Dynamic movement primitives

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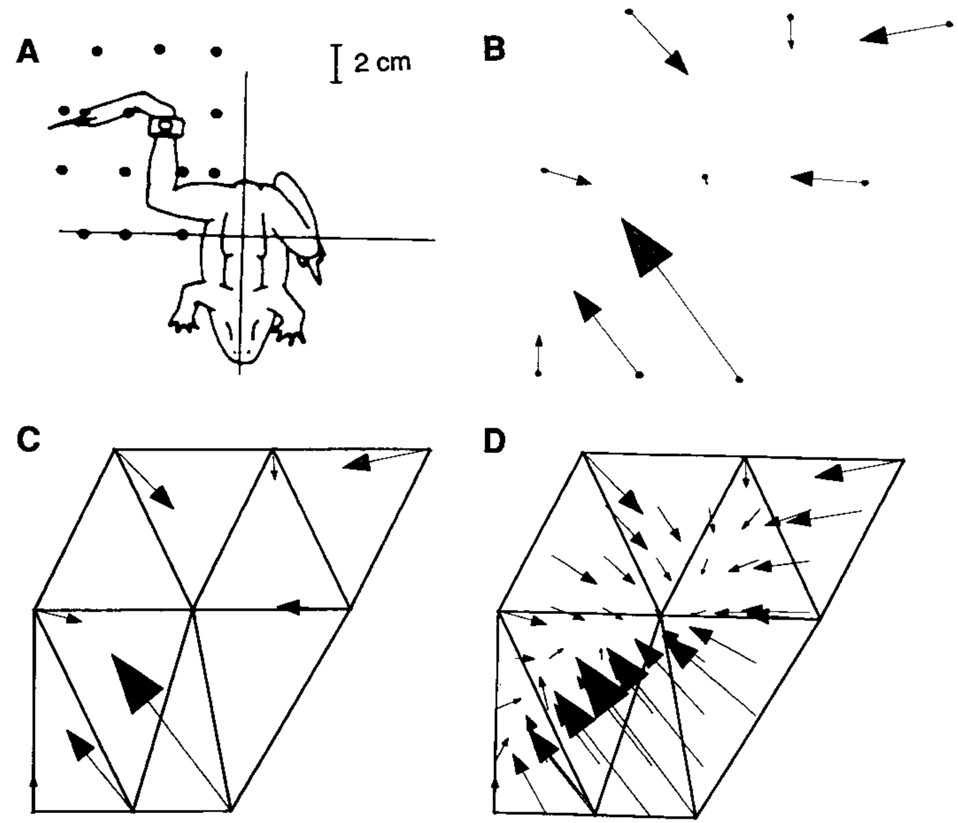
Neural motivation

- Notion that neural networks in the brain and spinal cord generated a limited set of temporal templates
- whose weighted superposition is used to generate any given movement

Evidence for "primitives" in frog spinal cord

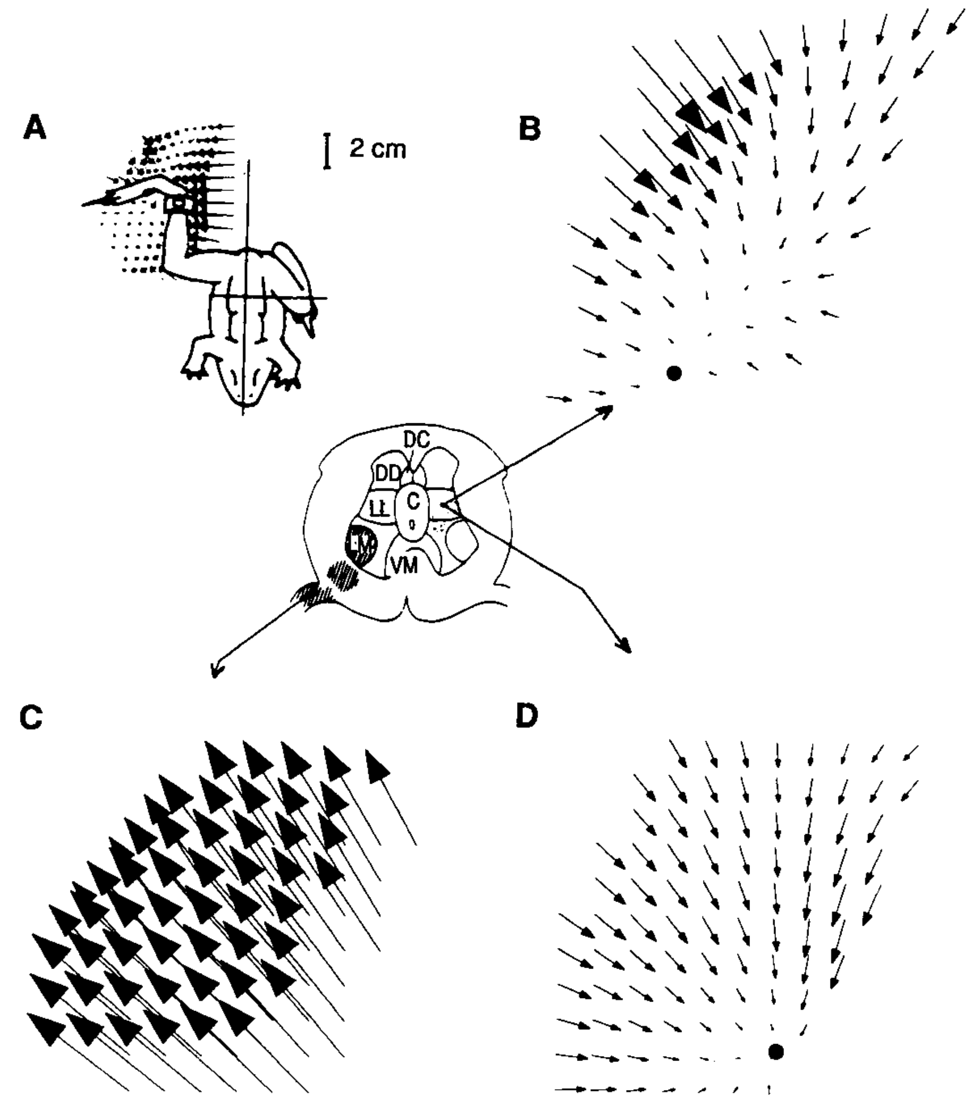
- electrical simulation in premotor spinal cord
- measure forces of resulted muscle activation pattern at different postures of limb
- interpolate force-field



[Bizzi, Mussa-Ivaldi, Gizter, 1991]

Evidence for “primitives” in frog spinal cord

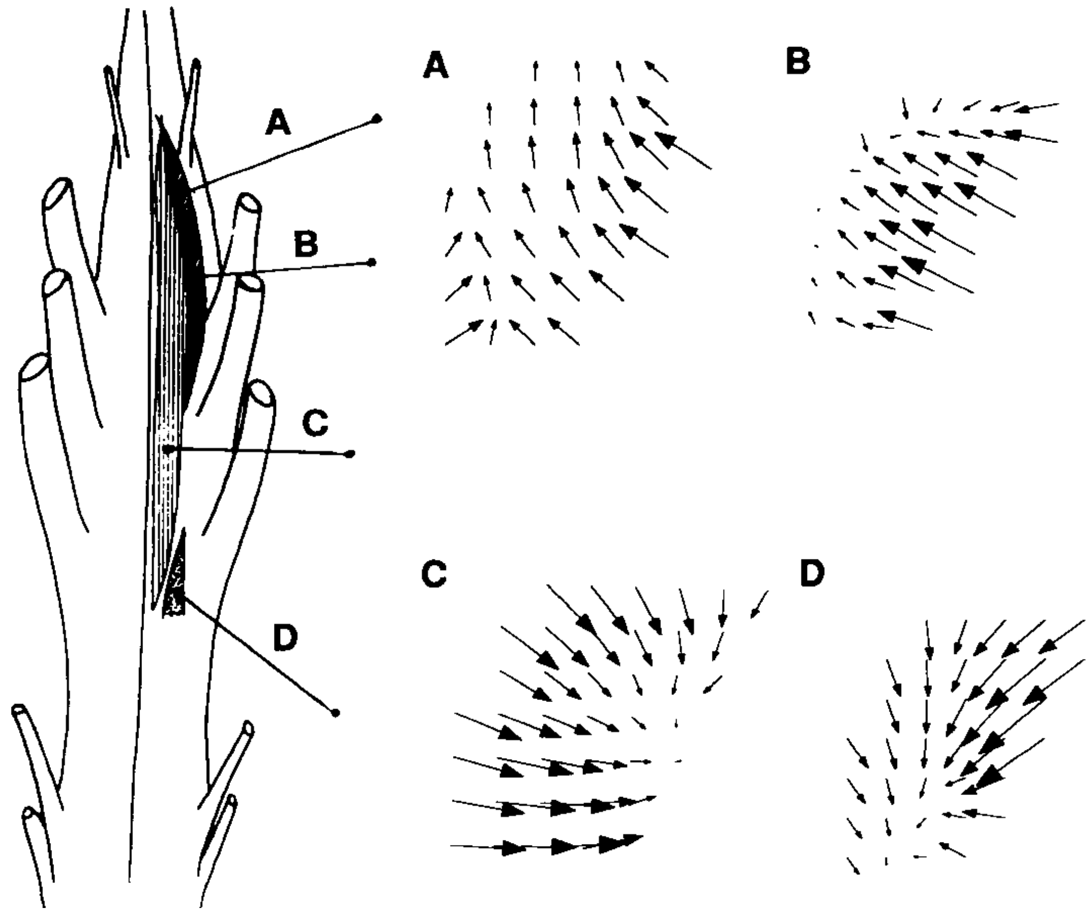
- parallel force-fields in premotor ares vs. convergent force fields from interneurons…



[Bizzi, Mussa-Ivaldi, Gizter, 1991]

Evidence for "primitives" in frog spinal cord

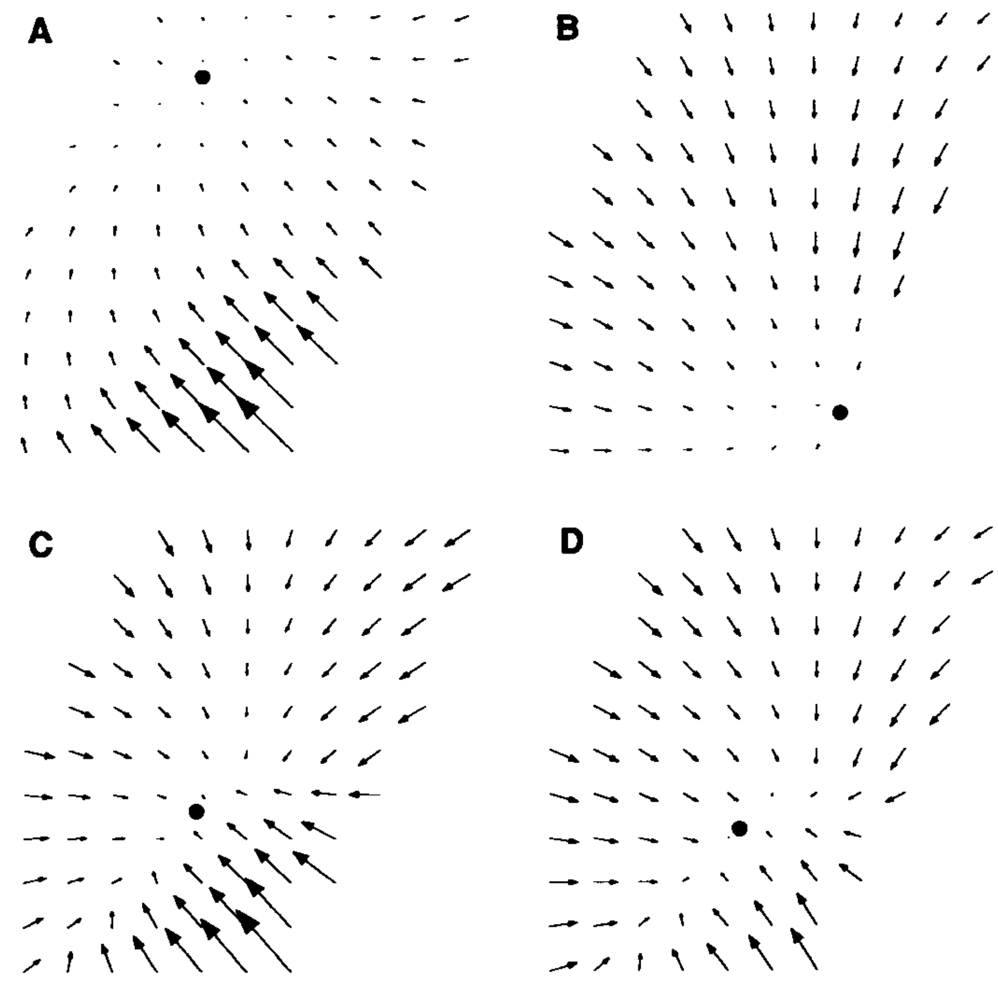
- convergent force-fields occur more often than expected by chance



[Bizzi, Mussa-Ivaldi, Gizter, 1991]

Evidence for “primitives” in frog spinal cord

- superposition of force-fields from joint stimulation



superposition of A and B

stimulating both A and B locations

[Bizzi, Mussa-Ivaldi, Gizter, 1991]

Mathematical abstraction

- (we'll criticize later the lack of analogy to the cited neurophysiology)

[Ijspeert et al., Neural Computation 25:328-373 (2013)]

