Prof. Dr. G. Schöner, Institut für Neuroinformatik gregor.schoener@rub.de

Autonomous robotics

April 30, 2020

Exercise 2 Attractor Dynamics for vehicle motion

Note The following textbook is freely available online and is useful introduction into the qualitative theory of dynamical systems: Scheinermann, E.R.,: "Invitation to Dynamical Systems"
https://github.com/scheinerman/InvitationToDynamicalSystems
Focus on the continuous time aspect of the book.

1 Obstacle avoidance

In the lecture, we saw how to generate movement by generating a time course of vehicle heading, $\phi(t)$, from a dynamical system defined over ϕ. The contribution of a single obstacle to this dynamics is given by

$$\dot{\phi} = \alpha(\phi - \psi) \exp\left[-\frac{(\phi - \psi)^2}{2\sigma^2}\right]$$

where ψ is the direction in which an obstacle lies, α is the strength of repulsion, and σ determines the width of this contribution. [For background, read through the first part of the paper Schöner, Dose, Engels (1995) made available on the web page.]

1. Plot the first factor and describe the geometrical meaning of the two parameters, ψ and α.
2. Plot the second factor and describe the geometrical meaning of the two parameters, ψ and σ.
3. Plot the product. Is the slope of the dynamics at $\phi = \psi$ affected by the second factor? Why or why not?
4. Plot the time course of heading direction that results from this dynamics when the initial heading direction, $\phi(0)$ is (a) $< \psi$, (b) $> \psi$, (c) $= \psi$. These plots are qualitative based on your mental "simulation" of the dynamics.
5. Plot the same time courses when α is larger.
6. State what happens when the initial heading, $\phi(0)$ is far from ψ: $|\phi(0) - \psi| >> \sigma$?

2 Human movement

The work reported in: B R Fajen, W H Warren, S. Temizer, L P Kaebling: "A Dynamical Model of Visually-Guided Steering, Obstacle Avoidance, and Route Selection" (*International Journal of Computer Vision* **54**:1334 (2003)) is based on the approach of Schöner, Dose, Engels (1995). That first paper is also on available on the web page.

Read the paper, focussing on the first part, around the dynamics listed in Equation (4). For the following, take only the attraction term into account:

$$\ddot{\phi} = -b\dot{\phi} - k(\phi - \psi_g)$$

where the various constants have been contracted into k. You can further simplify this by introducing a shifted variable $\theta = \phi - \psi_g$.

1. Compute the fixed point. [Hint: Transform the second order equation into a first order equation by introducing an auxiliarly variable $\omega = \dot{\phi}$]

2. Write this linear dynamics as

$$\begin{pmatrix} \dot{\theta} \\ \dot{\omega} \end{pmatrix} = \mathbf{A} \begin{pmatrix} \theta \\ \omega \end{pmatrix}$$

   where **A** is a matrix. Compute the eigenvalues of that matrix. [Hint: If you don't know how to do this, there is an example in Scheinermann's book, e.g., on page 62, after Eq. 2.10 there. There is also an appendix that states this in general on page 266, A.1.3., after Eq. A.2]

3. Determine the stability of the fixed point by determining if the Eigenvalues have negative real parts.