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Autonomous robotics

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Exercise 3

This is about the degree of freedom problem. We will make a "toy" version of that. Consider an arm that only moves within a plane. Assume the arm has two joints (a "shoulder" and an "elbow"). You want to control the tip of the arm (the "hand"), but only along the horizontal axis, x. You do not care about the axis, y, that points away from the body.

1. Make a drawing of the arm with coordinate axes and mark two joint angles, θ_1 and θ_2. Introduce two parameters, l_1 and l_2, for the length of the upper and the lower arm, respectively.

2. Write down the equation that determines where the horizontal position position of the hand is as a function of the two joint angles (you can adapt this from the lecture, but you can also figure this out yourself).

3. Draw two or more examples of arm configurations that have the same x value. How does this relate to "self-motion" within the "uncontrolled manifold" (UCM)? Try to guess the UCM based on the few examples you drew and sketch that guess that as a line in the space of the two joint angles (roughly).

4. Compute the UCM by solving for one of the two joint angles in that equation. For very value of x, you have a different UCM. [Hint: introduce a "segment angle" $\psi_2 = \theta_2 + \theta_1$ and solve the x-equation of the kinematics for ψ_2 as a function of x and θ_1, then replace again ψ_2 and solve for θ_2 as a function of x and θ_1.]

5. Make a drawing of that manifold (approximate) as a line in the space of the two joint angles. [Hint: It might be easiest to do this numerically... or you could take the core function, plot it, and then step-wise apply the remaining functions to that...]. Compare to your guess in number 3.

(These are ordered in increasing level of difficulty.)