COMPUTER VISION: DEEP LEARNING LAB COURSE

DAY 2 – FEATURE-BASED IMAGE CLASSIFICATION

SEBASTIAN HOUBEN

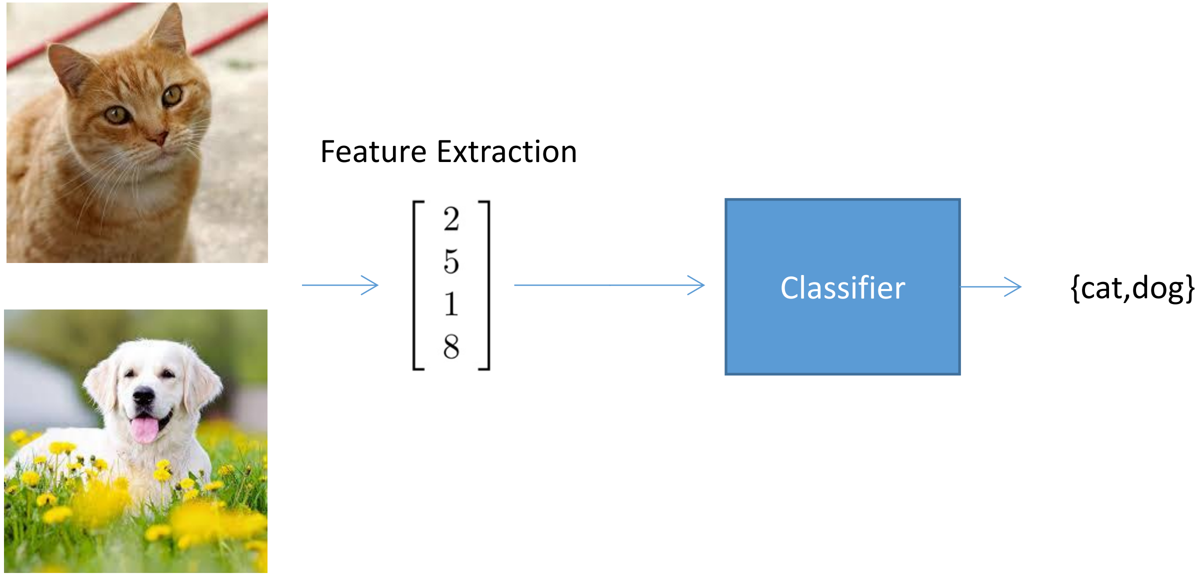
Schedule

Today

- Histogram of Oriented Gradients (HOG)
- Dimensionality Reduction with Principal Component Analysis (PCA)
- Going Deeper into Classification
    - Underfitting / Overfitting
    - Training-Test-Validation
- Support Vector Machine (SVM)
- Multi-Class SVM

RUHR UNIVERSITÄT BOCHUM RUB

Classification pipeline



Classification pipeline (Multi-class)

<u>Feature Extraction</u>

Histogram-of-Oriented-Gradients

$$\begin{bmatrix} 2 \\ 5 \\ 1 \\ 8 \end{bmatrix}$$

Multi-class SVM

{speed limit 20, speed limit 30, …, derestriction, yield way, …}

Histogram-of-Oriented-Gradients

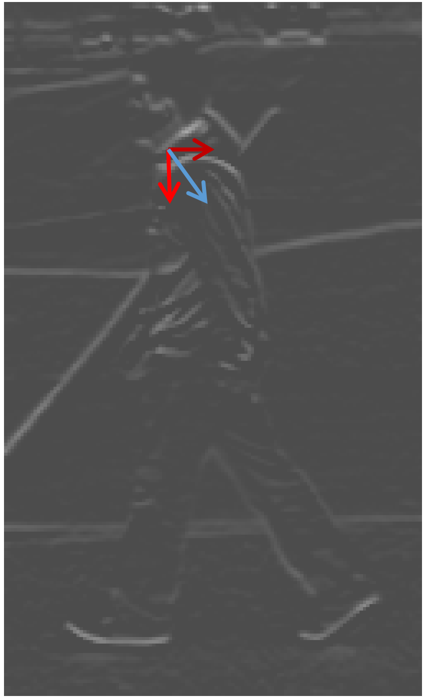
- Dalal & Triggs 2005
- Initially used for pedestrian detection
- Describes local gradient orientation distribution

Histogram-of-Oriented-Gradients

- Dalal & Triggs 2005
- Initially used for pedestrian detection
- Describes local gradient orientation distribution
- Compute gradients
    - Convolute image with

      $\begin{bmatrix} -1 & 0 & 1 \end{bmatrix}$ and $\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$

    - Yields pixel-wise orientation

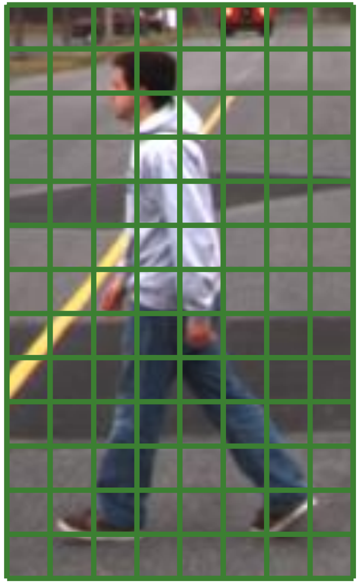


Histogram-of-Oriented-Gradients

- Dalal & Triggs 2005
- Initially used for pedestrian detection
- Describes local gradient orientation distribution
- Compute gradients
    - Convolute image with

$$\begin{bmatrix} -1 & 0 & 1 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