Base oscillator

- damped harmonic oscillator

$$\tau \ddot{y} = \alpha_z(\beta_z(g - y) - \dot{y}) + f,$$

y: position

- written as two first order equations

$$\tau \dot{z} = \alpha_z(\beta_z(g - y) - z) + f,$$

$$\tau \dot{y} = z,$$

z: velocity

- has fixed point attractor

$$(z, y) = (0, g)$$

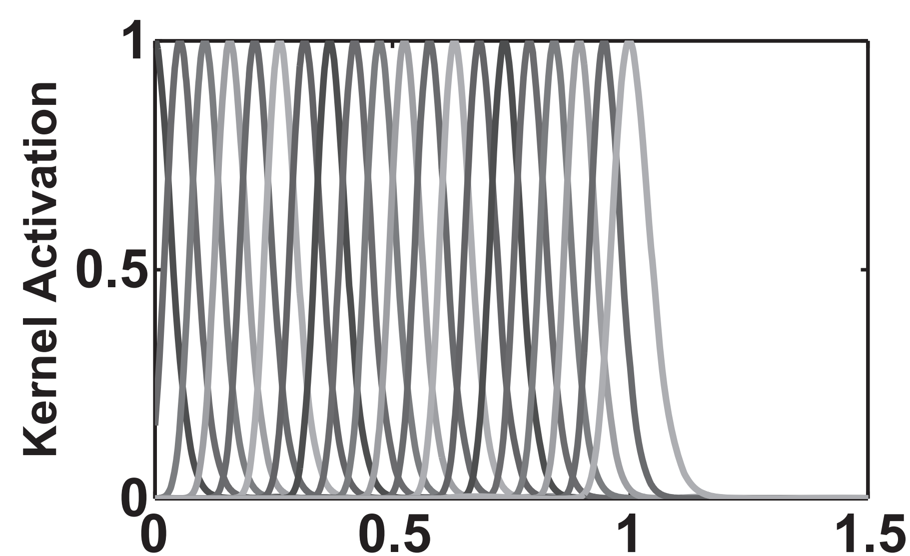
g: goal point

Forcing function

- base functions
- weighted superposition makes forcing function
- which are explicit functions of time!
- => non-autonomous
- and, through c_i, also staggered in time, so there is a "score" being kept in time

$$\Psi_i(x) = \exp\left(-\frac{1}{2\sigma_i^2}(x - c_i)^2\right),$$

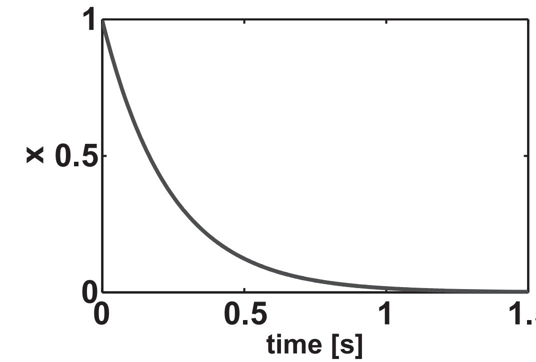
$$f(t) = \frac{\sum_{i=1}^{N} \Psi_i(t) w_i}{\sum_{i=1}^{N} \Psi_i(t)}$$



"Canonical system"

- "phase" variable, x, to (seemingly) get rid of non-autonomous character of dynamics

$$\tau \dot{x} = -\alpha_x x,$$



- but: "fake".. as x is reset to an initial condition at each new movement initiation x(0)=1

$$f(x) = \frac{\sum_{i=1}^{N} \Psi_i(x) w_i}{\sum_{i=1}^{N} \Psi_i(x)} x(g - y_0)$$

- new: scale forcing functions with amplitude and with temporal distance from end of mov

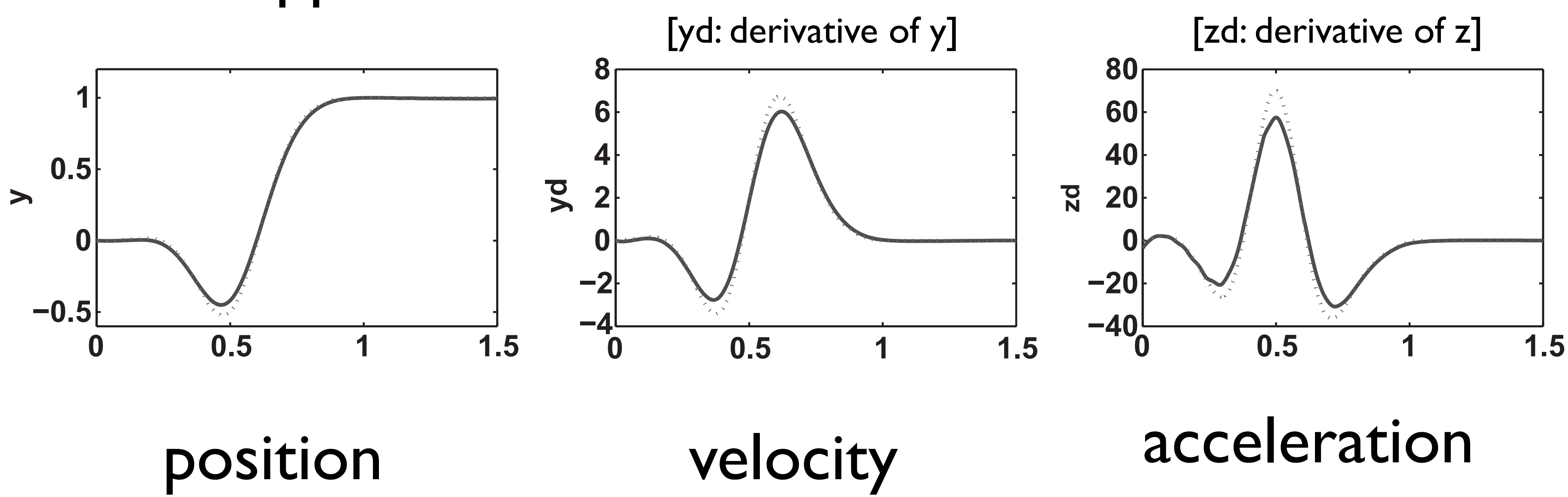
y_0 initial position

$g - y_0$ amplitude

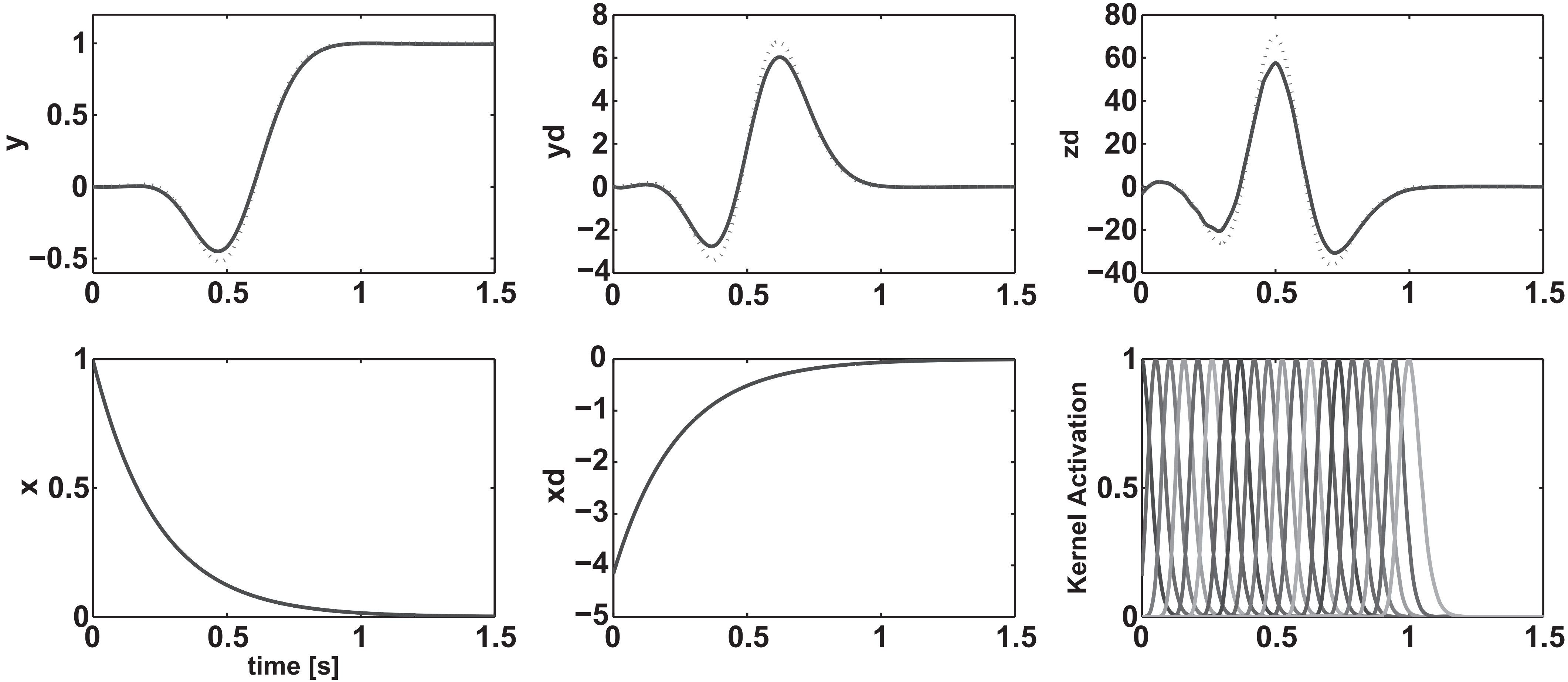
Example 1D

- weights fitted to track dotted trajectory (=5th order polynomial)… with first goes in the negative direction
- 20 kernels…

dotted: target
solid: approximation



Example 1D



y
1
0.5
0
−0.5
0
0.5
1
1.5
yd
8
6
4
2
0
−2
−4
zd
80
60
40
20
0
−20
−40
x
1
0.5
0
time [s]
xd
0
−1
−2
−3
−4
−5
Kernel Activation

The planning problem

- is to make sure the movement plan arrives at the target in a given time…
- the spatial goal is implemented by setting an attractor at the goal state
- the movement time is implicitly encoded in the tau/time scale of the "timing" variable…
  - but that relies on cutting off the timing variable, x, as some threshold level… as exponential time course never reaches zero…
  - quite sensitive to that threshold…

Periodic movement

- trivial phase oscillator (cycle time, tau) $\tau\dot{\phi} = 1,$
- trivial amplitude, r (constant), can be modulated by explicit time dependence
- forcing-function are functions of phase and amplitude

$$f(\phi, r) = \frac{\sum_{i=1}^{N} \Psi_i w_i}{\sum_{i=1}^{N} \Psi_i} r,$$

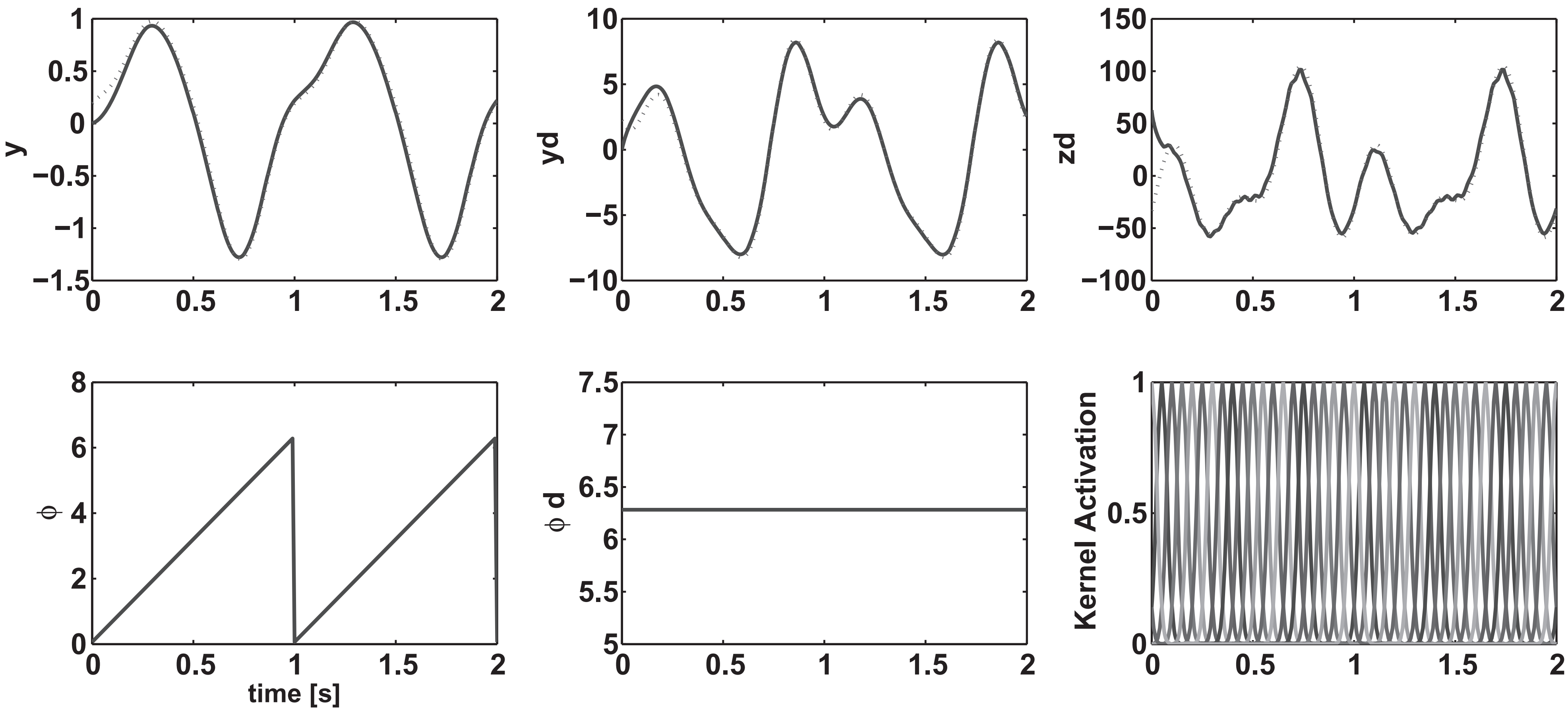
$$\Psi_i = \exp(h_i(\cos(\phi - c_i) - 1))$$

