Prof. Dr. G. Schöner, Institut für Neuroinformatik gregor.schoener@rub.de

Autonomous robotics

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Exercise 5: Kinematics and Degree of freedom problem

Note: You can use this book for help with kinematics (although it is quite advanced, so you'll have to search for simple examples to get help): A Mathematical Introduction to Robotic Manipulation by RM Murray, Z Li, SS Sastry, CRC Press, Boca Raton FL USA, available from RM Murray at

`www.cds.caltech.edu/~murray/books/MLS/pdf/mls94-complete.pdf`

You won't need this book for the simple exercises here, however.

Kinematics

Consider a two-point planar arm with a "shoulder" and an "elbow" joint, controlling an end-effector, the tip of the "hand" that is rigidly attached to the elbow.

1. Make a drawing of the arm with the two coordinate axes and mark the two joint angles, θ_1 and θ_2 in the way done in the lectures (counting the shoulder angle from the horizontal, the elbow angle from the continuation of the upper arm). Denote the segment lengths, l_1 and l_2. [You can copy the figure from a source that you cite correctly.]

2. Derive the forward kinematics that links the joint angles to the end-effector. The result was given in the lecture. [The derivation makes use of the definitions of the geometrical meaning of sin and cos.]

3. Determine the Jacobian from this by computing the partial derivates. [The result is also listed in the lecture slides, but make the computation.] Write down the differential forward kinematics as a vector-valued equation as in the lecture.

4. Now consider a velocity vector in joint space in which only the shoulder joint moves. Compute the end-effector velocity vector. Evaluate that function when the arm is initially horizontal and interpret the geometrical meaning of that vector. The same for the arm initially pointing away from the x-axis and being all stretched.

5. Make a sketch of the reachable workshop of the arm (assume the upper arm is longer than the lower arm, no joint limits).

6. Look up the inverse kinematics in the lecture slides. Insert the end effector velocity vectors you obtained from the second task and try to see if you can obtain the required velocity vector in joint space. [Caution, something is wrong at that point!]

Toy UCM

This is about the degree of freedom problem and the "uncontrolled manifold". We use the same arm as in the previous task. We will make a "toy" version of the UCM in which only the horizontal position, x, of the end-effector matters, while the position along the other axis, y, does not matter.

1. Draw two or more examples of arm configurations that have the same x value. Write down an equation that is characterizes all these configurations, that together form the "uncontrolled manifold" (UCM).

2. Solve that equation, thus computing the UCM. For very value of x, you have a different UCM. [Hint: Introduce a "segment angle" $\psi_2 = \theta_2 + \theta_1$ and write that angle as a function of x. Do not forget to return in the end to the original joint angles.]

3. Make a drawing of the UCM for different values of x.

4. For a ramp-like movement of $x(t)$ (draw that), proposed graphically different possible paths in the space of the joint angles.