Summary: main conceptual points

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Dynamical systems

functional link between state and its rate of change
Dynamical system

 presente determines the future

\[ \frac{dx}{dt} = f(x) \]
Dynamical systems

- fixed point = constant solution
- neighboring initial conditions converge = attractor
Bifurcations are instabilities

In families of dynamical systems, which depend (smoothly) on parameters, the solutions change qualitatively at bifurcations at which fixed points change stability.

\[ \dot{x} = \alpha - x^2 \]

- **\( \alpha \) positive**
- **\( \alpha = 0 \)**
- **\( \alpha \) negative**

**fixed point**

\[ x_0 = \sqrt{\alpha} \]

- **stable**
- **unstable**
Basic ideas of attractor dynamics approach

- behavioral variables
- time courses from dynamical system: attractors
- tracking attractors
- bifurcations for flexibility
Behavioral variables: example

- vehicle moving in 2D: heading direction
- constraints: obstacle avoidance and target acquisition

![Diagram showing vehicle movement and behavioral variables]

- $\Delta \psi$: angle change
- $\Psi_{\text{obs}}$: obstacle angle
- $\Psi_{\text{tar}}$: target angle
- $\text{robot}$
- $\text{target}$
- arbitrary, but fixed reference axis
Behavioral dynamics: example

- Behavioral constraint: target acquisition

![Diagram showing a vehicle and a target with an attractor](image)
behavioral constraint: obstacle avoidance

Behavioral dynamics: example

robot

obstacle

arbitrary, but fixed reference axis

$d\phi/dt$

repellor

$\psi_{\text{obs}}$
Behavioral dynamics

- Each contribution is a “force-let” with
  - Specified value
  - Strength
  - Range

\[ \frac{\text{d}\phi}{\text{d}t} \sim \text{strength} \]

\[ \Psi_{\text{tar}} \text{ specified value} \]

\[ \sim \text{strength} \]

Range
Behavioral dynamics: bifurcations

- constraints not in conflict
Behavioral dynamics

- constraints in conflict

![Diagram showing behavioral dynamics with obstacles, target, and dφ/dt graph.]

\[ \frac{d\phi}{dt} \]

\[ \phi \]
transition from “constraints not in conflict” to “constraints in conflict” is a bifurcation
In a stable state at all times

\[
\frac{d\phi}{dt} = F(\text{vehicle}, \text{obstacle}, \text{heading direction})
\]

\[
F = \frac{d\phi}{dt}
\]
Obstacle avoidance: sub-symbolic

- obstacles need not be segmented
- do not care if obstacles are one or multiple: avoid them anyway…

\[ \Delta \psi \]

\[ \Psi_{\text{obs}} \]

\[ \theta_{\text{obs}} \]

\[ d\phi/dt \]

repellor
In contrast to higher-level implementations where one obstacle with increasing distance sensed at the sensor sector to warrant clear passage.

Thus, when no obstacle is within the range of the distance sensitivity and the net contribution is zero.

The range of the repulsive forcelet is limited based on sensor range and on the constraint of passing next to the virtual obstacle without the net contribution is exactly one obstacle, we are in-

The constant $\psi$, is a decreasing function of the distance, $\psi = 2\pi - \arctan \frac{d}{R}$, from the virtual obstacle at direction $\psi$.

From this rotation results three virtual obstacles now at directions $\psi_1, \psi_2, \psi_3$.

The bold line represents the resultant obstacle avoidance dynamics. The resultant repeller is at $\psi = \pi/2$.

Figures 3.

Figure 4.

Figures 6.

In that figure exemplary illustrate that the summed obstacles $\Delta \theta_i$.

Each sensor serves as a local attractor is near $\psi = \pi/2$ and an-

$\phi$.

$\phi$.

$d\phi/dt$

resultant repeller

resultant repeller

$d\phi/dt$

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Bifurcations
2nd order attractor dynamics to explain human navigation

\[ \dot{\phi} = -b\dot{\phi} - k_g (\phi - \psi_g) (e^{-c_1d_g} + c_2) + k_o (\phi - \psi_o) (e^{-c_3|\phi - \psi_o|}) (e^{-c_4d_o}) \]

inertial term

\[ \ddot{\phi} \]

damping term

attractor goal heading

repellor obstacle heading

[Fajen Warren…]
model-experiment match: goal

experiment

model
model-experiment match: obstacle

**Experiment**

- **Figure 3**: Pathways produced by the model around obstacles located at 4 m ahead exert an appreciable influence on steering behavior. Note that the exponential terms introduce nonlinearity into the system.

**Model**

- **Figure 4**: Curves correspond to (a) initial obstacle angle in the 4 m condition and (b) initial obstacle distance in the 4 m condition. The parameter $\phi$ is a gain term for the obstacle component, while $\psi$ is a function that influences the obstacle's angular acceleration decreases with both obstacle angle and distance:

  - $\psi(\theta) = \begin{cases} 
  \theta & \text{if } \theta < \phi \psi \\
  \phi \psi & \text{otherwise}
  \end{cases}

  This function is used to model how goal and obstacle components interact to perform route selection.