- base oscillator

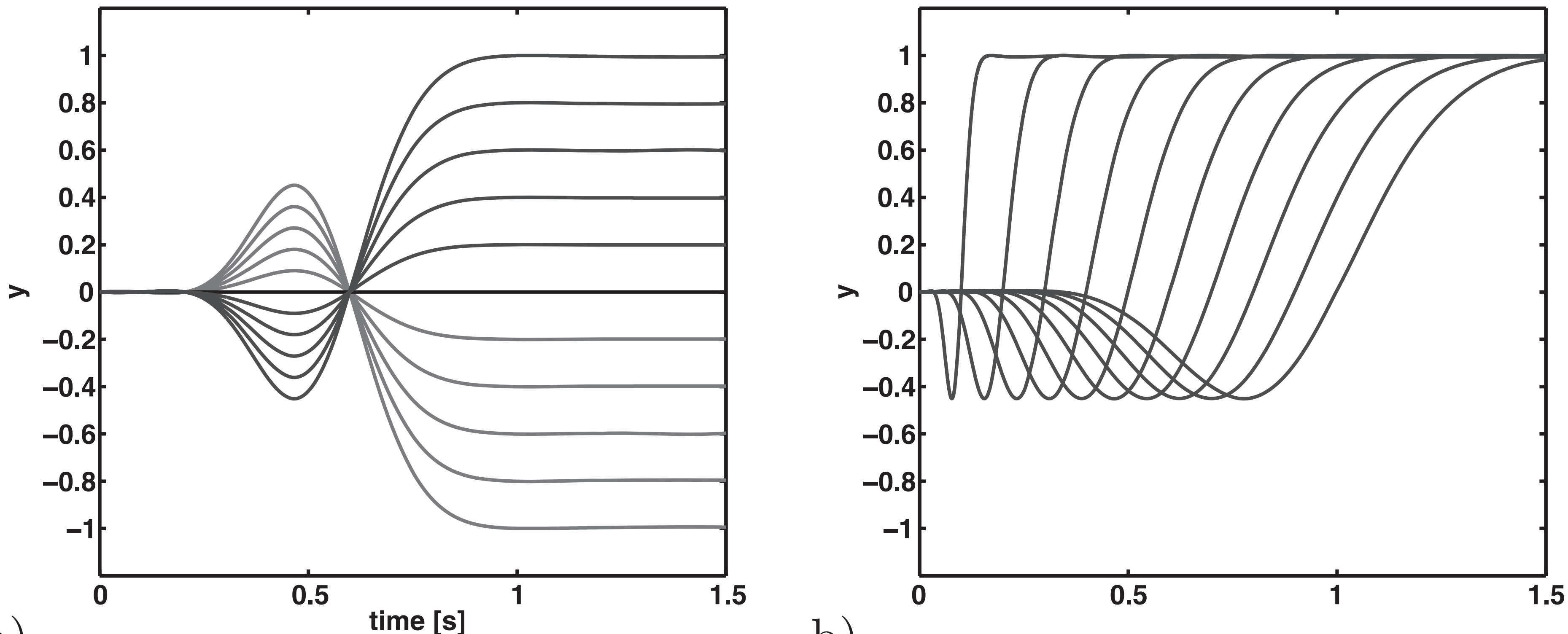
$$\tau\dot{z} = \alpha_z(\beta_z(g - y) - z) + f,$$

$$\tau\dot{y} = z,$$

Example: rhythmic movement



Scaling primitives



a)

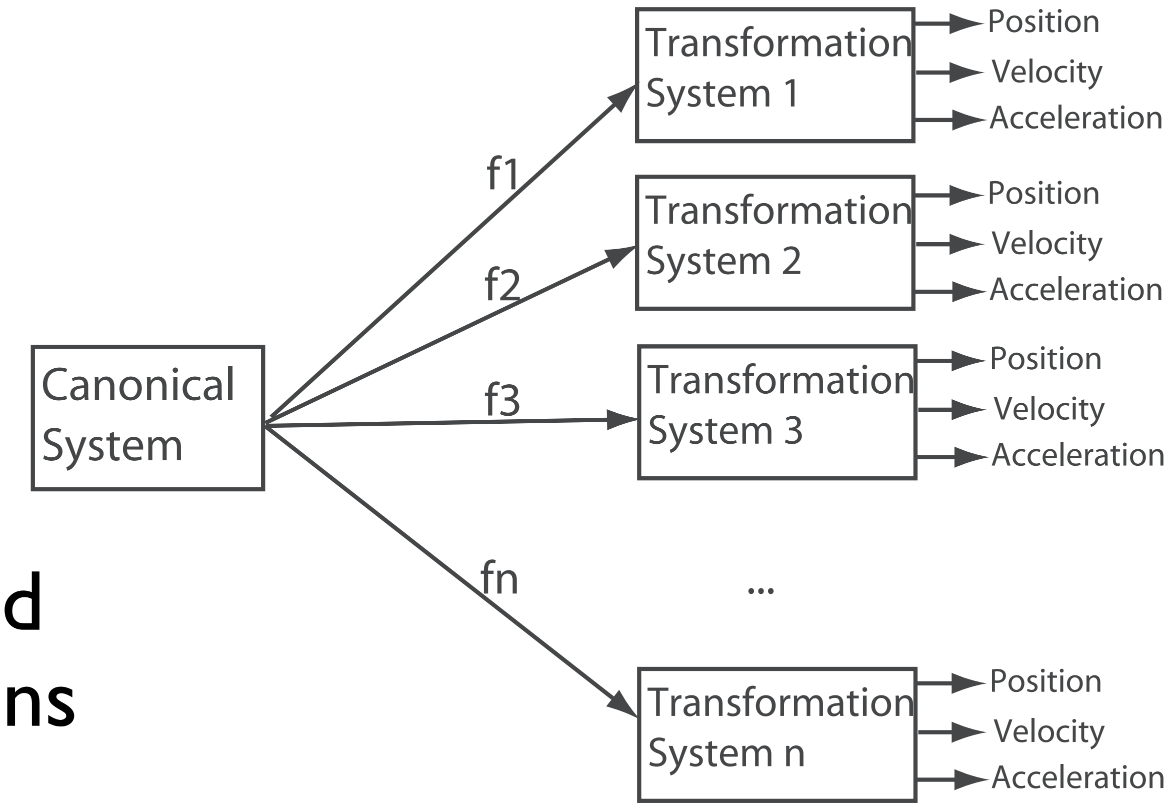
b)

scale in space from -1 to 1

scale time from 0.15 to 1.7
but: not trivially right

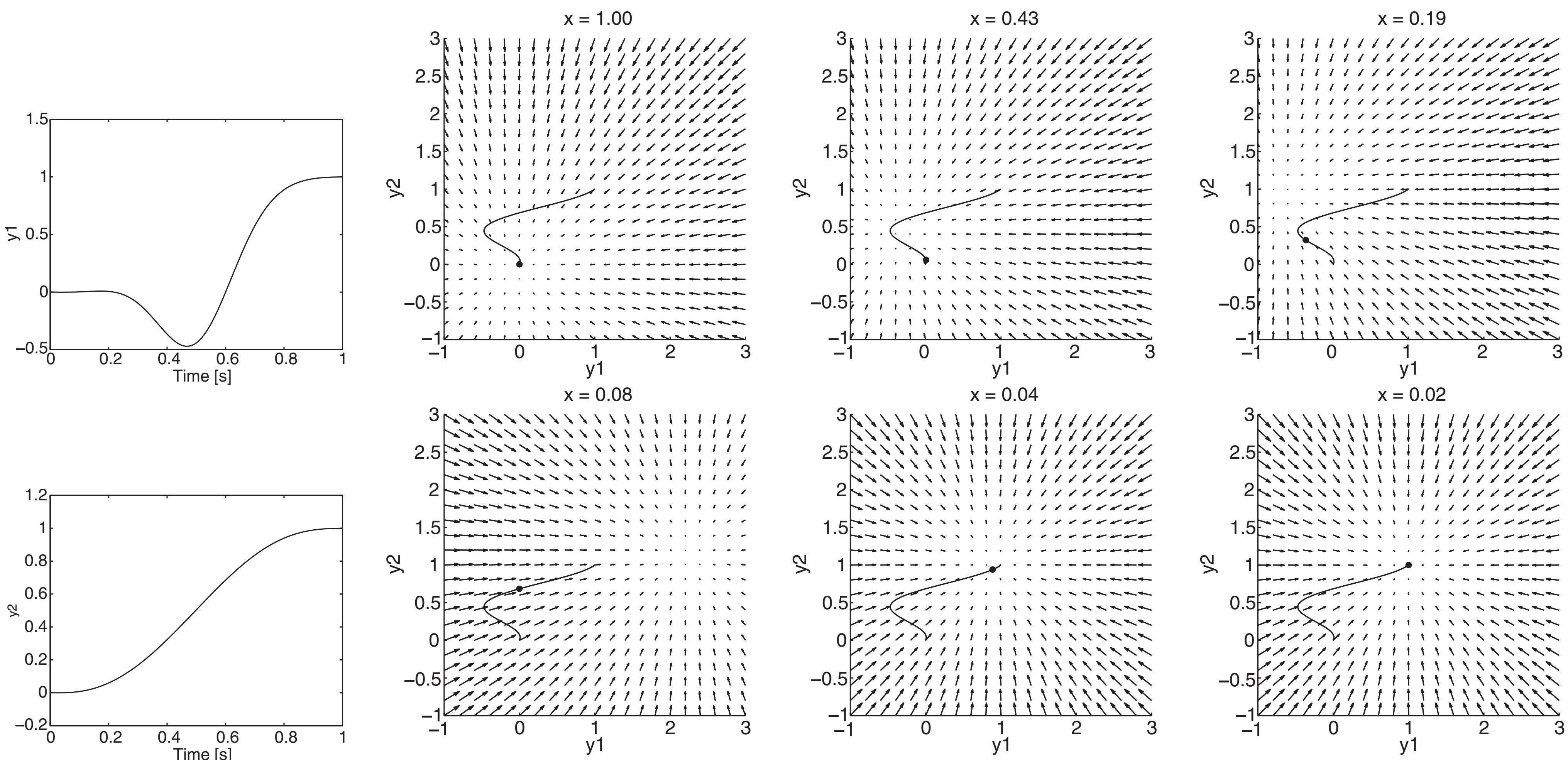
Multi-dimensional trajectories

- rather than couple multiple movement generator (deemed “complicated”)…
- only one central harmonic oscillator and multiple transformations of that…



Example 2D

- single “phase” x
- two base oscillator systems y1, y2
- with two sets of forcing functions



Learning the weights

- base oscillator

$$[\ \tau\ddot{y} = \alpha_z(\beta_z(g - y) - \dot{y}) + f,\ ]$$

$$f_{target} = \tau^2\ddot{y}_{demo} - \alpha_z(\beta_z(g - y_{demo}) - \tau\dot{y}_{demo}).$$

- forcing function from sample trajectory

$$[\quad f(x) = \frac{\sum_{i=1}^{N} \Psi_i(x) w_i}{\sum_{i=1}^{N} \Psi_i(x)} x(g - y_0) \quad]$$

- weights by minimizing error J

$$J_i = \sum_{t=1}^{P} \Psi_i(t)(f_{target}(t) - w_i \xi(t))^2,$$

$$\xi(t) = x(t)(g - y_0)$$ for discrete mov

$$\xi(t) = r$$ for rhythmic mov

Learning the weights

- can be solved analytically

minimum of

$$J_i = \sum_{t=1}^{P} \Psi_i(t)\left(f_{target}(t) - w_i \xi(t)\right)^2,$$

$\xi(t) = x(t)(g - y_0)$

$\xi(t) = r$

is

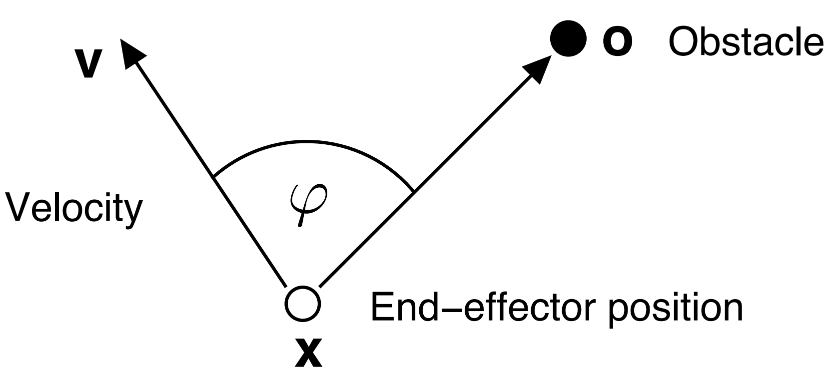
$$w_i = \frac{\mathbf{s}^T \boldsymbol{\Gamma}_i \mathbf{f}_{target}}{\mathbf{s}^T \boldsymbol{\Gamma}_i \mathbf{s}},$$

where (P=# sample times in demo trajectories):

$$\mathbf{s} = \begin{pmatrix} \xi(1) \\ \xi(2) \\ \dots \\ \xi(P) \end{pmatrix} \quad \boldsymbol{\Gamma}_i = \begin{pmatrix} \Psi_i(1) & & & 0 \\ & \Psi_i(2) & & \\ & & \dots & \\ 0 & & & \Psi_i(P) \end{pmatrix} \quad \mathbf{f}_{target} = \begin{pmatrix} f_{target}(1) \\ f_{target}(2) \\ \dots \\ f_{target}(P) \end{pmatrix}$$

Obstacle avoidance

- inspired by Schöner/ Dose (in Fajen Warren form)
- obstacle avoidance force-let



$$\tau \dot{z} = \alpha_z(\beta_z(g - y) - z) + f + C_t,$$

$$\tau \dot{y} = z.$$

$$\mathbf{C}_t = \gamma \mathbf{R}\dot{\mathbf{y}}\, \theta \exp(-\beta\theta),$$

