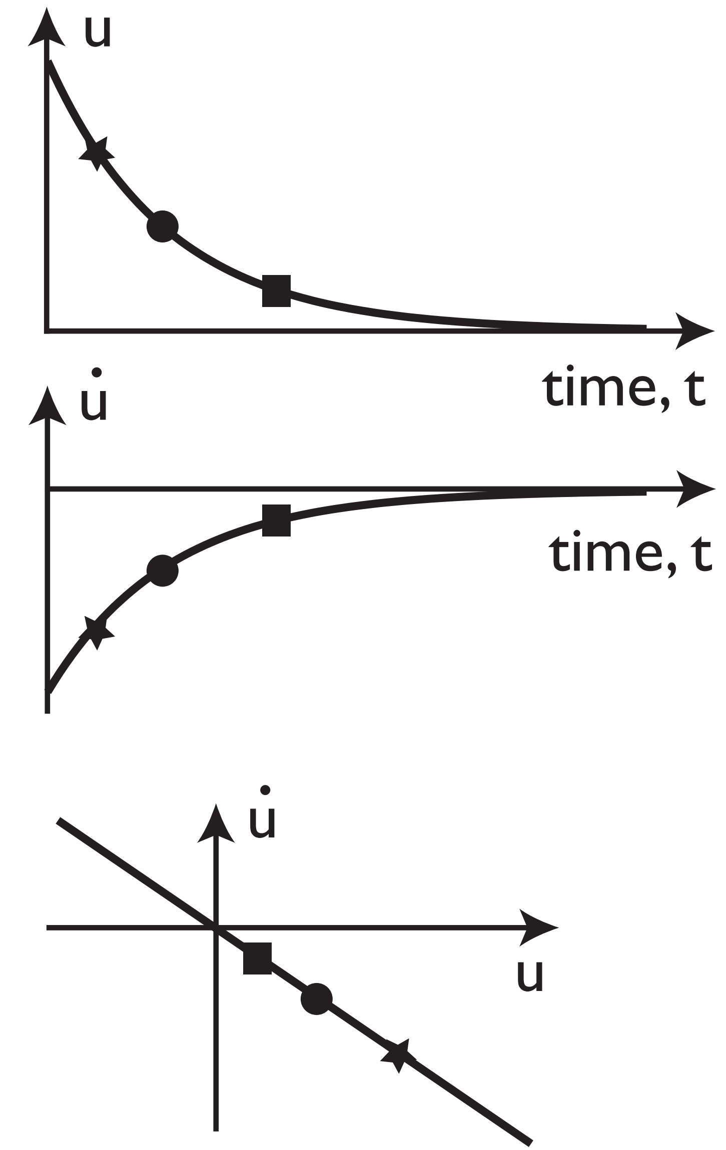
Summary: main conceptual points

Gregor Schöner, INI, RUB

Dynamical systems

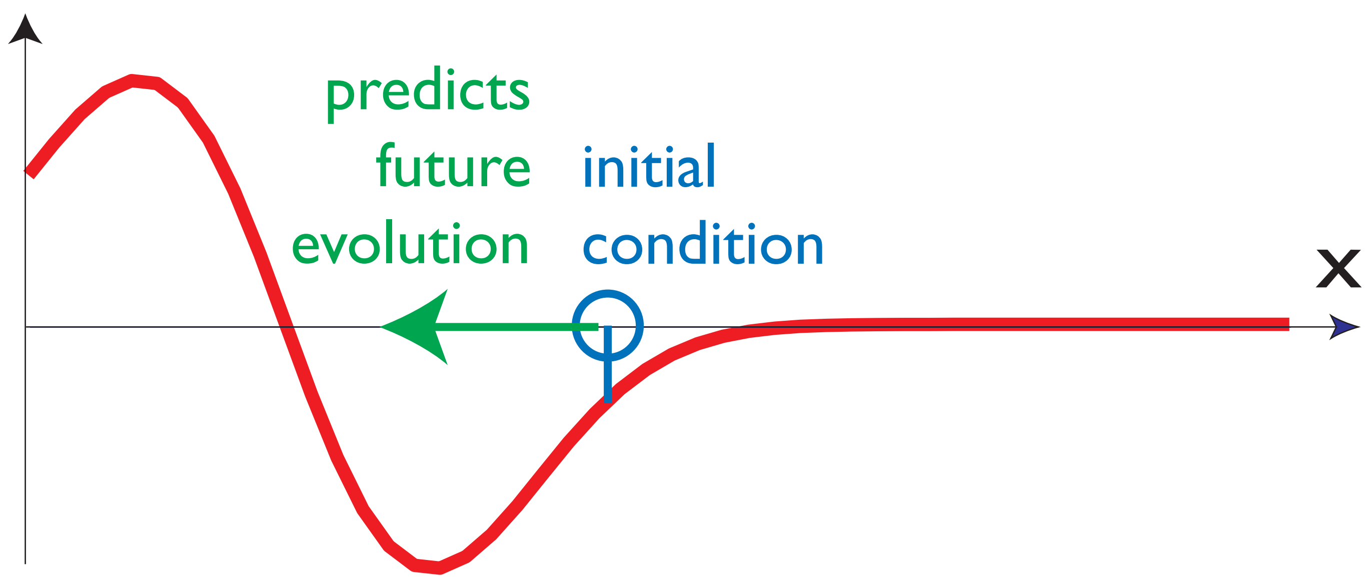
- functional link between state and its rate of change



Dynamical system

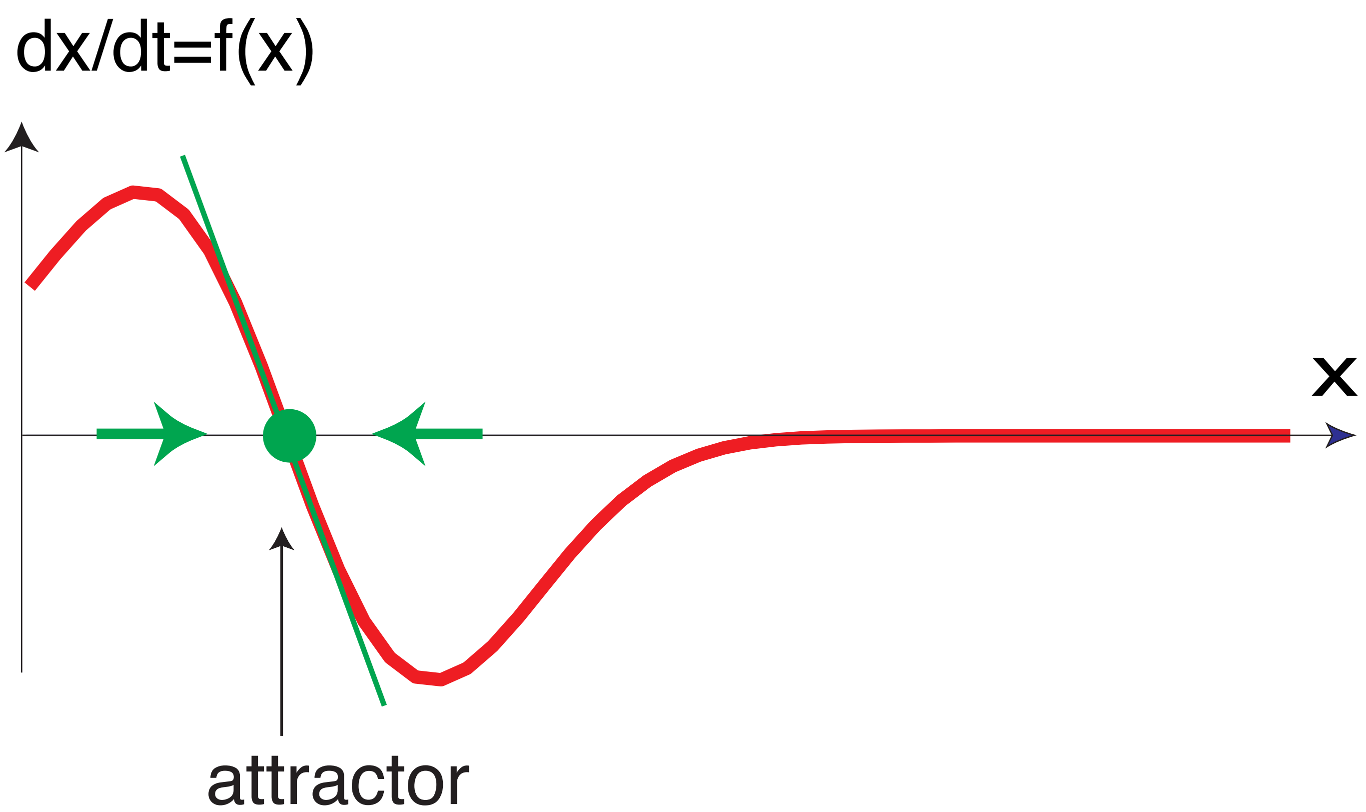
- present determines the future

dx/dt=f(x)



Dynamical systems

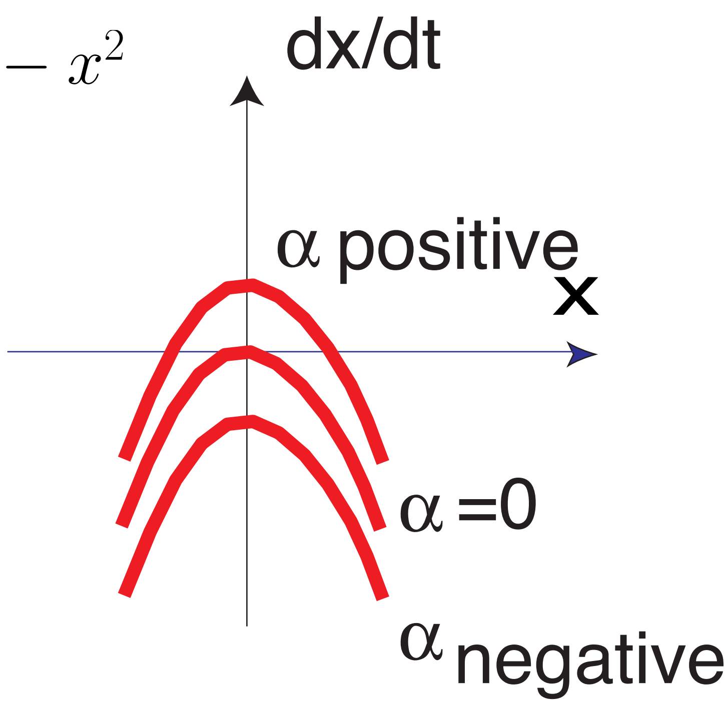
- fixed point = constant solution
- neighboring initial conditions converge = attractor

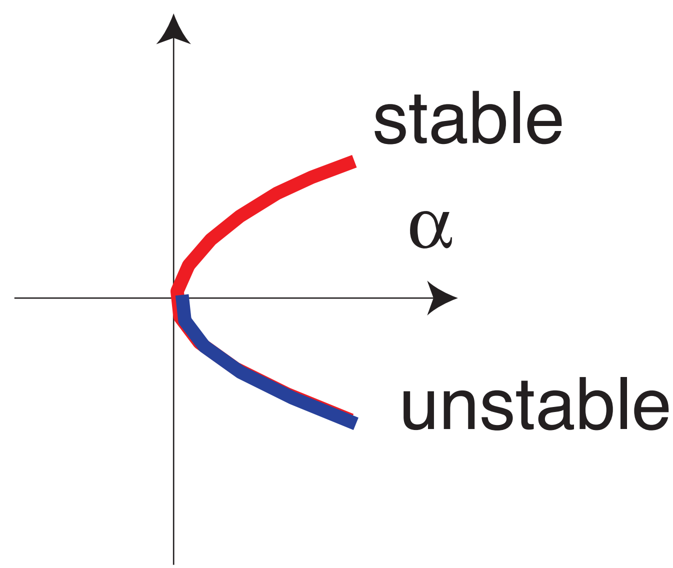


Bifurcations are instabilities

- In families of dynamical systems, which depend (smoothly) on parameters, the solutions change qualitatively at bifurcations
- at which fixed points change stability

$$\dot{x} = \alpha - x^2$$



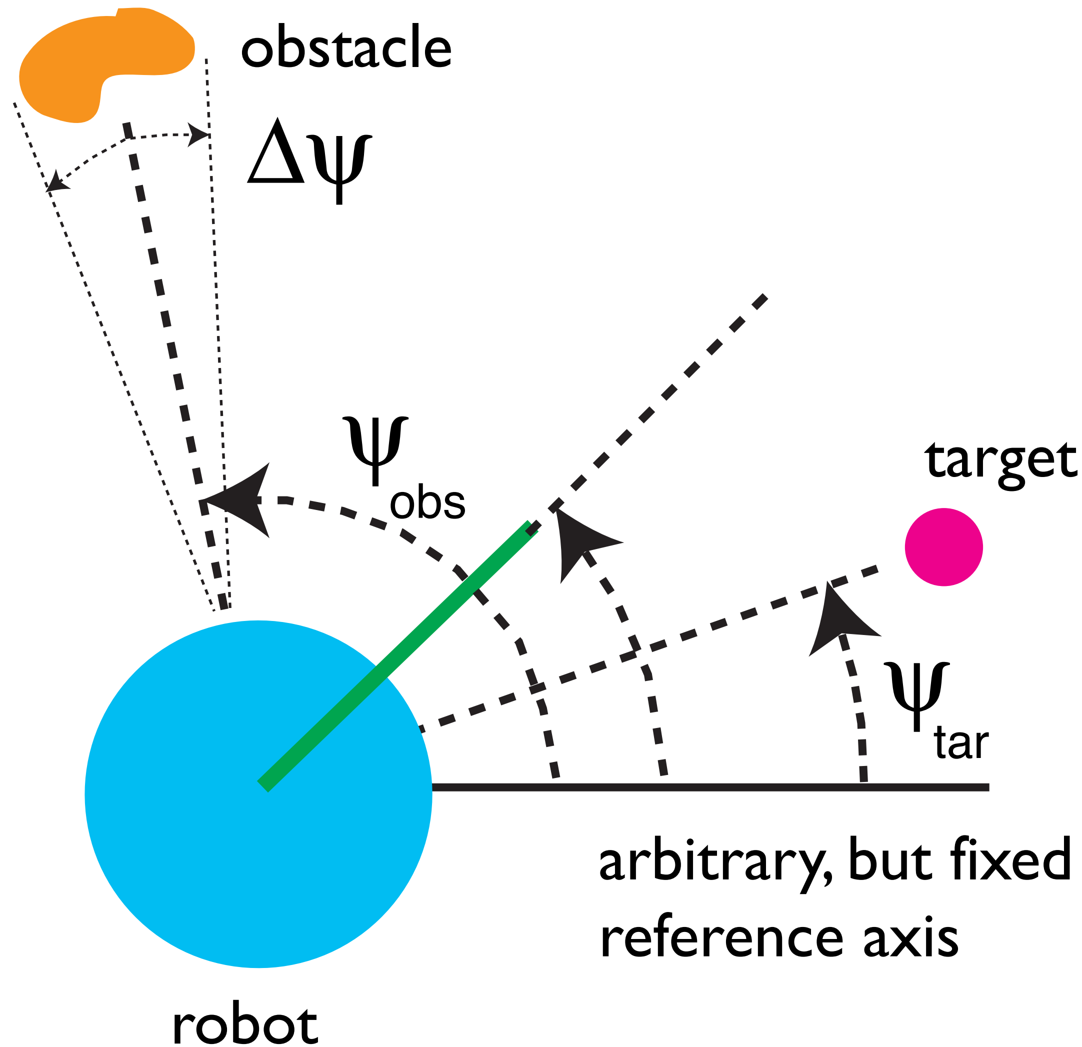


Basic ideas of attractor dynamics approach

- behavioral variables
- time courses from dynamical system: attractors
- tracking attractors
- bifurcations for flexibility

