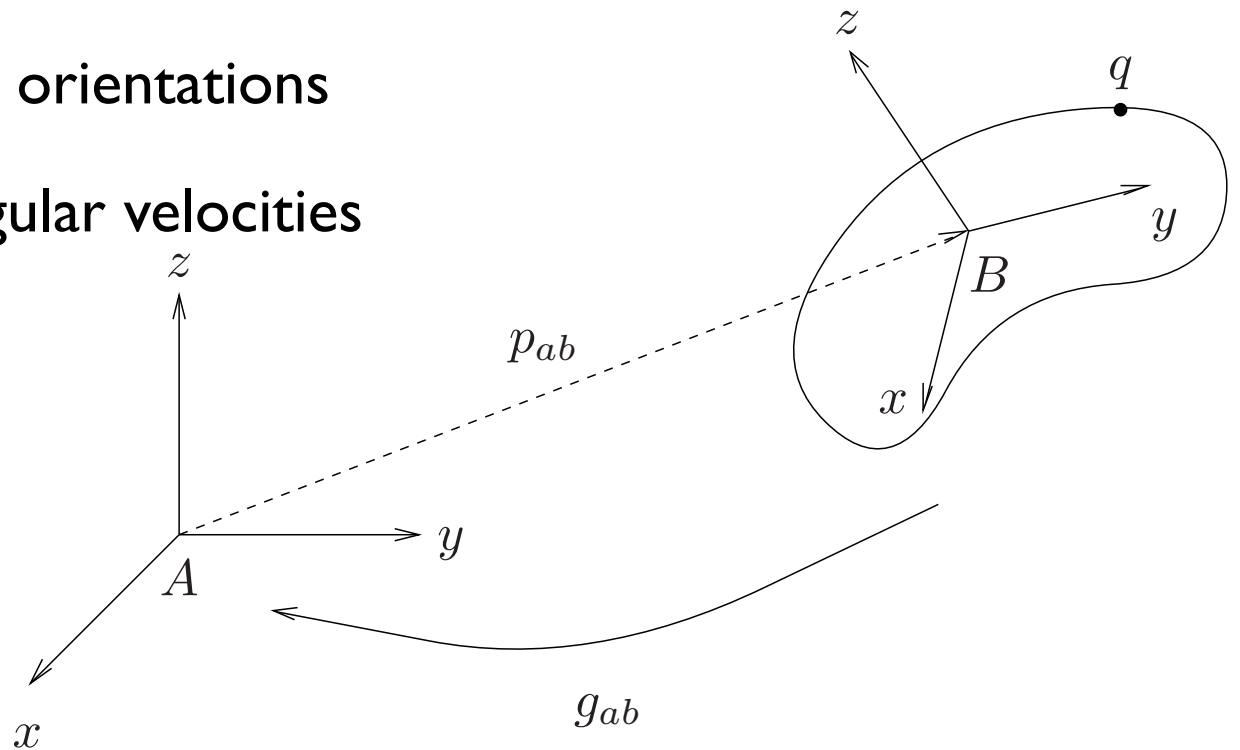
[Murray, Li, Sastry, 1994]

Rigid body motion

- a rigid body performs motion in 6D

- three positions, three orientations

- three linear, three angular velocities



- description of such motion by

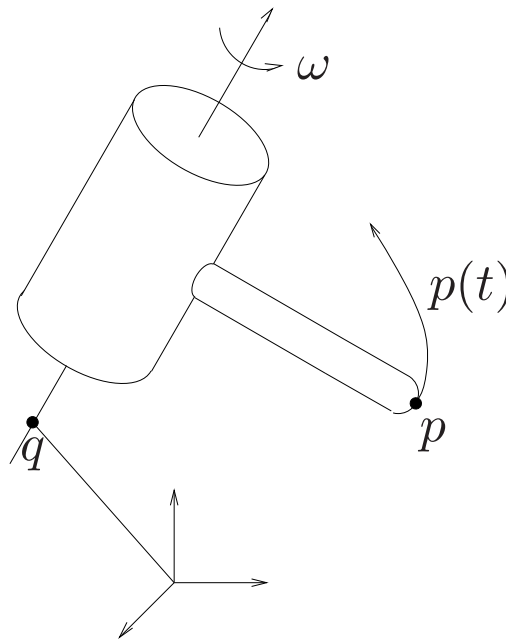
[Murray, Li, Sastry, 1994]

- the position vector

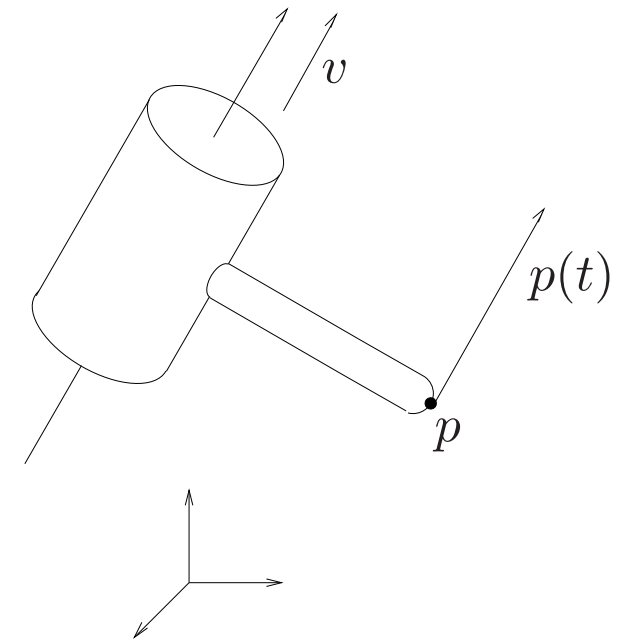
- a representation of rotation (Euler angles, Rotation matrix, generator of Lie group)

Rigid body motion

- constraints... revolute, prismatic, spherical.. joints
- reduce the number of degrees of freedom
- holonomic: can be formalized by reducing the number of variables



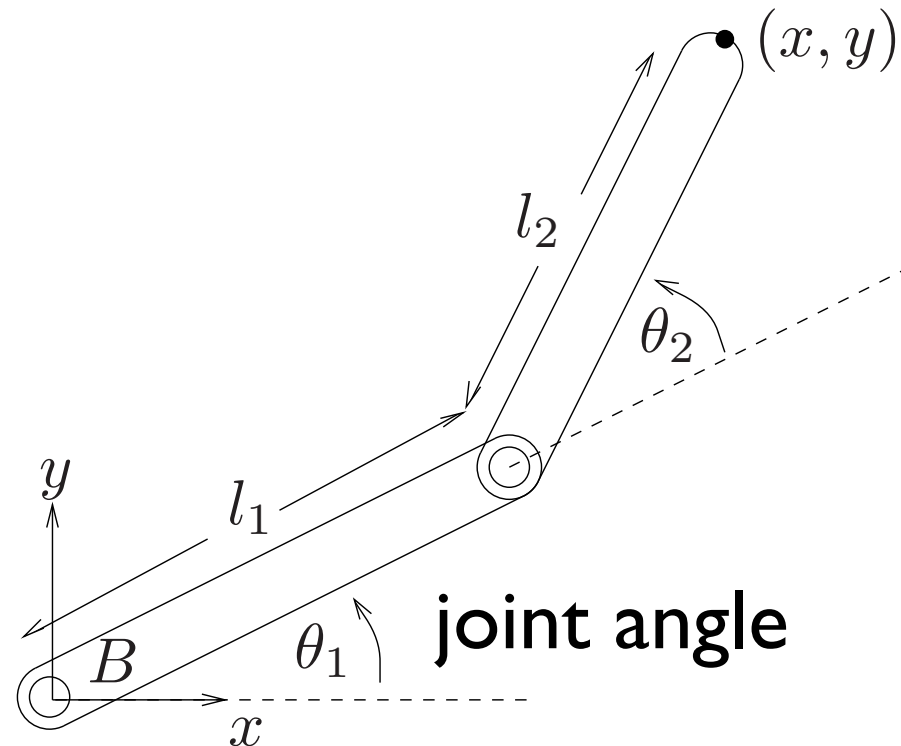
revolute joint



prismatic joint

Rigid body motion

- in a in a kinematic chain, the degrees of freedom of each rigid segment is reduced
- for revolute prismatic joints to a single(!) degree of freedom captured



[Murray, Li, Sastry, 1994]

Kinematics vs Kinetics

- kinematics: the description of the possible spatial (and velocity space) configurations of an arm taking into account the constraints

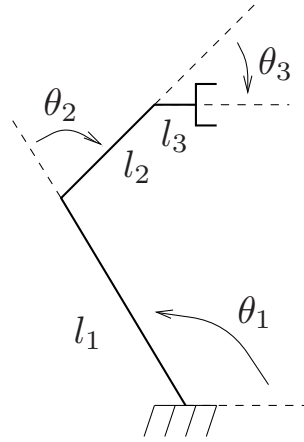
- treated now

- kinetics: the dynamic equations of motion of an arm taking into account the constraints, gravity, and actuators mounted on the joints

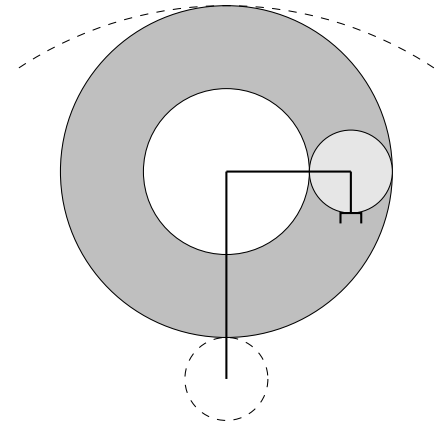
- (later in the lecture series)

Kinematic chain

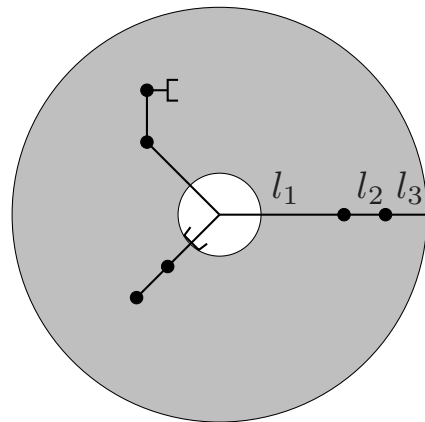
■ notion of work space



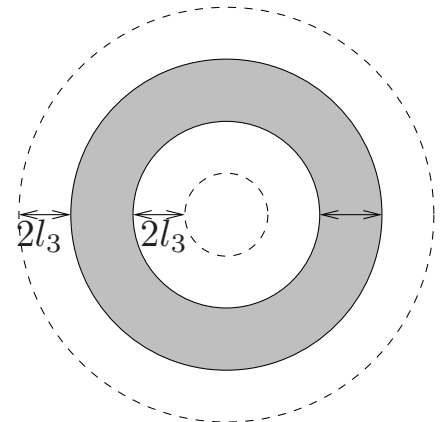
(a)



(b)



reachable space



dexterous space

[Murray, Li, Sastry, 1994]