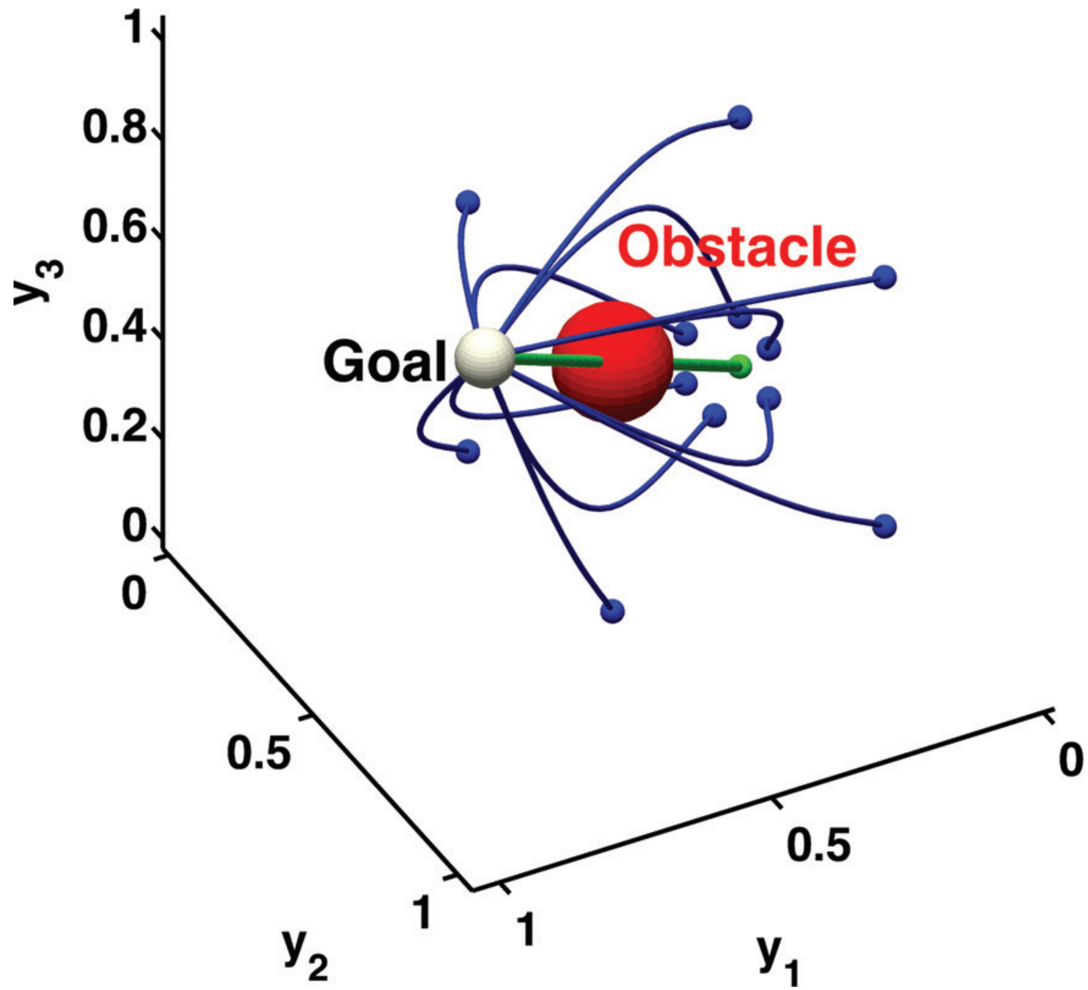
where

$$\theta = \arccos\left(\frac{(\mathbf{o} - \mathbf{y})^T \dot{\mathbf{y}}}{|\mathbf{o} - \mathbf{y}||\dot{\mathbf{y}}|}\right),$$

$$\mathbf{r} = (\mathbf{o} - \mathbf{y}) \times \dot{\mathbf{y}}.$$

[actually this is: Reimann, Iossifidis, Schöner, 2010]

Obstacle avoidance

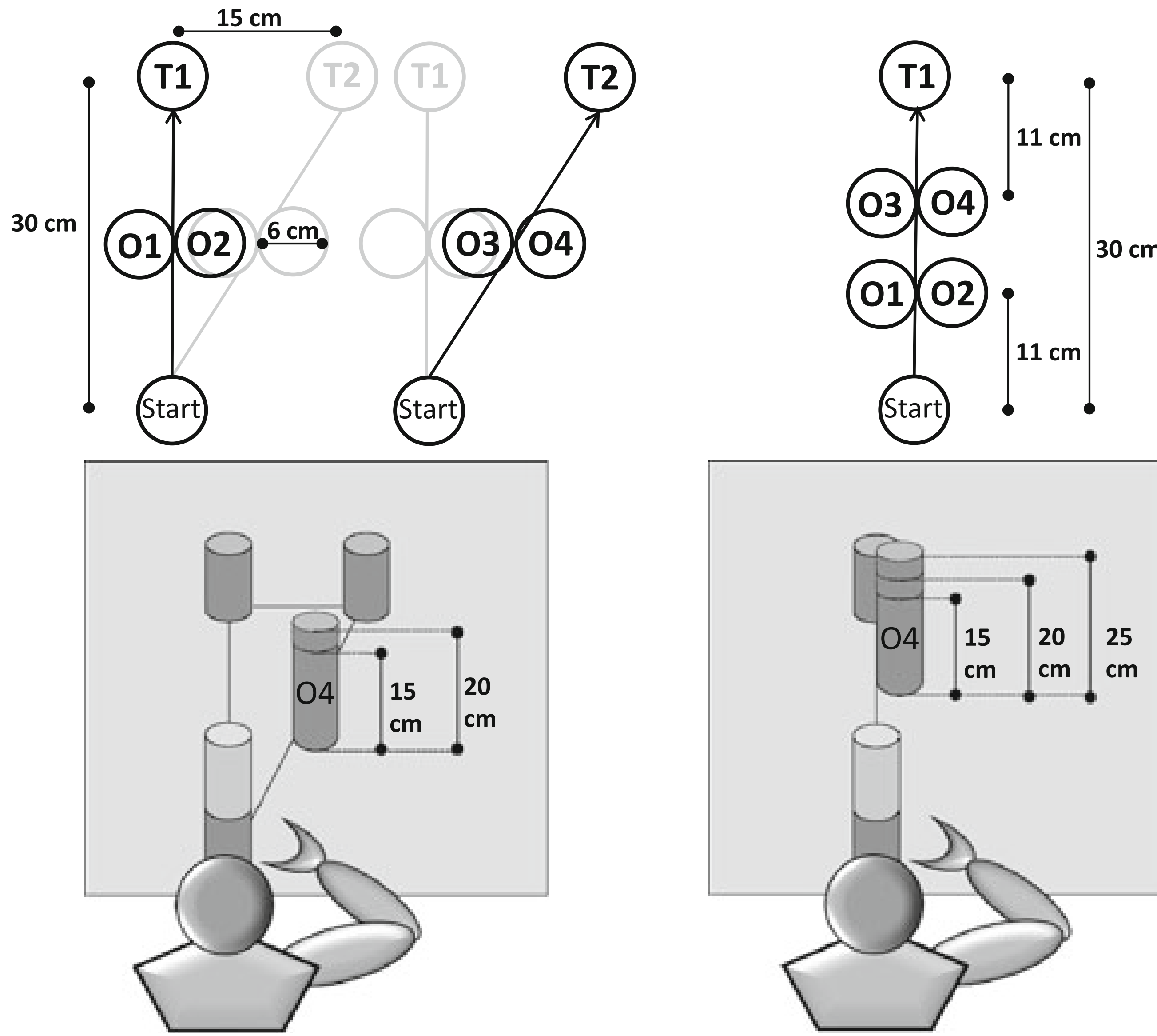


But: human obstacle avoidance is not really like that…

- => Grimme, Lipinski, Schöner, 2012

Experiment

- naturalistic movements: hand moving objects to targets while avoiding obstacles
- spatial arrangement of obstacles is varied…
- may that apparent complexity of movements emerge from simple invariant elementary movements?



[Grimme, Lipinski, Schöner, EBR 2012]

paths

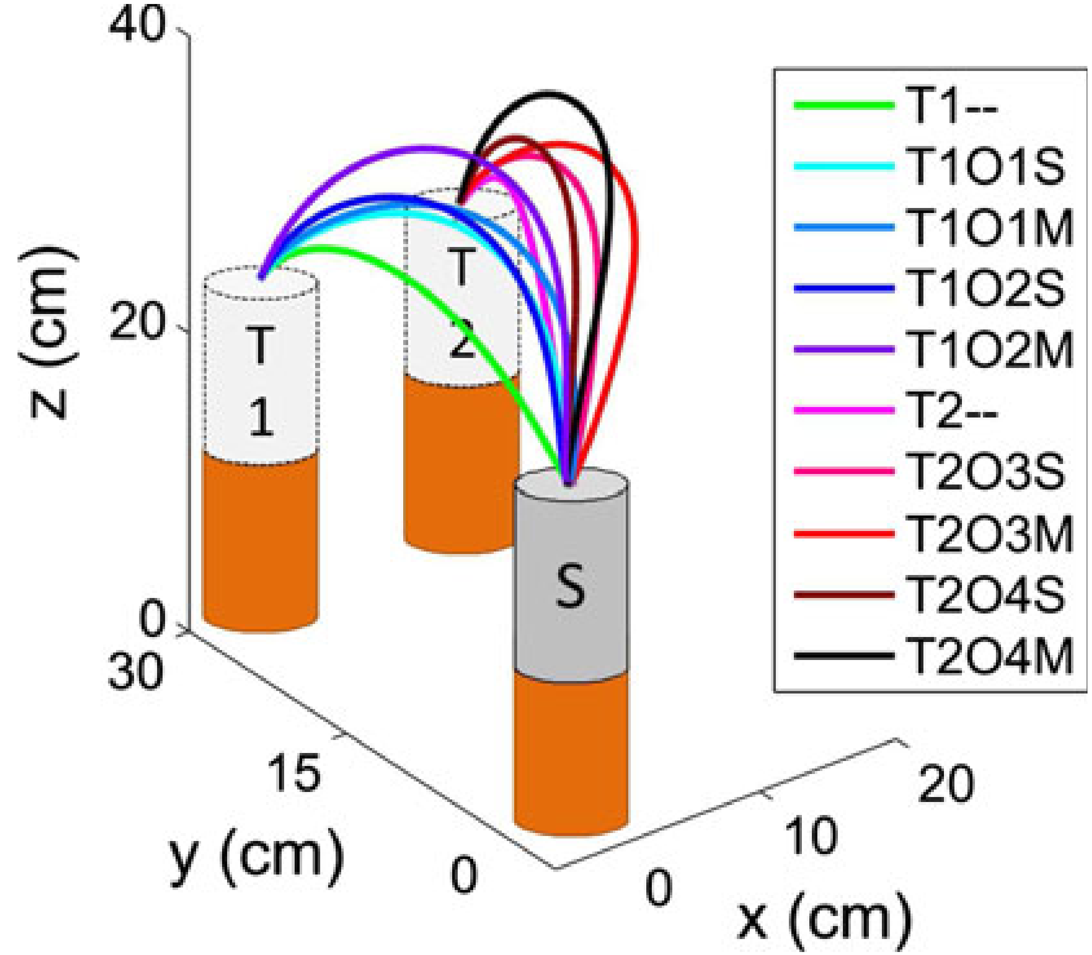
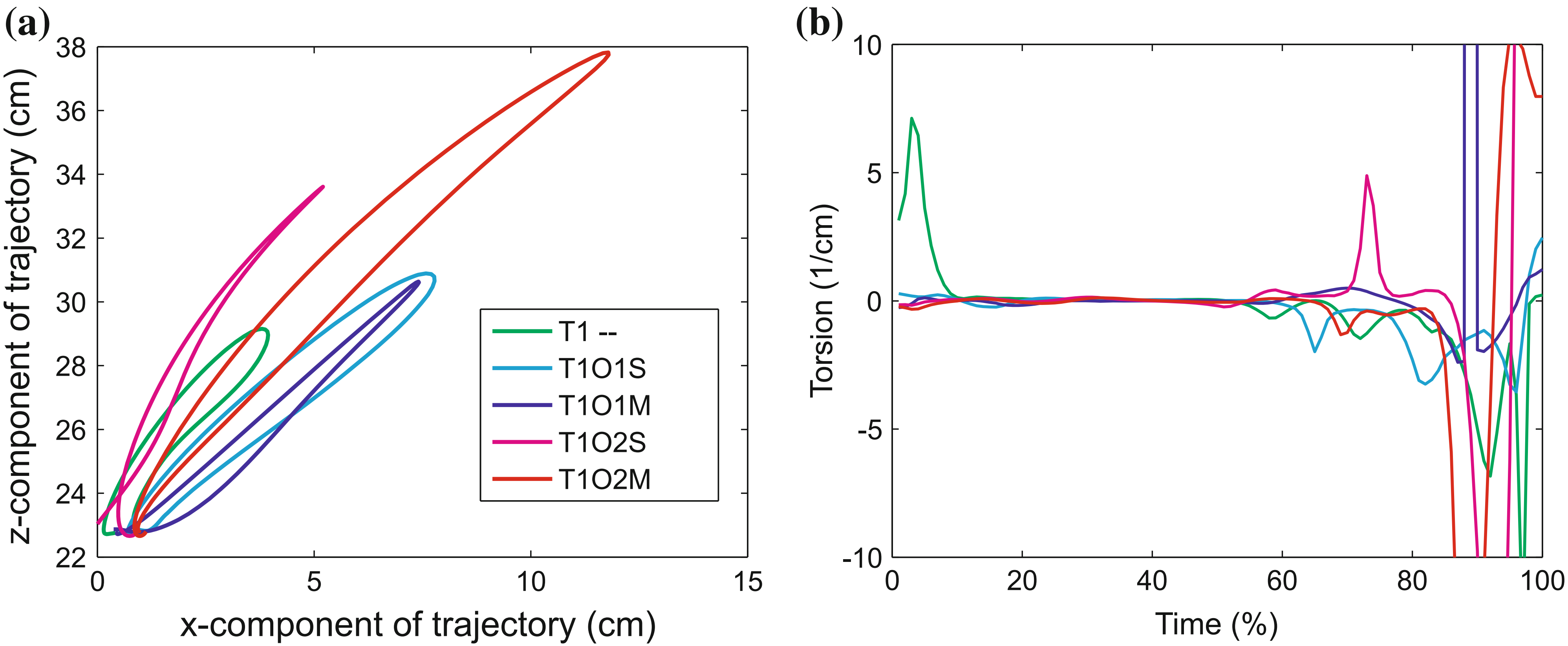
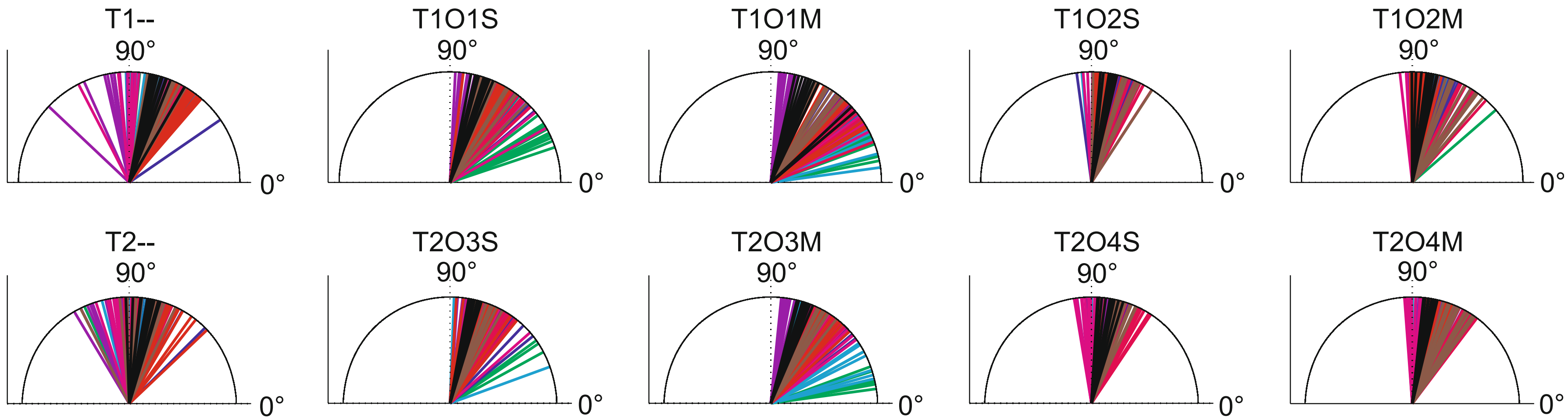