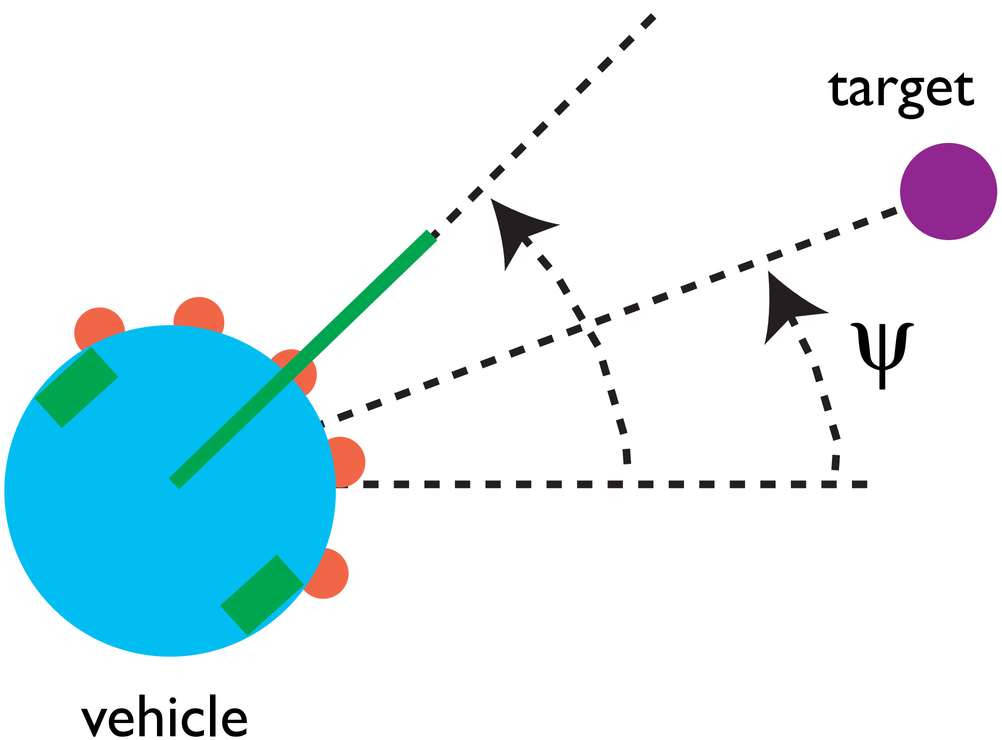
Behavioral variables: example

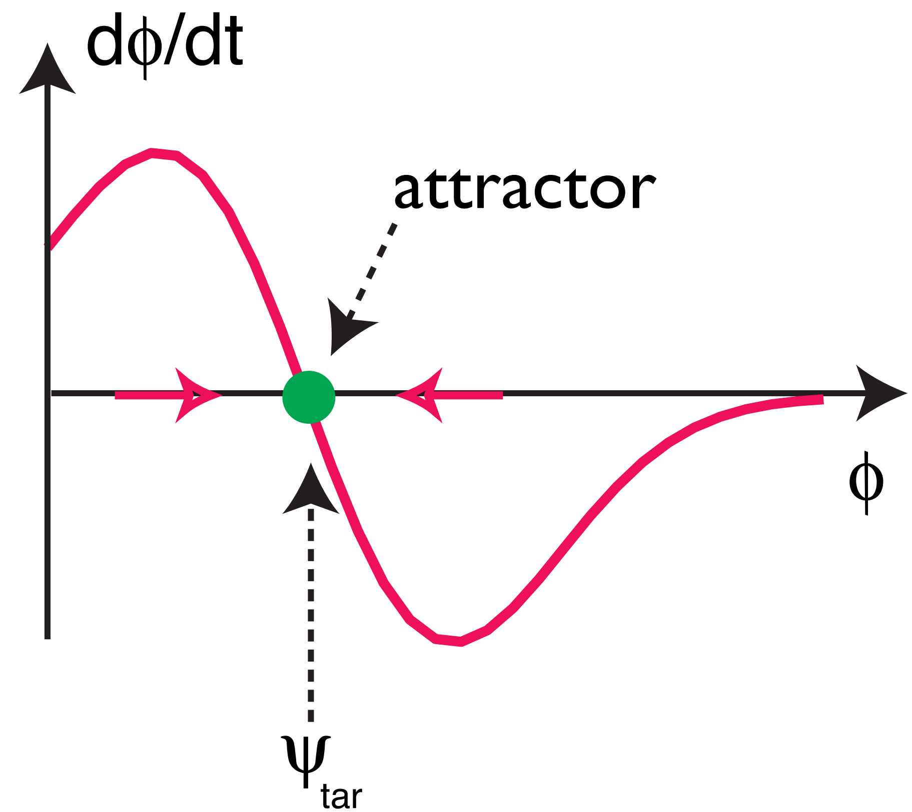
- vehicle moving in 2D: heading direction
- constraints: obstacle avoidance and target acquisition



Behavioral dynamics: example

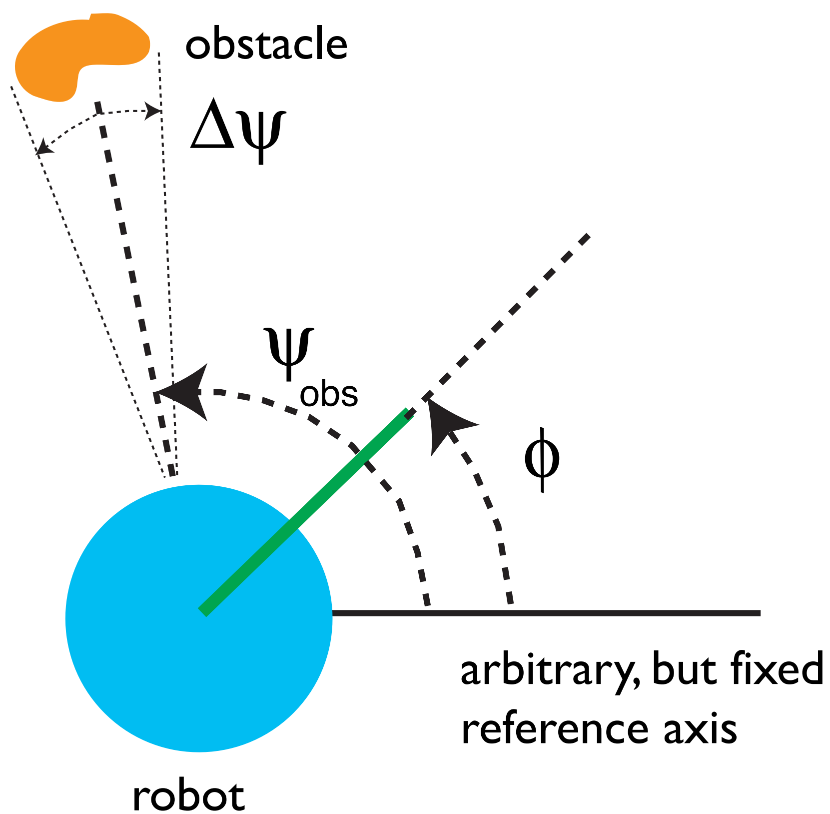
- behavioral constraint: target acquisition

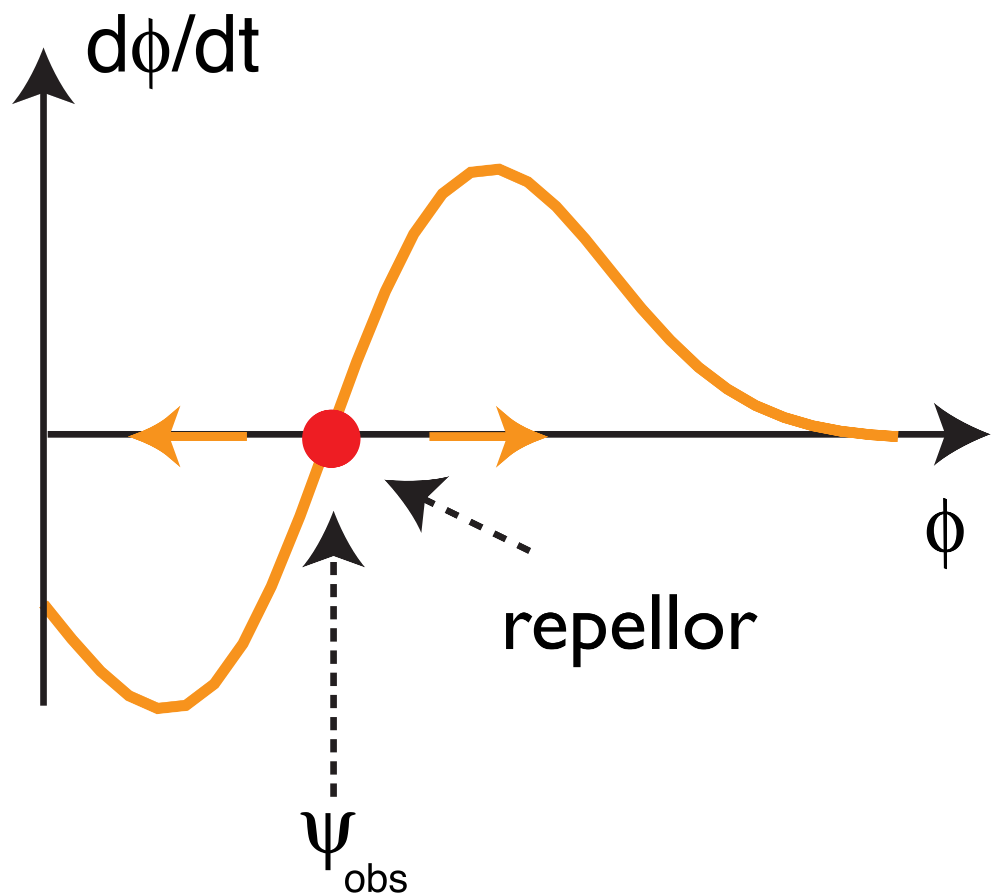




Behavioral dynamics: example

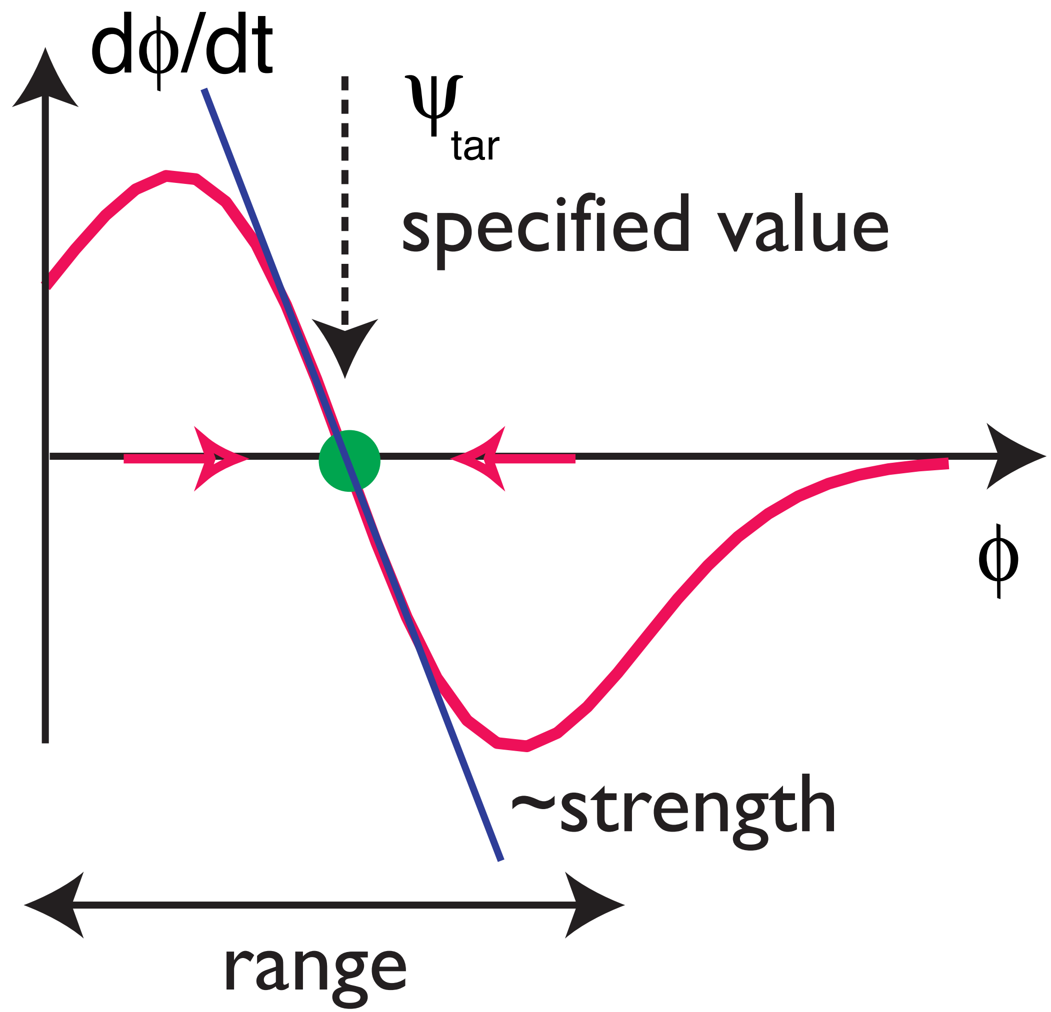
- behavioral constraint: obstacle avoidance





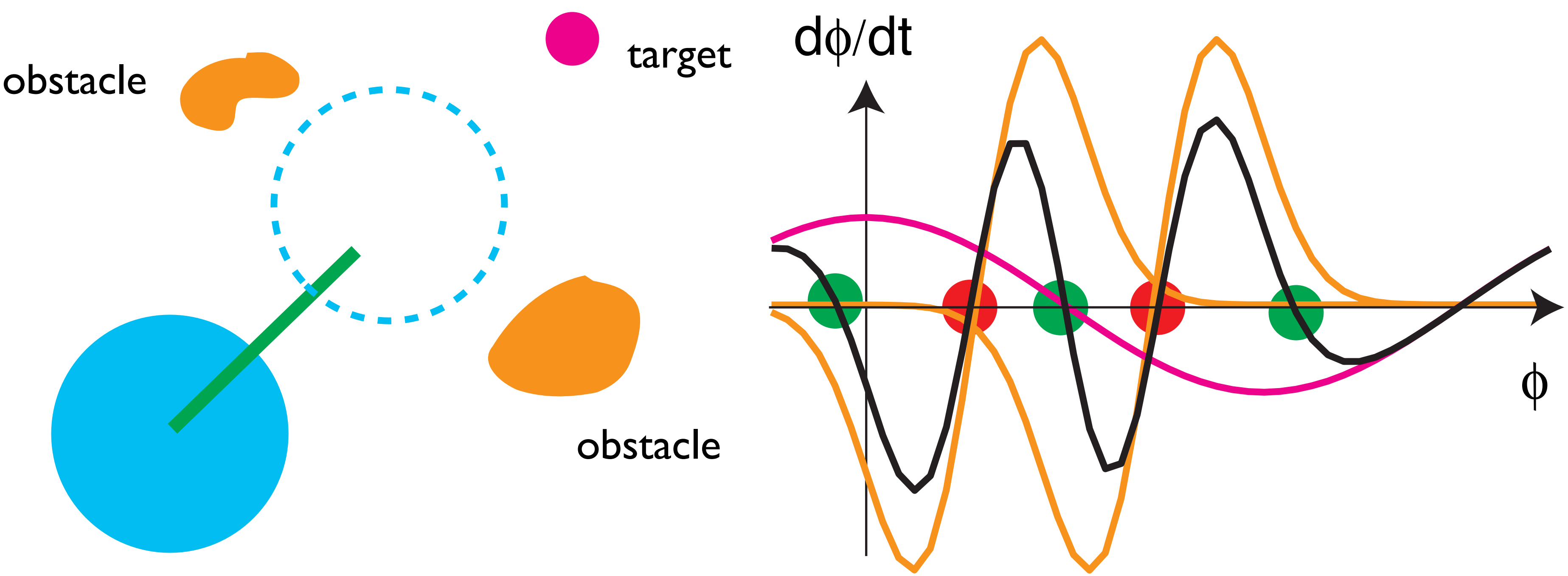
Behavioral dynamics

- each constribution is a “force-let” with
  - specified value
  - strength
  - range