    - Yields pixel-wise orientation
- Divide image into cells (e.g., 8x8 pixels)

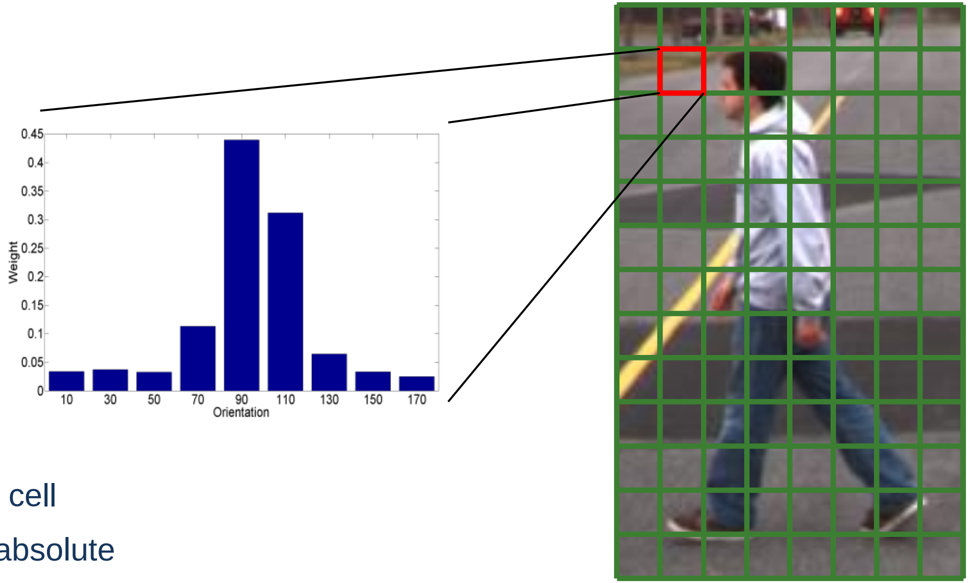


Histogram-of-Oriented-Gradients

- Dalal & Triggs 2005
- Initially used for pedestrian detection
- Describes local gradient orientation distribution
- Compute gradients
  - Convolute image with

    $$\begin{bmatrix}-1 & 0 & 1\end{bmatrix} \quad \text{and} \quad \begin{bmatrix}-1 \\ 0 \\ 1\end{bmatrix}$$

  - Yields pixel-wise orientation
- Divide image into cells (e.g., 8x8 pixels)
- Compute a histogram of all orientations present in each cell
  - Weigh the contribution of each pixel with its absolute gradient magnitude



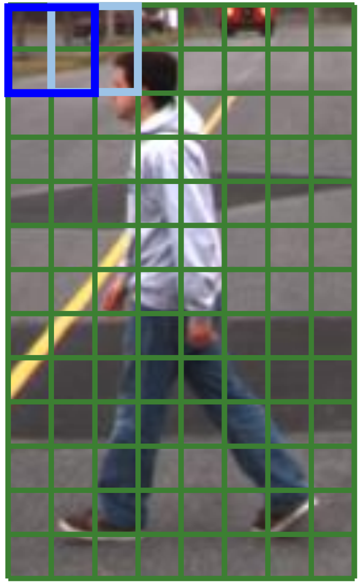
RUHR UNIVERSITÄT BOCHUM RUB

Histogram-of-Oriented-Gradients

- Dalal & Triggs 2005
- Initially used for pedestrian detection
- Describes local gradient orientation distribution
- Compute gradients
    - Convolute image with

      $$\begin{bmatrix} -1 & 0 & 1 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

    - Yields pixel-wise orientation
- Divide image into cells (e.g., 8x8 pixels)
- Compute a histogram of all orientations present in each cell
    - Weigh the contribution of each pixel with its absolute gradient magnitude
- Combine neighbouring cells to blocks (e.g. 2x2 cells) and normalize histograms with respect to sum of all pixel gradients magnitudes

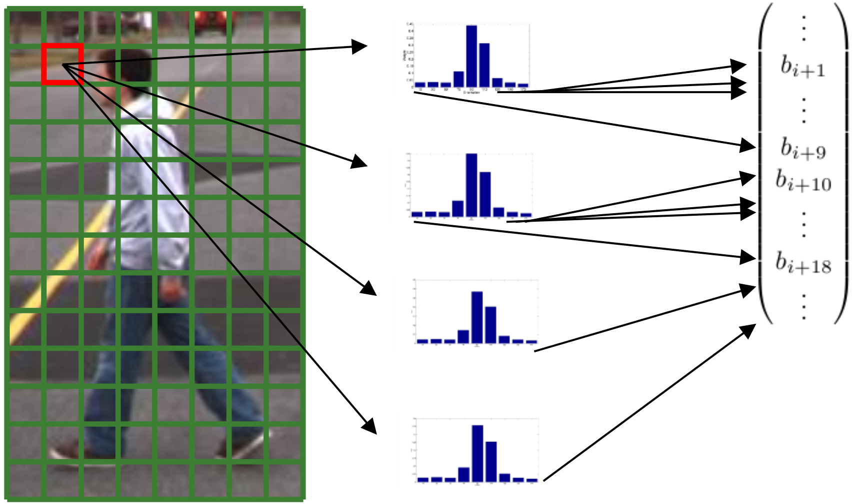


Histogram-of-Oriented-Gradients

- Dalal & Triggs 2005
- Initially used for pedestrian detection
- Describes local gradient orientation distribution
- Compute gradients
    - Convolute image with