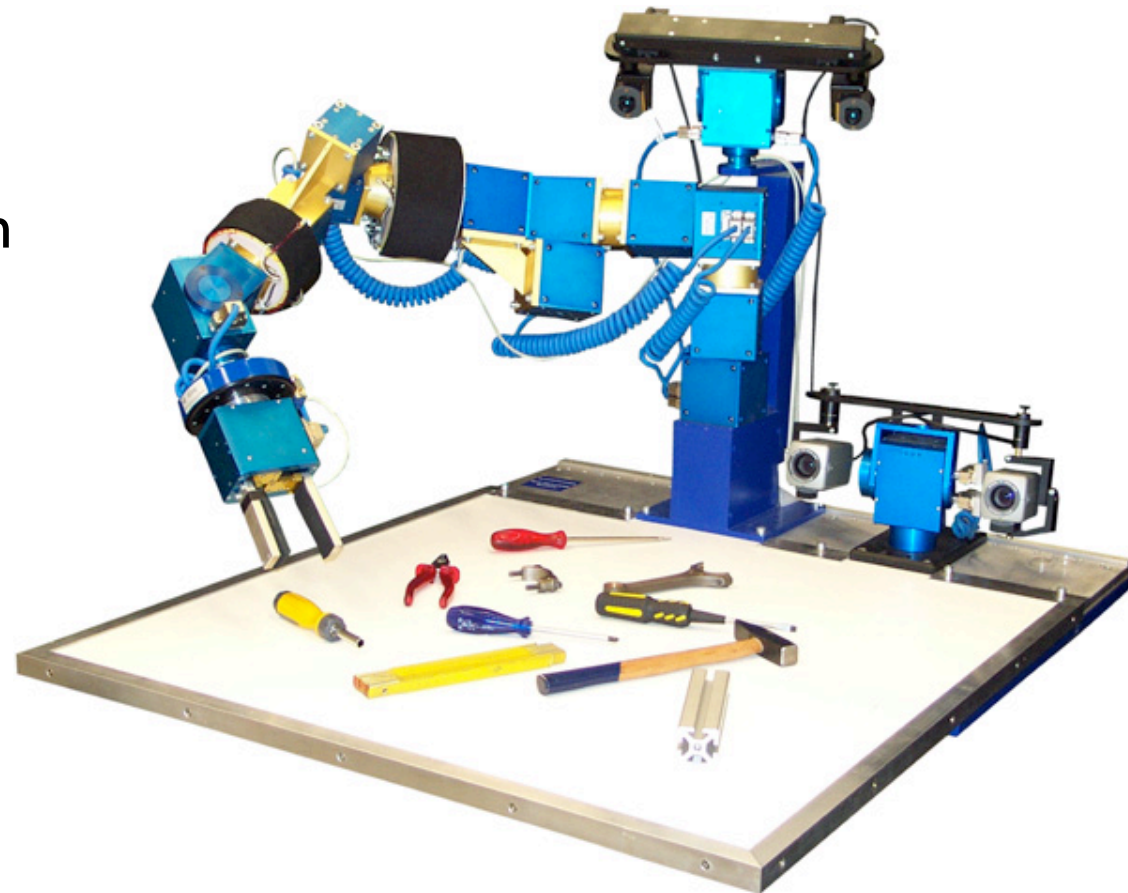
Basic concepts manipulator kinematics

■ end-effector

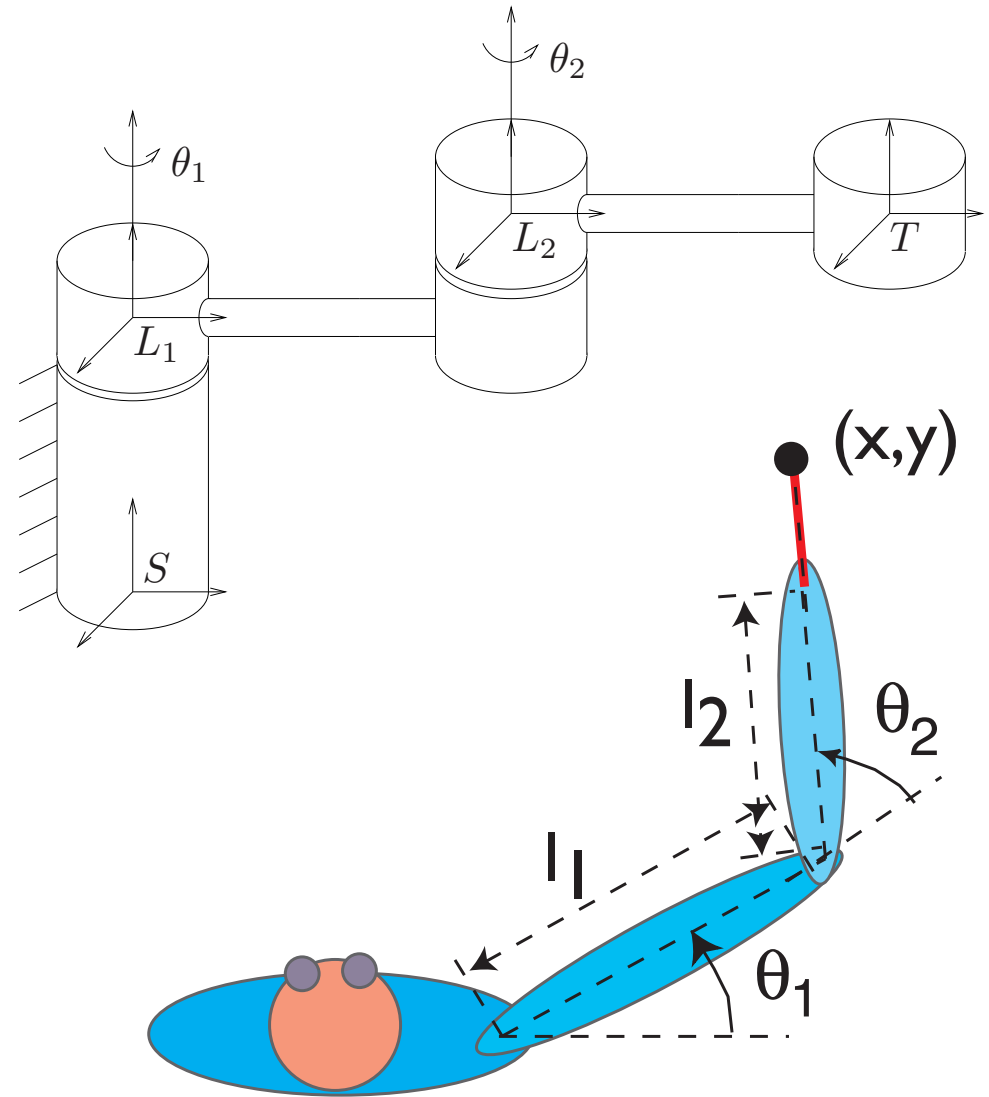
■ e.g. with 3 translational and 3 rotational degrees of freedom

■ configuration space

■ e.g. 7 actuated joint angles



Forward kinematics



- where is the hand, given the joint angles..

$$\mathbf{x} = \mathbf{f}(\theta)$$

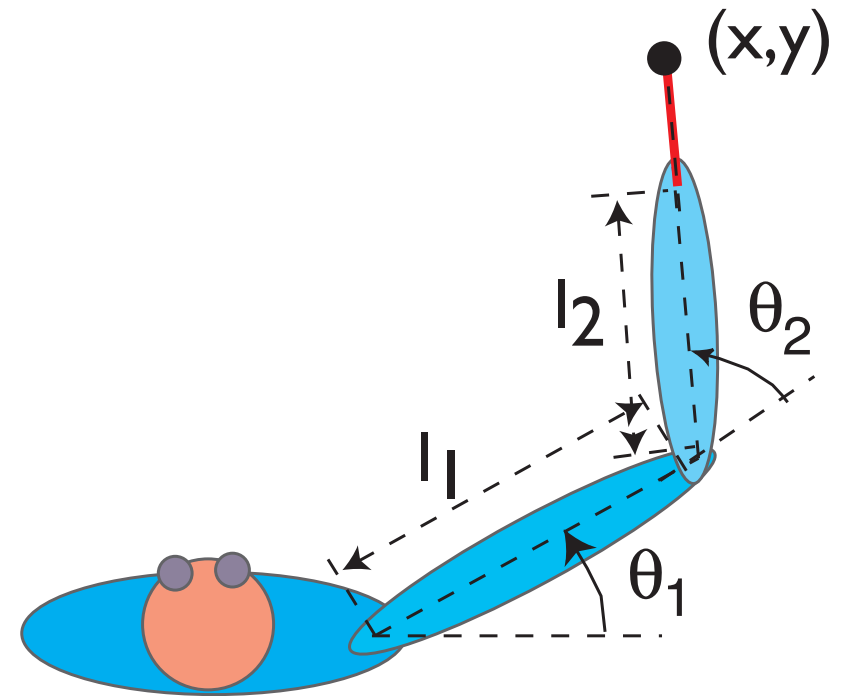
$$x = l_1 \cos(\theta_1) + l_2 \cos(\theta_1 + \theta_2)$$

$$y = l_1 \sin(\theta_1) + l_2 \sin(\theta_1 + \theta_2)$$

Differential forward kinematics

- where is the hand moving, given the joint angles and velocities

$$\dot{\mathbf{x}} = \mathbf{J}(\theta)\dot{\theta}$$



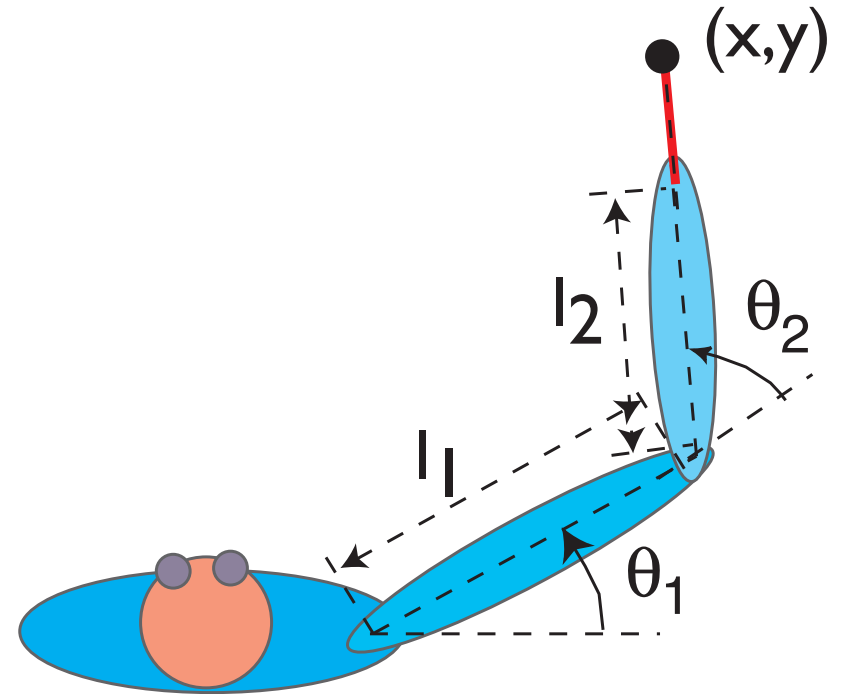
$$\dot{x} = -l_1 \sin(\theta_1)\dot{\theta}_1 - l_2 \sin(\theta_1 + \theta_2)\dot{\theta}_1 - l_2 \sin(\theta_1 + \theta_2)\dot{\theta}_2$$

$$\dot{y} = l_1 \cos(\theta_1)\dot{\theta}_1 + l_2 \cos(\theta_1 + \theta_2)\dot{\theta}_1 + l_2 \cos(\theta_1 + \theta_2)\dot{\theta}_2$$

Differential forward kinematics

- where is the hand moving, given the joint angles and velocities

$$\dot{\mathbf{x}} = \mathbf{J}(\theta)\dot{\theta}$$

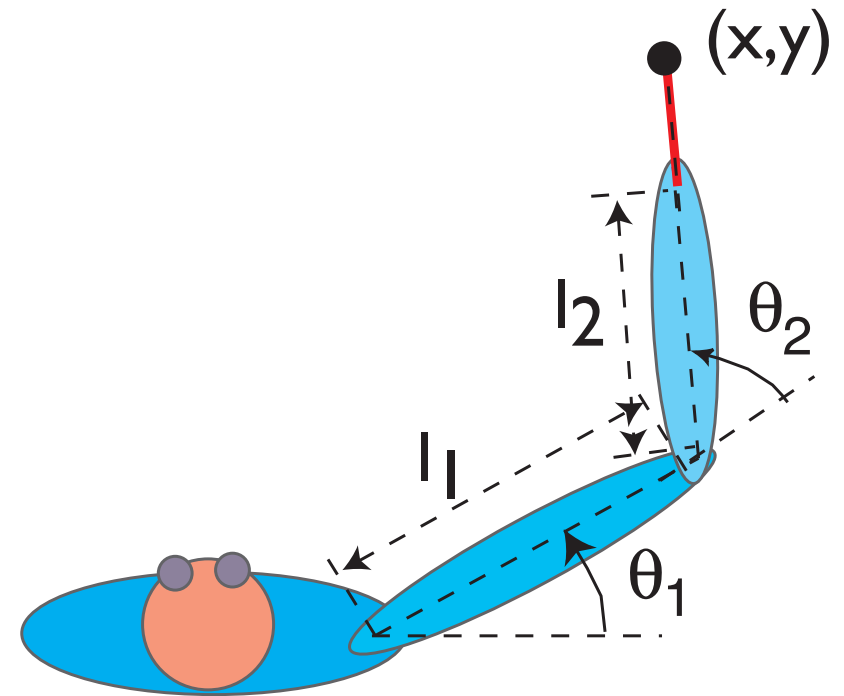


$$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} -l_1 \cos(\theta_1) - l_2 \cos(\theta_1 + \theta_2) & -l_2 \cos(\theta_1 + \theta_2) \\ l_1 \sin(\theta_1) + l_2 \sin(\theta_1 + \theta_2) & l_2 \sin(\theta_1 + \theta_2) \end{pmatrix} \cdot \begin{pmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{pmatrix}$$

Inverse kinematics

- what joint angles are needed to put the hand at a given location
- exact solution:

$$\theta = \mathbf{f}^{-1}(\mathbf{x})$$



Inverse kinematics

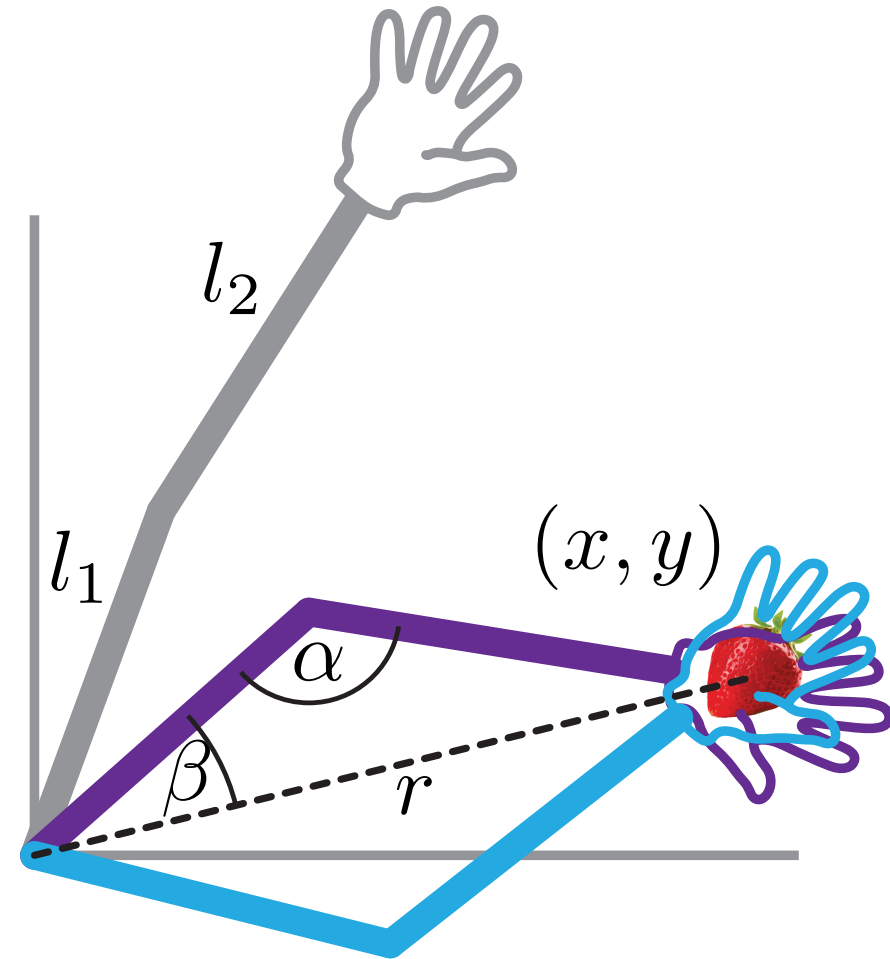
$$\theta_1 = \arctan_2(y, x) \pm \beta$$

$$\theta_2 = \pi \pm \alpha$$

$$\alpha = \cos^{-1} \left(\frac{l_1^2 + l_2^2 - r^2}{2l_1l_2} \right)$$

$$\beta = \cos^{-1} \left(\frac{r^2 + l_1^2 - l_2^2}{2l_1l_2} \right)$$

where $r^2 = x^2 + y^2$



[thanks to Jean-Stéphane Jokeit]

■ => multiple “leafs” of the inverse kinematics

Differential inverse kinematics

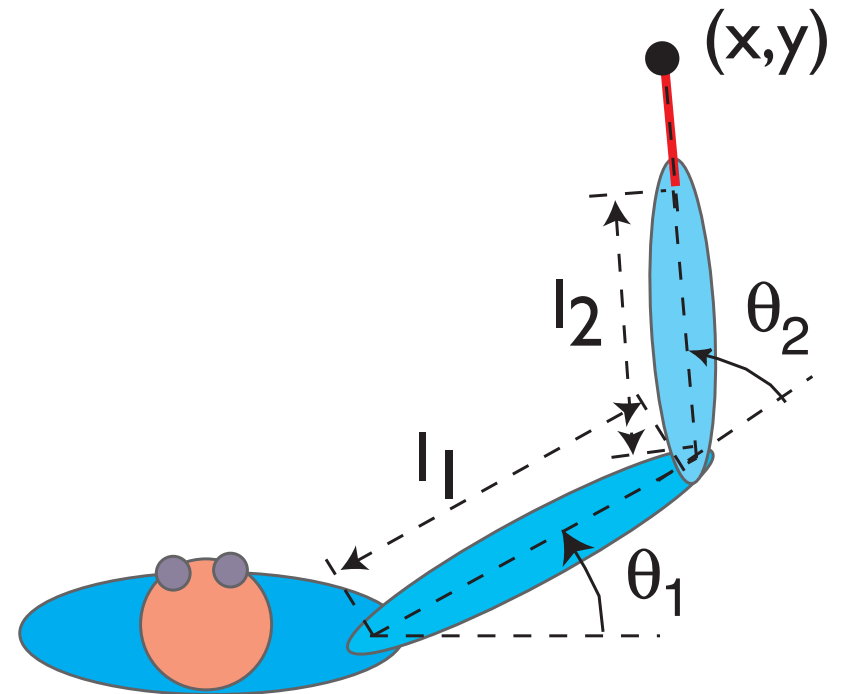
- which joint velocities to move the hand in a particular way

$$\dot{\theta} = \mathbf{J}^{-1}(\theta)\dot{\mathbf{x}}$$

with the inverse of

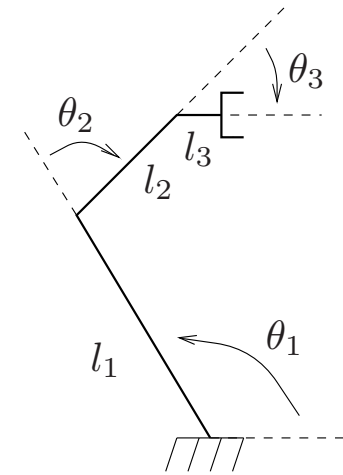
$$\mathbf{J}(\theta) = \begin{pmatrix} -l_1 \cos(\theta_1) - l_2 \cos(\theta_1 + \theta_2) & -l_2 \cos(\theta_1 + \theta_2) \\ l_1 \sin(\theta_1) + l_2 \sin(\theta_1 + \theta_2) & l_2 \sin(\theta_1 + \theta_2) \end{pmatrix}$$

if it exists!