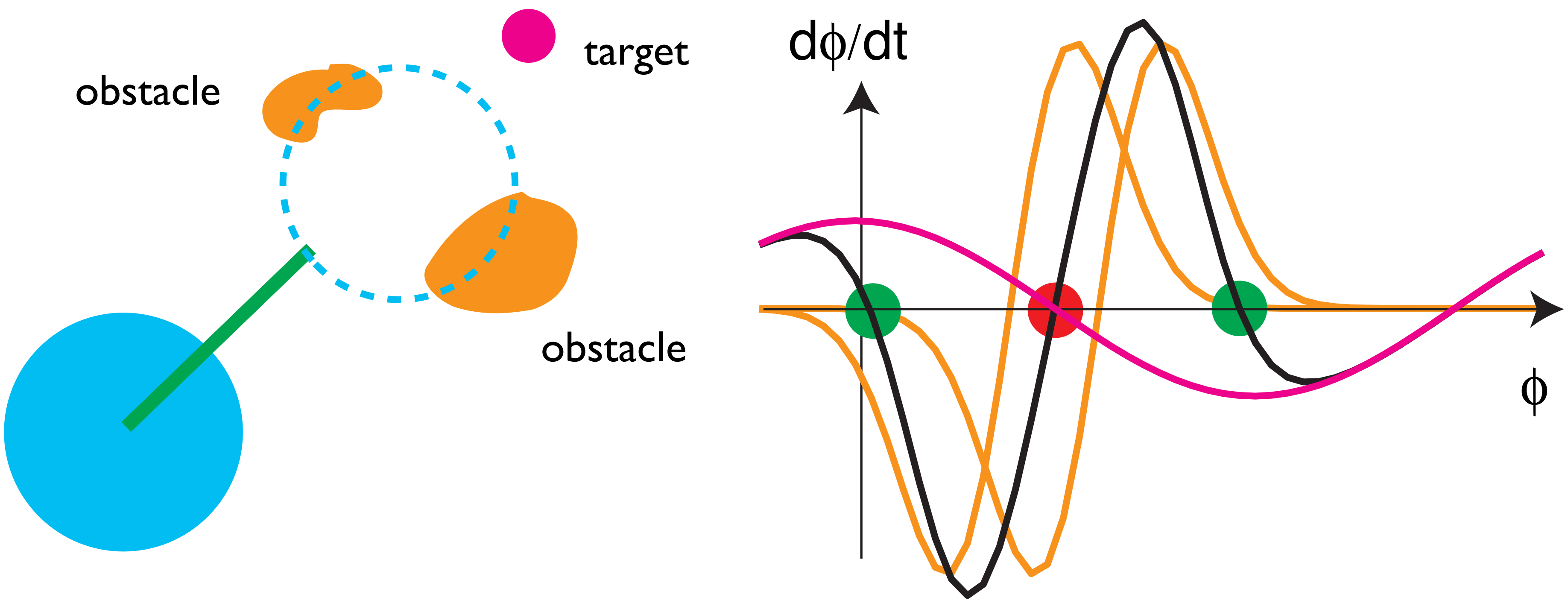
Behavioral dynamics: bifurcations

- constraints not in conflict



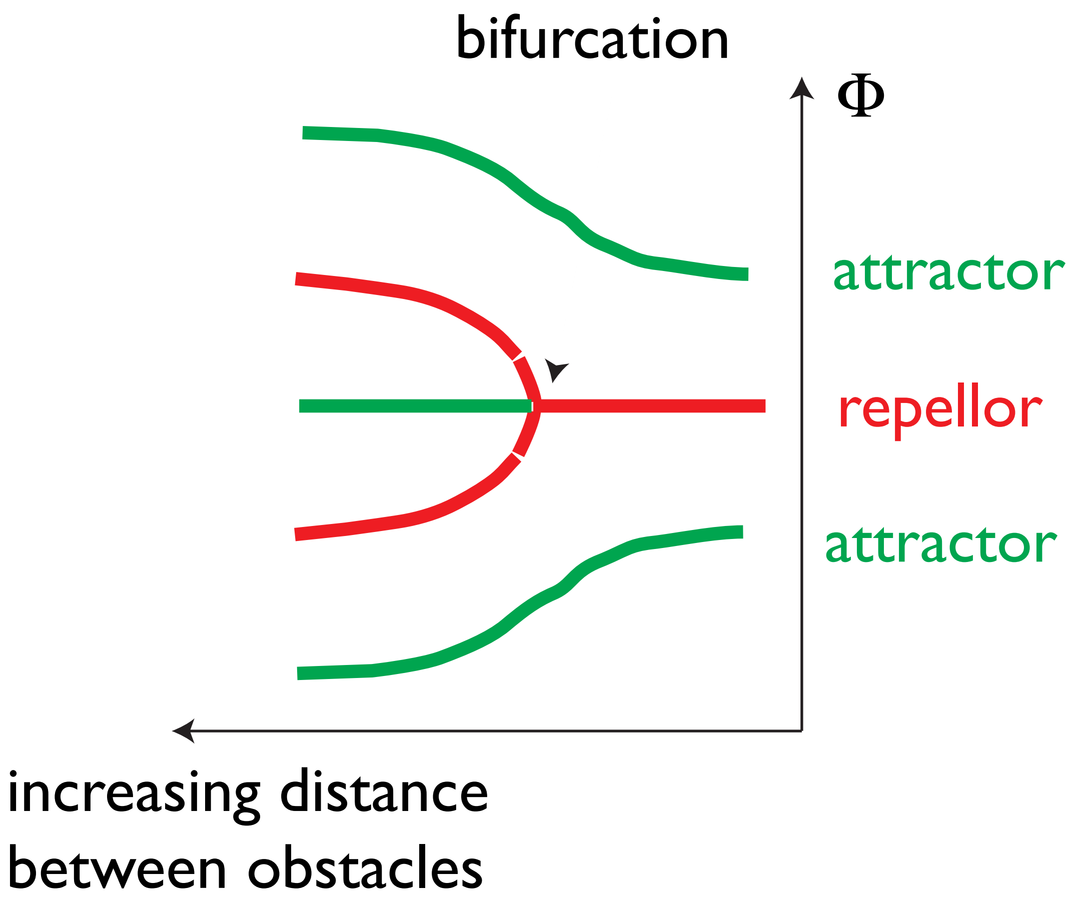
Behavioral dynamics

- constraints in conflict

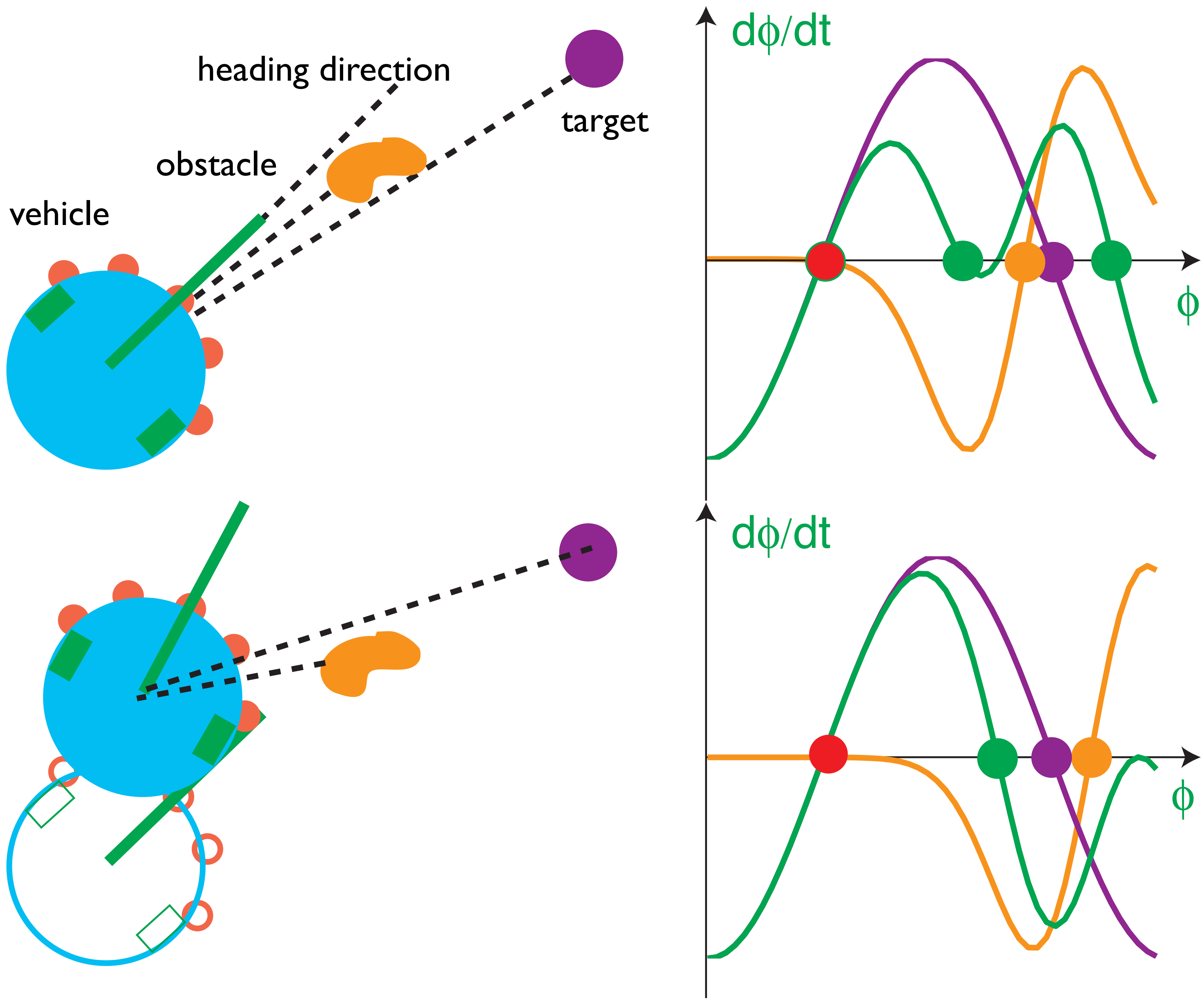


Behavioral dynamics

- transition from "constraints not in conflict" to "constraints in conflict" is a bifurcation