Fig. 3 Mean (over all participants) 3D obstacle avoidance paths from the starting position (*S*) to both target positions (*T1* and *T2*)

paths are planar

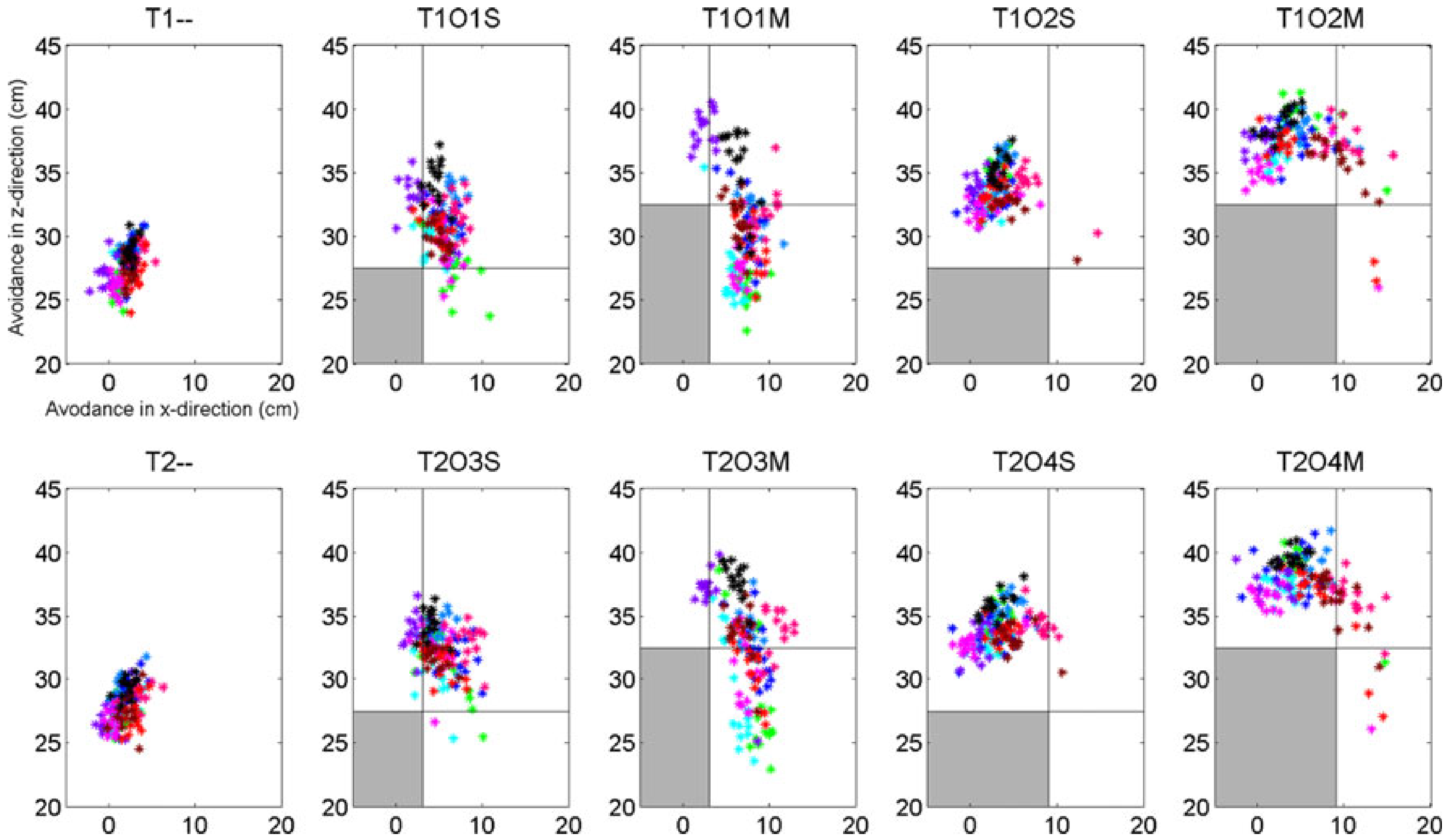


the plane of movement depends on the obstacle height



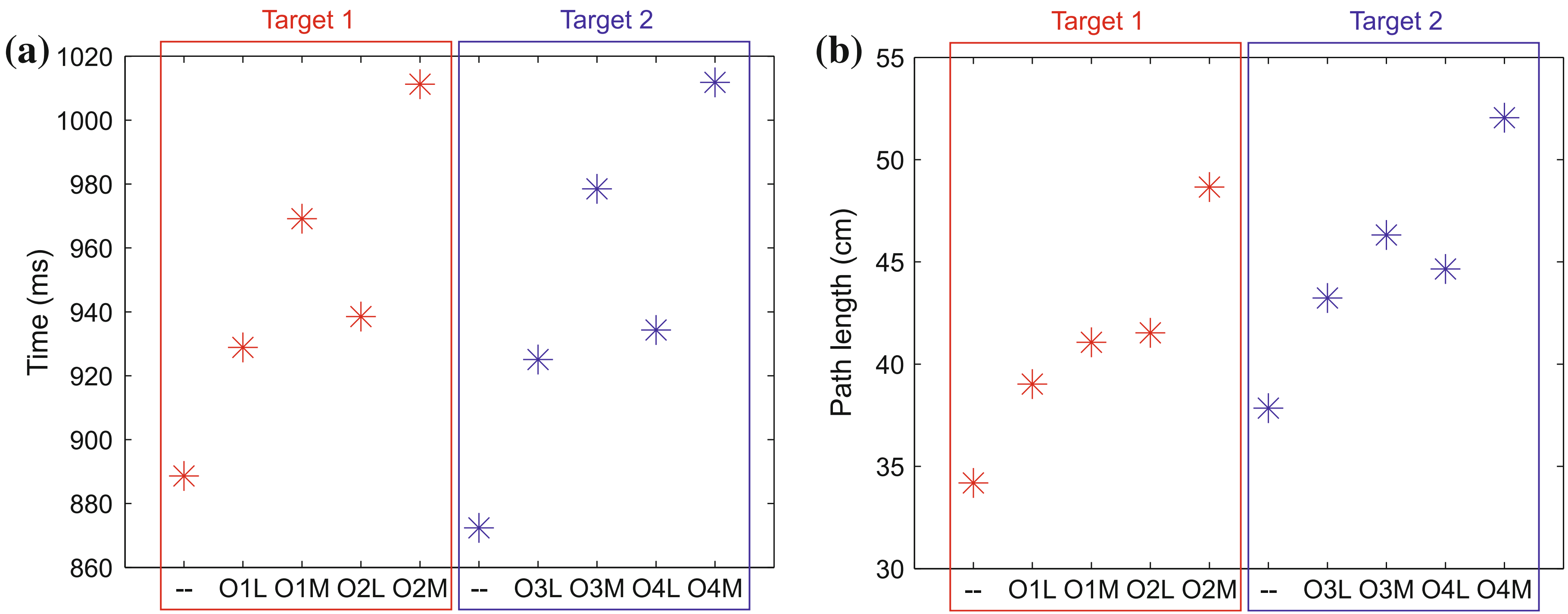
colors: participants…

the plane of movement depends on the obstacle height



colors: participants…

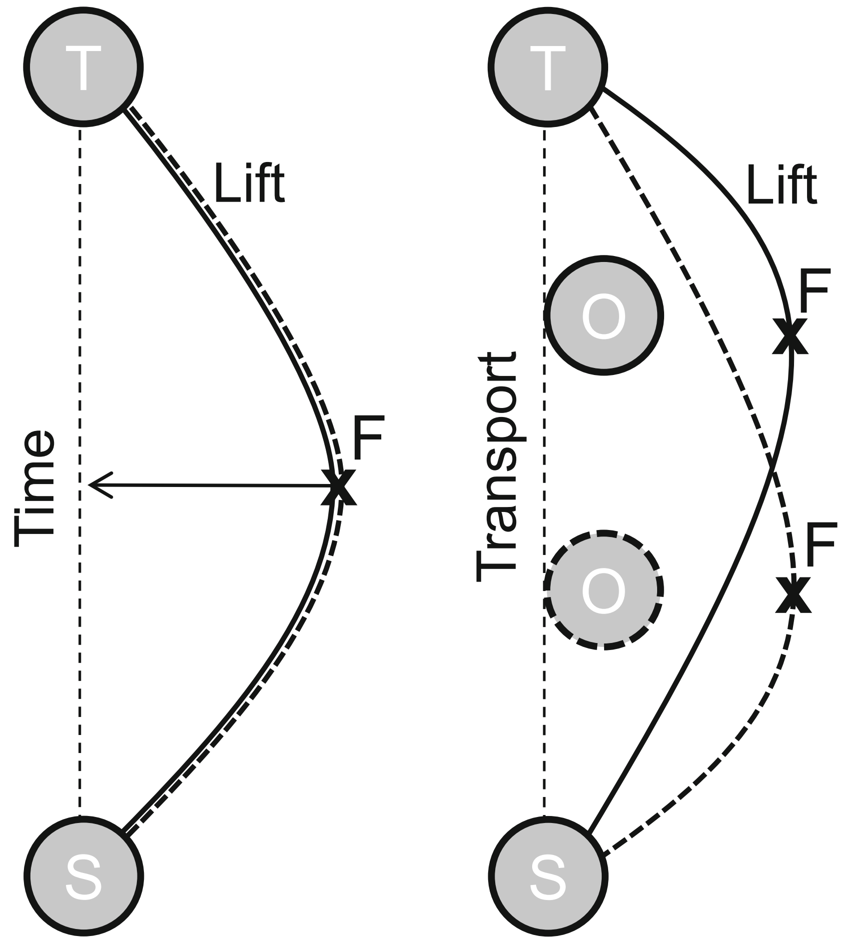
trajectories are isochronous



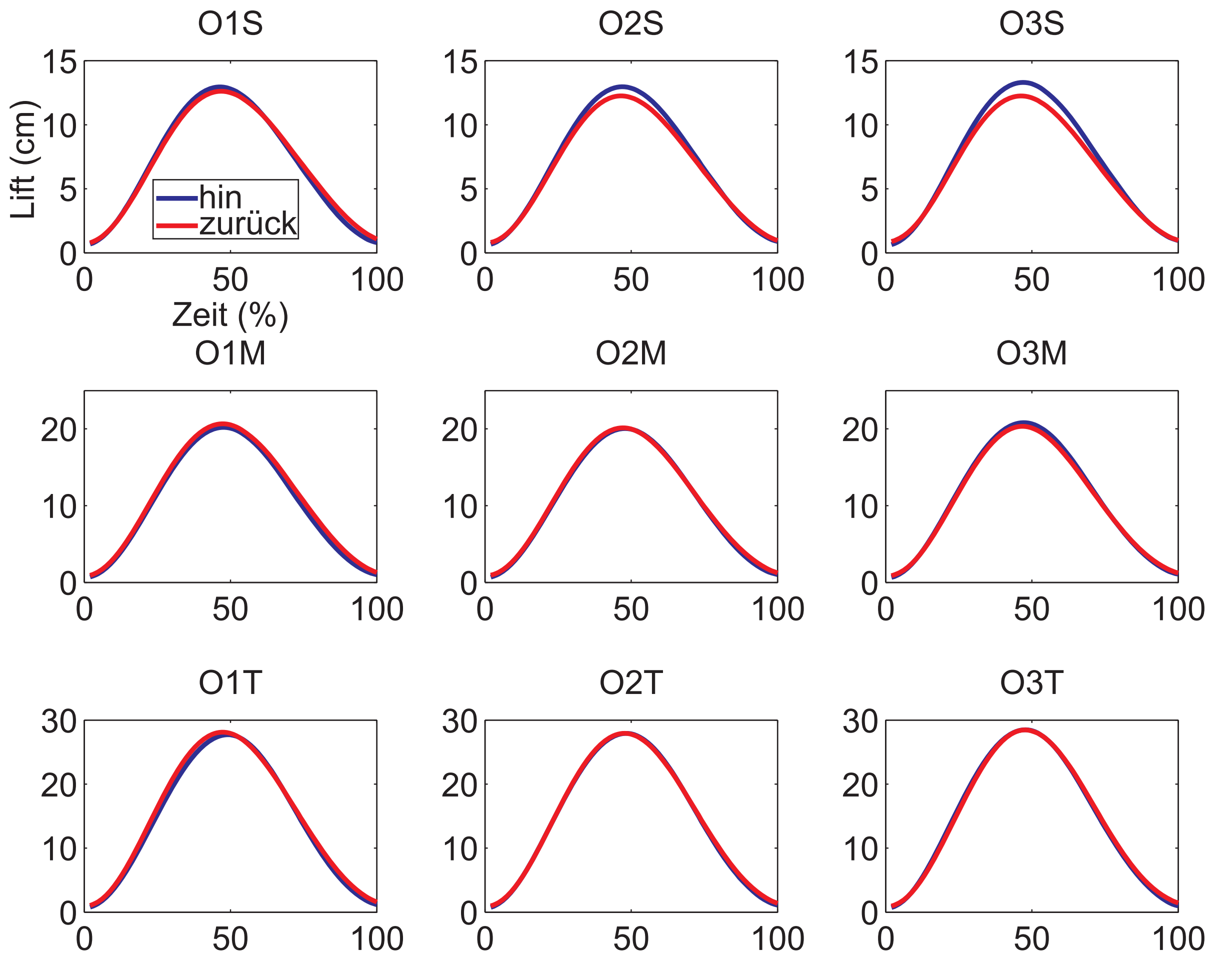
same movement time

different path length

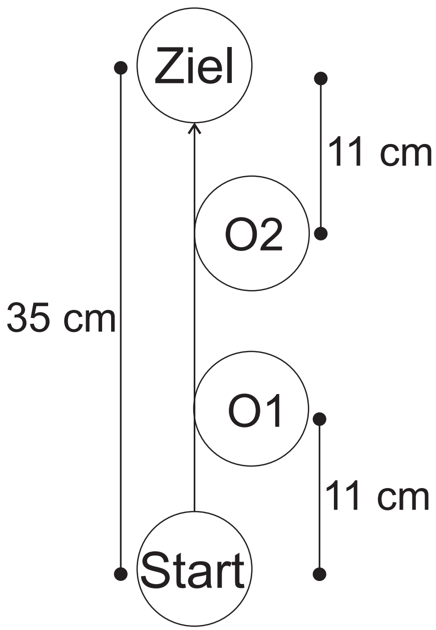