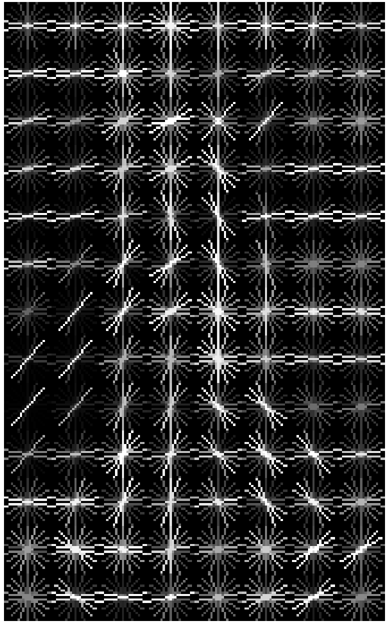
      $$\begin{bmatrix} -1 & 0 & 1 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

    - Yields pixel-wise orientation
- Divide image into cells (e.g., 8x8 pixels)
- Compute a histogram of all orientations present in each cell
    - Weigh the contribution of each pixel with its absolute gradient magnitude
- Combine neighbouring cells to blocks (e.g. 2x2 cells) and normalize histograms with respect to sum of all pixel gradients magnitudes
- For all blocks for all cells concatenate the histograms



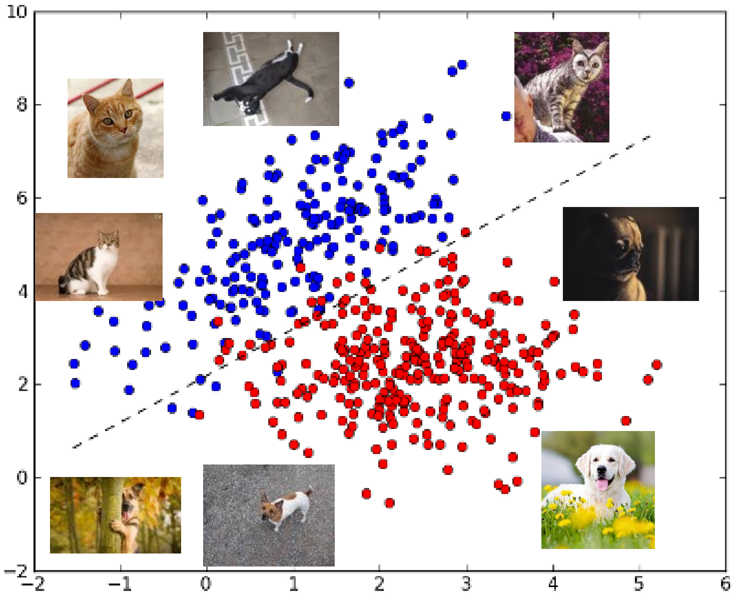
RUHR UNIVERSITÄT BOCHUM RUB

Histogram-of-Oriented-Gradients

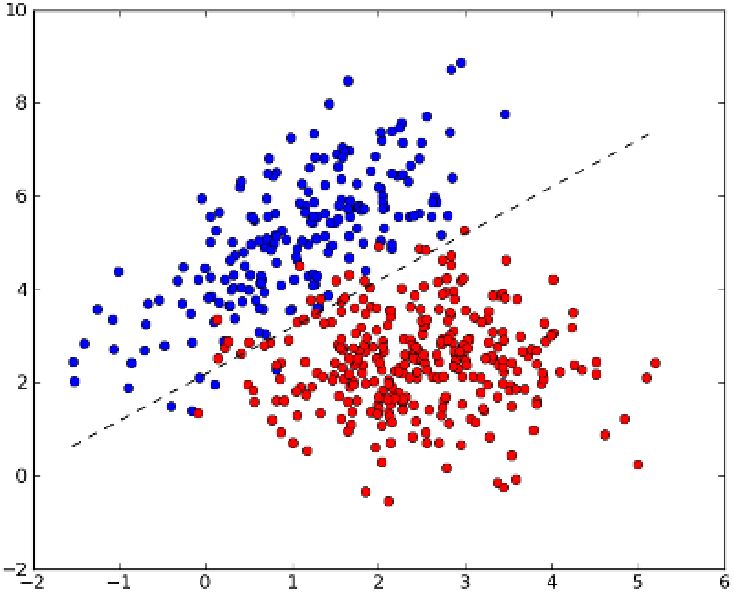


Visualizing High-dimensional Feature Spaces

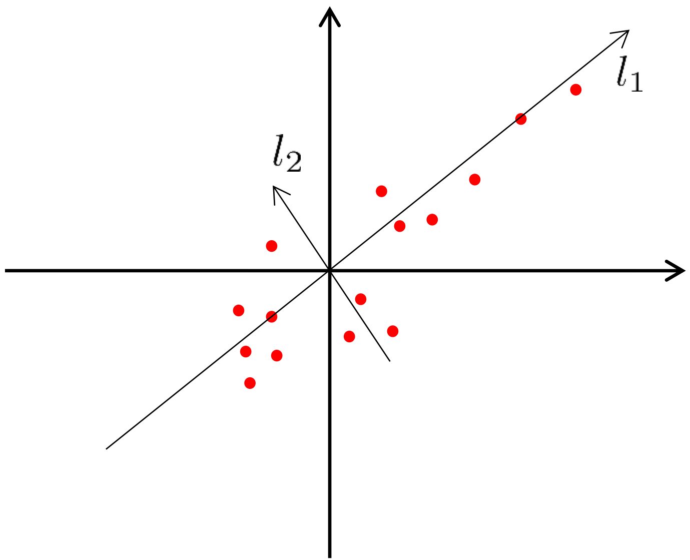
- High-dimensional vectors are hard to interpret
- Visualizing in 2d or 3d is preferable
- Dimensionality reduction / embedding
- Several methods:
  - PCA (Principal Component Analysis)
  - t-SNE (t-distributed Stochastic Nearest-Neighbour Embedding)
  - LLE (Locally-Linear Embedding)
  - MDS (Multi-Dimensional Scaling)