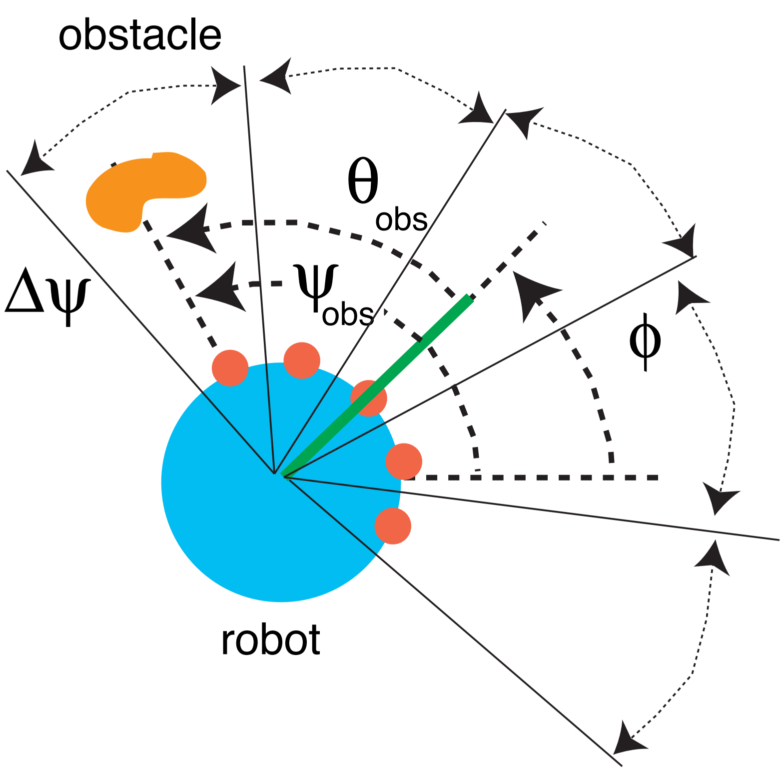


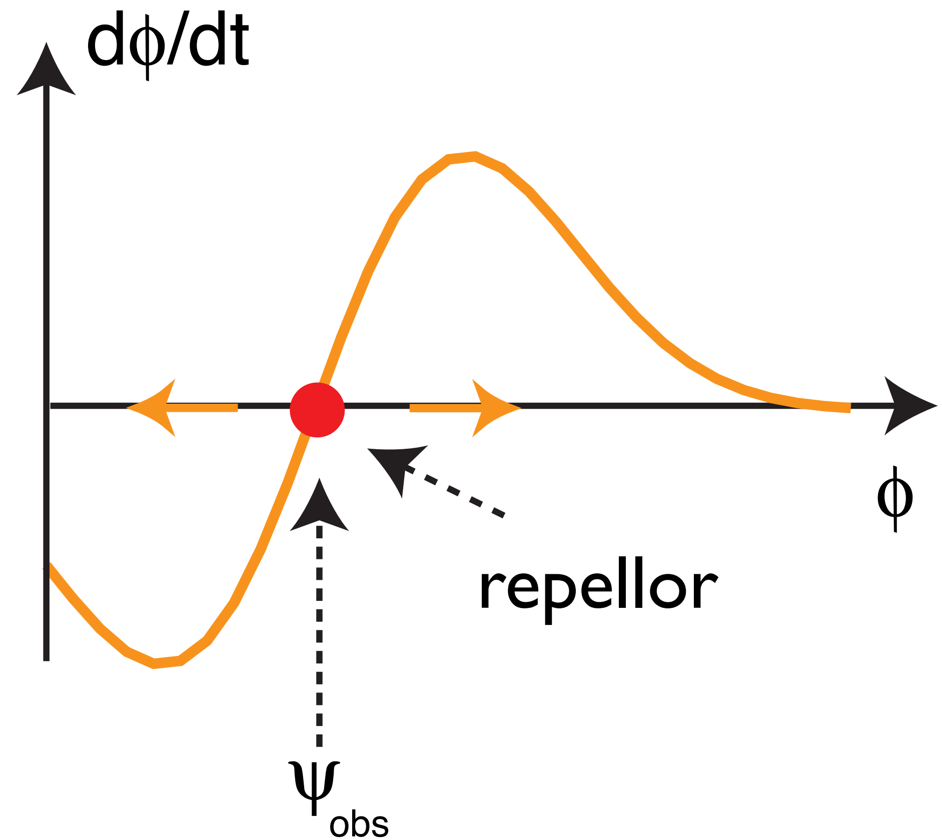
In a stable state at all times

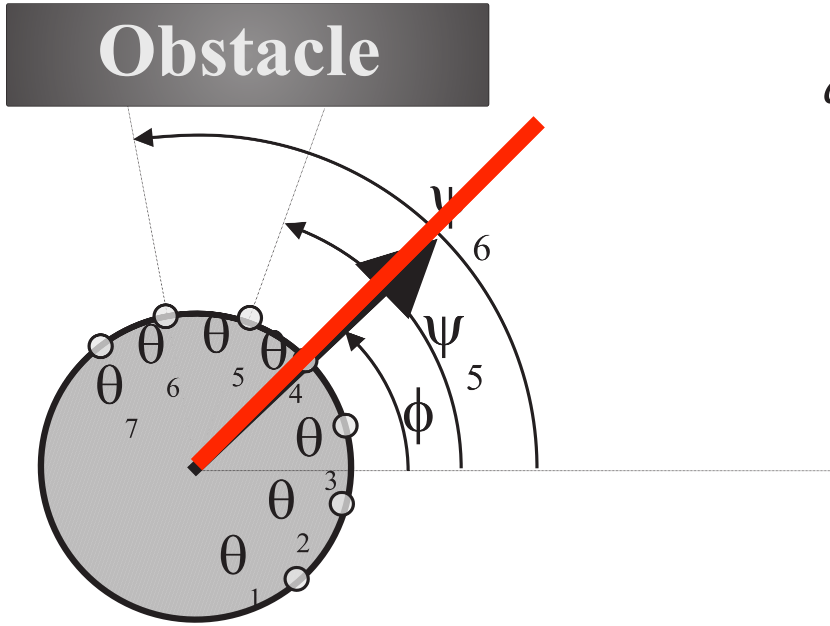


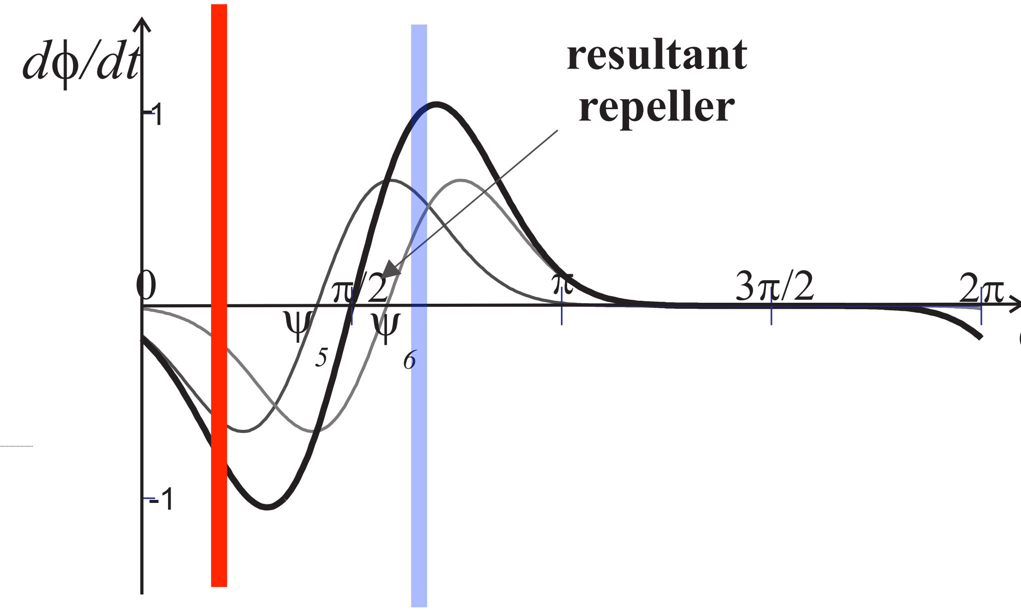
Obstacle avoidance: sub-symbolic

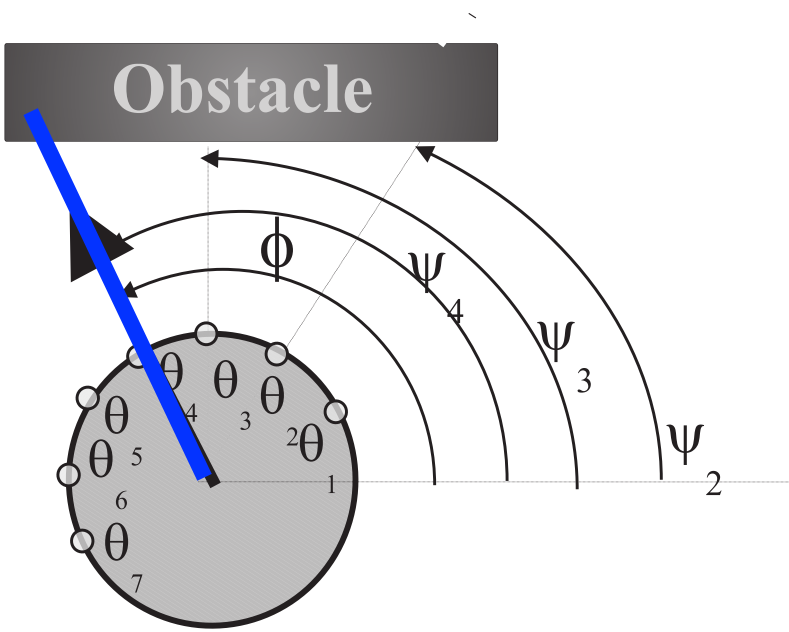
- obstacles need not be segmented
- do not care if obstacles are one or multiple: avoid them anyway…

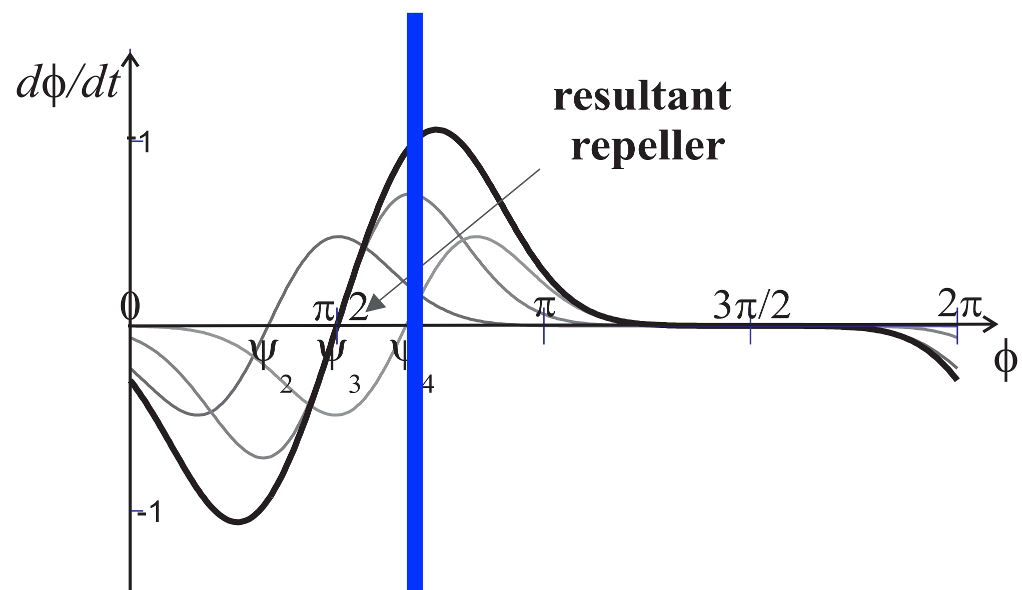








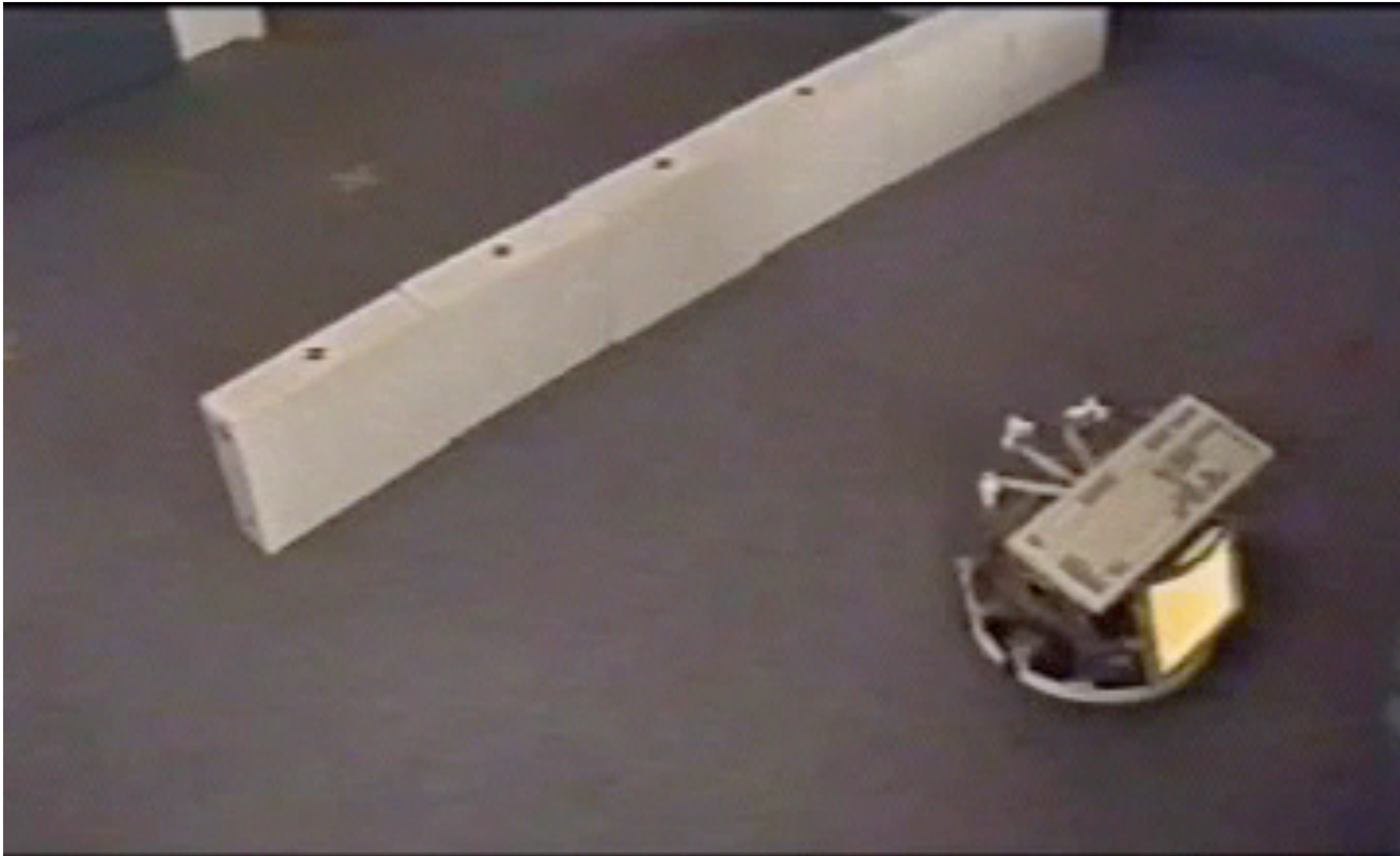




=> dynamics invariant!

[from: Bicho, Jokeit, Schöner]

Bifurcations



2nd order attractor dynamics to explain human navigation

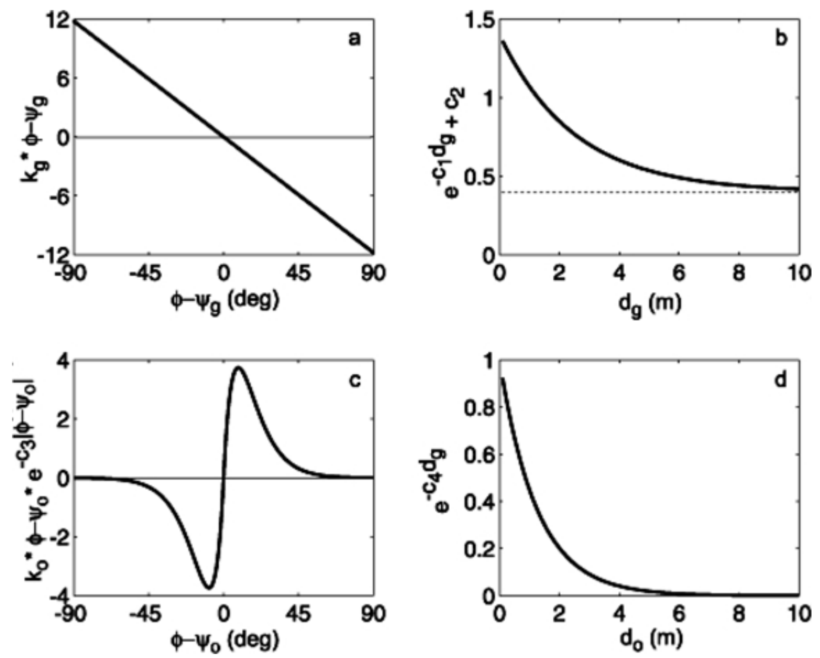
inertial term

damping term

attractor goal heading

$$\ddot{\phi} = -b\dot{\phi} - k_g(\phi - \psi_g)(e^{-c_1 d_g} + c_2) + k_o(\phi - \psi_o)\left(e^{-c_3|\phi - \psi_o|}\right)(e^{-c_4 d_o})$$

repellor obstacle heading

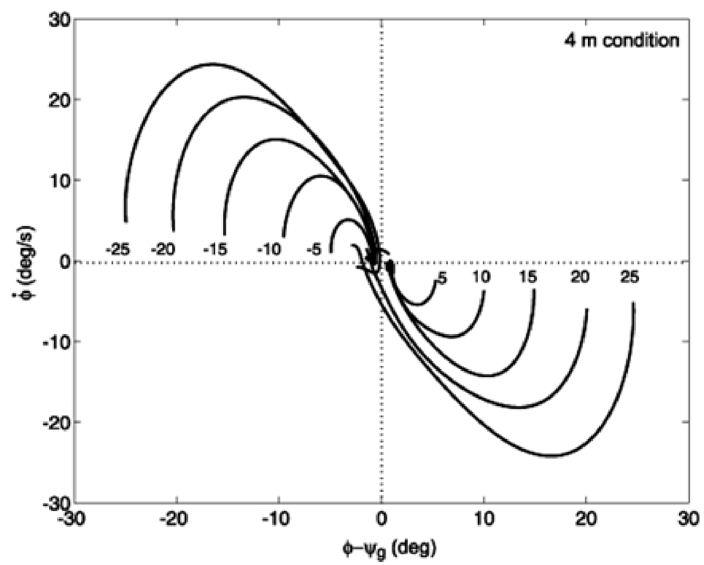


[Fajen Warren…]

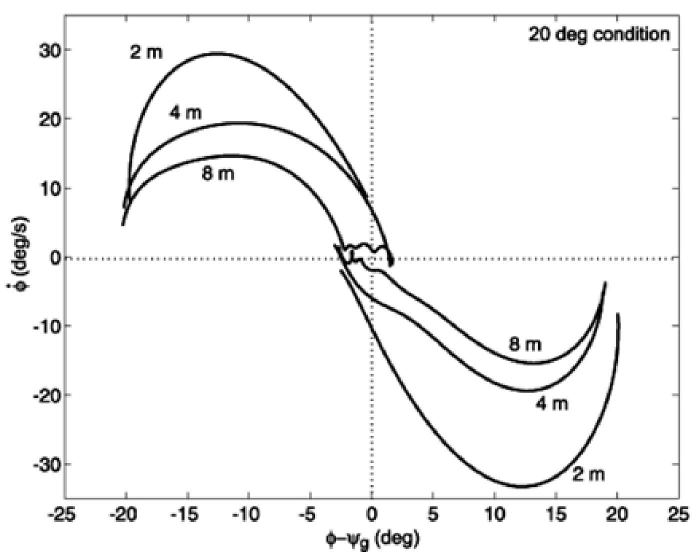
model-experiment match: goal

5

experiment

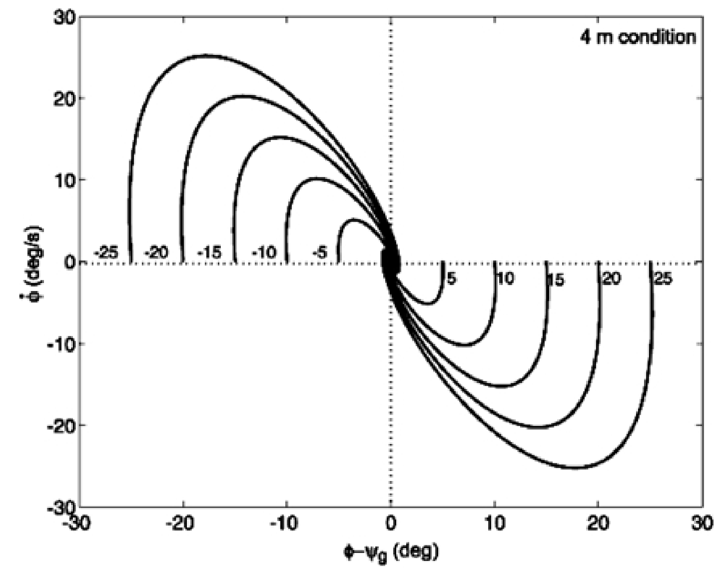


(a)