local isochrony

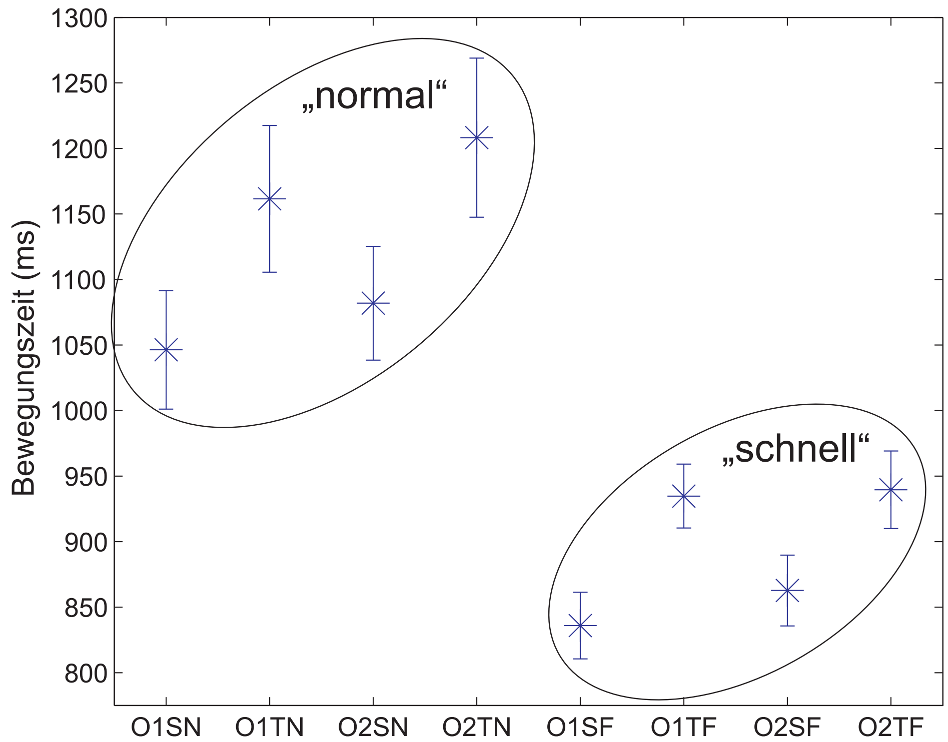


invariance of lift across space

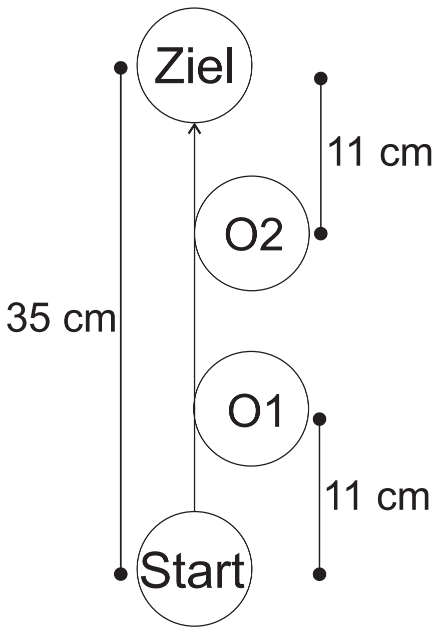


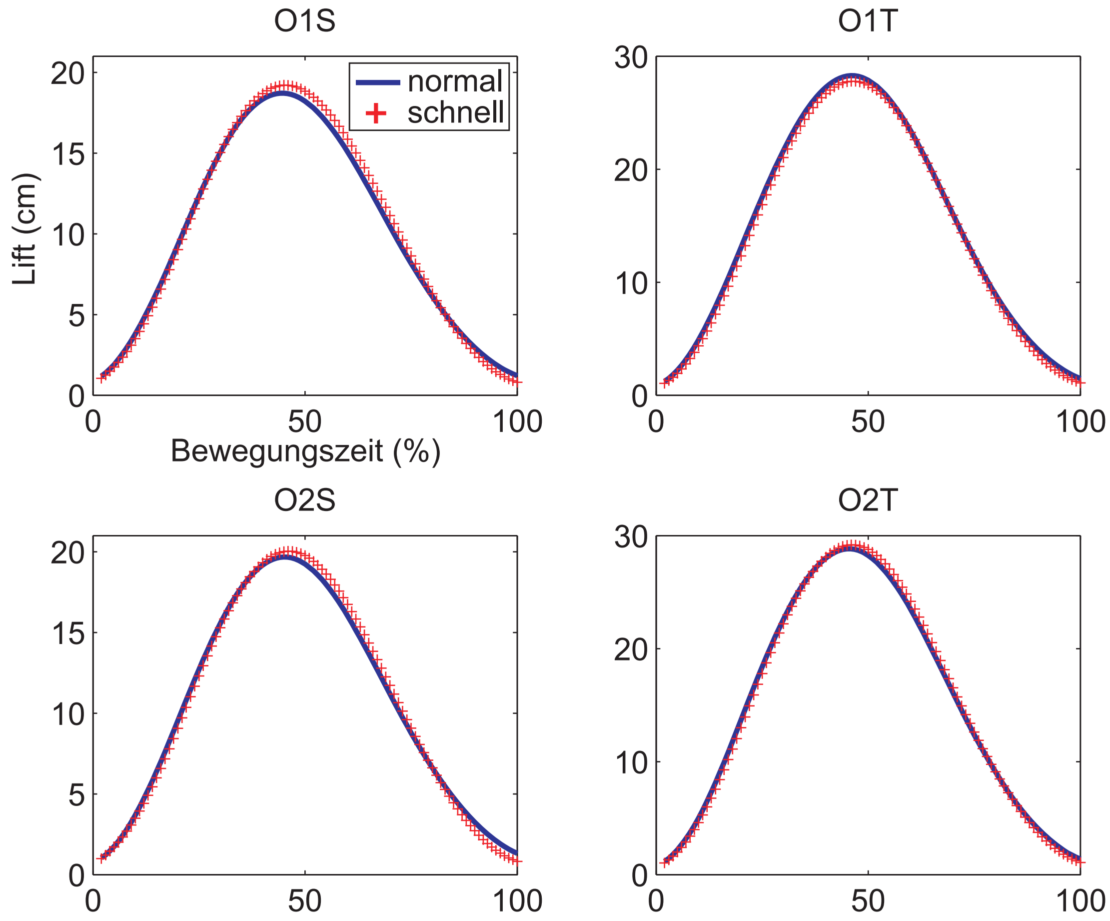
scaling with movement time





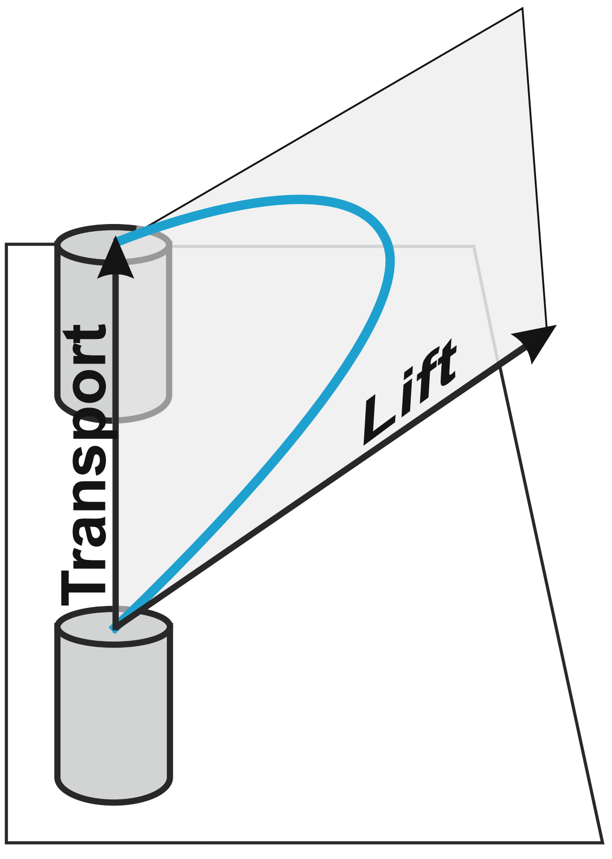
scaling with movement time



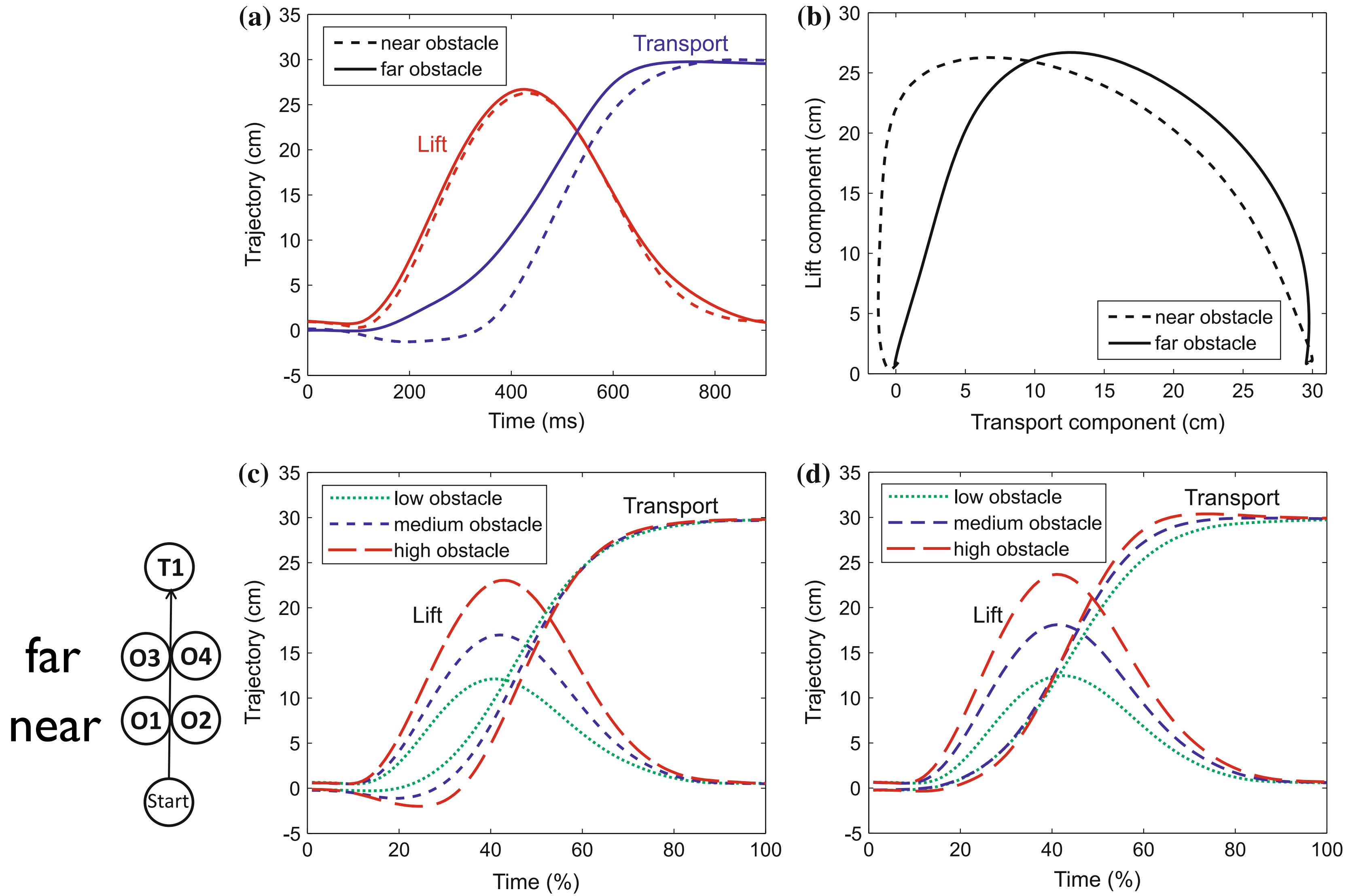


elementary behaviors

- based on planarity
- decompose movement into transport and lift component
- => a different sense of "primitives"…
  - not to span learning data/fitting movement
  - but to express different tasks/ constraints…