Visualizing High-dimensional Feature Spaces



Visualizing High-dimensional Feature Spaces

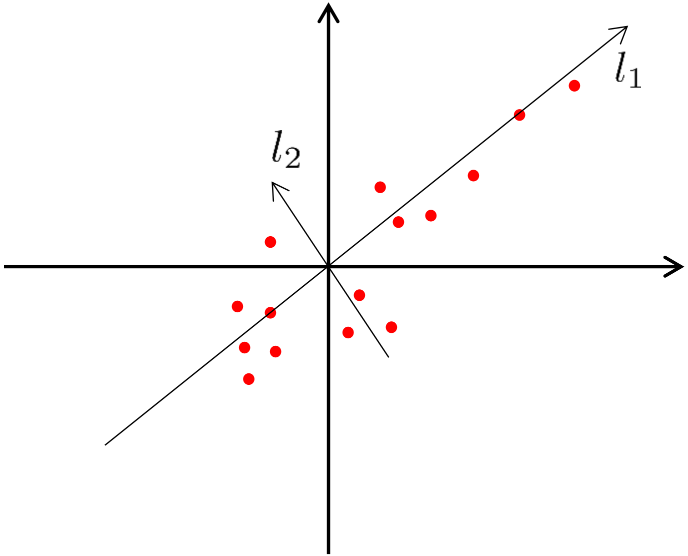


- Find function that maps data points to 2 dimensions: $f : \mathbb{R}^n \to \mathbb{R}^2$
- Make it easy: Linear
- Thus, can be represented by a 2 x n matrix

$$f(x) = Lx = \begin{pmatrix} l_1 \\ l_2 \end{pmatrix} x$$

- But linear means 0 is mapped to 0
  - Subtract mean value from dataset beforehand
- Consists of two rows
- Rows represent the axes of main variance (principal axes)

Visualizing High-dimensional Feature Spaces



- Rows represent the axes of main variance (principal axes)
- $$\min_{l_1, ||l_1||^2=1} \sum_i (x_i^T l_1)^2$$
- Row vector maximizing this, is given by eigenvector of
  $$C = \sum_i x_i x_i^T$$
  w.r.t. largest eigenvalue (covariance matrix *C*)
- Generally: Take the eigenvectors corresponding to the largest eigenvalues of the covariance matrix and project the zero-mean dataset to these vectors

$$\min_{l_1, ||l_1||^2=1} \sum_i (x_i^T l_1)^2$$

$$= \min_{l_1, ||l_1||^2=1} \sum_i l_1^T x_i x_i^T l_1$$

$$= \min_{l_1} \max_{\lambda \geq 0} \sum_i l_1^T x_i x_i^T l_1 + \lambda(||l_1||^2 - 1)$$

$$\min_{\lambda \geq 0} 2 \sum_i x_i x_i^T l_1 + 2\lambda l_1 = 0$$

$$\min_{\lambda \geq 0} \left(\sum_i x_i x_i^T \right) l_1 = \lambda l_1$$

$$y^T A y = y^T \begin{pmatrix} \sum_j A_{1j} y_j \\ \cdots \\ \sum_j A_{nj} y_j \end{pmatrix}$$

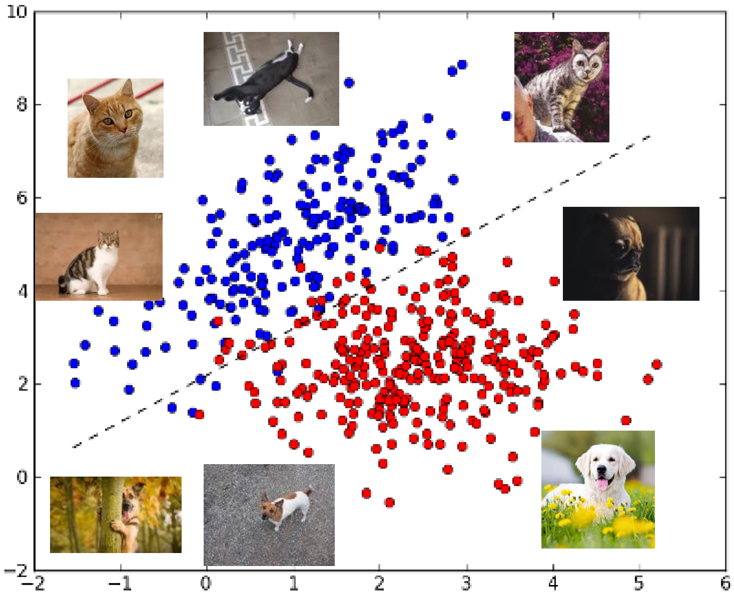
$$= \sum_i \sum_j A_{ij} y_i y_j$$

$$\frac{\partial}{\partial y_k} \sum_{ij} A_{ij} y_i y_j$$

$$= \sum_{i \neq k} A_{ik} y_i + \sum_{i \neq k} \underbrace{A_{ki}}_{=A_{ik}?} y_i + 2 A_{kk} y_i$$