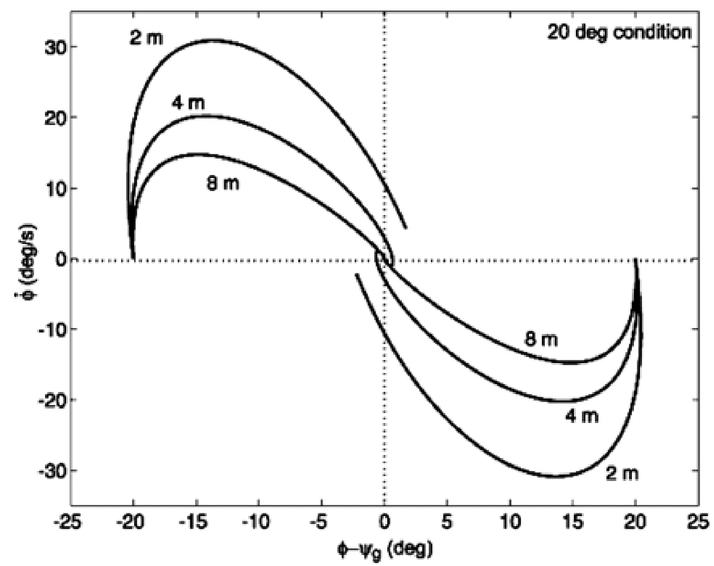


(b)

model



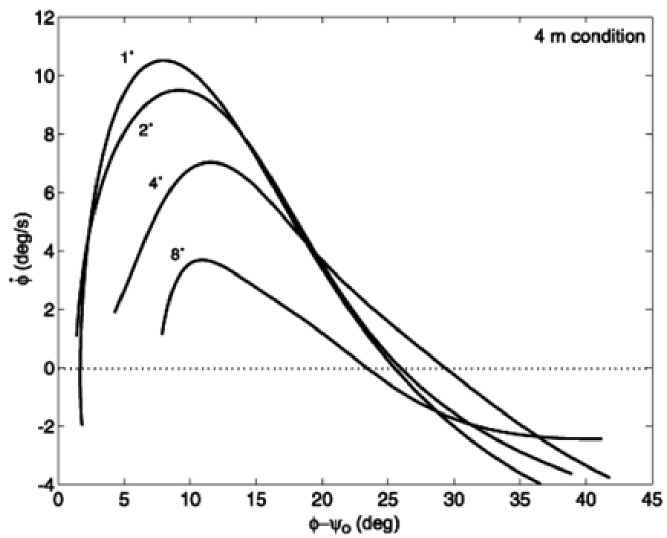
(a)



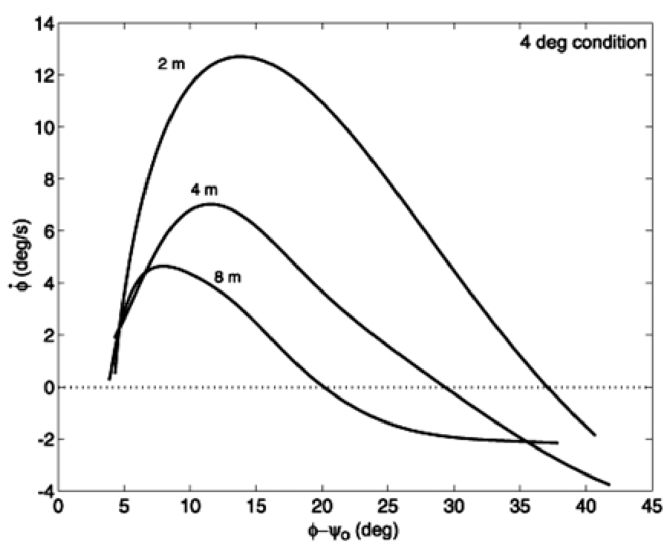
(b)

model-experiment match: obstacle

experiment

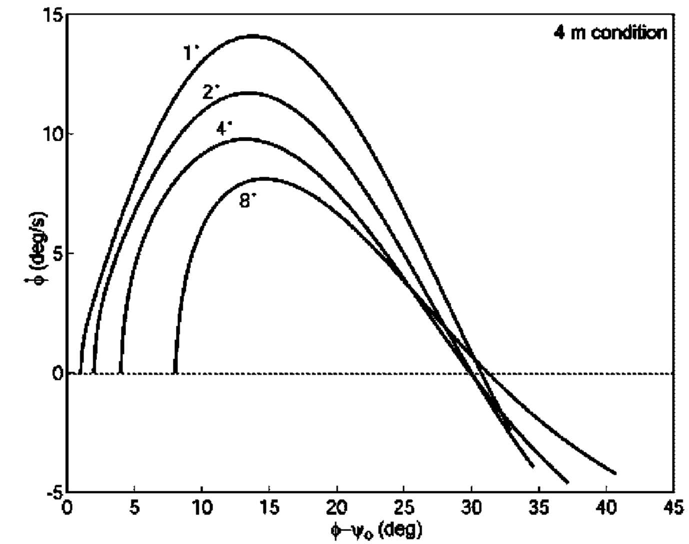


(a)

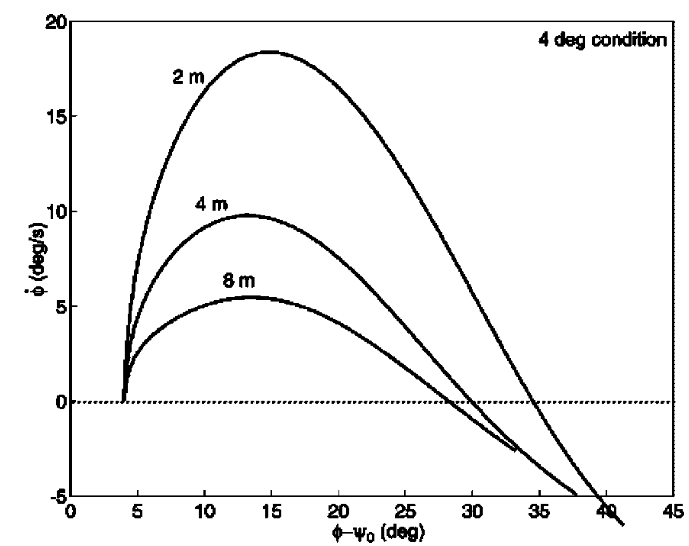


(b)

model



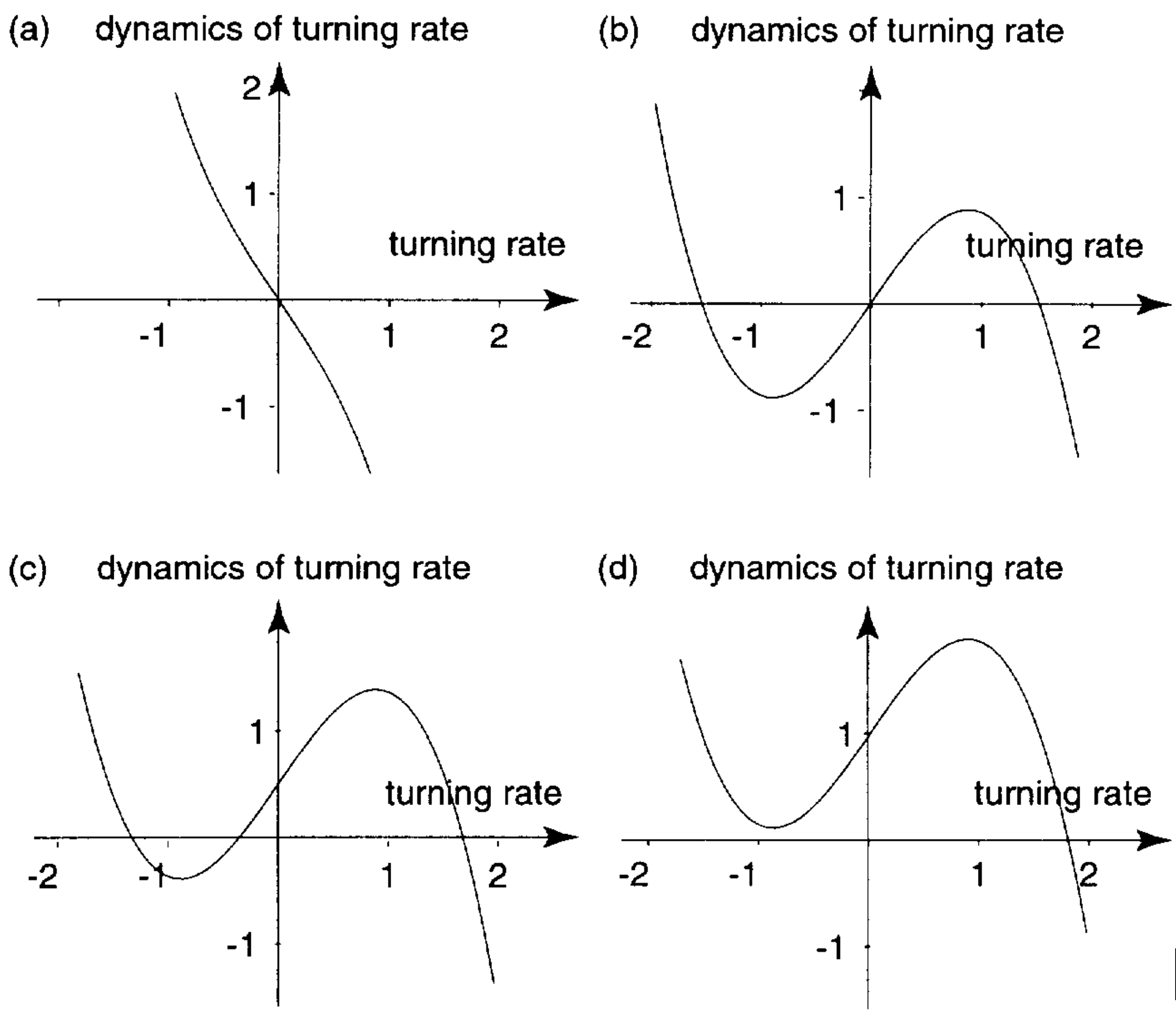
(a)



(b)

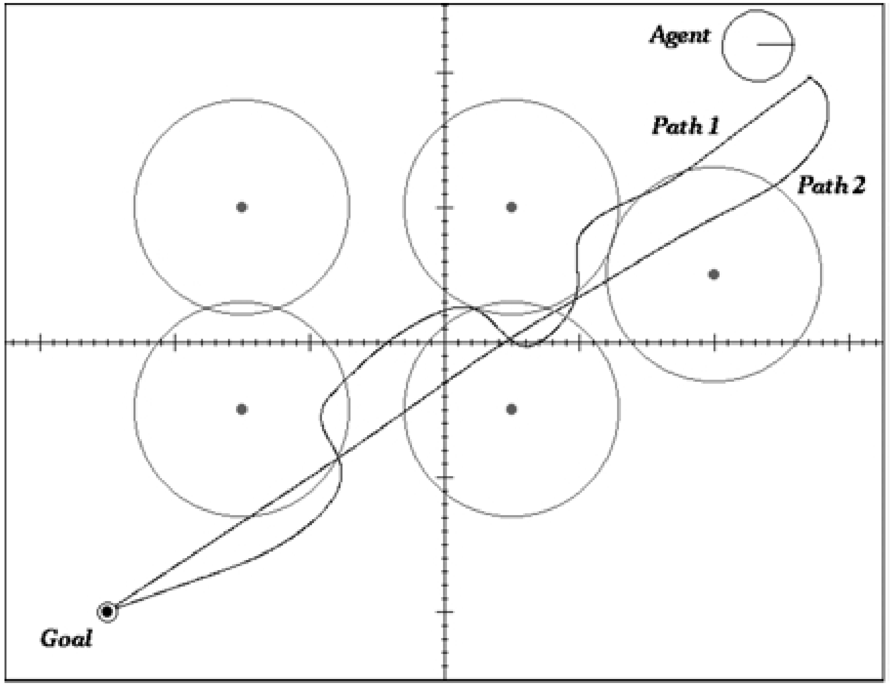
Alternative 2nd oder approach

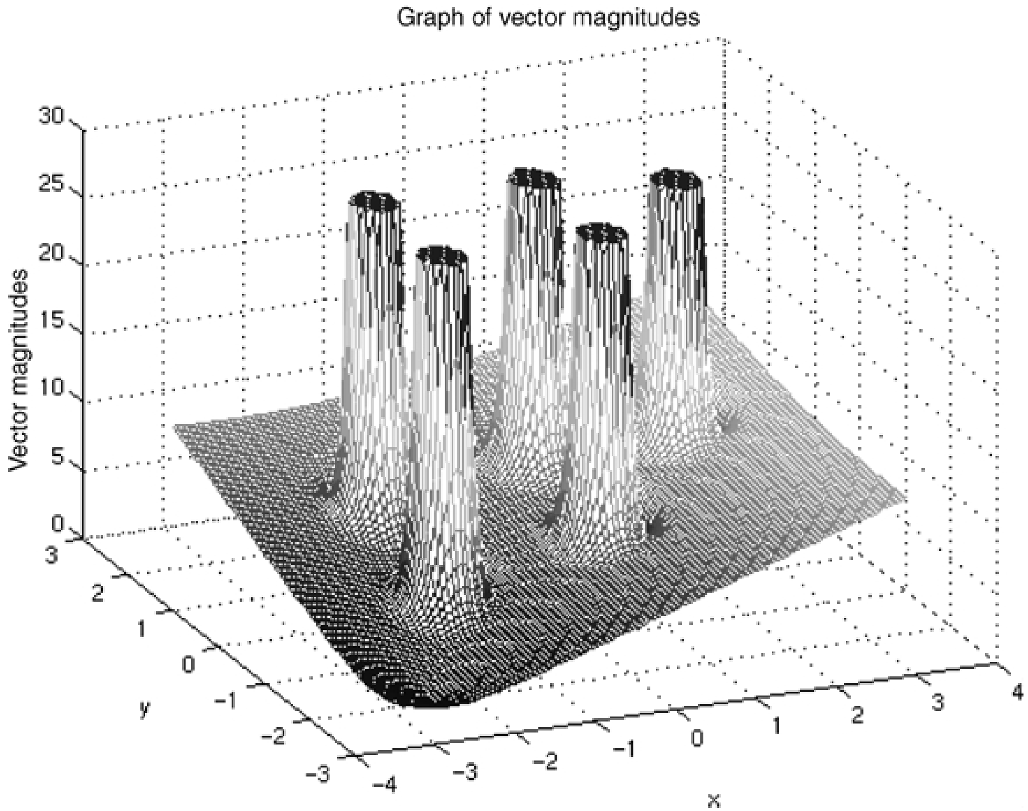
$$\dot{\omega} = (\alpha + \tfrac{1}{2}\pi)c_{\text{obs}}F_{\text{obs}} + \alpha\omega - \gamma\omega^3$$