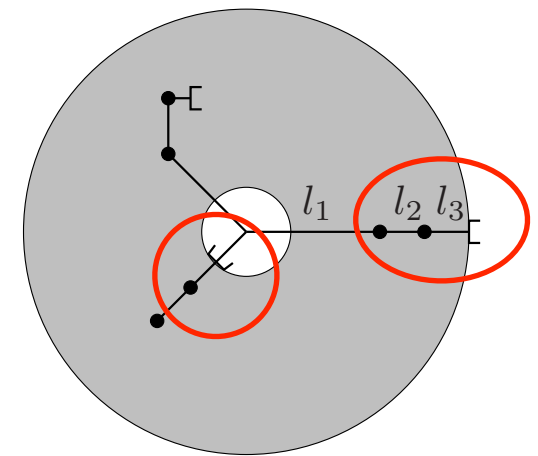


Singularities

- where the Eigenvalue of the Jacobian becomes zero (real part)...
- so that movement in a particular direction is not possible...
- typically at extended postures or inverted postures
- at limit of workspace



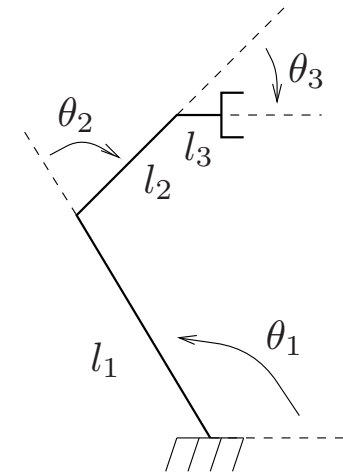
(a)



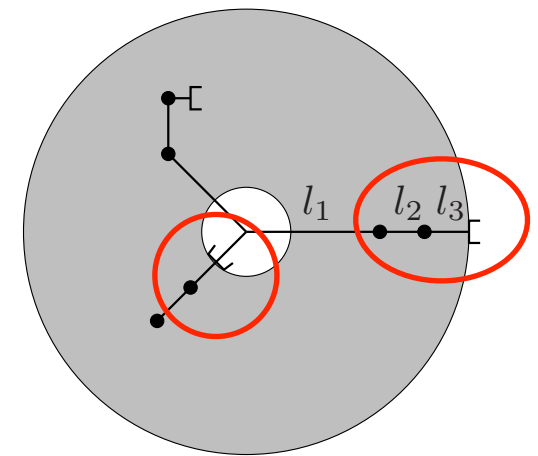
(c)

Singularities

- leading to non-invertability!
- and to sensitive dependence on parameters
- => avoid singularities in motor planning... major effort in robotics
- humans: joint angles prevent us from getting near singularities (for the most part)



(a)

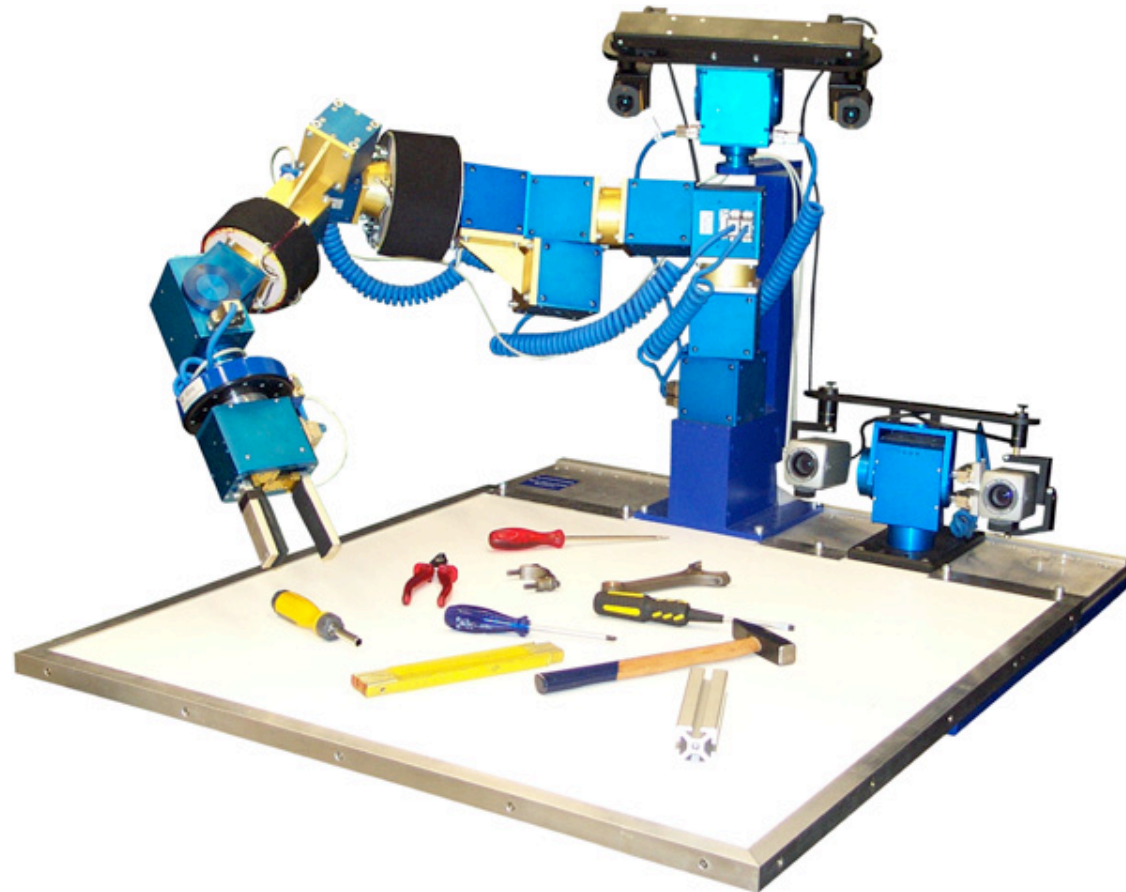


(c)

Summary arm kinematics

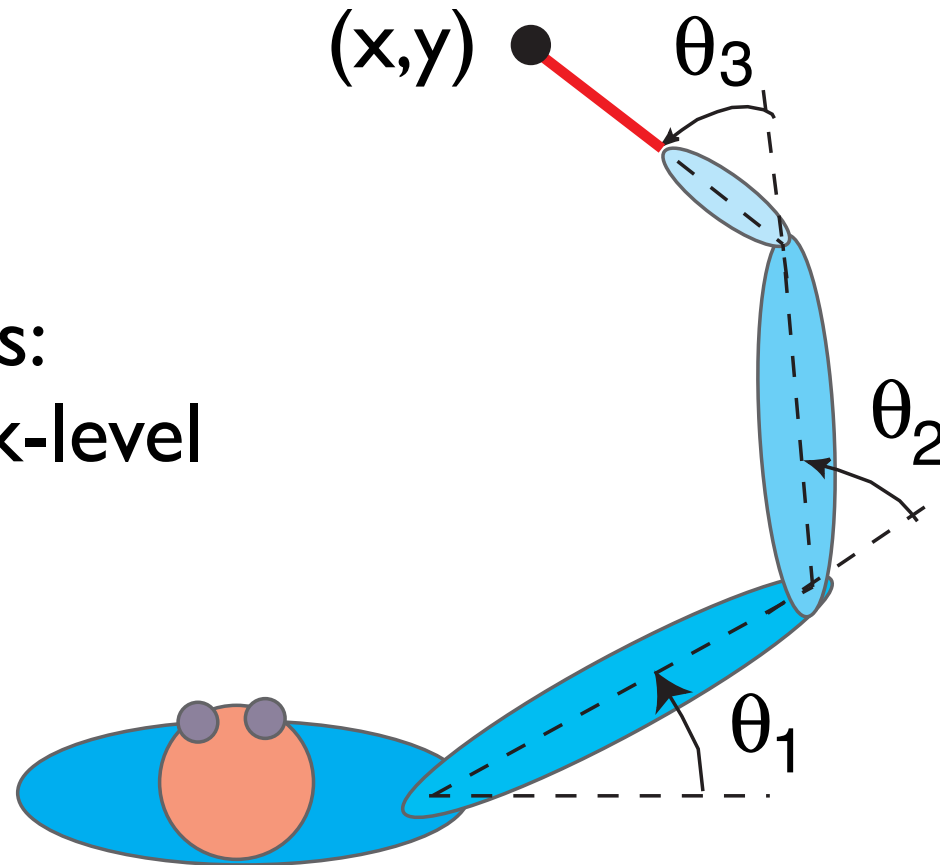
kinematic model $\mathbf{x} = \mathbf{f}(\theta)$ $\dot{\mathbf{x}} = \mathbf{J}(\theta)\dot{\theta}$

inverse kinematic model $\theta = \mathbf{f}^{-1}(\mathbf{x})$ $\dot{\theta} = \mathbf{J}^{-1}(\theta)\dot{\mathbf{x}}$



Redundant kinematics

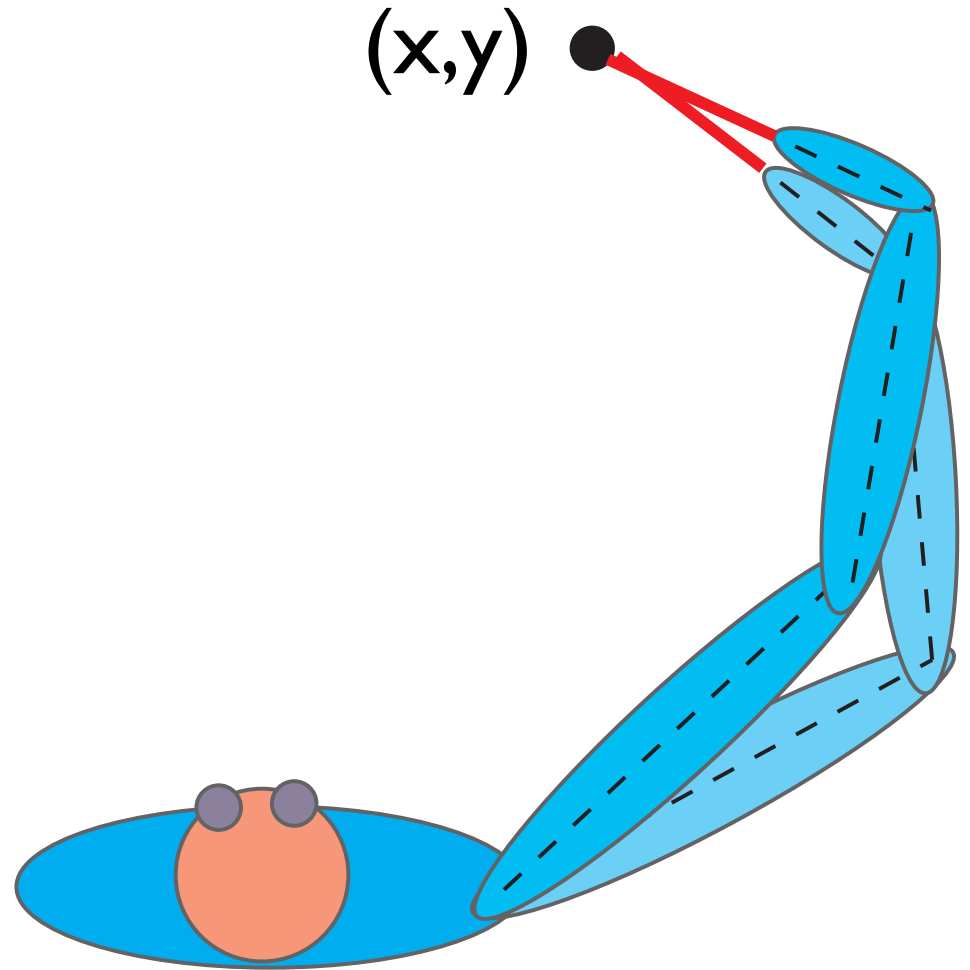
- redundant arms/tasks:
more joints than task-level
degrees of freedom



$$x = l_1 \cos(\theta_1) + l_2 \cos(\theta_1 + \theta_2) + l_3 \cos(\theta_1 + \theta_2 + \theta_3)$$
$$y = l_1 \sin(\theta_1) + l_2 \sin(\theta_1 + \theta_2) + l_3 \sin(\theta_1 + \theta_2 + \theta_3)$$

Redundant kinematics

■ => (continuously) many inverse solutions...