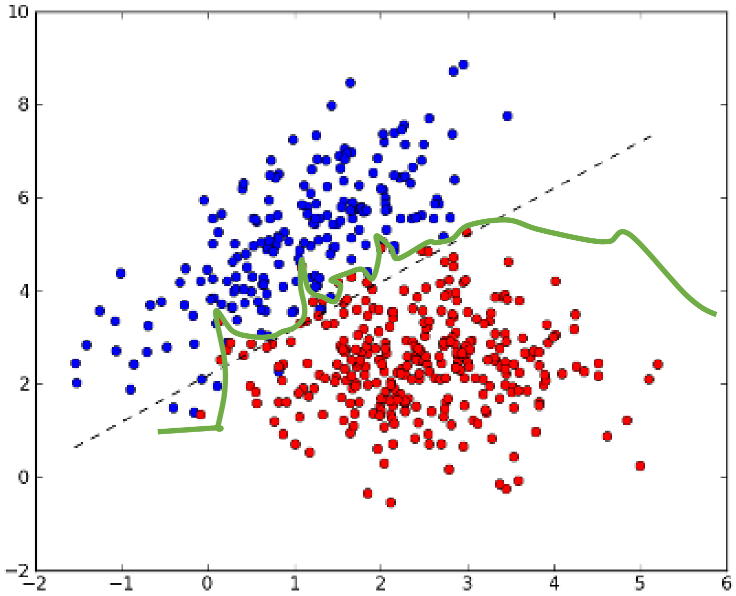
$$= \sum_i 2 A_{ik} y_i = 2Ay$$

Image Classification



- Linear classifier finds hyperplane to seperate sets of points
- A more complex classifier might find a better way to seperate the two datasets

Image Classification



- Linear classifier finds hyperplane to seperate sets of points
- A more complex classifier might find a better way to seperate the two datasets
- Many ML methods have hyper-parameters that control the complexity of the function to fit
- But: In general, very complex functions tend to perform worse on unseen data

RUHR UNIVERSITÄT BOCHUM RUB

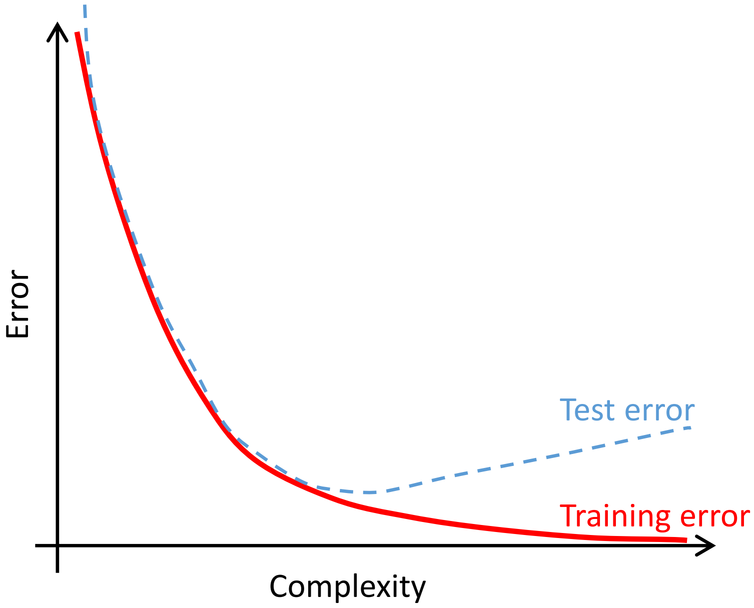
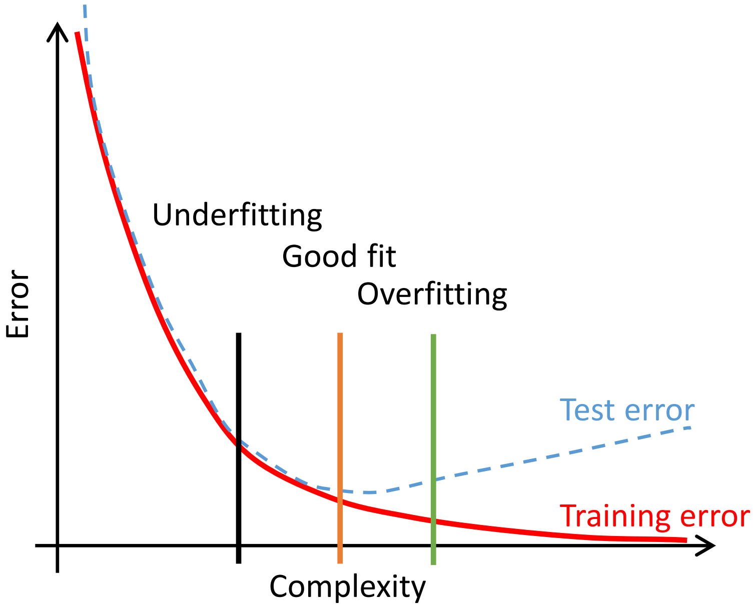


