Motor control

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Motor control

is about the processes of bringing about the physical movement of an arm (robot or human)

this entails

- the mechanical dynamics of an arm
- control principles



actuators

Resource

- R M Murray, Z Li, S S. Sastry: A mathematical introduction to robotic manipulation. CRC Press, 1994
- online version available

Newton's law

- for a mass, m, described by a variable, x, in an inertial frame: $m\ddot{x} = f(x, t)$ where f is a force
- in non-inertial frames, e.g. rotating or accelerating frames:
 - 🧲 centripetal forces
 - coriolis forces

Rigid bodies: constraints

constraints reduce the effective numbers of degrees of freedom...



$$F_i = m_i \ddot{r}_i \qquad r_i \in \mathbb{R}^3, i = 1, \dots, n.$$
$$g_j(r_1, \dots, r_n) = 0 \qquad j = 1, \dots, k.$$

Rigid bodies: constraints

generalized coordinates capture the remaining, free degrees of freedom



$$r_i = f_i(q_1, \dots, q_m)$$
$$i = 1, \dots, n$$

$$g_j(r_1, \dots, r_n) = 0$$

$$j = 1, \dots, k.$$

Lagrangian mechanics

- The Lagrangian framework makes it possible to capture dynamics in generalized coordinates that reflect constraints
- Lagrange function L = kineticpotential energy $L(q, \dot{q}) = T(q, \dot{q}) - V(q),$

Least action principle: The integral of L over time=action is minimal $\delta A = \delta \int L(q, \dot{q}, t) dt = 0$

[Murray, Sastry, Li, 94]

Lagrangian mechanics

Least action principle: The integral of L over
time=action is minimal
$$\delta A = \delta \int L(q, \dot{q}, t) dt = 0$$

[Murray, Sastry, Li, 94]

Euler-Lagrange equation

$$\delta A = \int (\frac{\partial L}{\partial q} \delta q + \frac{\partial L}{\partial \dot{q}} \delta \dot{q}) dt = 0$$

with $\delta \dot{q} = d\delta q/dt$

and with partial integration

$$\delta A = \left[\frac{\partial L}{\partial \dot{q}}\delta q\right] + \int \left(\frac{\partial L}{\partial q} - \frac{d}{dt}\frac{\partial L}{\partial \dot{q}}\right)\delta q \, dt = 0$$

first term vanishes: no variation at start/end points

Euler-Lagrange equation

$$=>\frac{d}{dt}\frac{\partial L}{\partial \dot{q}} - \frac{\partial L}{\partial q} = 0$$

 \blacksquare ... plus generalized external forces, γ

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}} - \frac{\partial L}{\partial q} = \gamma$$

in component form:

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = \Upsilon_i \qquad i = 1, \dots, m,$$

Example: pendulum

d generalized coordinates: $heta, \phi$

$$T = \frac{1}{2}ml^2 \|\dot{r}\|^2 = \frac{1}{2}ml^2 \left(\dot{\theta}^2 + (1 - \cos^2\theta)\dot{\phi}^2\right)$$

$$V = -mgl\cos\theta,$$

mq

$$L(q, \dot{q}) = \frac{1}{2}ml^2 \left(\dot{\theta}^2 + (1 - \cos^2 \theta)\dot{\phi}^2\right) + mgl\cos\theta$$
position relative to base
$$r(\theta, \phi) = \begin{bmatrix} l\sin\theta\cos\phi\\ l\sin\theta\sin\phi\\ -l\cos\theta \end{bmatrix}$$