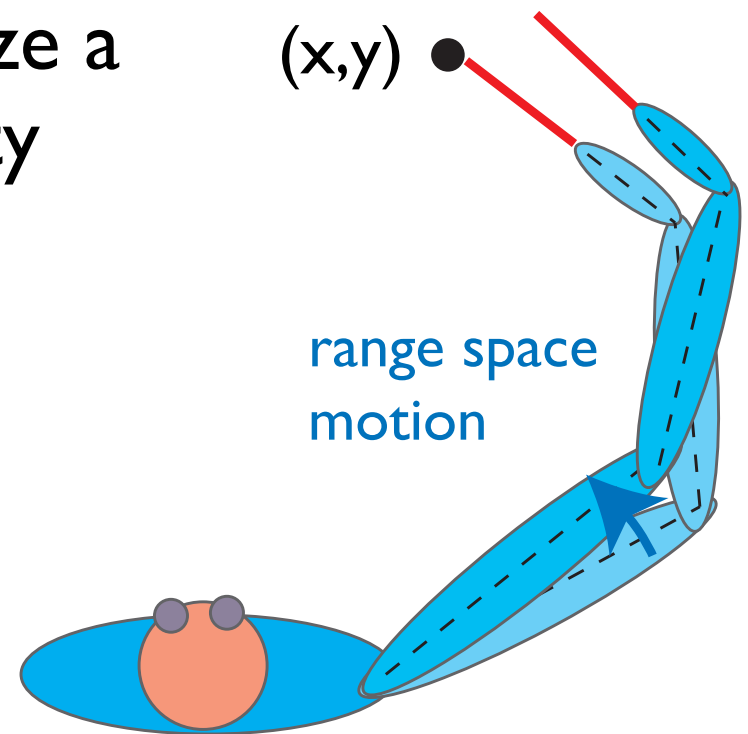
Redundant kinematics

- use pseudo-inverses that minimize a functional (e.g., total joint velocity or total momentum)

$$\dot{\mathbf{x}} = \mathbf{J}(\theta)\dot{\theta}$$

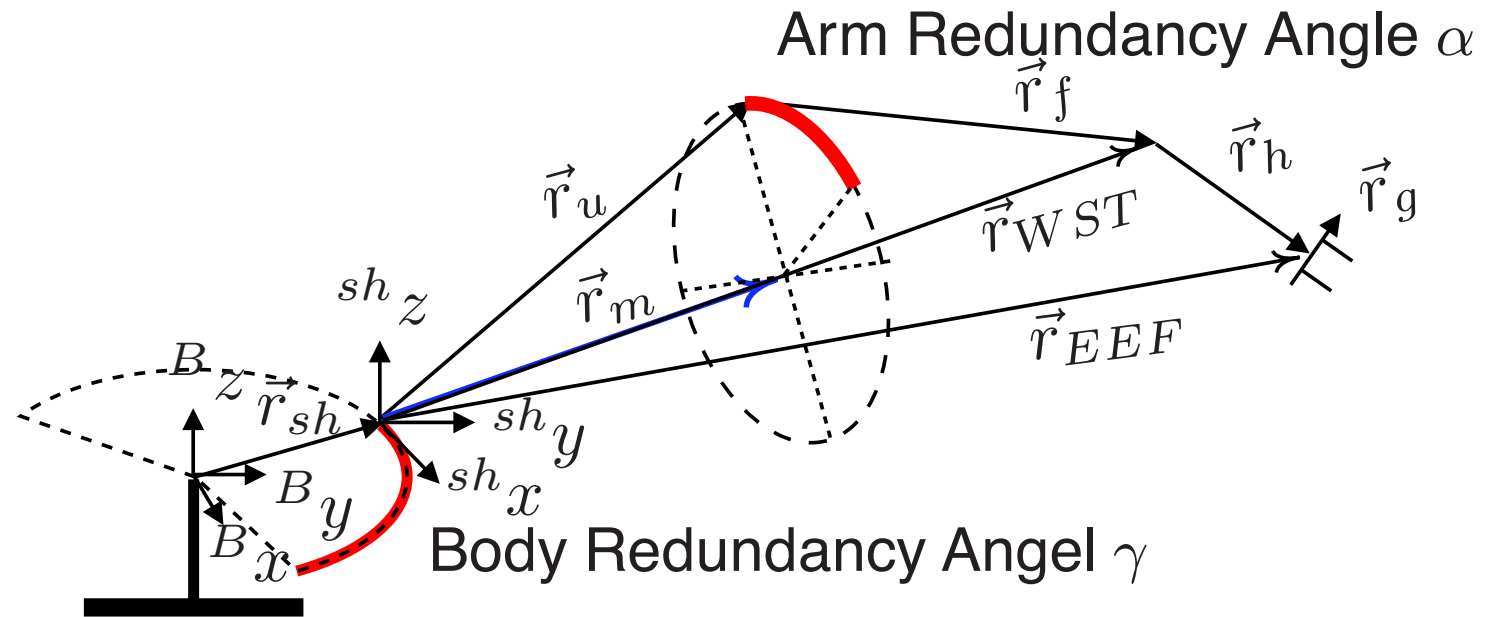
$$\dot{\theta} = \mathbf{J}^+(\theta)\dot{\mathbf{x}}$$

$$\mathbf{J}^+(\theta) = \mathbf{J}^T(\mathbf{J}\mathbf{J}^T)^{-1} \quad \text{pseudo-inverse}$$



Spaces for robotic motion planning

- or use extra degrees of freedom for additional tasks



[Iossifidis, Schöner, ICRA 2004]

Degree of freedom problem in human movement

■ what is a DoF?

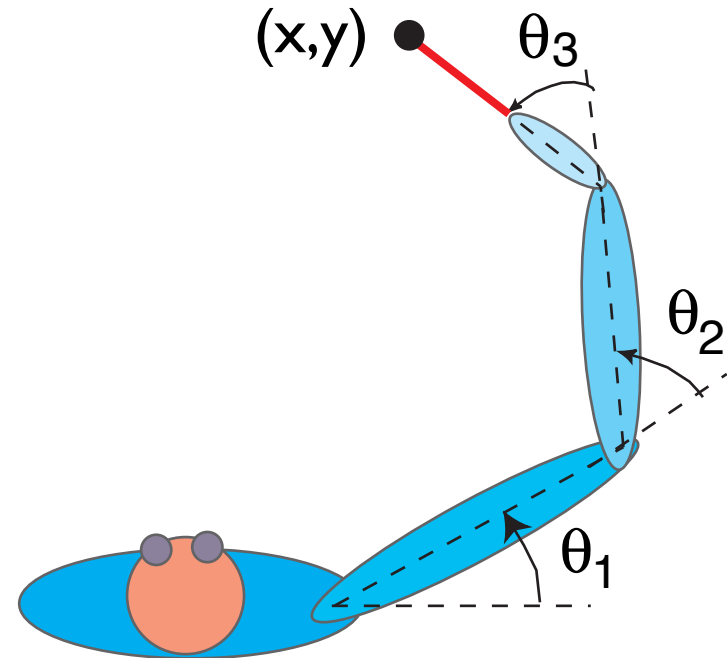
■ variable that can be independently varied

■ e.g. joint angles

■ muscles/muscle groups

■ but: assess to which extent they can be activated independently...

■ .. mode picture



$$x = l_1 \cos(\theta_1) + l_2 \cos(\theta_1 + \theta_2) + l_3 \cos(\theta_1 + \theta_2 + \theta_3)$$
$$y = l_1 \sin(\theta_1) + l_2 \sin(\theta_1 + \theta_2) + l_3 \sin(\theta_1 + \theta_2 + \theta_3)$$

Degree of freedom problem in human movement

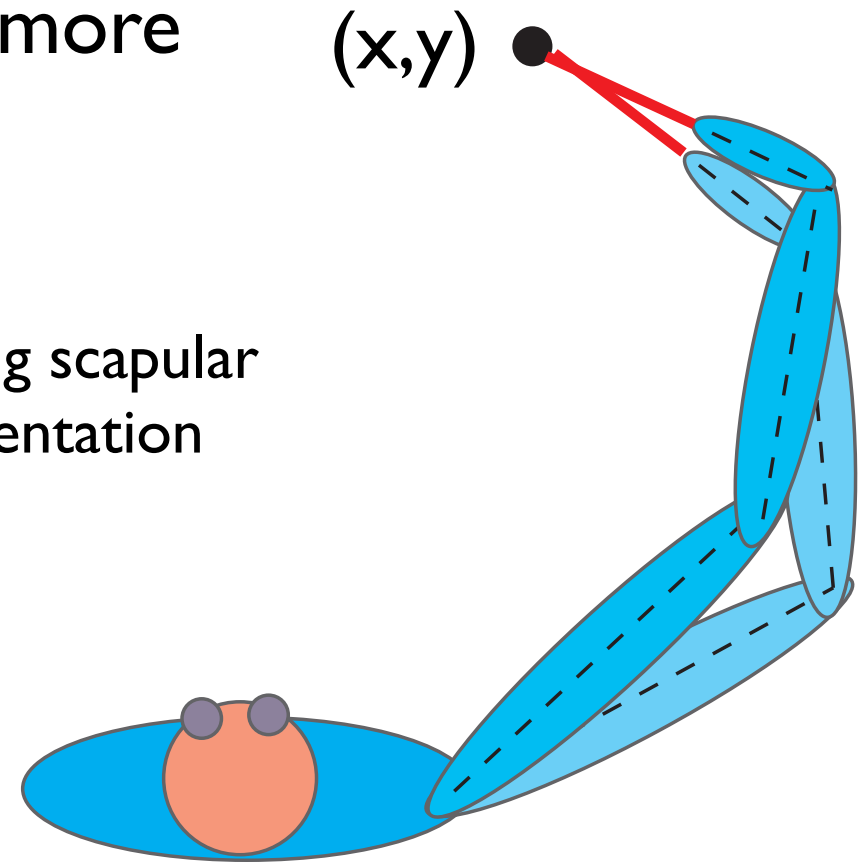
- for most tasks, there are many more degrees of freedom than task constraints...

- e.g., 10 joints in the upper arm including scapular joints to control hand position and orientation (3 to 5 or 6 DoF)

- but typically more: involve upper trunk movements

- or even make a step to move

- many muscles per joint (e.g. about 750 muscles in the human body vs. about 50 DoF)

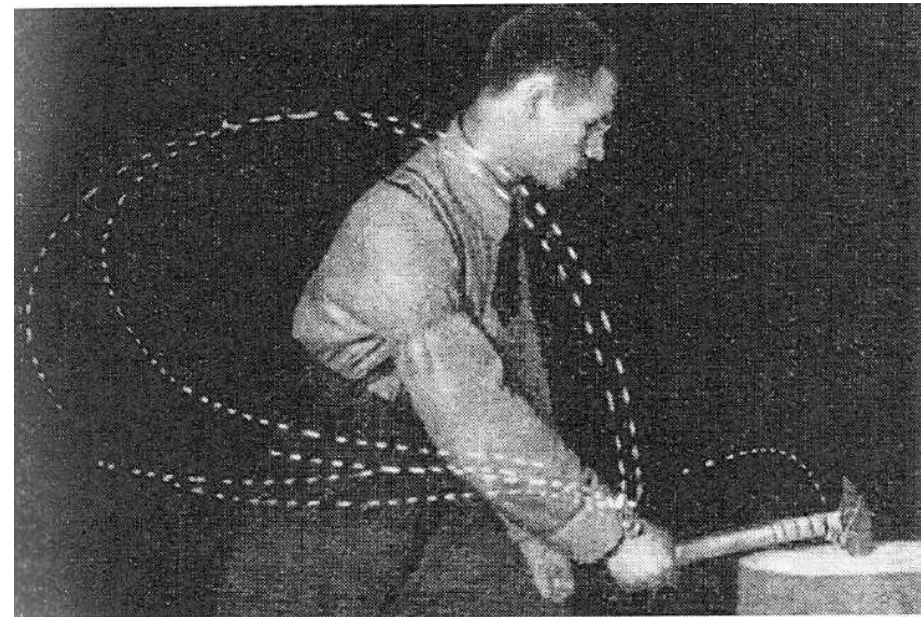


Degree of freedom problem in human movement

- Nikolai Bernstein... 1930's... in the Soviet Union
- “how to harness the many DoF to achieve the task”

Bernstein's workers

- highly skilled workers wielding a hammer to hit a nail... => hammer trajectory in space less variable than body configuration
- as detected in superposing spatial trajectories of lights on hammer vs. on body..
- but: camera frame anchored to nail/space, while initial body configuration varied

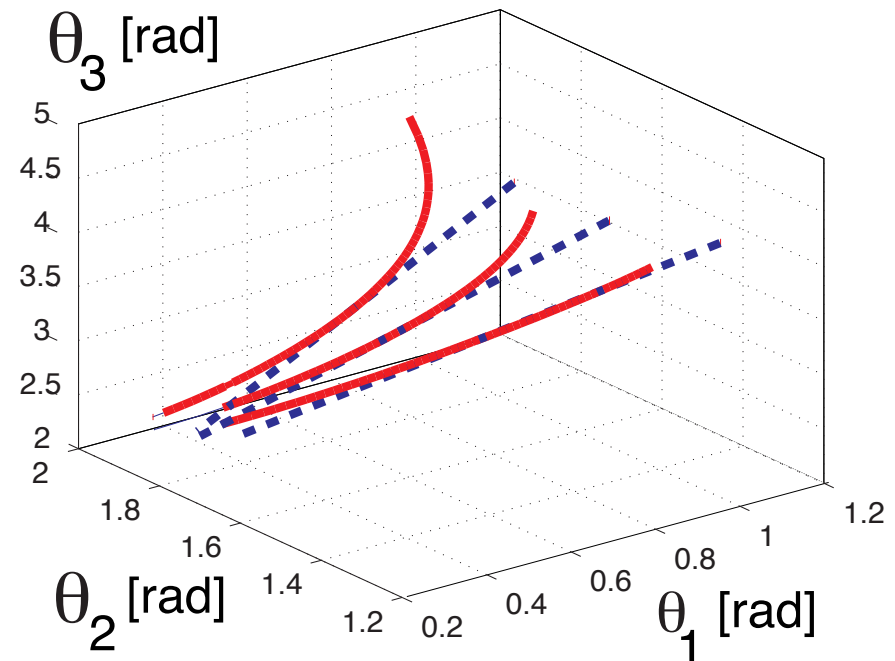
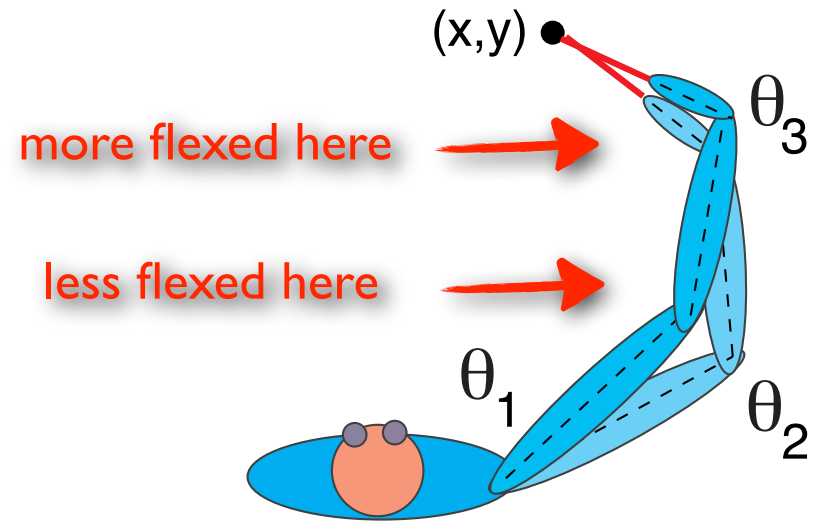


Bernstein's workers

- was the hammer position in space less variable than the joint configuration?
 - that is, does the task structure variance?
 - so that the solution to the degree of freedom problem lies in the variance/stability of the joint configuration?
- but: does this make any sense?
 - different reference frames for body vs. task
 - different units in the task vs joint space