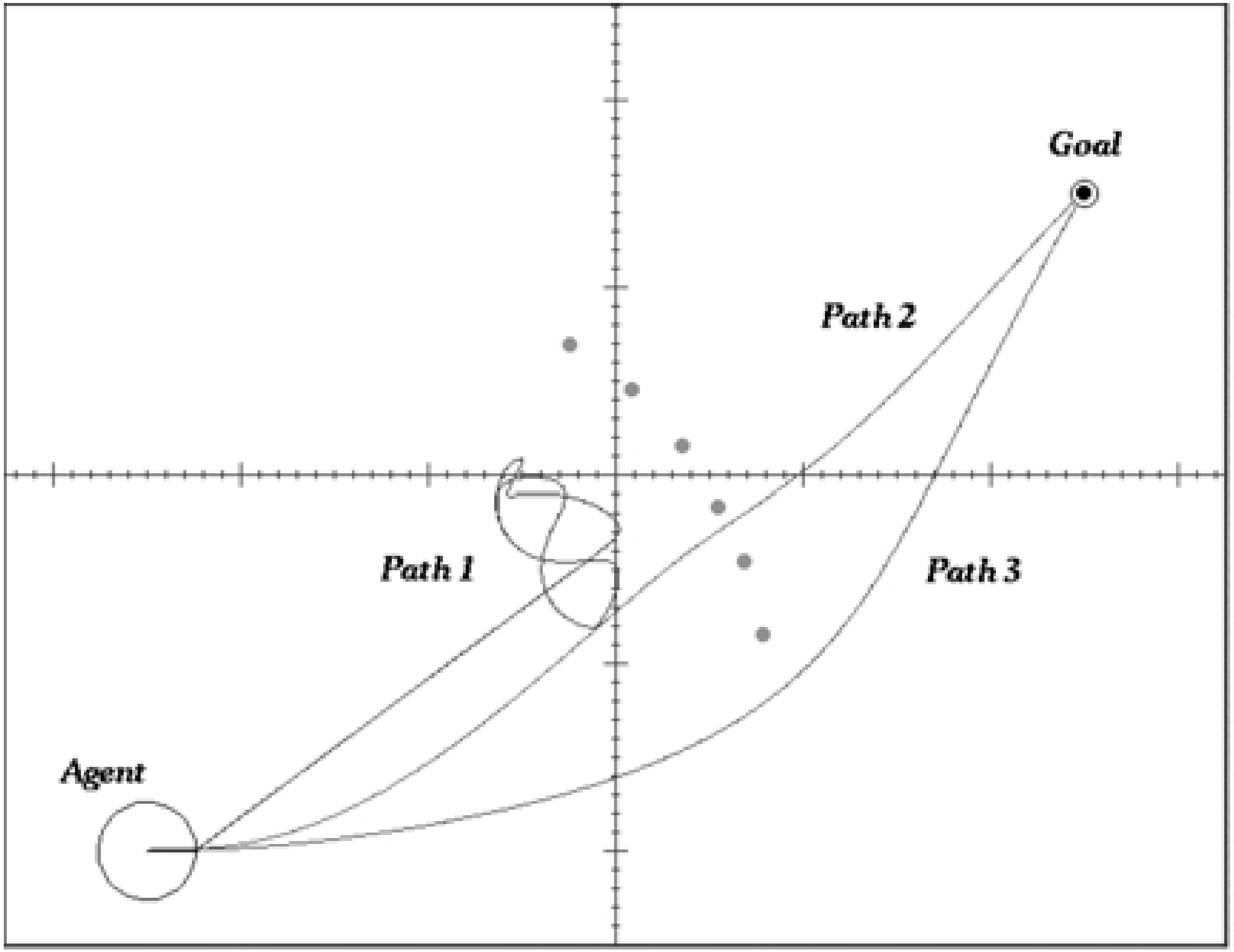
[Bicho, Schöner, 97]

Potential field approach





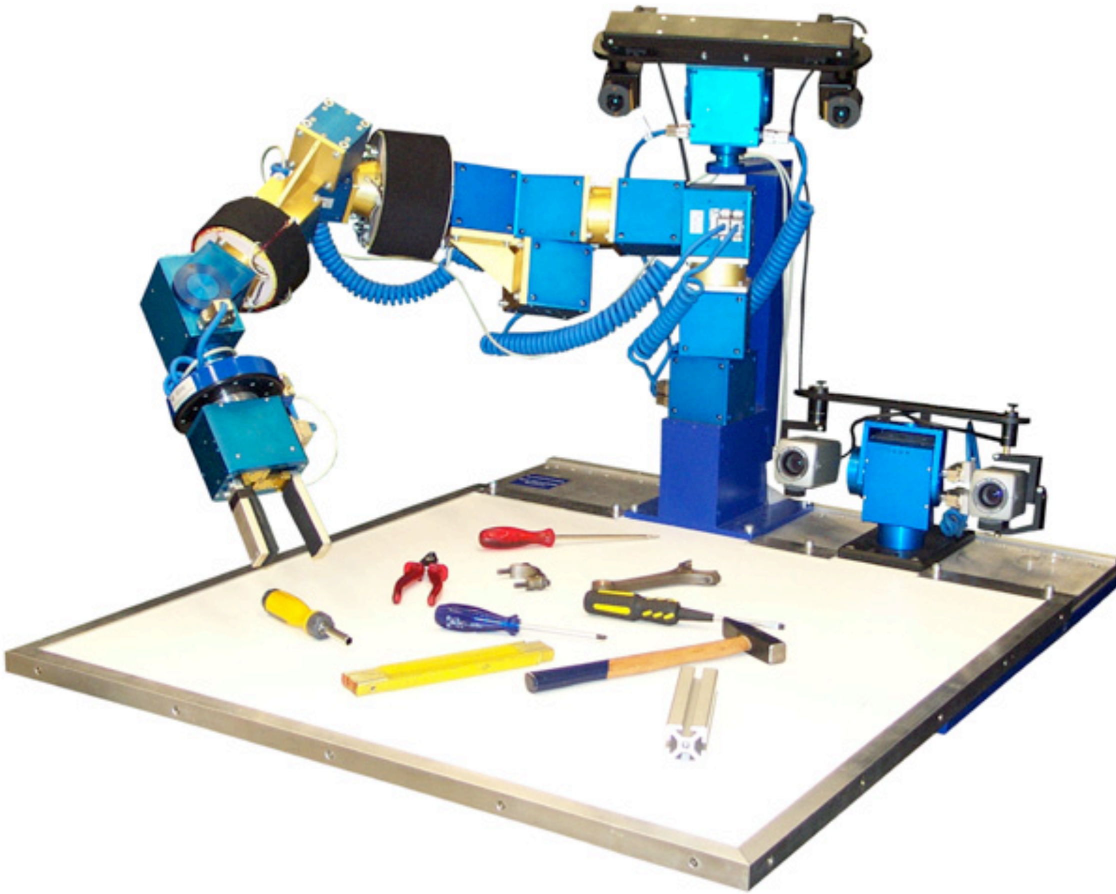
spurious attractors in potential field approach



Spaces for robotic motion planning 8/9

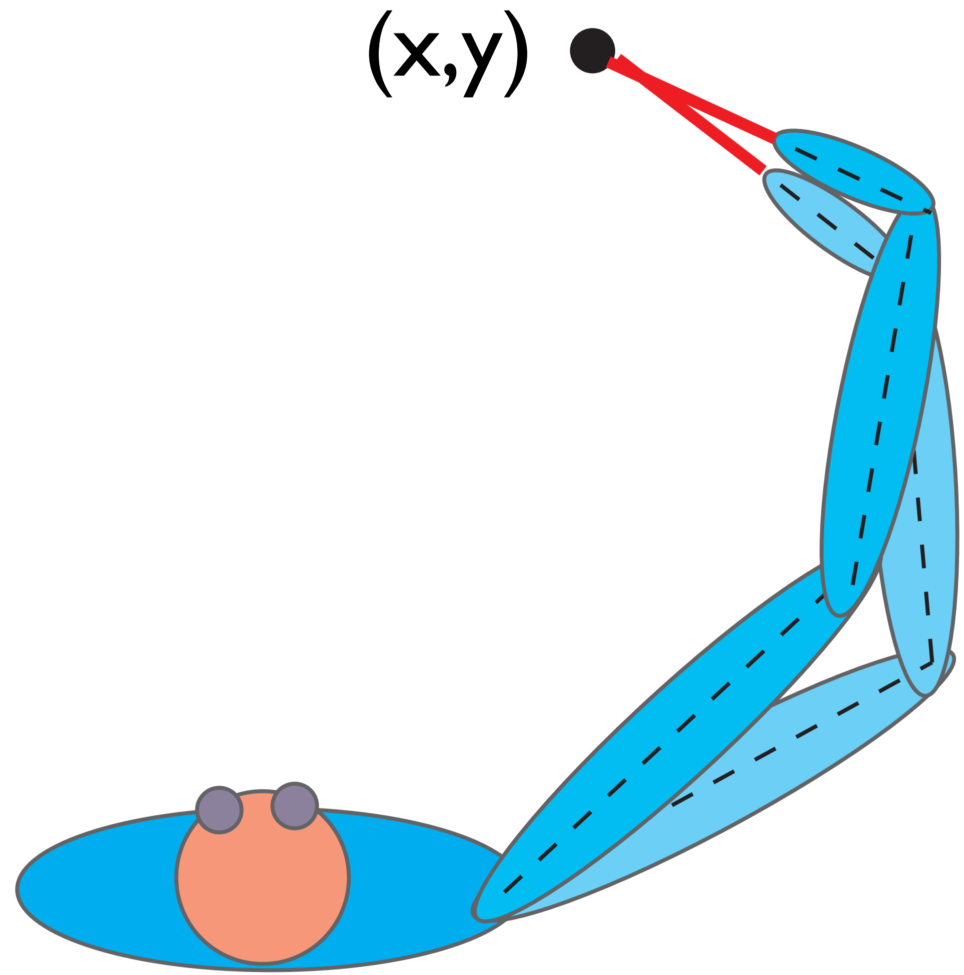
kinematic model	$\mathbf{x} = \mathbf{f}(\theta)$	$\dot{\mathbf{x}} = \mathbf{J}(\theta)\dot{\theta}$
inverse kinematic model	$\theta = \mathbf{f}^{-1}(\mathbf{x})$	$\dot{\theta} = \mathbf{J}^{-1}(\theta)\dot{\mathbf{x}}$

- transform end-effector to configuration space through inverse kinematics
- problems of singularities and multiple "leafs" of inverse…



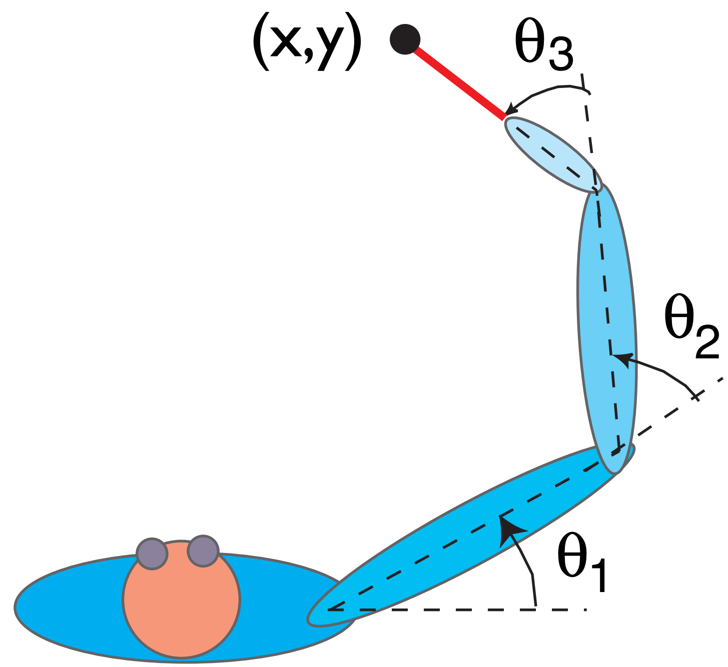
Redundant kinematics

- redundant arms/tasks: more joints than task-level degrees of freedom
- => (continuously) many inverse solutions…



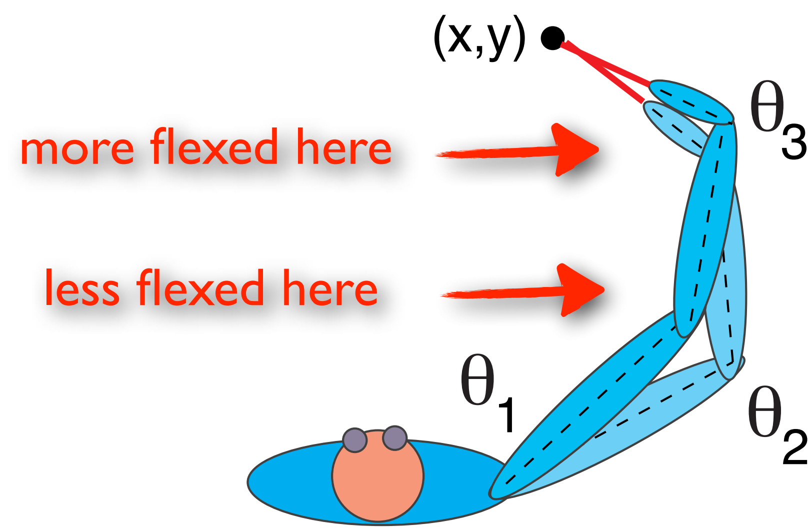
Degree of freedom problem in human movement

- what is a DoF?
  - variable that can be independently varied
  - e.g. joint angles
- muscles/muscle groups
  - but: assess to which extent they can be activated independently…
  - .. mode picture

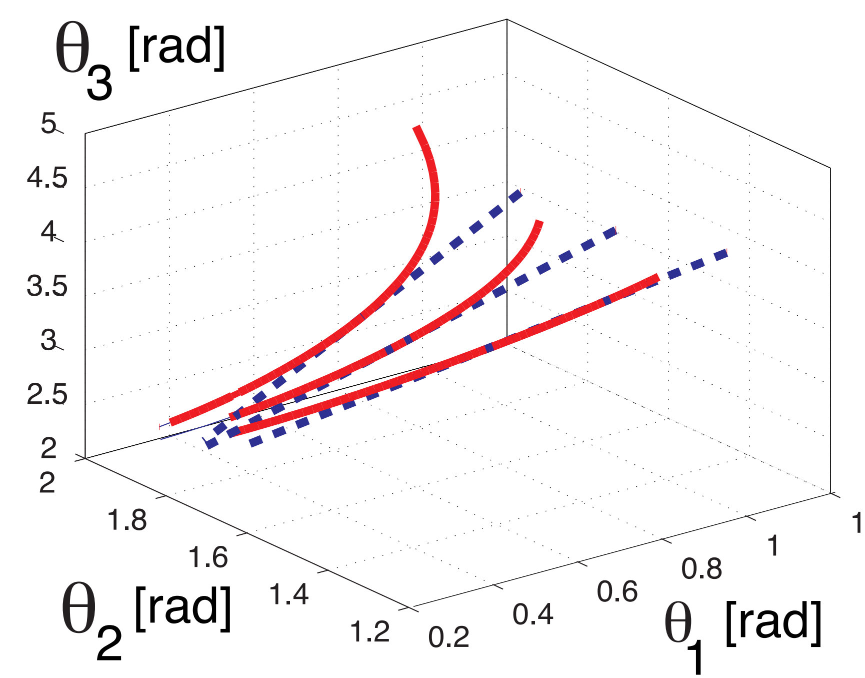


$$x = l_1 \cos(\theta_1) + l_2 \cos(\theta_1+\theta_2) + l_3 \cos(\theta_1+\theta_2+\theta_3)$$
$$y = l_2 \sin(\theta_1\ ) + l_2 \sin(\theta_1+\theta_2) + l_3 \sin(\theta_1+\theta_2+\theta_3)$$