- **Figure 5**: Paths produced by the model to goals located at 15 deg. Configurations of goal and obstacle used in Simulation #2. Note that the exponential terms introduce nonlinearity into the system. Terms $\phi$ and $\psi$ influence the obstacle's angular acceleration decreases with both obstacle angle and distance.

- **Figure 6**: Curves correspond to (a) initial obstacle distance in the 4 m condition and (b) initial obstacle distance in the 4 deg condition. The parameter $\phi$ is a gain term for the obstacle component, while $\psi$ is a function that influences the obstacle's angular acceleration decreases with both obstacle angle and distance:

  - $\psi(\theta) = \begin{cases} 
  \theta & \text{if } \theta < \phi \psi \\
  \phi \psi & \text{otherwise}
  \end{cases}

  This function is used to model how goal and obstacle components interact to perform route selection.
Alternative 2nd order approach

\[ \dot{\omega} = (\alpha + \frac{1}{2}\pi) c_{obs} F_{obs} + \alpha \omega - \gamma \omega^3 \]

(a) dynamics of turning rate
(b) dynamics of turning rate
(c) dynamics of turning rate
(d) dynamics of turning rate

[Bicho, Schöner, 97]
Potential field approach

A Typical Performance Example

Large tick marks indicate the limit distance of repulsive interaction (shown as dots). The circles around the obstacles labeled goal, and the agent was assumed to avoid them. The environment consisted of a 5 m × 6.5 m room with a start-point and a goal location. The agent moved in a straight line path. In contrast, the potential field method produced a resultant vector that was smoother and shorter than the dynamical model. The damping term constrains the agent's angular acceleration and deceleration, resulting in frequent sharp turns. The dynamical model produces an angular acceleration that varies with the direction of the local force, traversing a short linear path. In this section, we consider high-level conceptual differences between the dynamical model and the potential field method. A low-level quantitative comparison reveals that the potential field approach generates taller cones that extend to infinity, whereas the dynamical model produces a resultant vector that is closer to the agent. This is partially due to the importance of rotational acceleration, which also acts to smooth the agent's speed. Differences between the two methods are evident in the example in Fig. 14, where the dynamical model has a smoother, shorter path than the potential field method. The actual trajectories observed in the simulation closely resemble the predicted paths. The 3D plots in Fig. 15 represent the artificial potential field and the resultant force vectors for the example scene. The top graph (Fig. 15(a)) shows the artificial potential field inside the room and (b) and vector magnitudes. The graph of vector magnitudes is shown in Fig. 15(b). The agent's speed and direction are controlled by the potential field function, which depends only on the shortest distance to the nearest obstacle. The effect of the target is similar in both methods, serving to draw the agent toward the goal. The potential field approach is smoother, shorter, and more accurate than the dynamical model.
spurious attractors in potential field approach

Advantage.
The potential field approach is a local obstacle avoidance method, and local minima are a serious problem. An agent using the potential field method alone without a high level path planner can easily get stuck in local minima, even in the simplest scenes. The dynamical model, in contrast, has few such problems, at least in simple scenes. Because it only controls angular acceleration and not the agent's speed (never stopping the agent), local minima are avoided in two ways: the agent either takes advantage of the canceling effect (described below) and passes between the obstacles (if the distance decay parameter $c_4$ is big), or it takes a path around the obstacle cluster (if $c_4$ is small). In the latter case it may overshoot the target, but it easily homes in from another direction. Thus, with appropriate parameter settings the dynamical model can avoid local minima in simple scenes.

Disadvantage.
However, if the locations of the obstacles are symmetrical about the agent's path to the target, then their contributions to the angular acceleration will have similar magnitudes but opposite signs, and therefore cancel each other. This canceling effect creates a spurious attractor in the center of the obstacle array, which may lead the agent into a gap that is too small, or even to crash into an obstacle at the center of a perfectly symmetrical array. As noted above, one way to avoid the canceling effect is to increase obstacle repulsion with distance by reducing the exponential decay term $c_4$, thereby inducing an outside path around the entire array. In cases with only a few obstacles, adding a noise term to the model may allow it to escape unstable fixed points.

These advantages and disadvantages are illustrated in Fig. 20. In this example the agent starts in the lower left corner with an initial heading of $0^\circ$, and moves at a constant translation speed of 1 m/s. Path 1 shows a sample local minimum for the potential field method. The agent is stuck in a bowl (a region of small outward-pointing resultant vectors surrounded by large inward-pointing vectors) and is reduced to oscillating back and forth. Another type of local minimum is being frozen in a location where the attractive and repulsive forces can cancel each other, producing a resultant force of zero magnitude. Path 2 is traversed with the dynamical model ($c_4 = 1.6$). Since there are obstacles on both sides of the agent, their combined contribution to the angular acceleration demonstrates the canceling effect along the path, and the agent passes between them. Path 3 is also traversed by the dynamical model using a more gradual exponential decay with distance ($c_4 = 0.4$). The repulsive regions of the obstacles are larger, and therefore they force the agent to take an outside path.