Image Classification

- Linear classifier finds hyperplane to seperate sets of points
- A more complex classifier might find a better way to seperate the two datasets
- Many ML methods have hyper-parameters that control the complexity of the function to fit
- But: In general, very complex functions tend to perform <u>worse on unseen data</u>

Image Classification



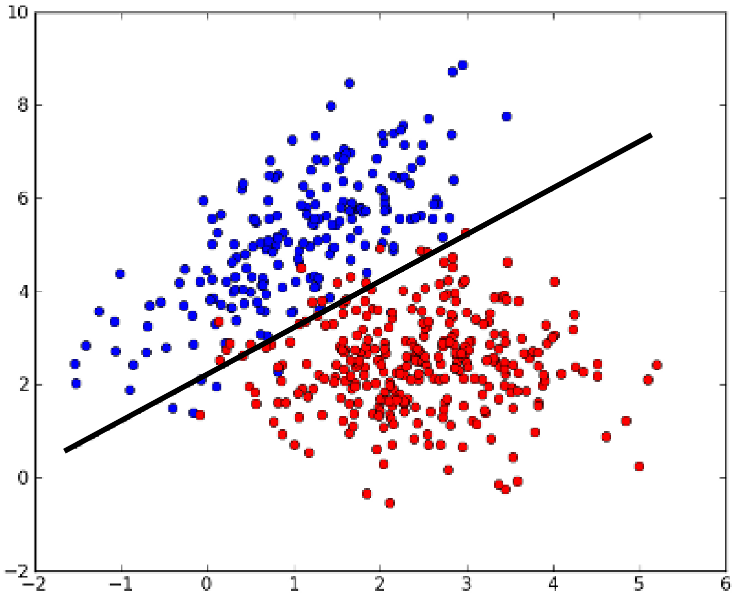
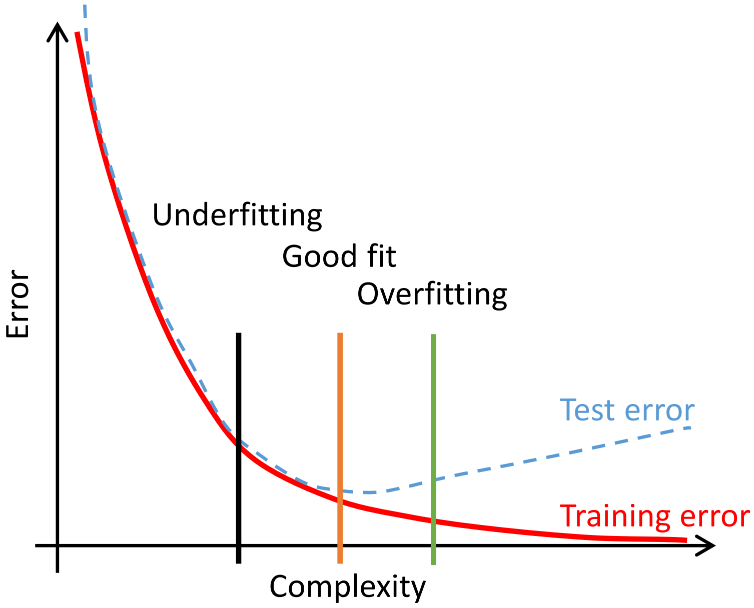