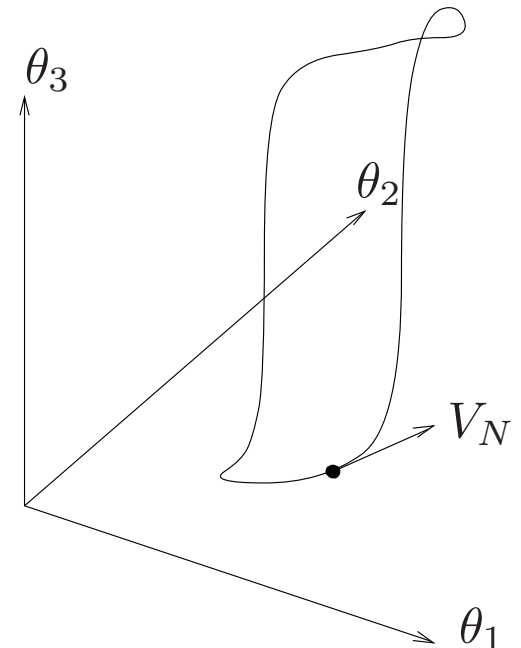
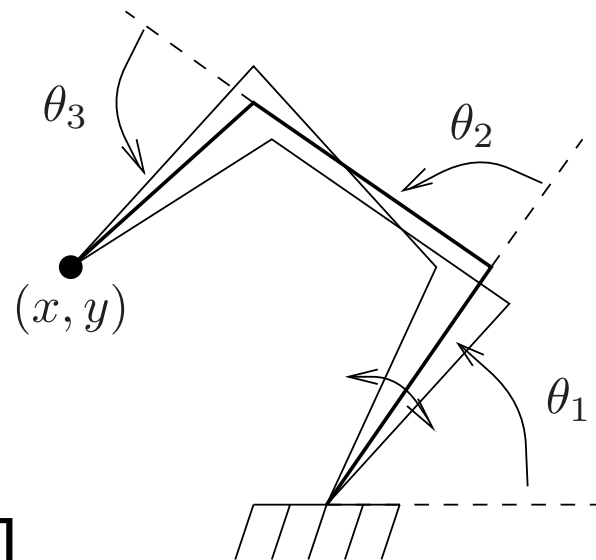
Concept of the UnControlled Manifold

- the many DoF are coordinated such that changes that affect the task-relevant dimensions are resisted against more than changes that do not affect task relevant dimension
- leading to compensation



Robotic concept

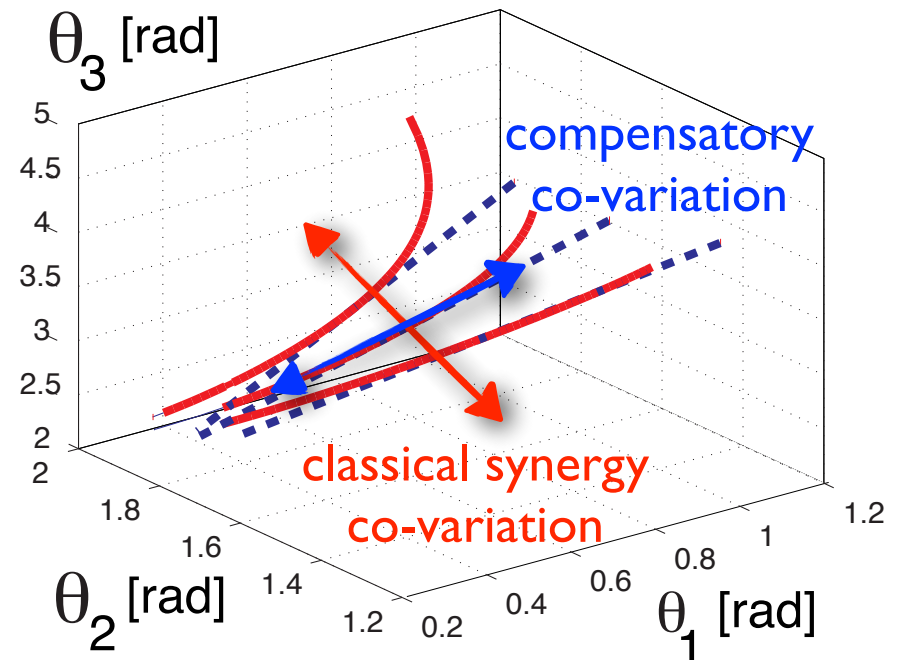
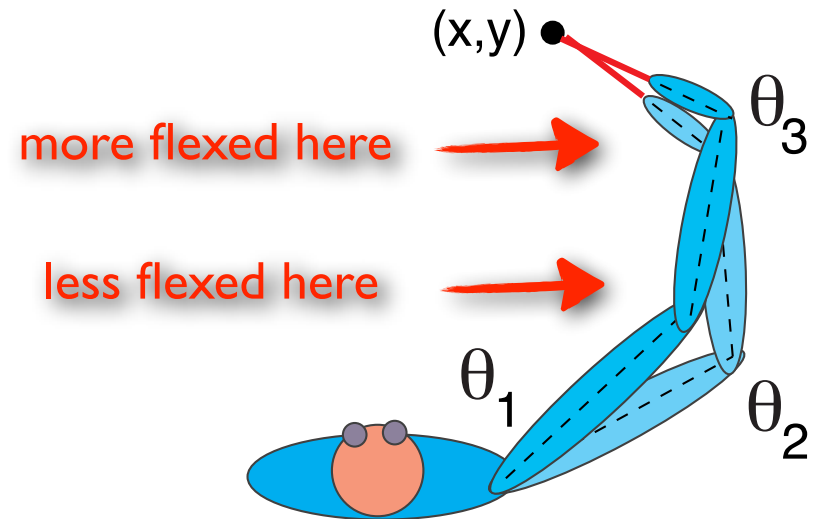
- self-motion manifold: within which end-effector does not move
- the pseudo-inverse is locally orthogonal to the self-motion manifold
- self-motion is needed to avoid hysteresis in redundant manipulators



[Murray, Li, Sastry, 1994]

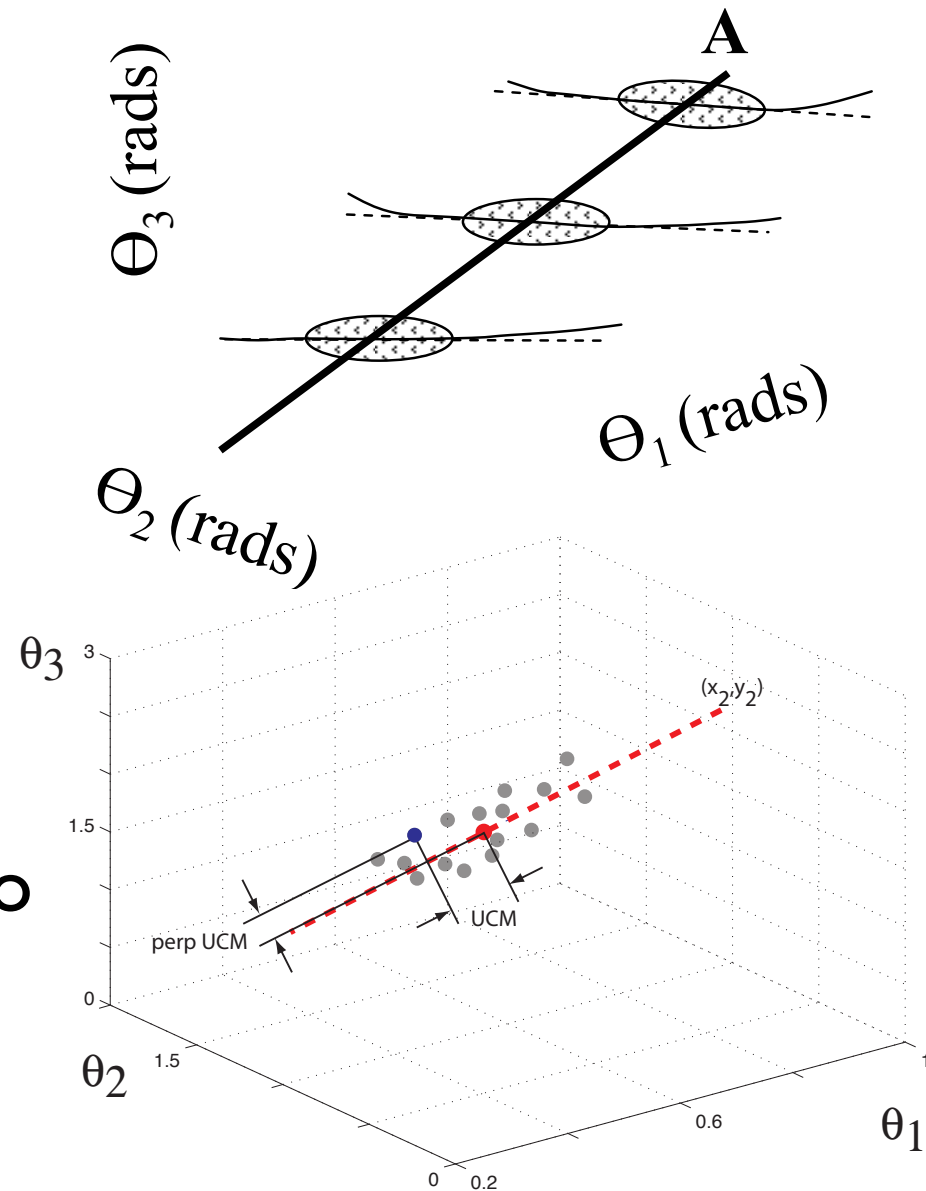
Structure of variance: UCM effect

- variance in movement lies primarily within the UCM, leaving the end-effector invariant!
- => measuring variance of end-effector in the same (embedding) space as the joint configuration



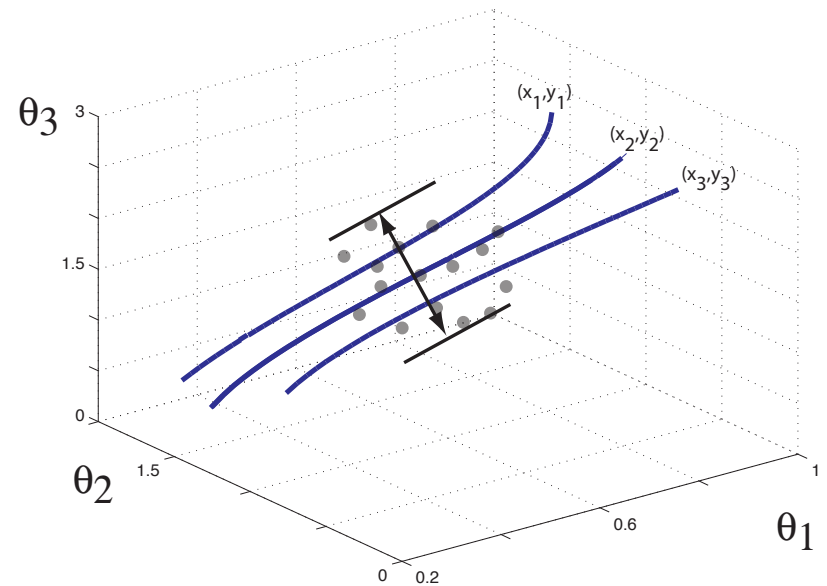
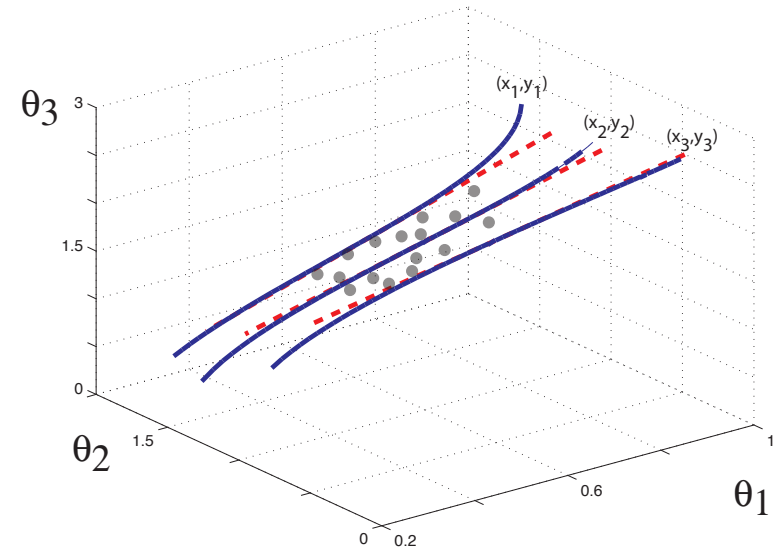
UCM synergy: data analysis

- align trials in time
- hypothesis about task variable
- compute null-space (tangent to the UCM)
- predict more variance within null space than perpendicular to it