Concept of the UnControlled Manifold 9



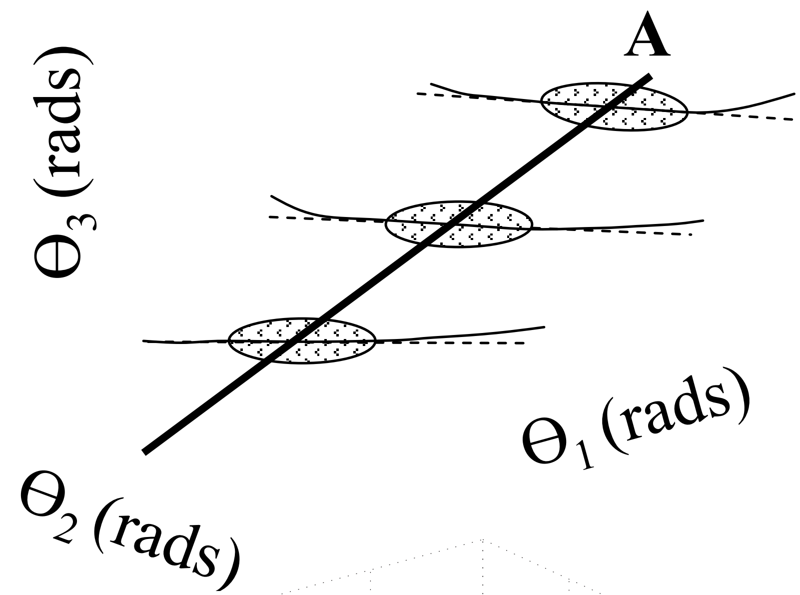
- the many DoF are coordinated such that changes that affect the task-relevant dimensions are resisted against more than changes that do not affect task relevant dimension
- leading to compensation

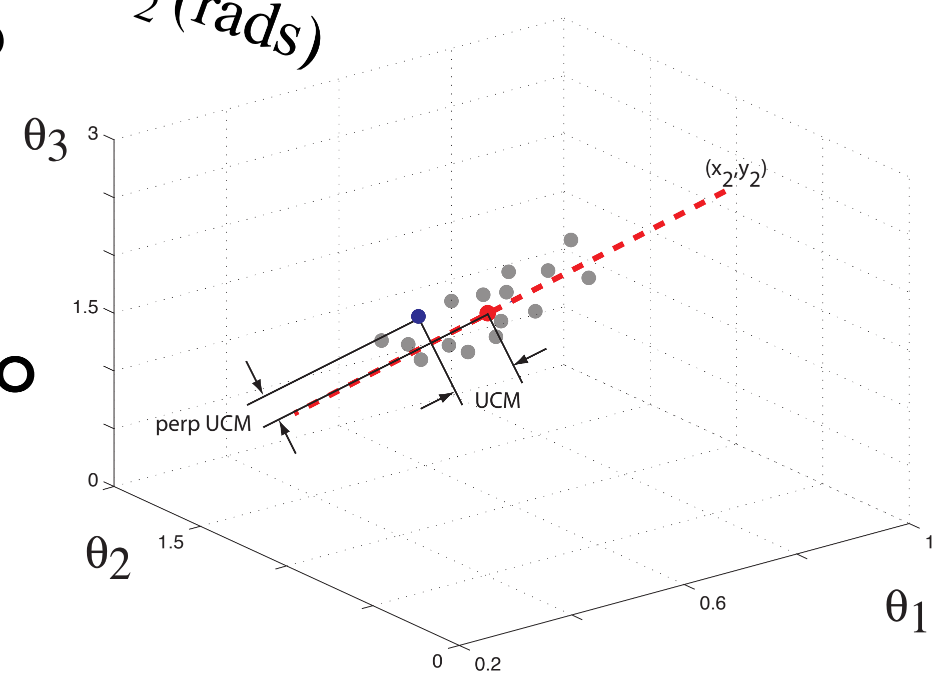


[Scholz, Schöner, EBR 126:289 (99)]

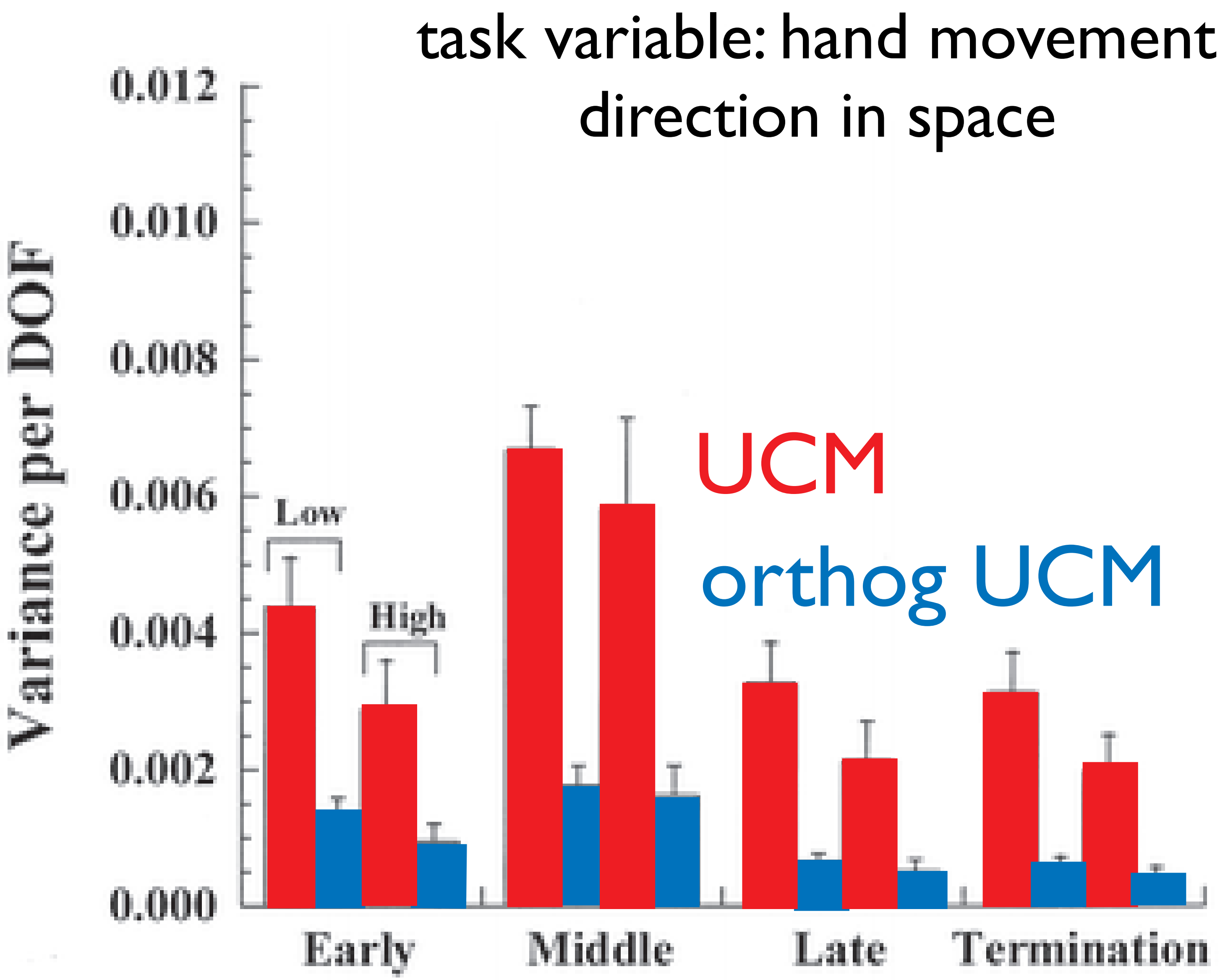
UCM synergy: data analysis

- align trials in time
- hypothesis about task variable
- compute null-space (tangent to the UCM)
- predict more variance within null space than perpendicular to it

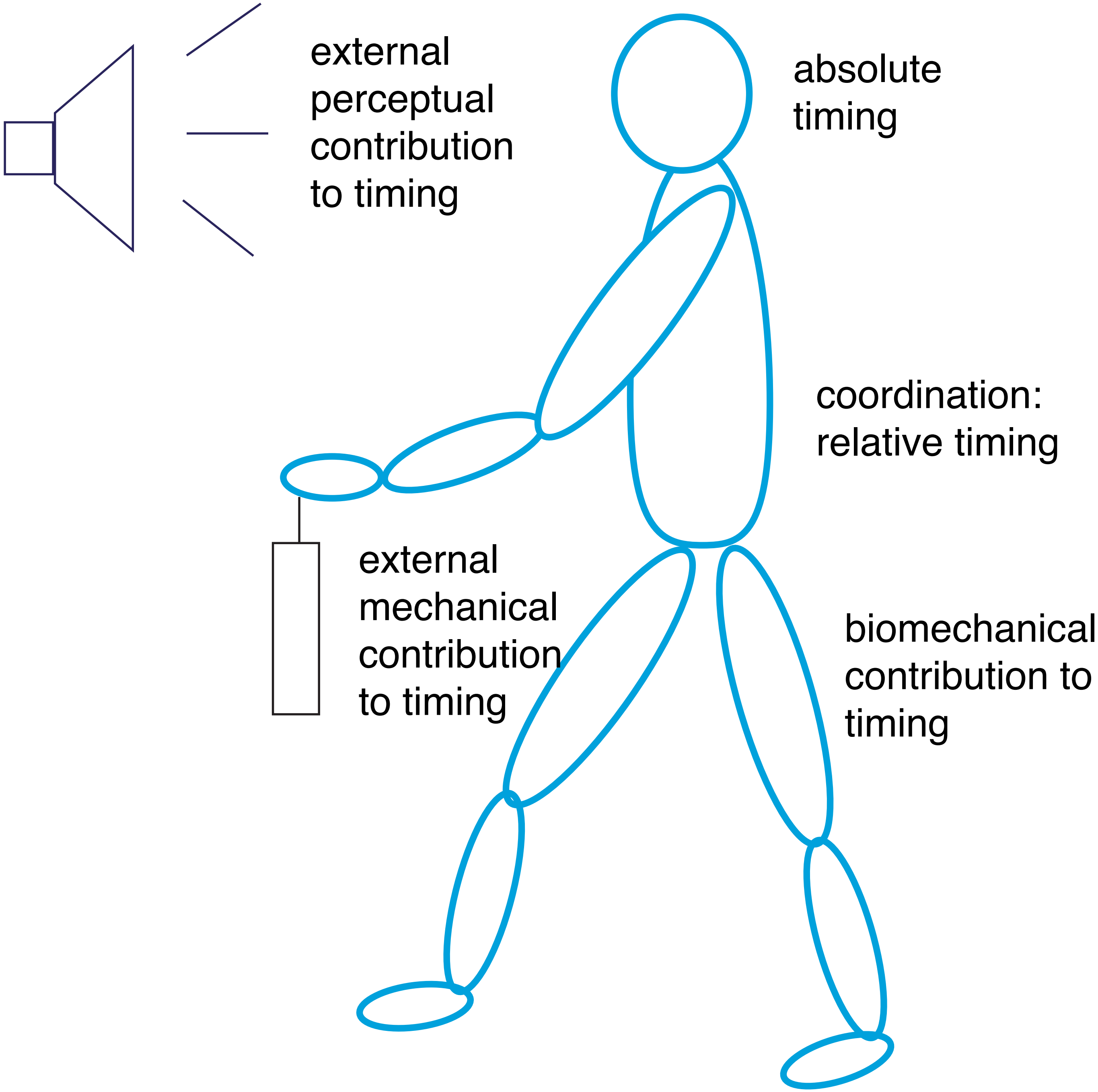




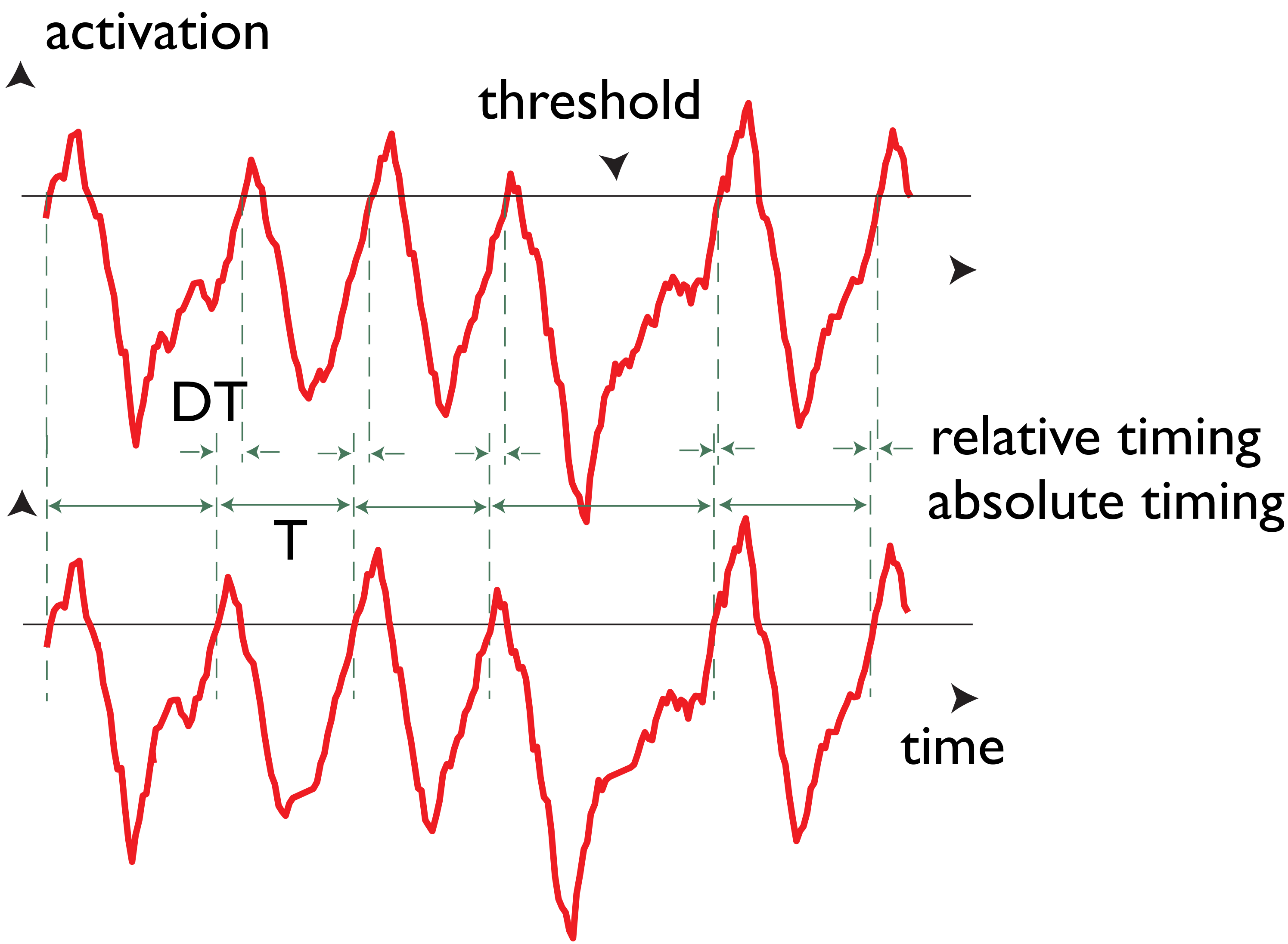
Example : pointing with 10 DoF arm at targets in 3D