Image Classification



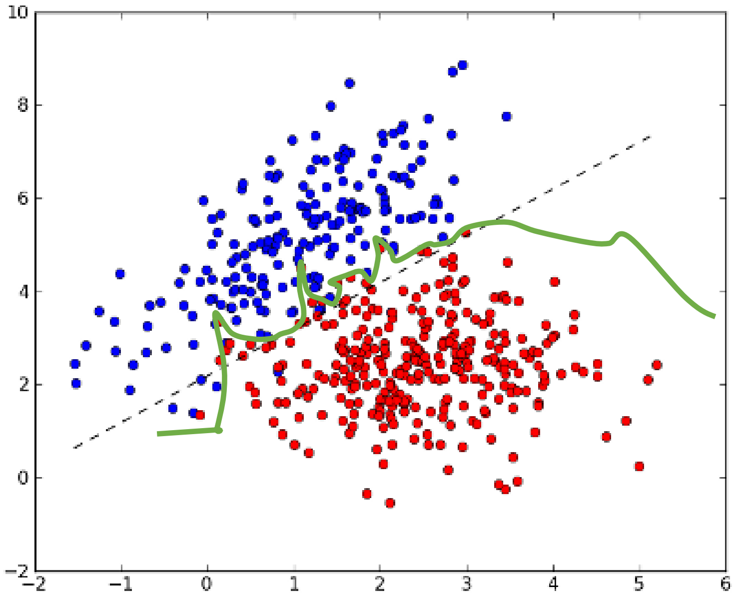
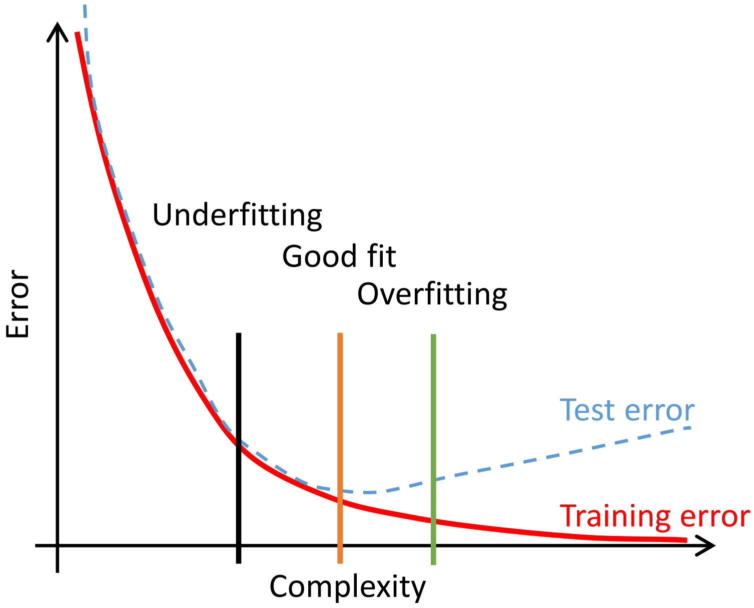
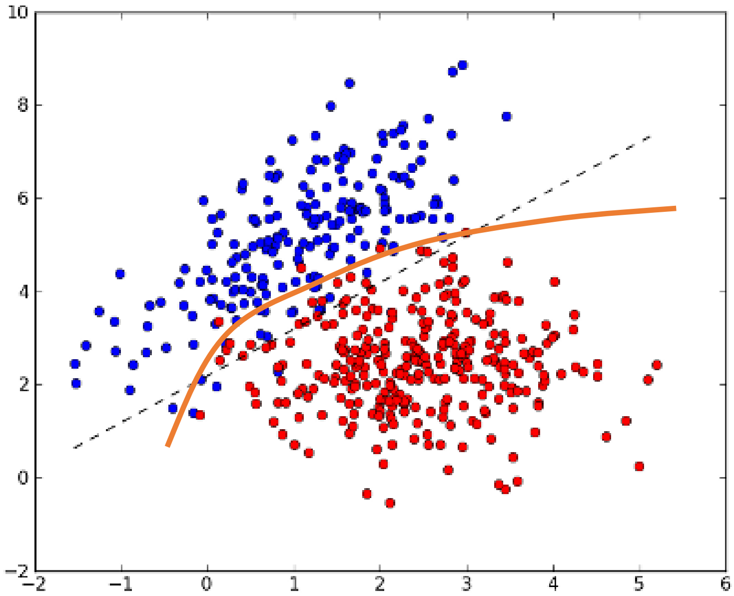


Image Classification





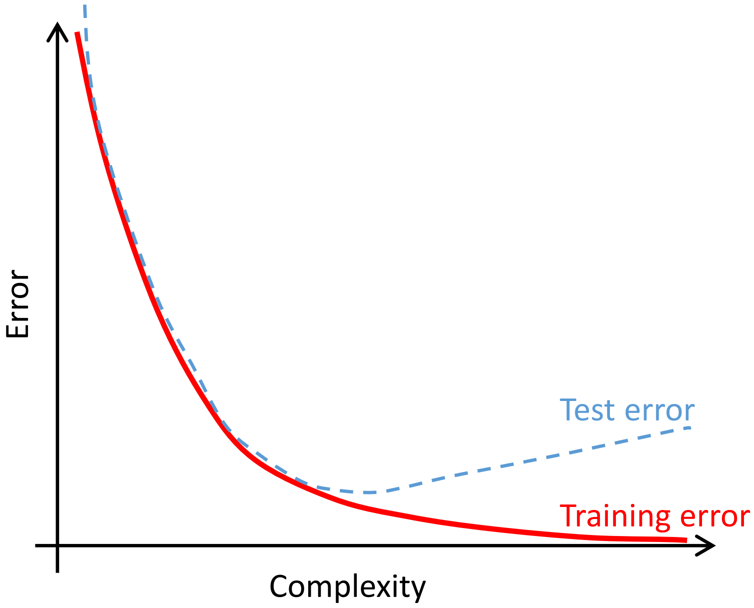
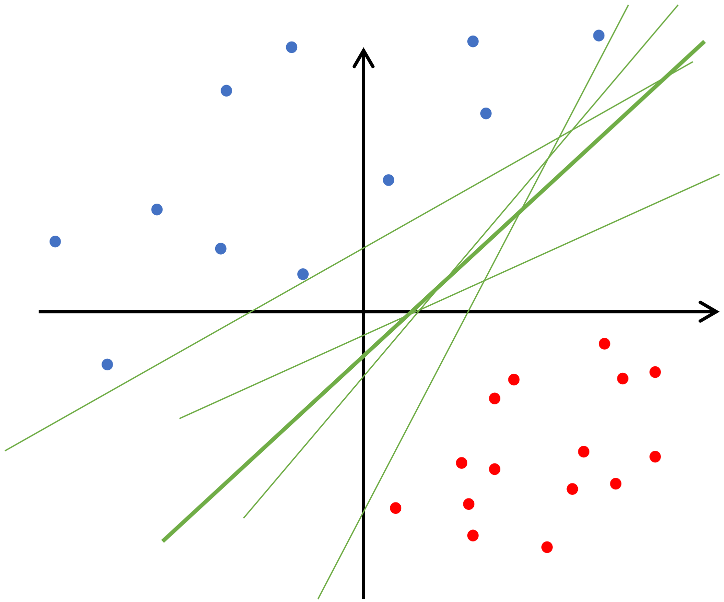


Image Classification

- Linear classifier finds hyperplane to seperate sets of points
- A more complex classifier might find a better way to seperate the two datasets
- Many ML methods have hyper-parameters that control the complexity of the function to fit
- But: In general, very complex functions tend to perform worse on unseen data
- Need to estimate the training error: split dataset into training-validation-test