Example: pendulum

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{\theta}} = \frac{d}{dt}\left(ml^2\dot{\theta}\right) = ml^2\ddot{\theta}$$
$$\frac{\partial L}{\partial \theta} = ml^2\sin\theta\cos\theta\,\dot{\phi}^2 - mgl\sin\theta$$
$$\frac{d}{dt}\frac{\partial L}{\partial \dot{\phi}} = \frac{d}{dt}\left(ml^2\sin^2\theta\,\dot{\phi}\right) = ml^2\sin^2\theta\,\ddot{\phi} + 2ml^2\sin\theta\cos\theta\,\dot{\theta}\dot{\phi}$$
$$\frac{\partial L}{\partial \phi} = 0$$

$$\begin{bmatrix} ml^2 & 0 \\ 0 & ml^2 \sin^2 \theta \end{bmatrix} \begin{bmatrix} \ddot{\theta} \\ \ddot{\phi} \end{bmatrix} + \begin{bmatrix} -ml^2 \sin \theta \cos \theta \, \dot{\phi}^2 \\ 2ml^2 \sin \theta \cos \theta \, \dot{\theta} \dot{\phi} \end{bmatrix} + \begin{bmatrix} mgl \sin \theta \\ 0 \end{bmatrix} = 0.$$

inertial centrifugal gravitational (coriolis)

Example: two-link planar robot

generalized coordinates: θ_1, θ_2

where $s_i = \sin(\theta_i), c_i = \cos(\theta_i)$

$$\begin{bmatrix} \alpha + 2\beta c_2 & \delta + \beta c_2 \\ \delta + \beta c_2 & \delta \end{bmatrix} \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} + \begin{bmatrix} -\beta s_2 \dot{\theta}_2 & -\beta s_2 (\dot{\theta}_1 + \dot{\theta}_2) \\ \beta s_2 \dot{\theta}_1 & 0 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix}$$

inertial centrifugal/coriolis active

torques

 θ_1

 \mathcal{X}

Open-chain manipulator

$$M(\theta)\ddot{\theta} + C(\theta,\dot{\theta})\dot{\theta} + N(\theta,\dot{\theta}) = \tau$$

inertial centrifugal/ gravitational active torques

generate joint torques that produce a desired motion... θ_d

PD control
$$\tau = -K_v \dot{e} - K_p e$$
,

where $e = \theta - \theta_d$.

$$M(\theta)\ddot{\theta} + C(\theta,\dot{\theta})\dot{\theta} + N(\theta,\dot{\theta}) = \tau$$

most commonly used: individual joint PD error $e = \theta - \theta_d$ $\mathbf{T} = \ddot{\theta}_d + K_v \dot{e} + K_p e + K_i \int e dt$

$$M(\theta)\ddot{\theta} + C(\theta,\dot{\theta})\dot{\theta} + N(\theta,\dot{\theta}) = \tau$$

there are many more sophisticated models that compensate for interaction torques/ inertial coupling... e.g.

computed torque control

$$\tau = \underbrace{M(\theta)\ddot{\theta}_d + C\dot{\theta} + N}_{\tau_{\rm ff}} + \underbrace{M(\theta)\left(-K_v\dot{e} - K_pe\right)}_{\tau_{\rm fb}}$$

$$M(\theta)\ddot{\theta} + C(\theta,\dot{\theta})\dot{\theta} + N(\theta,\dot{\theta}) = \tau$$

$$\Rightarrow (\ddot{\theta} - \ddot{\theta}_d) = \ddot{e} = -K_v \dot{e} - K_p e$$

augmented PD control

$$\tau = M(\theta)\ddot{\theta}_d + C(\theta,\dot{\theta})\dot{\theta}_d + N(\theta,\dot{\theta}) - K_v\dot{e} - K_pe$$

$$M(\theta)\ddot{\theta} + C(\theta,\dot{\theta})\dot{\theta} + N(\theta,\dot{\theta}) = \tau$$

Problems

real-time computation.. discretization error

imprecise model of arm (and object that is handled)

Control systems

robotic motion as a special case of control



[Dorf, Bischop, 2011]