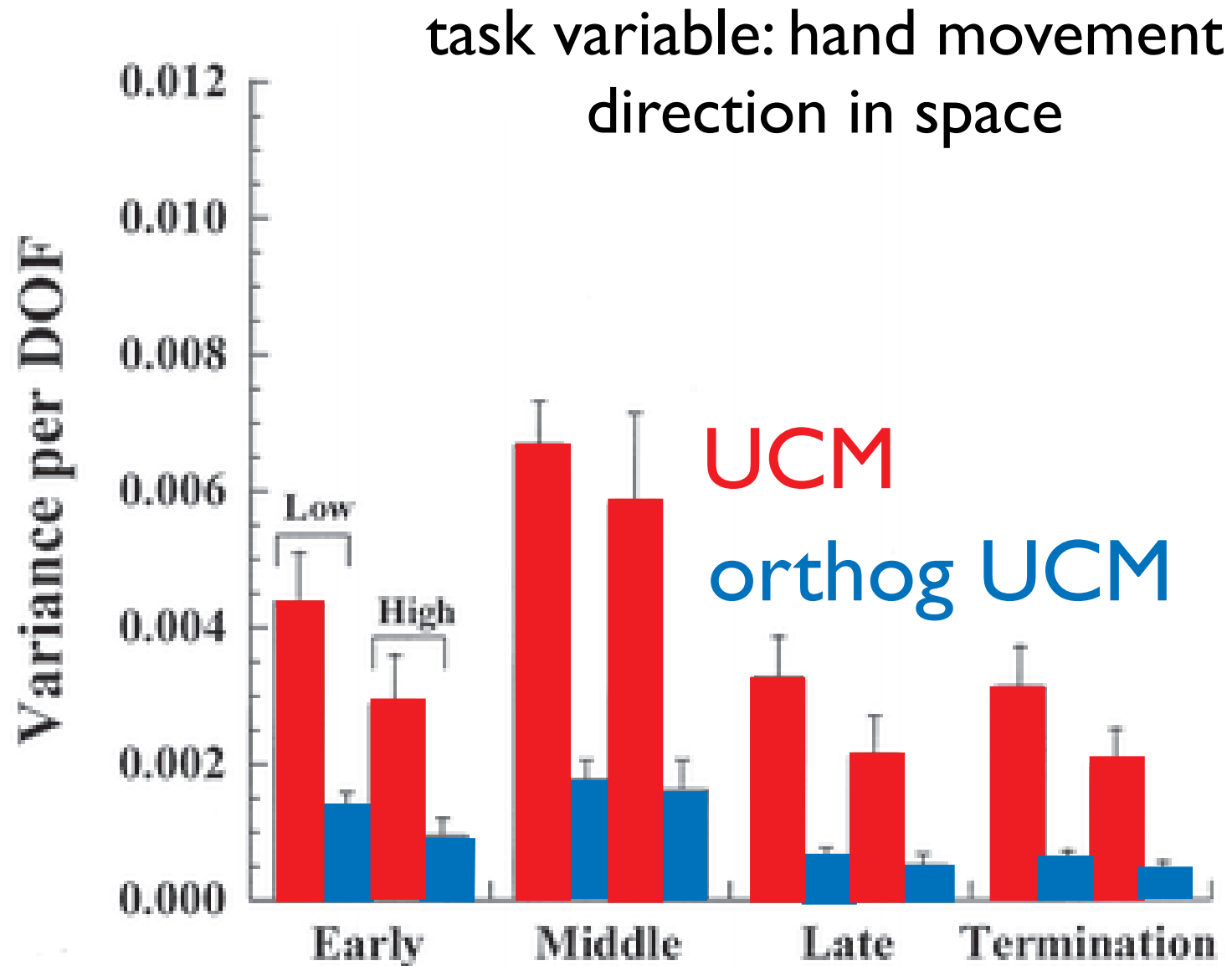


UCM synergy: data analysis

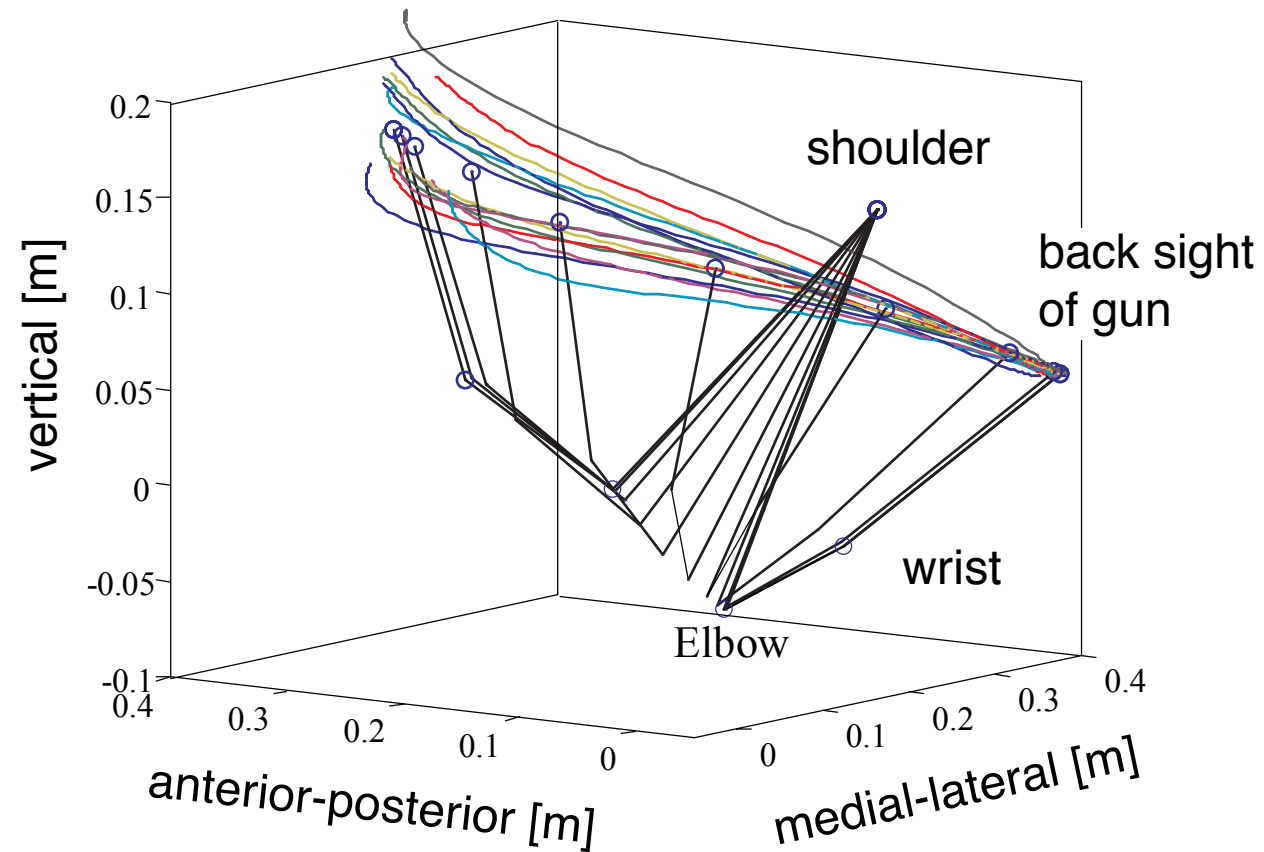
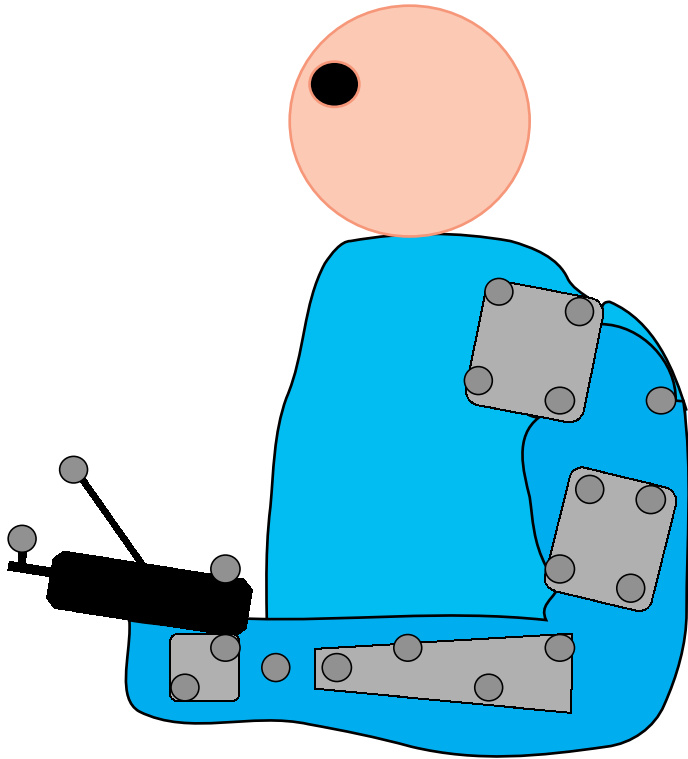
- supplement hypothesis testing by checking for correlation (Hermann, Sternad...)
- look for increase in variance of task variable when correlation within data is destroyed



Example 1: pointing with 10 DoF arm at targets in 3D



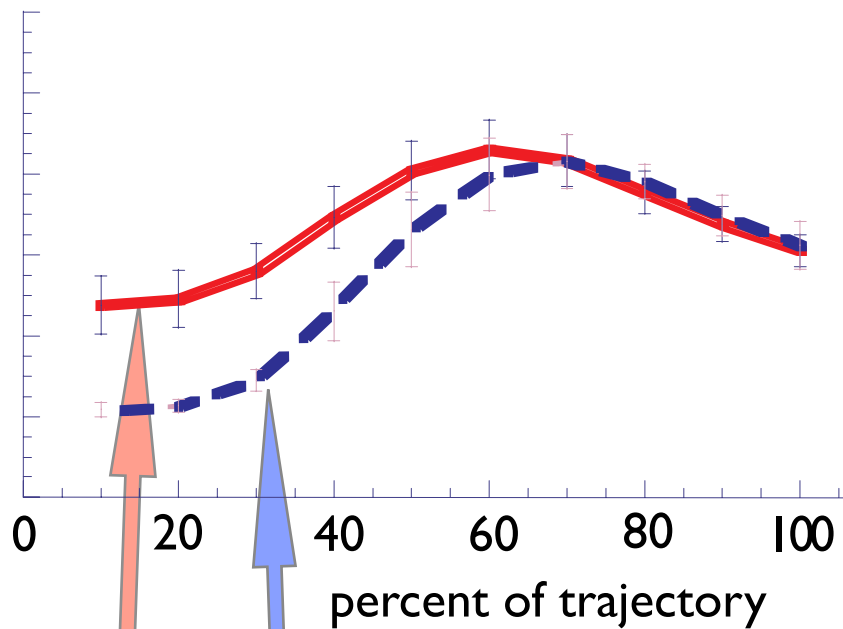
Example 2: shooting with 7 DoF arm at targets in 3D



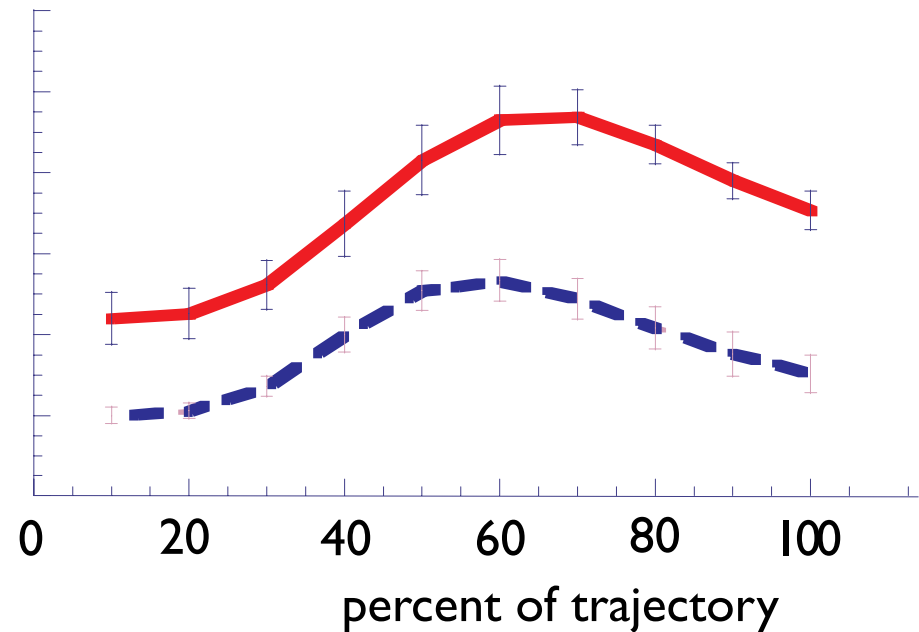
[from Scholz, Schöner, Latash: EBR 135:382 (2000)]

Example 2: shooting with 7 DoF arm at targets in 3D

gun spatial position



gun orientation to target



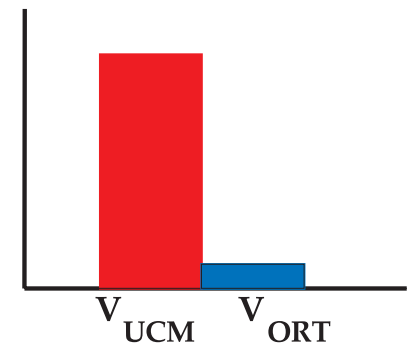
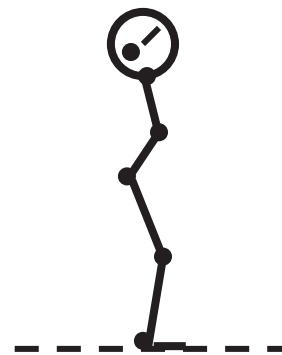
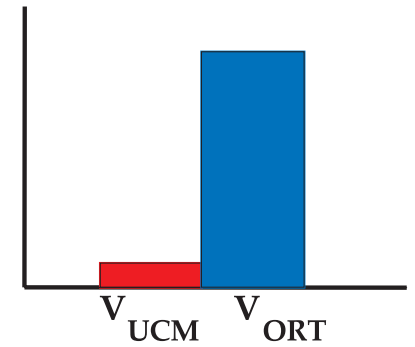
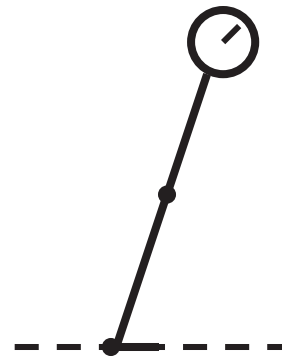
[from Scholz, Schöner, Latash: EBR 135:382 (2000)]

variance
within
UCM

variance
perpendicular
to UCM

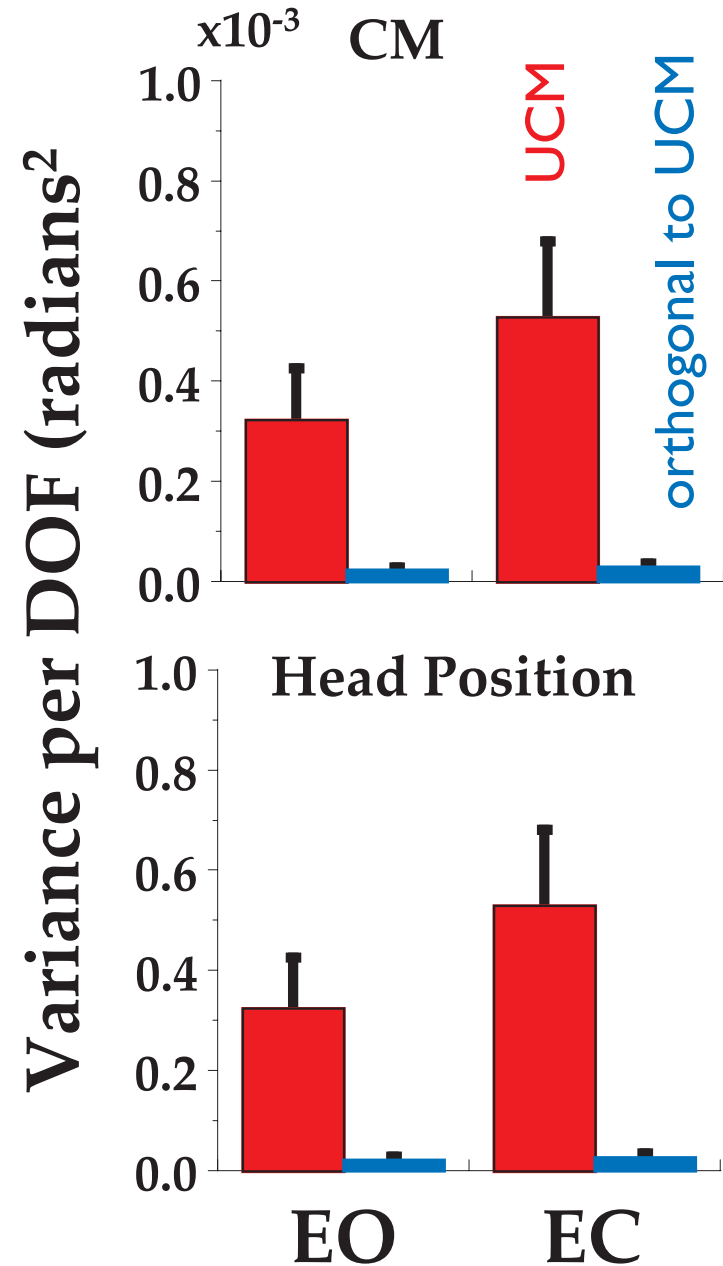
Example 3: posture

- Inverted pendulum hypothesis predicts the opposite than UCM



Example 3: posture

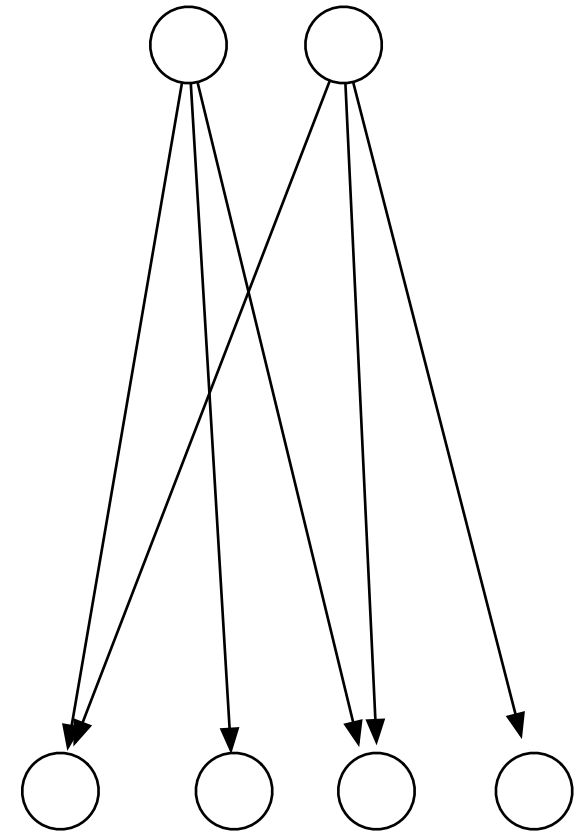
■ but: find signature of UCM synergy



Classical synergy

- a feedforward neural network from end-effector level representation to muscle level representation