Support Vector Machines

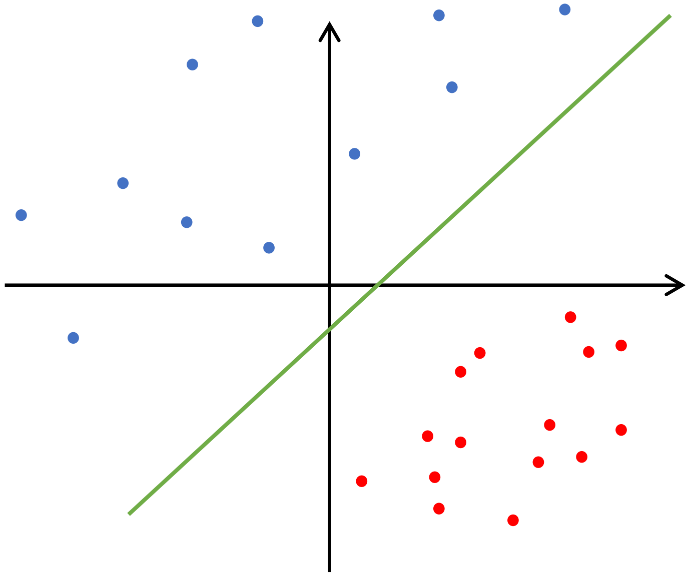


Support Vector Machines

- Labelled Data: $(x_i, y_i), x_i \in \mathbb{R}^n, y_i \in \{-1, 1\}$
- Solve:

$$\min_{w,b} \; w^T w$$

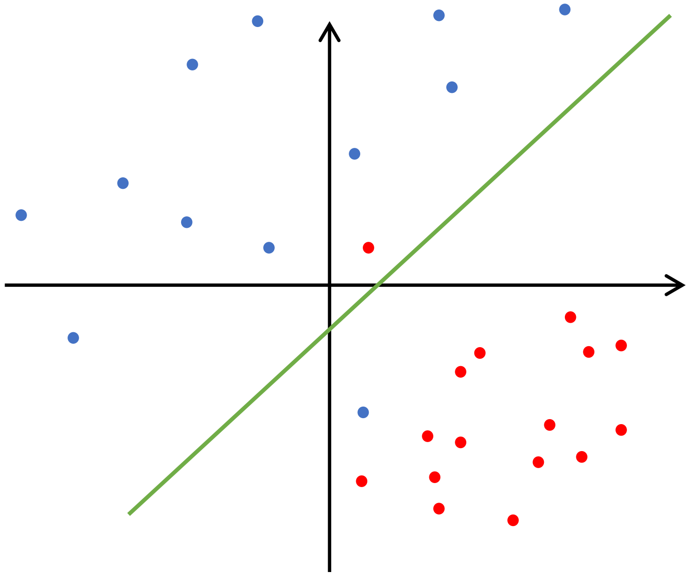
$$\text{s.t.} \; y_i \cdot \left(w^T x_i + b\right) \geq 1$$



Support Vector Machines

- Labelled Data: $(x_i, y_i), x_i \in \mathbb{R}^n, y_i \in \{-1, 1\}$
- Solve:

$$\begin{aligned} \min_{w,b} \ & w^T w + C \sum_i \xi_i \\ \text{s.t.} \ & y_i \cdot \left(w^T x_i + b\right) \geq 1 - \xi_i \\ & \xi_i \geq 0 \end{aligned}$$

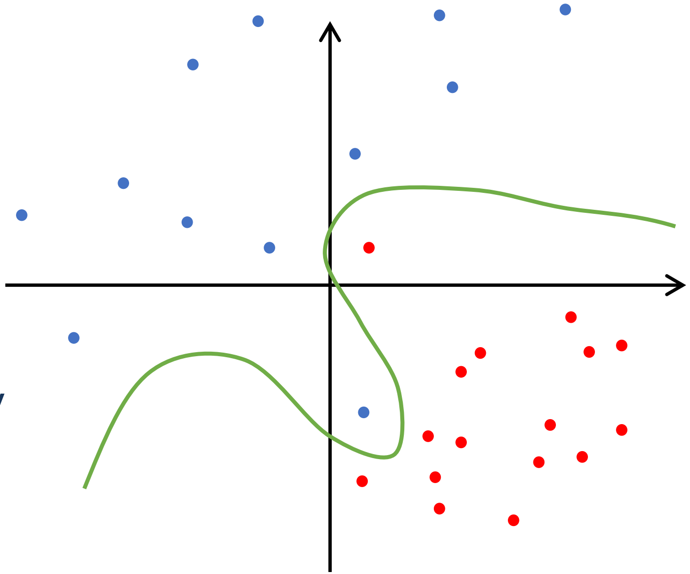


Support Vector Machines

- Labelled Data: $(x_i, y_i), x_i \in \mathbb{R}^n, y_i \in \{-1, 1\}$
- Solve:

$$\min_{w,b} \; w^T w + C \sum_i \xi_i$$
$$\text{s.t. } \; y_i \cdot \left(w^T \phi_\gamma(x_i) + b\right) \geq 1 - \xi_i$$
$$\xi_i \geq 0$$

- C, γ are a hyper-parameter that control complexity
- Multiclass: One-vs-All
  - most confident classifier wins
  - Confidence ist given by distance to border
- Multiclass: One-vs-One



QUESTIONS?
EXERCISES.