Neural Dynamics

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Activation

- How to represent the inner state of the Central Nervous System?

- \( \Rightarrow \) Activation concept
Activation

- neural state variables
  - membrane potential of neurons?
  - spiking rate?
  - ... population activation...
Activation

- activation as a real number, abstracting from biophysical details
  - low levels of activation: not transmitted to other systems (e.g., to motor systems)
  - high levels of activation: transmitted to other systems
  - as described by sigmoidal threshold function
  - zero activation defined as threshold of that function
Activation

compare to connectionist notion of activation:
- same idea, but tied to individual neurons

compare to abstract activation of production systems (ACT-R, SOAR)
- quite different... really a function that measures how far a module is from emitting its output...
Neurons as input-output threshold elements that form feed-forward neural networks.

\[
\text{output} = g \left( \sum \text{(inputs)} \right)
\]
Recurrent neural networks

- require a concept of time
- time is not discrete (spiking is asynchronous) \( \Rightarrow \) neural dynamics…
Activation dynamics

- activation variables $u(t)$ as time continuous functions...

\[ \tau \frac{du(t)}{dt} = f(u) \]

- what function $f$?
Activation dynamics

\[ \tau \dot{u} = \xi_t \]

start with \( f=0 \)

Probability distribution of perturbations

resting level
Activation dynamics

\[ \tau \dot{u} = -u + h + \xi_t. \]
In a dynamical system, the present predicts the future: given the initial level of activation $u(0)$, the activation at time $t$: $u(t)$ is uniquely determined.

\[
\frac{du(t)}{dt} = \dot{u}(t) = -u(t) + h \quad (h < 0)
\]
mental simulation

=> dynamical systems tutorial Mathis Richter
Neural dynamics

- Stationary state = fixed point = constant solution
- Stable fixed point: nearby solutions converge to the fixed point = attractor
Neural dynamics

"attractor structures ensemble of solutions = flow"

\[ \tau \dot{u}(t) = -u(t) + h \]
Neuronal dynamics

- inputs = contributions to the rate of change
  - positive: excitatory
  - negative: inhibitory
- => shifts the attractor
- activation tracks this shift (stability)

\[ \tau \dot{u}(t) = -u(t) + h + \text{inputs}(t) \]
=> simulation
tutorial on numerics

- dynamical system
  continuous time
- differential
  quotient
  approximates the derivative in discrete time
- Euler iteration
  equation in discrete time

\[ \dot{u} = f(u). \]

\[ \dot{u}(t_i) \approx \frac{u(t_i) - u(t_{i-1})}{\Delta t} \]

\[ u(t_i) = u(t_{i-1}) + \Delta t f(u(t_{i-1})). \]
Matlab code
Neuronal dynamics with self-excitation

\[ \tau \dot{u}(t) = -u(t) + h + S(t) + c \sigma(u(t)) \]
Neuronal dynamics with self-excitation

\[ \tau \dot{u}(t) = -u(t) + h + S(t) + c\sigma(u(t)) \]
Neuronal dynamics with self-excitation

\[ \tau \dot{u}(t) = -u(t) + h + S(t) + c\sigma(u(t)) \]

\[ \Rightarrow \text{nonlinear dynamics!} \]
Neuronal dynamics with self-excitation

- at intermediate stimulus strength: bistable
- “on” vs “off” state

\[ \tau \dot{u}(t) = -u(t) + h + S(t) + c\sigma(u(t)) \]
Neuronal dynamics with self-excitation

- Increasing input strength => detection instability
Neuronal dynamics with self-excitation

- Decreasing input strength => reverse detection instability
Neuronal dynamics with self-excitation

The detection and the reverse detection instability create discrete events out of input that changes continuously in time.
Simulation
Neuronal dynamics with competition

\[ \tau \dot{u}_1(t) = -u_1(t) + h - \sigma(u_2(t)) + S_1 \]
\[ \tau \dot{u}_2(t) = -u_2(t) + h - \sigma(u_1(t)) + S_2 \]
Neuronal dynamics with competition

- The rate of change of activation at one site depends on the level of activation at the other site.
- Mutual inhibition

\[ \tau \dot{u}_1(t) = -u_1(t) + h - \sigma(u_2(t)) + S_1 \]
\[ \tau \dot{u}_2(t) = -u_2(t) + h - \sigma(u_1(t)) + S_2 \]

Sigmoidal nonlinearity
to visualize, assume that $u_2$ has been activated by input to positive level

$\Rightarrow$ then $u_1$ is suppressed
why would $u_2$ be positive before $u_1$ is? E.g., it grew faster than $u_1$ because its inputs are stronger/inputs match better

=> input advantage translates into time advantage which translates into competitive advantage
Neuronal dynamics with competition

vector-field in the absence of input

resting state

ID cut through vector-field

du/dt = f(u)

resting level
Neuronal dynamics with competition

vector-field (without interaction) when both neurons receive input

\[ \frac{du}{dt} = f(u) \]

1D cut through vector-field

stimulus determined state

input

activated level
only activated neurons participate in interaction!

sigmoidal nonlinearity
Neuronal dynamics with competition

- vector-field of mutual inhibition

site 1 inhibits site 2

site 2 inhibits site 1

interaction combined
Neuronal dynamics with competition

vector-field with strong mutual inhibition: bistable
Neuronal dynamics with competition

before input is presented

after input is presented
Neuronal dynamics with competition

=> biased competition

stronger input to site 1:
attractor with activated $u_1$ stronger,
attractor with activated $u_2$ weaker, may become unstable
Neuronal dynamics with competition

=> biased competition

before input is presented

after input is presented
=> simulation
=> hands-on exercise NOW

in the robotics lab..
where do activation variables come from?

=> DFT lecture