Control systems
$$\dot{x} = f(t, x, u)$$
 $y = \eta(t, x, u)$

state of process/actuator x

output, y

control signal, u



$$\begin{aligned} & \textbf{Control systems} \\ & \dot{x} = f(t, x, u) \qquad \qquad y = \eta(t, x, u) \end{aligned}$$

control law: u as a function of y (or ^y), desired response, y_d

disturbances modeled stochastically



- theoretical core of robotic control theory: proving stability/asymptotic stability...
- and realizing that stability in numerical implementation

Problem: actuators

- actuators do not generate a precise/desired torque... true torque depends on the load
- difficult to measure torque during movement... so not easy/practical to control torque
- actuators are generally way stronger than actual loads... so that the position/velocity feedback loop can compensate for any deviation of real from required torque

Problem: contact forces

- as soon as the robot arm makes contact, a host of problems arise from the contact forces and their effect on the arm and controller...
- need compliance... resisting to a welldefined degree
- => impedance control... research frontier

Link to movement planning

- where does "desired trajectory" come from?
- typically from end-effector level movement planning
 - then add an inverse kinematic...
 - which can be problematic
- alternative: planning and control in endeffector space

Operational space formulation Euler-Langrage in end-effector space $\Lambda(x)\ddot{x} + \mu(x, \dot{x}) + p(x) = F$ with F forces acting on the end-effector equivalent dynamics in joint space $A(q)\ddot{q} + b(q, \dot{q}) + g(q) = \Gamma$ with joint torques $\Gamma = J^T(q)F$

[Khatib, 1987]

Operational space formulation

in end-effector space add constraints as contributions to the "virtual forces"

$$F_{\mathbf{x}_d}^* = -\operatorname{grad}[U_{\mathbf{x}_d}(\mathbf{x})],$$

$$F_O^* = -\operatorname{grad}[U_O(\mathbf{x})].$$



 $\Lambda(x)\ddot{x} + \mu(x, \dot{x}) + p(x) = F$

[Khatib, 1986, 1987]

Optimal control

given a plant
$$\dot{x} = f(x, u)$$

find a control signal u(t)

that moves the state from an final position $x_i(0)$ to a terminal position $x_f(t_f)$ within the time t_f

a (difficult) planning problem!

minimize a cost function to find such a signal

How does the human (or other animal) movement system generate movement?

mechanics:... biomechanics

- actuator: muscle
- control?
- optimal control?

Human motor control

human movement is highly compliant...



Is posture "controlled"?

- the elbow does not behave like a passive mechanical system with a free joint at the elbow: $J\ddot{\theta}=0$
 - where J is inertial moment of forearm (if upper arm is held fixed)
- Instead, the elbow resists, when pushed => there is active control= stabilization of the joint





the mass spring model

 Anatol Feldman has figured out, what the macroscopic description of this stabilization is



the invariant characteristic

the mass-spring model

- this is an elastic force (because it is proportional to position)
- there is also a viscous component (resistance depends on joint velocity)

$$J\ddot{\theta} = \boxed{-k(\theta - \lambda) - \mu\dot{\theta}}$$

active torques generated by the muscle

agonist-antagonist action

- one lambda per muscle
- tested on muscles detached at one end
- co-contraction controls stiffness



stiffness

- the stiffness, k, can be measured from perturbations
- the viscosity "mu" is more difficult to determine



$$J\ddot{\theta} = -k(\theta {-}\lambda) {-} \mu \dot{\theta}$$

neural basis of EP model: spinal reflex loops



 alphagamma reflex loop generates the stretch reflex

[Kandel, Schartz, Jessell, Fig. 37-11]

spinal cord: reflex loops

the stretch reflex acts as a negative feedback loop



[Kandel, Schartz, Jessell, Fig. 31-12]

spinal cord: coordination

Ia inhibitory interneuron mediates reciprocal innervation in stretch reflex, leading to automatic relaxation of antagonist on activation of agonist



[Kandel, Schartz, Jessell, Fig. 38-2]