Timing in nervous systems



Relative vs. absolute timing

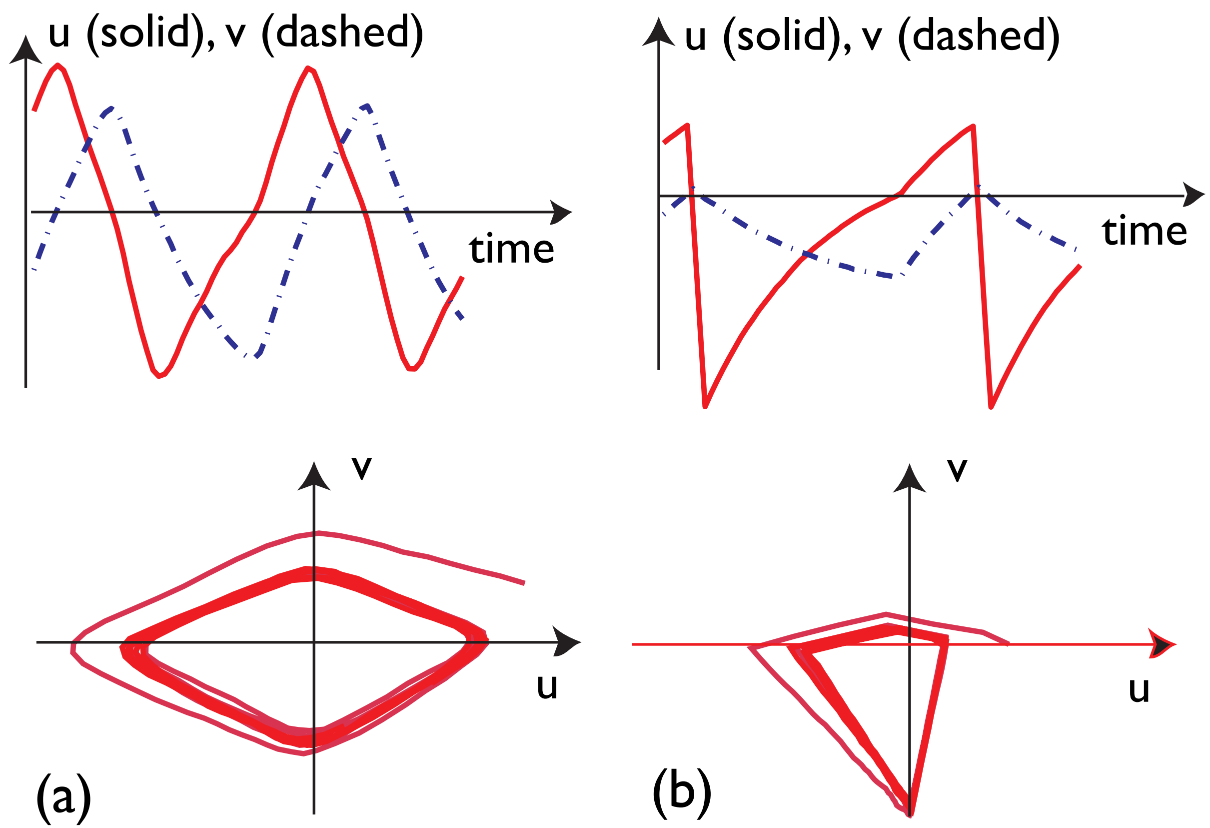


relative phase=DT/T

Neural oscillator

- relaxation oscillator

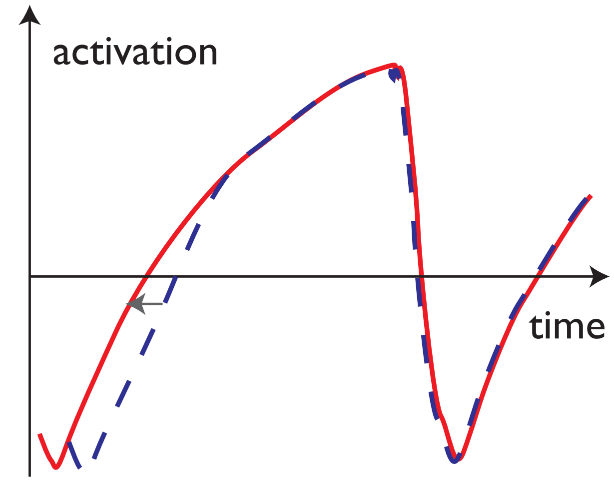
$$\tau\dot{u} = -u + h_u + w_{uu}f(u) - w_{uv}f(v)$$
$$\tau\dot{v} = -v + h_v + w_{vu}f(u),$$

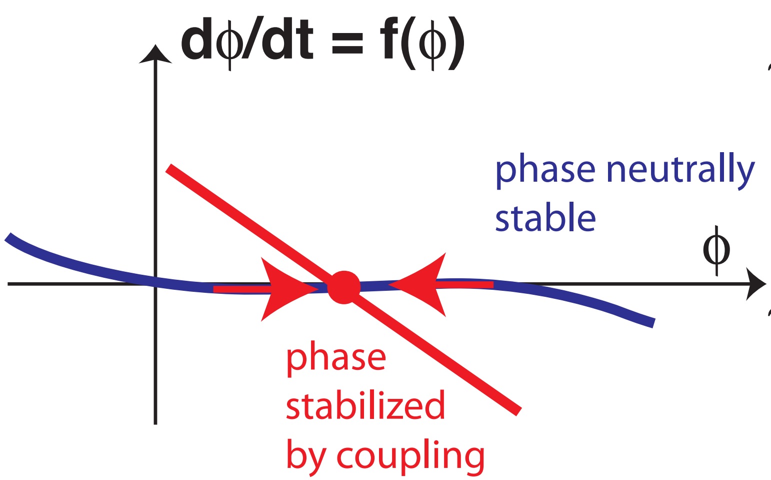


[Amari 77]

Coordination from coupling

- coordination=stable relative timing emerges from coupling of neural oscillators





$$\tau\dot{u}_1 = -u_1 + h_u + w_{uu}f(u_1) - w_{uv}f(v_1)$$
$$\tau\dot{v}_1 = -v_1 + h_v + w_{vu}f(u_1) \boxed{+\ cf(u_2)}$$
$$\tau\dot{u}_2 = -u_2 + h_u + w_{uu}f(u_2) - w_{uv}f(v_2)$$
$$\tau\dot{v}_2 = -v_2 + h_v + w_{vu}f(u_2) \boxed{+\ cf(u_1)}$$

[Schöner: Timing, Clocks, and Dynamical Systems. *Brain and Cognition* 48:31-51 (2002)]

Dynamics Movement Primitives

(not relevant for exam)

Open-chain manipulator

$$\boxed{M(\theta)\ddot{\theta} + C(\theta,\dot{\theta})\dot{\theta} + N(\theta,\dot{\theta}) = \tau}$$

inertial	centrifugal/ coriolis	gravitational	active torques

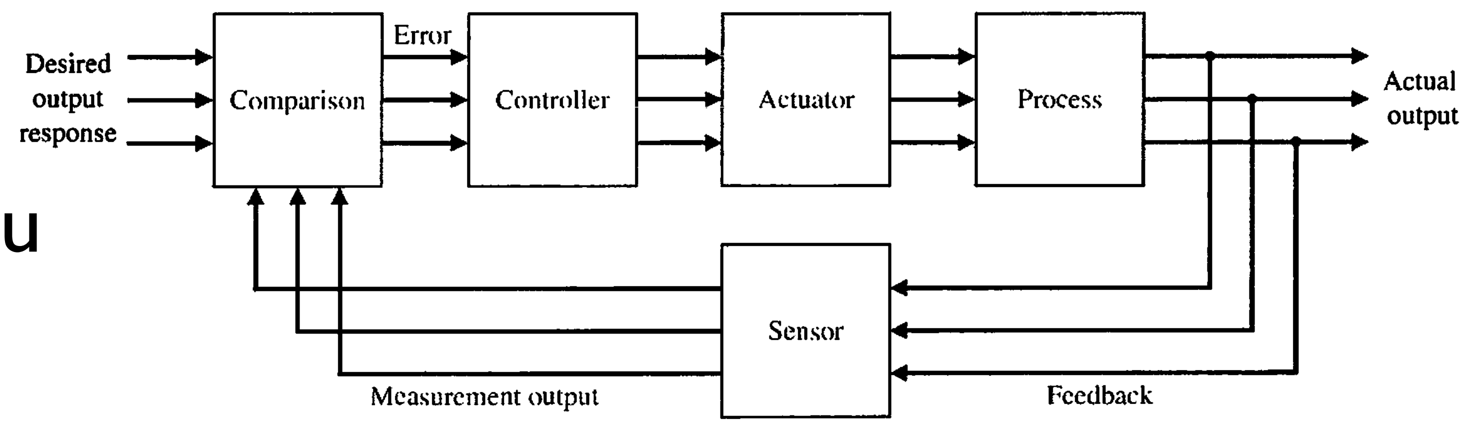
Control

- generate joint torques that produce a desired motion… θ_d
- PD control $\tau = -K_v \dot{e} - K_p e,$
- where $e = \theta - \theta_d.$

$$\boxed{M(\theta)\ddot{\theta} + C(\theta, \dot{\theta})\dot{\theta} + N(\theta, \dot{\theta}) = \tau}$$

Control systems

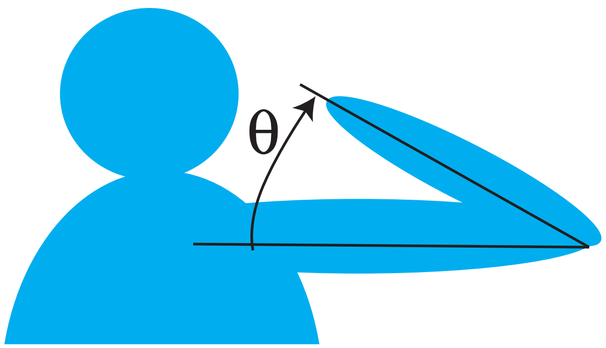
- $$\dot{x} = f(t, x, u) \qquad\qquad y = \eta(t, x, u)$$
- state of process, x
- output, y
- control signal, u
- control law:
  - u as a function of y (or ^y), desired response, y_d
- disturbances modeled stochastically

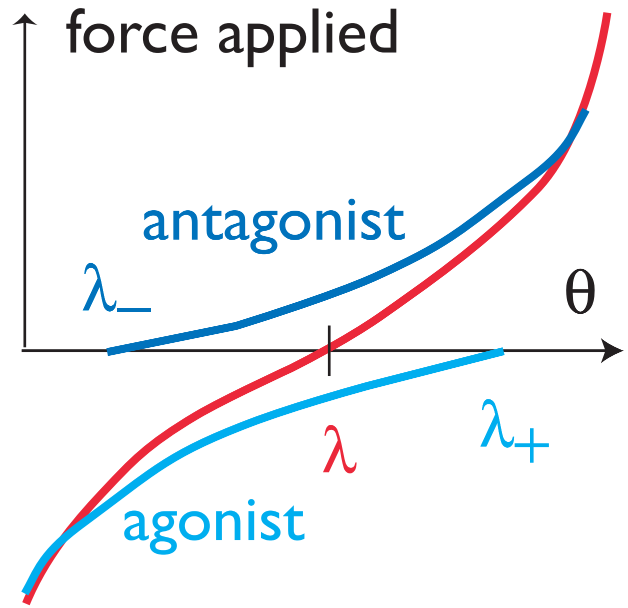


[Dorf, Bischop, 2011]

Human motor control

- posture resists when pushed => is actively controlled = stabilized by feedback
- invariant characteristic
  - one lambda per muscle
  - co-contraction controls stiffness

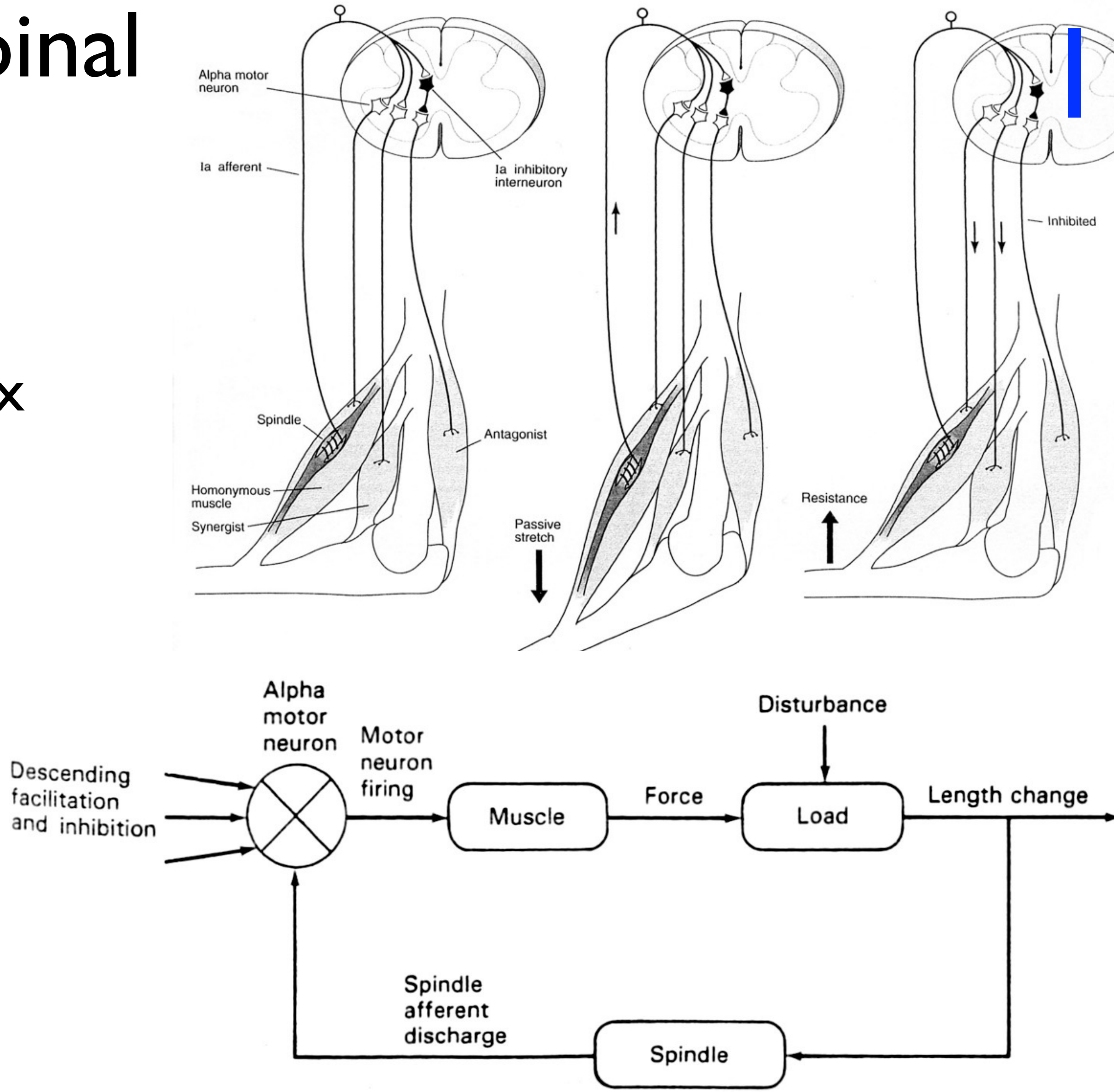




=>experiment

based on spinal reflexes

- stretch reflex



[Kandel, Schartz, Jessell, Fig. 37-11]

Exam

- concepts:
  - multiple choice questions
  - free text discussion questions
- dynamical systems concepts
  - graphically illustrating/interpreting dynamical systems
  - “mental simulation”
- using dynamical systems concepts to conceive of human/robotic behaviors