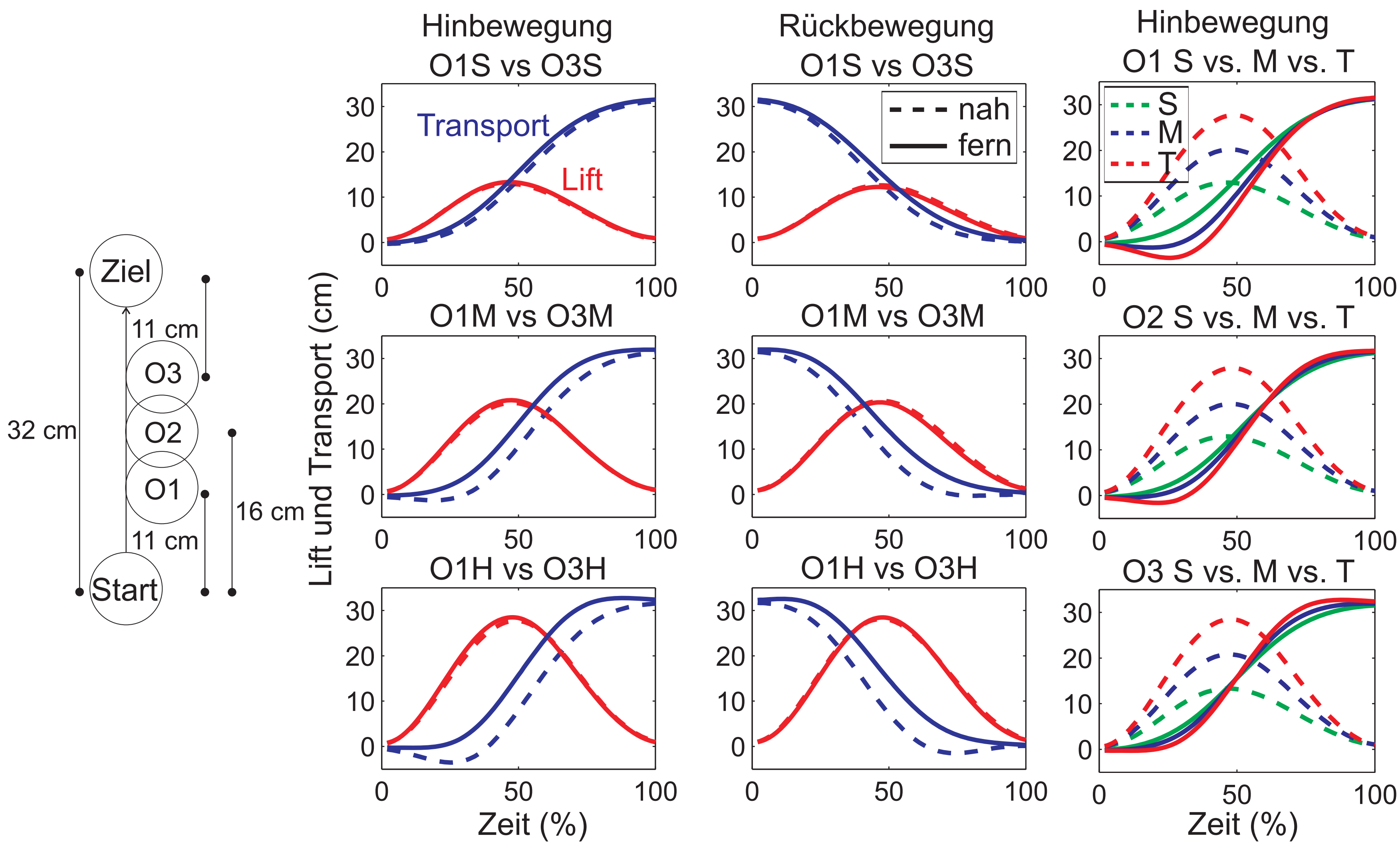
lift vs. transport



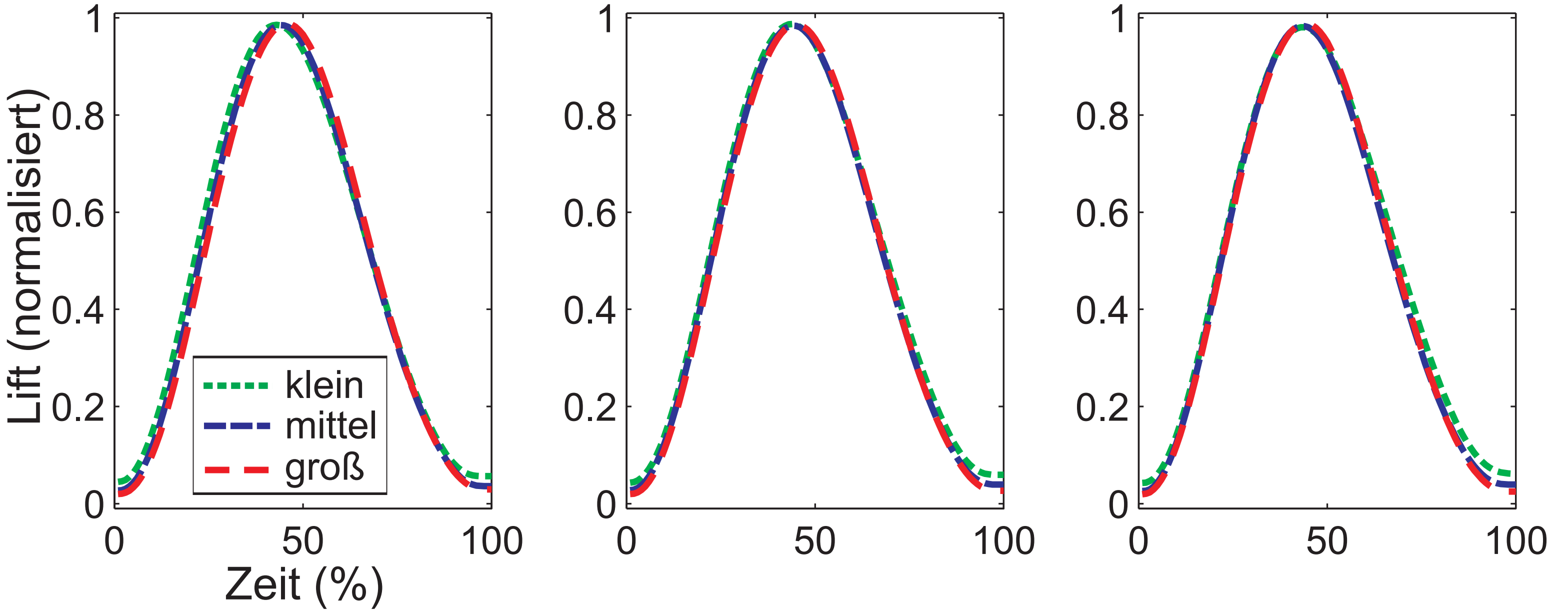
near

far

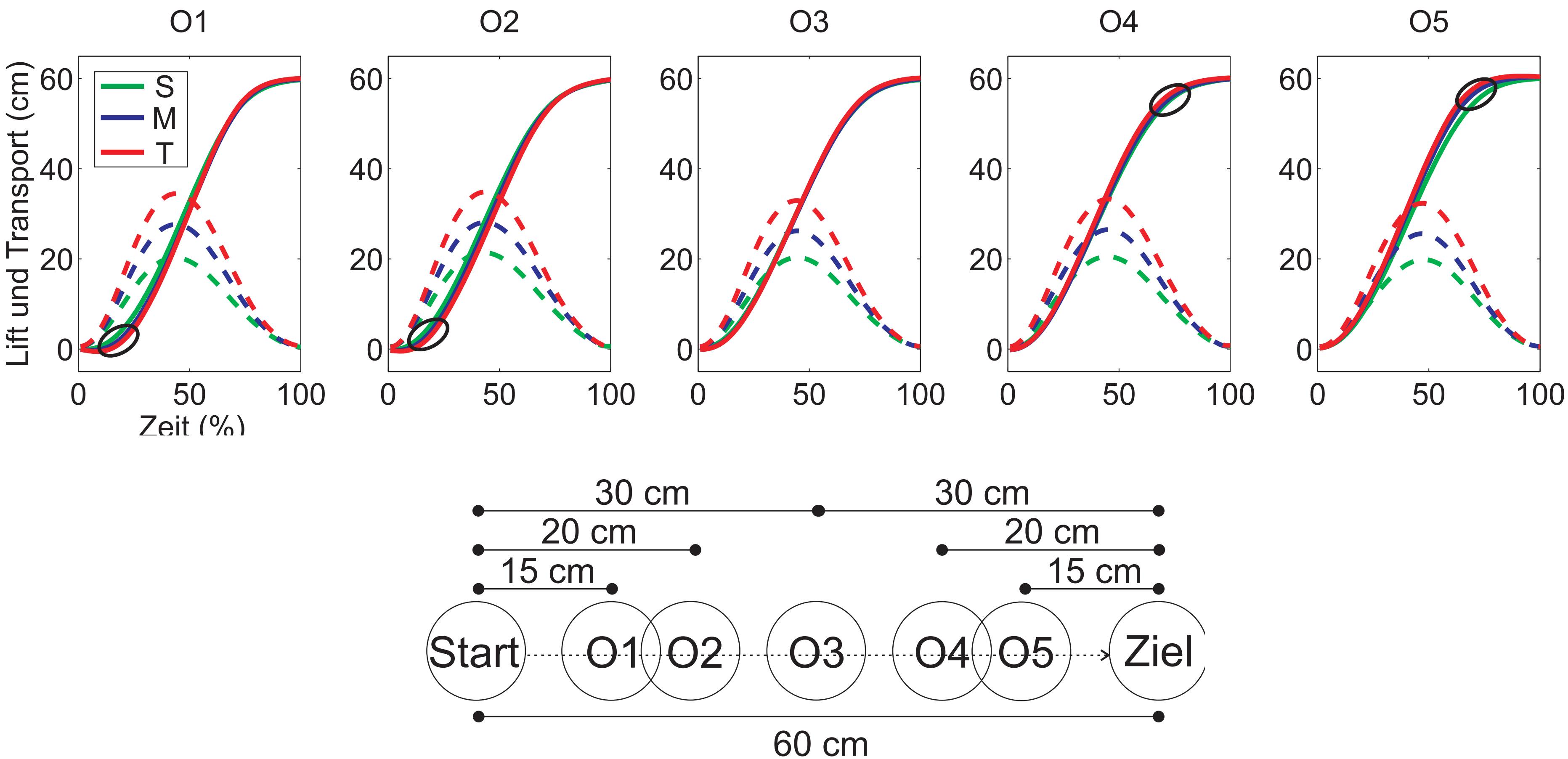
lift vs. transport



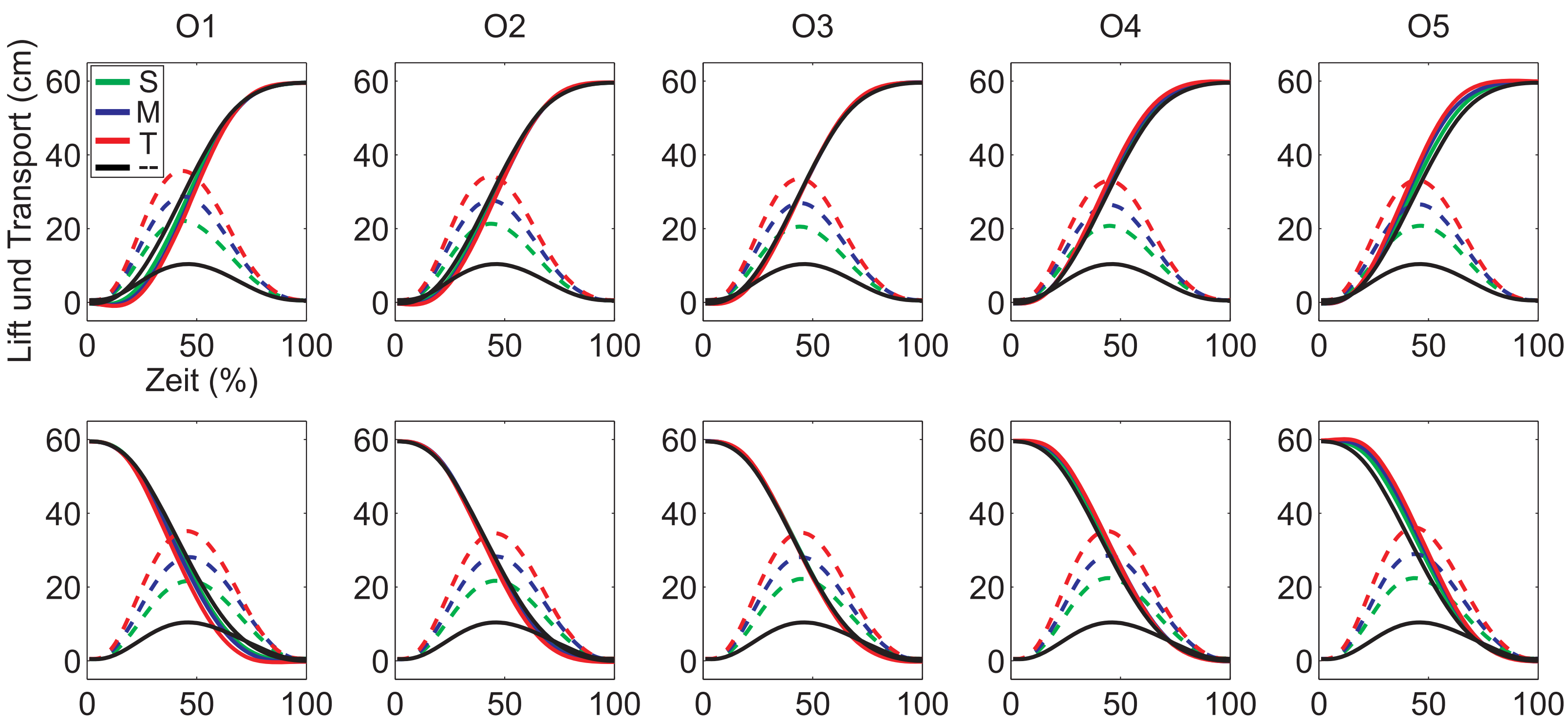
scaling lift to amplitude and time

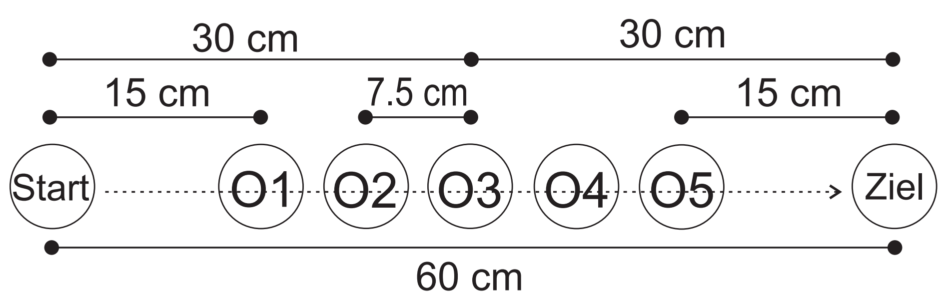


lift vs. transport



lift vs. transport

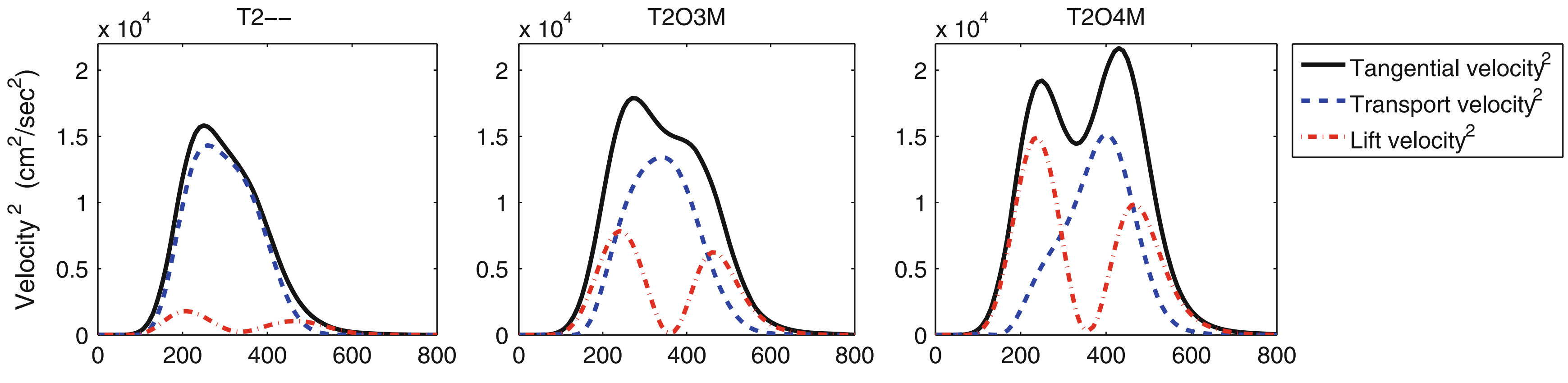


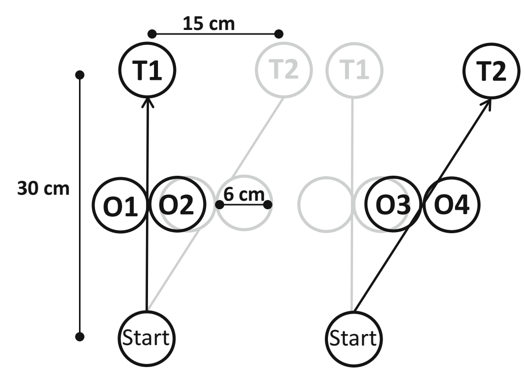


lift vs. transport

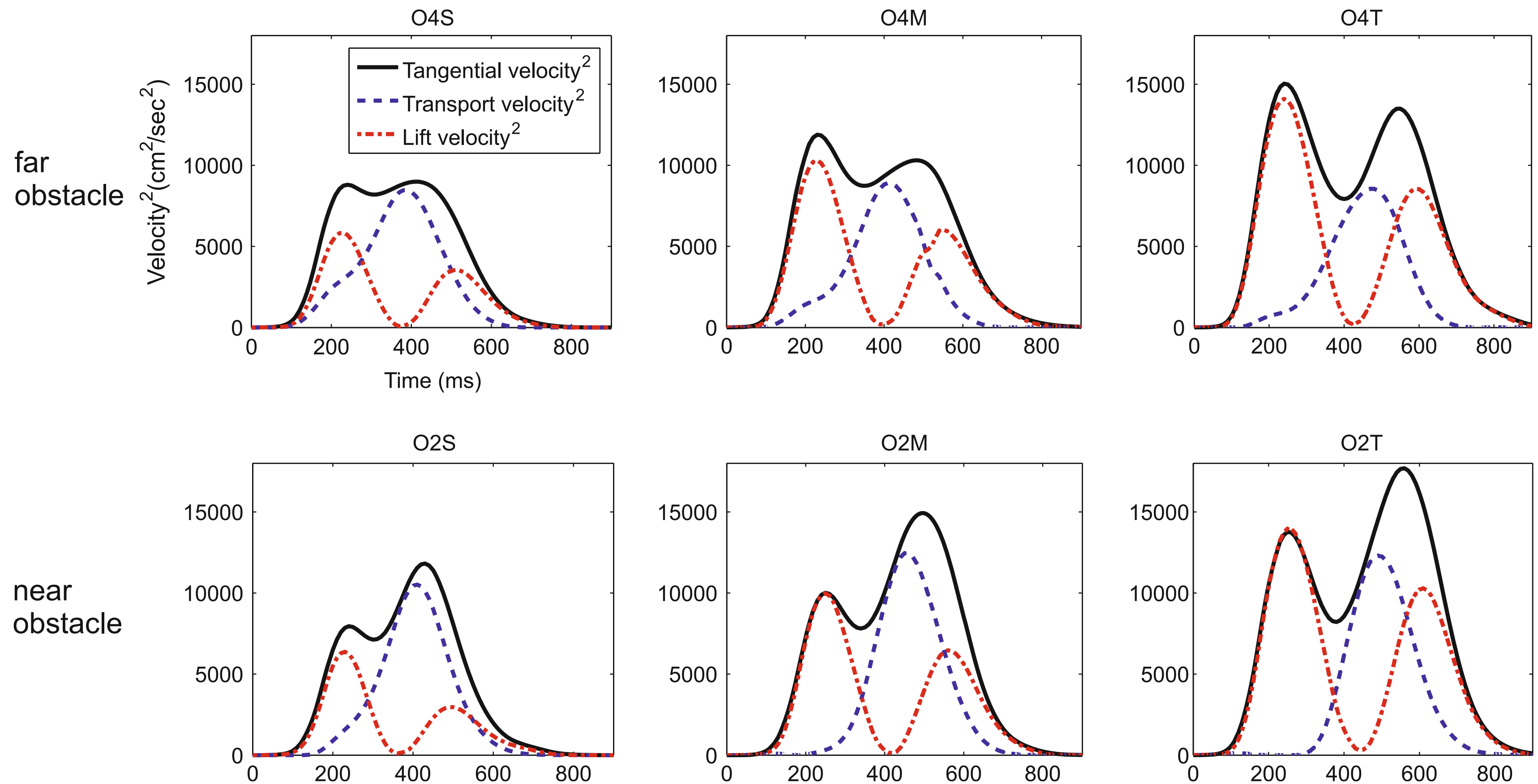
- invariance of lift under location of obstacle along transport
- approximate invariance of transport under height of obstacle
  - exact if obstacle is symmetrically half-way between start and target position of transport

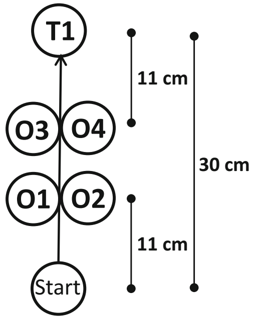
complexity from simple "primitives"



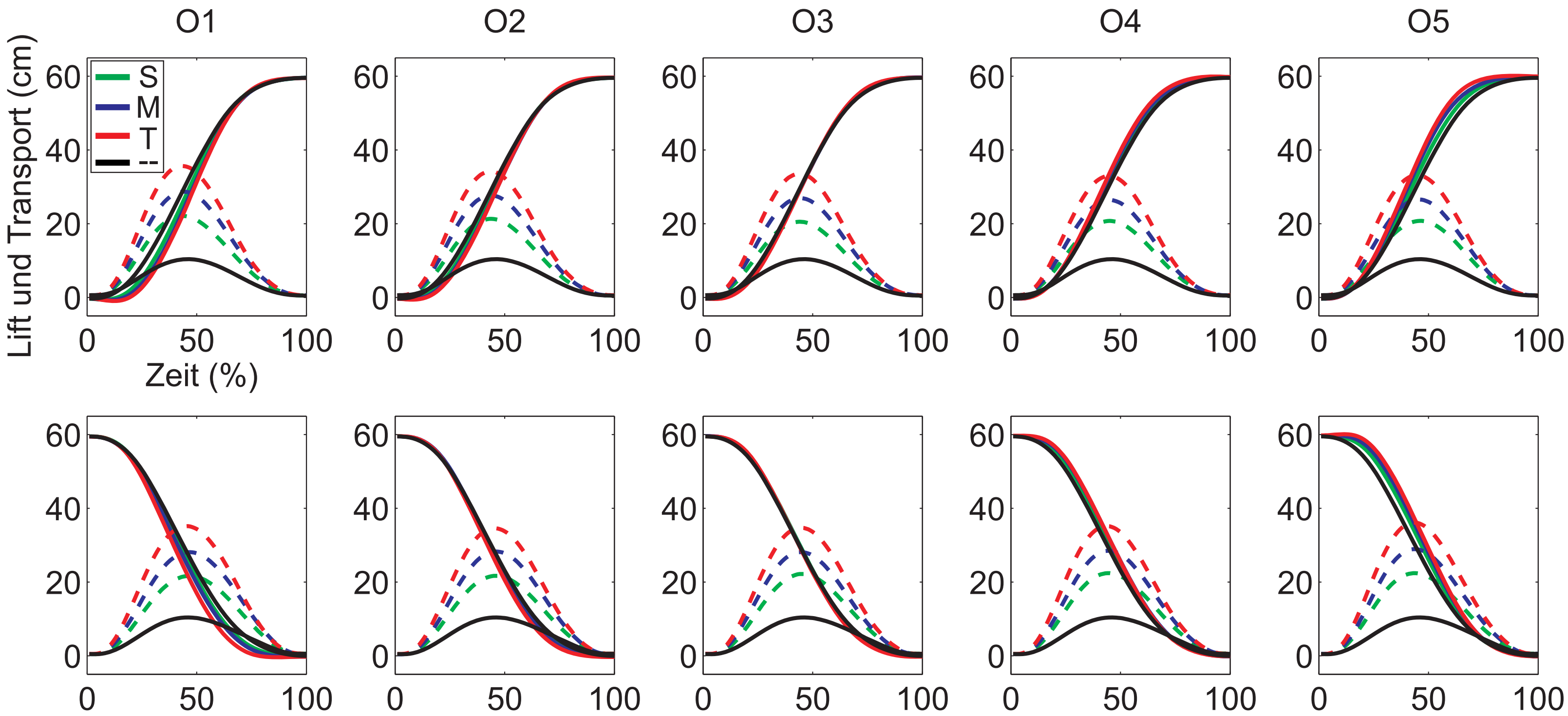


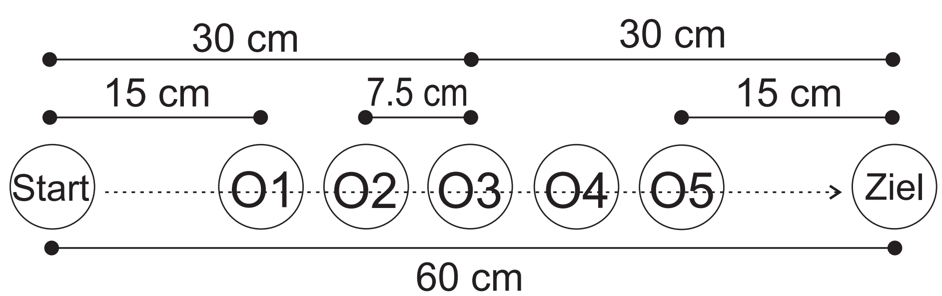
complexity from simple primitives





obstacle component





DMP and obstacle avoidance

- true nature of human movement
  - piecewise planar
  - isochronous
  - modulated in time to accomodate obstacle avoidance
- are not structural features of DMP

Coordination

- in phase dynamics: couple to external timers…
- but: issue of predicting such events and aligning the prediction to achieve synchronicity…

$$\tau \dot{x} = -\alpha_x x + C_c$$

$$\tau \dot{\phi} = 1 + C_c.$$

$$C_c = \alpha_c (\phi_{ext} - \phi).$$

Coordination

- coupling to spatial variables…
- … unclear what is new over classical work… much remains open..

Conclusion

- Simple DMP approach enables learning "movement styles" while imposing movement amplitude.. enabling generalization to new movement targets
  - it isn't clear how DMPs impose other constraints
  - obstacle avoidance by the end-effect…
  - obstacle avoidance by other parts of the effector? (solved in Reimann, Iossifidis, Schöner, 2010 etc.)
  - collision avoidance with a surface, avoidance only on the side of the arm.. etc..? (solved in Iossifidis, Schöner, 2004)

Conclusion

- DMP is a purely kinematic account
  - that includes kinematic constraints in very simple form
- => DMP has nothing to do with actual force-fields, that is, with how movement is physically generated!
  - DMP is not part of control

Conclusion

- DMP’s capacity to address timing is limited
  - movement time is not very well defined
  - the base oscillator is not a stable limit cycle oscillator, so the issue of decoupling timing from space is not addressed
  - DMP’s account for coordination is limited/not new
  - timing dimension of obstacle avoidance is not captured

Conclusion

- task dependence of primitives is not part of DMP's framework
  - => DMPs doe not account how different task with associated primitives are combined and integrated
- a different notion of primitives: elementary behaviors
  - e.g. lift and transport