Agent Speed.
A final difference between the two methods is that the dynamical model assumes a constant translational speed on the part of the agent. This is indeed the case in our human data: subjects tend to accelerate from a standstill and then maintain an approximately constant walking speed. However, the model produces different paths at different constant speeds, with all other parameters fixed. The reason for this behavior is that, when the agent enters a region that produces a non-zero angular acceleration, the accelerating effect lasts for a shorter time at higher speeds, inducing a smaller rotation. In contrast, since the potential field equations determine the direction of the agent's motion, it will always traverse the same path independent of speed. For any physical agent with mass and momentum, the responsiveness of trajectories to speed may actually be a desirable effect.

An example for the dynamical model is presented in Fig. 21. With a constant speed of 0.25 m/s, the model traverses path 1 to the left of the obstacle, but with a speed of 1.0 m/s it takes path 2 to the right. In these simulations, the agent's initial heading was $0^\circ$ (horizontal),
Spaces for robotic motion planning

kinematic model
\[ \mathbf{x} = f(\theta) \]
inverse kinematic model
\[ \theta = f^{-1}(\mathbf{x}) \]

- transform end-effector to configuration space through inverse kinematics
- problems of singularities and multiple “leafs” of inverse...
Redundant kinematics

- redundant arms/tasks: more joints than task-level degrees of freedom
- => (continuously) many inverse solutions…
what is a DoF?

- variable that can be independently varied
- e.g. joint angles

muscles/muscle groups

- but: assess to which extent they can be activated independently…
- .. mode picture

\[
x = l_1 \cos(\theta_1) + l_2 \cos(\theta_1 + \theta_2) + l_3 \cos(\theta_1 + \theta_2 + \theta_3)
\]

\[
y = l_2 \sin(\theta_1) + l_2 \sin(\theta_1 + \theta_2) + l_3 \sin(\theta_1 + \theta_2 + \theta_3)
\]
The concept of the UnControlled Manifold

- The many DoF are coordinated such that changes that affect the task-relevant dimensions are resisted against more than changes that do not affect task relevant dimension
- Leading to compensation

[Scholz, Schöner, EBR 126:289 (99)]
UCM synergy: data analysis

- align trials in time
- hypothesis about task variable
- compute null-space (tangent to the UCM)
- predict more variance within null space than perpendicular to it
Example: pointing with 10 DoF arm at targets in 3D

Task variable: hand movement direction in space
Timing in nervous systems

- External perceptual contribution to timing
- External mechanical contribution to timing
- Biomechanical contribution to timing
- Coordination: relative timing
- Absolute timing
Relative vs. absolute timing

relative phase = DT/T
Neural oscillator

Relaxation oscillator

\[
\begin{align*}
\tau \dot{u} &= -u + h_u + w_{uu}f(u) - w_{uv}f(v) \\
\tau \dot{v} &= -v + h_v + w_{vu}f(u),
\end{align*}
\]

Amari (1977)
Coordination from coupling

- Coordination = stable relative timing emerges from coupling of neural oscillators

\[ \frac{d\phi}{dt} = f(\phi) \]

\[
\begin{align*}
\tau \dot{u}_1 &= -u_1 + h_u + w_{uu}f(u_1) - w_{uv}f(v_1) \\
\tau \dot{v}_1 &= -v_1 + h_v + w_{vu}f(u_1) + cf(u_2) \\
\tau \dot{u}_2 &= -u_2 + h_u + w_{uu}f(u_2) - w_{uv}f(v_2) \\
\tau \dot{v}_2 &= -v_2 + h_v + w_{vu}f(u_2) + cf(u_1)
\end{align*}
\]

Dynamics Movement Primitives

(not relevant for exam)
Open-chain manipulator

\[ M(\theta)\ddot{\theta} + C(\theta, \dot{\theta})\dot{\theta} + N(\theta, \dot{\theta}) = \tau \]

- inertial
- centrifugal/coriolis
- gravitational
- active torques
generate joint torques that produce a desired motion... $\theta_d$

PD control

$$\tau = -K_v \dot{e} - K_p e,$$

where $e = \theta - \theta_d$.

$$M(\theta)\ddot{\theta} + C(\theta, \dot{\theta})\dot{\theta} + N(\theta, \dot{\theta}) = \tau$$
Control systems

\[ \dot{x} = f(t, x, u) \quad y = \eta(t, x, u) \]

- state of process, \( x \)
- output, \( y \)
- control signal, \( u \)
- control law:
  - \( u \) as a function of \( y \) (or \( \hat{y} \)), desired response, \( y_d \)
- disturbances modeled stochastically

[Dorf, Bischop, 2011]
Posture resists when pushed => is actively controlled = stabilized by feedback

- Invariant characteristic
  - One lambda per muscle
  - Co-contraction controls stiffness

=> Experiment
based on spinal reflexes

- stretch reflex

[Kandel, Schartz, Jessell, Fig. 37-11]
Exam

- concepts:
  - multiple choice questions
  - free text discussion questions

- dynamical systems concepts
  - graphically illustrating/interpreting dynamical systems
  - “mental simulation”

- using dynamical systems concepts to conceive of human/robotic behaviors