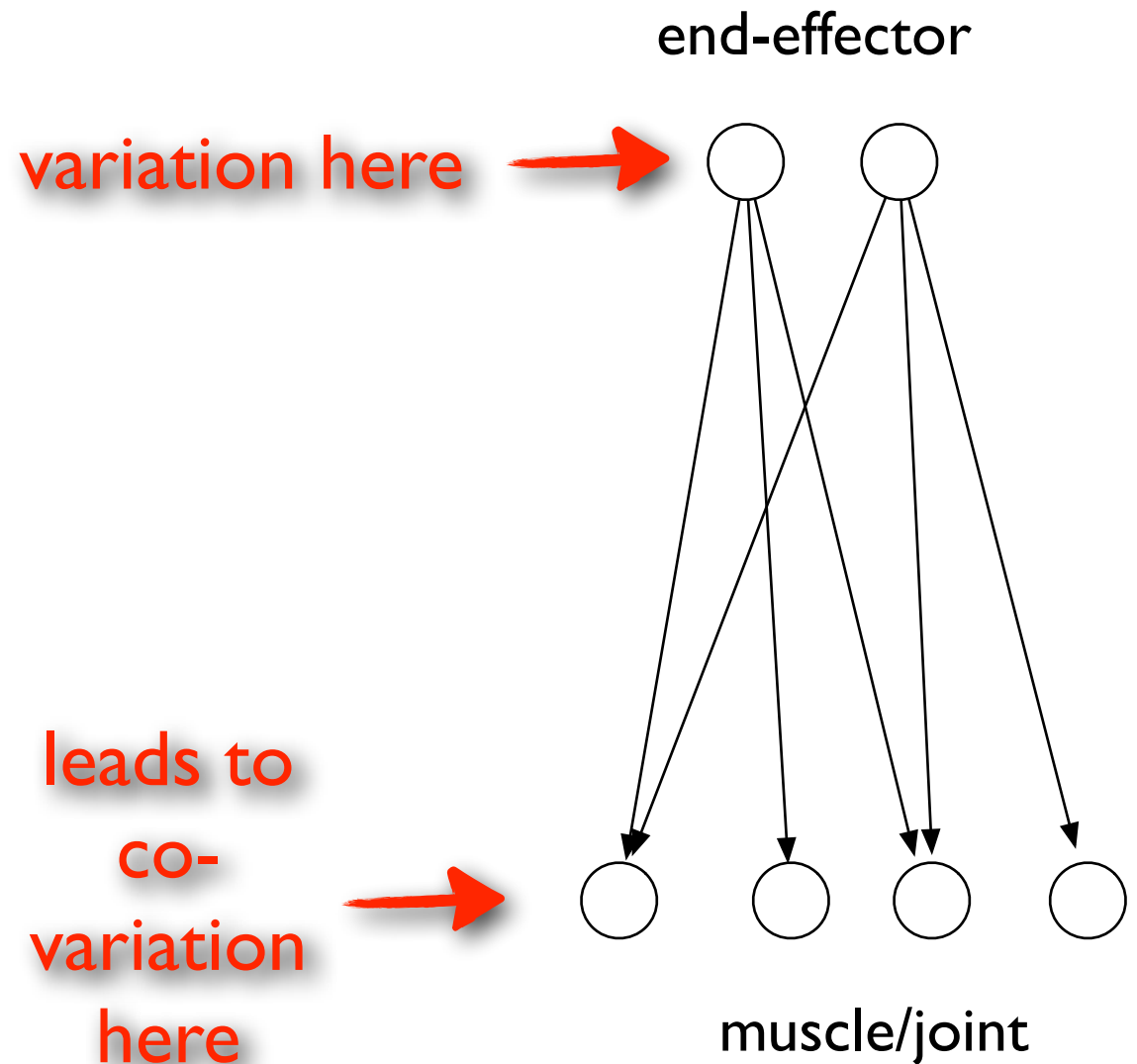
spatial (end-effector) level



muscle/joint level

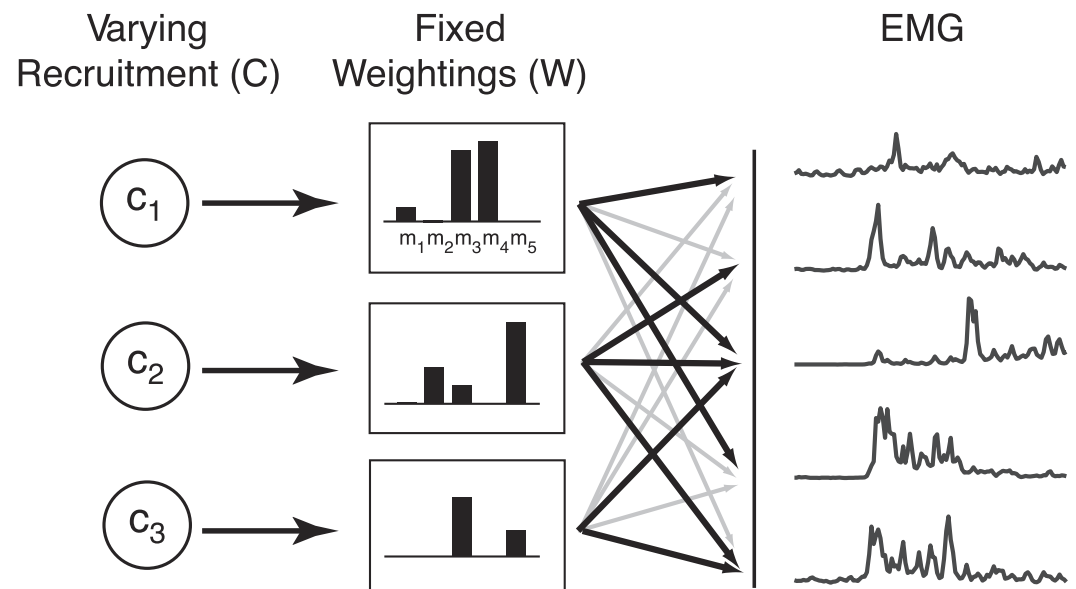
Classical synergy

- activation state at EE level varies in time or across tasks
- => covariation of muscle/joint descending activation



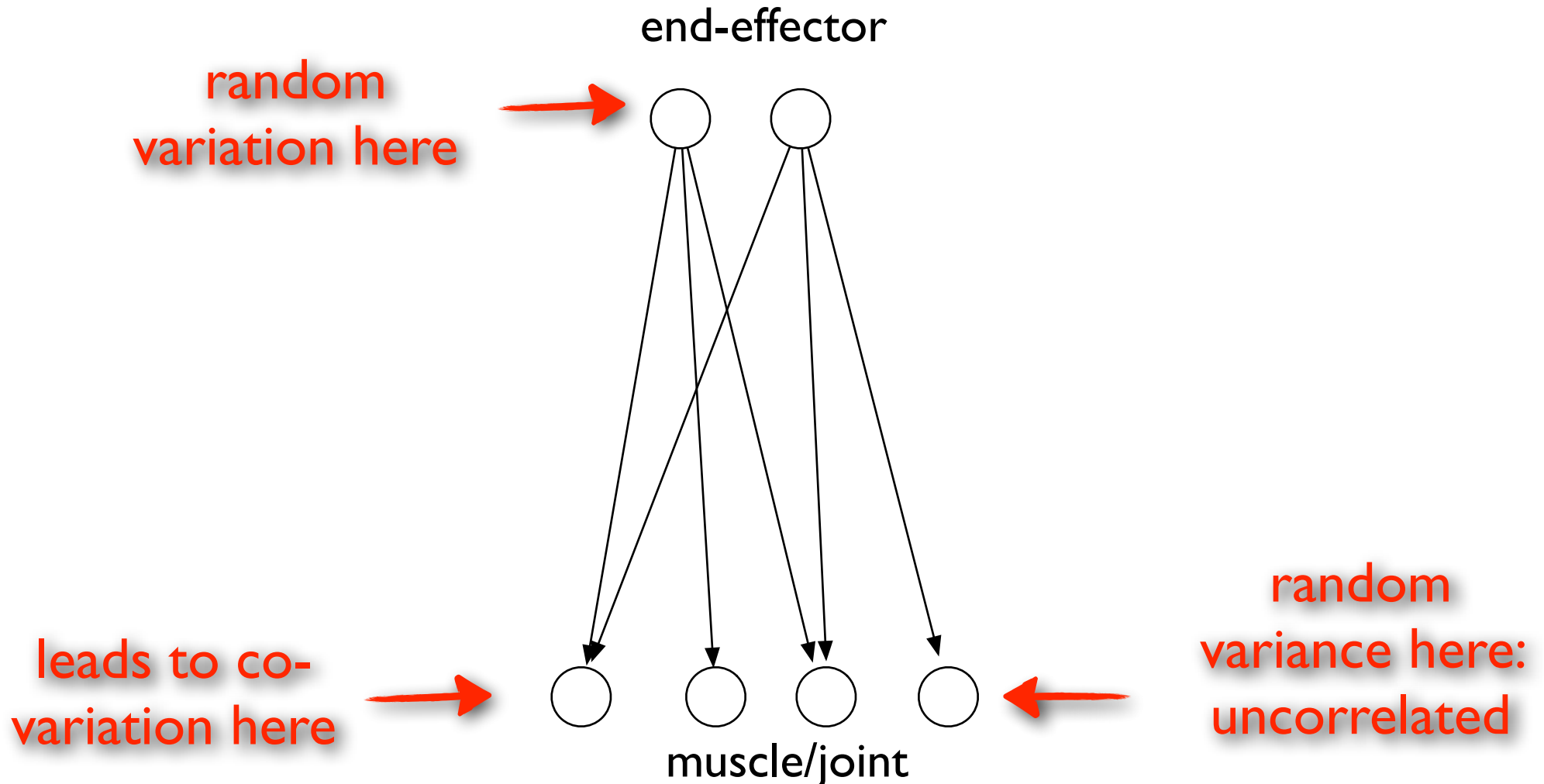
Classical synergy

- search for such covariation by looking for structure in data from many DoF across time and condition (e.g. by non-negative matrix factorization)
- of a small number of factors explain variance, conclude that classical synergies are at work



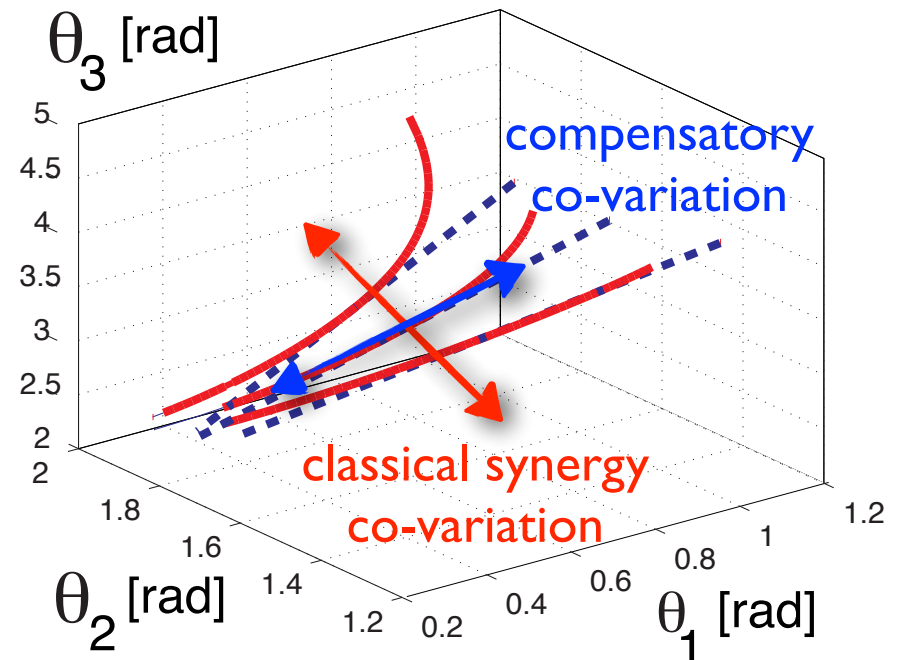
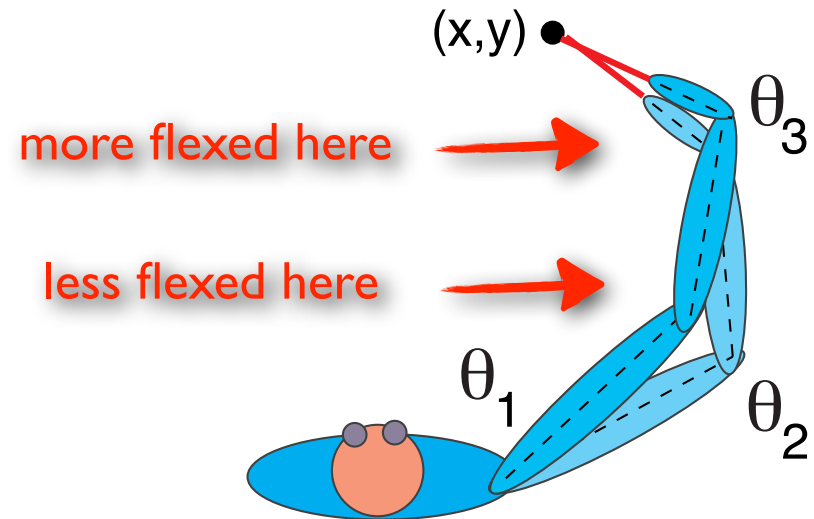
[Safavynia, Ting, 2012]

- the variance across repetitions for a given task at given point in time = **signature of stability**
- that variance is structured in the **OPPOSITE** way than predicted by the classical synergy!

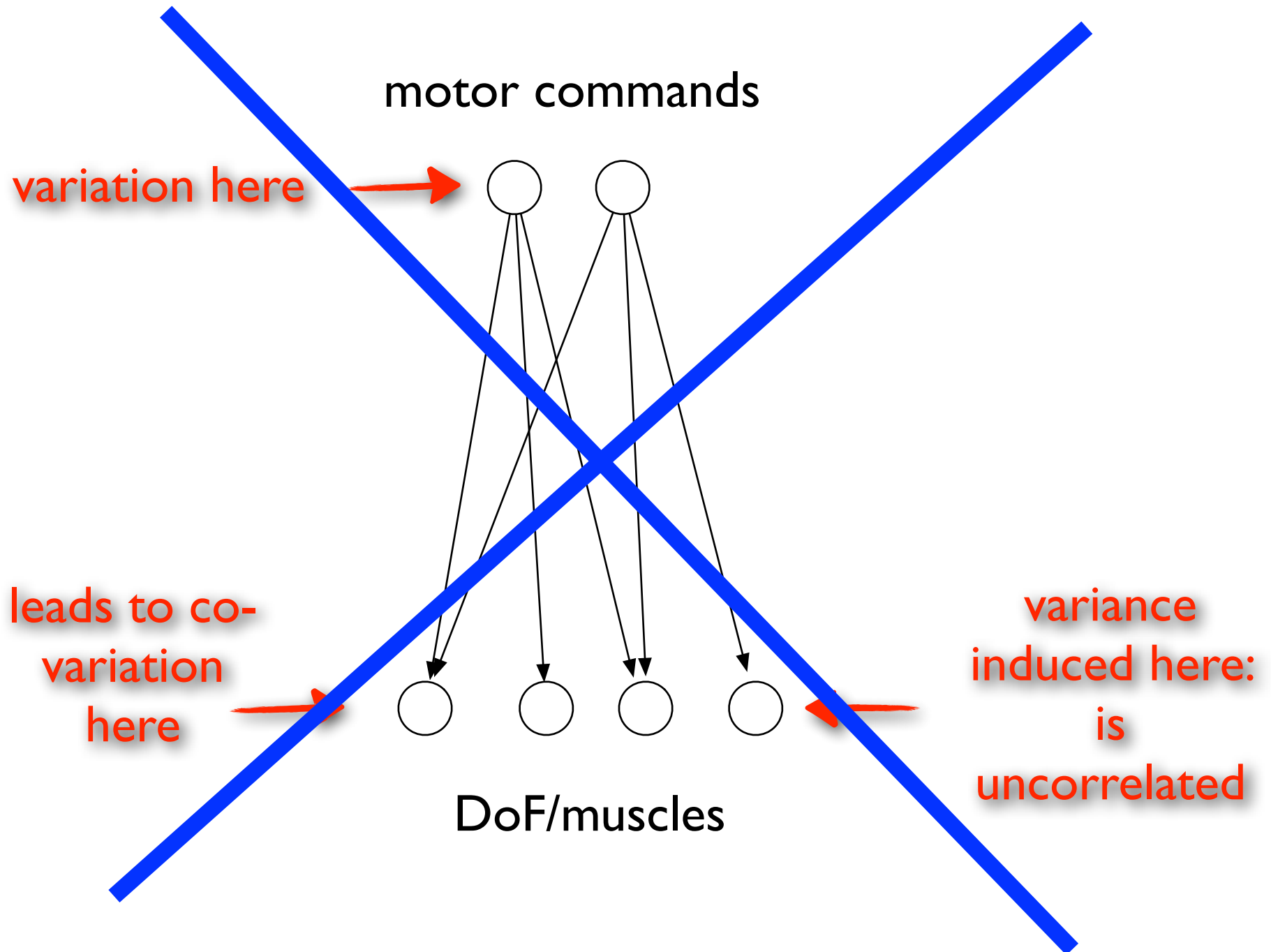


UCM effect

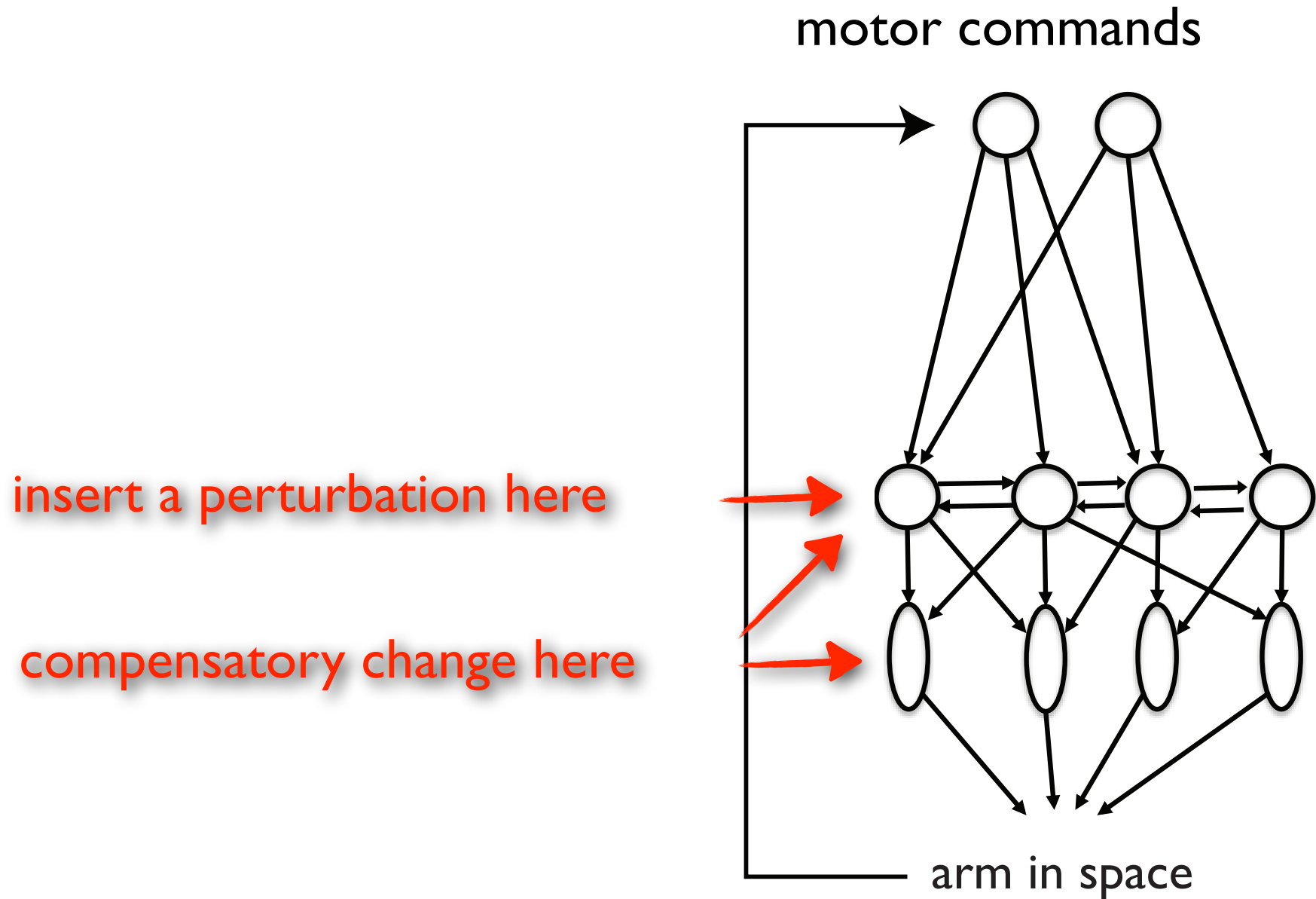
- the UCM effect reflects the opposite pattern of co-variation than the pattern predicted by the classical synergy concept



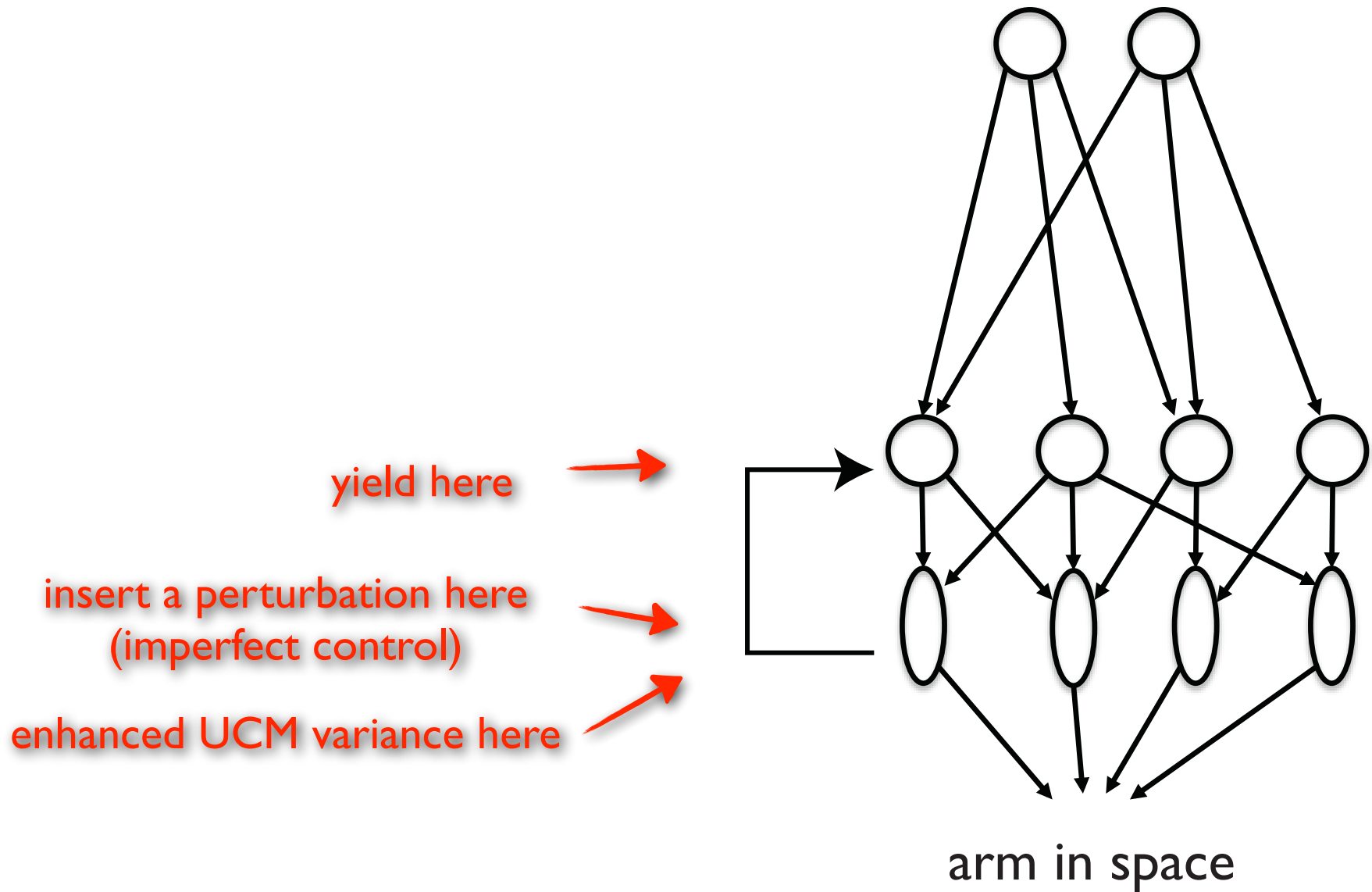
classical synergy is not sufficient



UCM synergy: decoupling



UCM synergy: back-coupling



[Martin, Scholz, Schöner *Neural Computation* 2009]

[Martin, Reimann, Schöner *Biological Cybernetics* 2019]

UCM synergy: from feedback

leads to change here



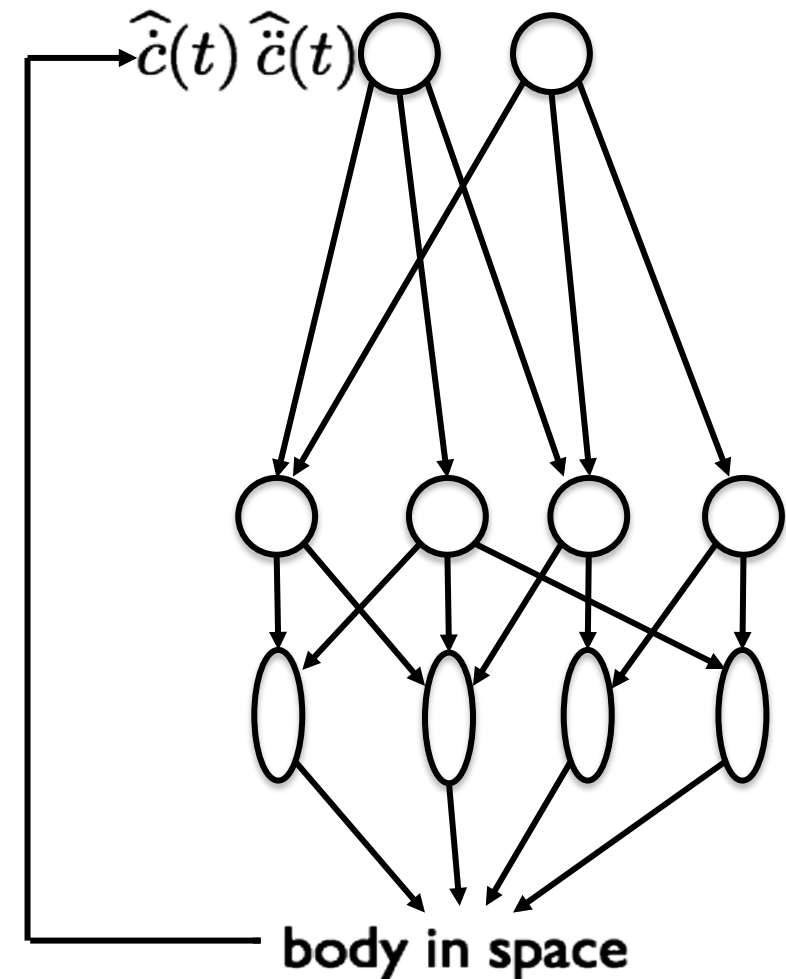
passes this to other DoF



insert a perturbation here



compensatory change here



Conclusion

- The problem of inverse kinematics is part of the broader “degree of freedom problem”
- Neither robots nor human movement systems can use a simple 1:1 optimal solution, but must allow self-motion to avoid drifts into singular configurations
- Humans have considerable self-motion and stabilize movement much less within the UCM (self-motion) space than orthogonal to it
- Beyond the feed-forward few-to-many mappings, this involves compensatory coupling among motor commands.