The degree of freedom problem

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Spaces for robotic motion planning

- Task level planning is about end-effector pose in space (e.g., 3 translational and 3 rotational degrees of freedom)
- Configuration space planning: joint angles of actuated degrees of freedom
Forward kinematics

where is the hand, given the joint angles..

\[ x = f(\theta) \]

\[ x = l_1 \cos(\theta_1) + l_2 \cos(\theta_1 + \theta_2) \]

\[ y = l_1 \sin(\theta_1) + l_2 \sin(\theta_1 + \theta_2) \]
Differential forward kinematics

where is the hand moving, given the joint angles and velocities

\[ \mathbf{x} = \mathbf{J}(\theta) \dot{\theta} \]

\[ \dot{x} = -l_1 \sin(\theta_1) \dot{\theta}_1 - l_2 \sin(\theta_1 + \theta_2) \dot{\theta}_1 - l_2 \sin(\theta_1 + \theta_2) \dot{\theta}_2 \]

\[ \dot{y} = l_1 \cos(\theta_1) \dot{\theta}_1 + l_2 \cos(\theta_1 + \theta_2) \dot{\theta}_1 + l_2 \cos(\theta_1 + \theta_2) \dot{\theta}_2 \]
where is the hand moving, given the joint angles and velocities

\[ \dot{x} = J(\theta) \dot{\theta} \]

\[
\begin{pmatrix}
\dot{x} \\
\dot{y}
\end{pmatrix}
= 
\begin{pmatrix}
-l_1 \cos(\theta_1) - l_1 \cos(\theta_1 + \theta_2) & -l_2 \cos(\theta_1 + \theta_2) \\
l_1 \sin(\theta_1) + l_2 \sin(\theta_1 + \theta_2) & l_2 \sin(\theta_1 + \theta_2)
\end{pmatrix} \cdot 
\begin{pmatrix}
\dot{\theta}_1 \\
\dot{\theta}_2
\end{pmatrix}
\]
Inverse kinematics

- What joint angles are needed to put the hand at a given location

- Exact solution:

\[ \theta = f^{-1}(x) \]
Inverse kinematics

\[ \theta_1 = \arctan_2(y, x) \pm \beta \]

\[ \theta_2 = \pi \pm \alpha \]

\[ \alpha = \cos^{-1} \left( \frac{l_1^2 + l_2^2 - r^2}{2l_1l_2} \right) \]

\[ \beta = \cos^{-1} \left( \frac{r^2 + l_1^2 - l_2^2}{2l_1l_2} \right) \]

where \[ r^2 = x^2 + y^2 \]

[thanks to Jean-Stéphane Jokeit]
Differential inverse kinematics

which joint velocities to move the hand in a particular way

\[ \dot{\theta} = J^{-1}(\theta) \dot{x} \]

with the inverse of

\[ J(\theta) = \begin{pmatrix} -l_1 \cos(\theta_1) - l_1 \cos(\theta_1 + \theta_2) & -l_2 \cos(\theta_1 + \theta_2) \\ l_1 \sin(\theta_1) + l_2 \sin(\theta_1 + \theta_2) & l_2 \sin(\theta_1 + \theta_2) \end{pmatrix} \]

if it exists!
Spaces for robotic motion planning

kinematic model

\[ x = f(\theta) \]
\[ \dot{x} = J(\theta) \dot{\theta} \]

inverse kinematic model

\[ \theta = f^{-1}(x) \]
\[ \dot{\theta} = J^{-1}(\theta) \dot{x} \]

- transform end-effector to configuration space through inverse kinematics
- problems of singularities and multiple “leafs” of inverse...
Redundant kinematics

Redundant arms/tasks: more joints than task-level degrees of freedom

\[ x = l_1 \cos(\theta_1) + l_2 \cos(\theta_1 + \theta_2) + l_3 \cos(\theta_1 + \theta_2 + \theta_3) \]
\[ y = l_2 \sin(\theta_1) + l_2 \sin(\theta_1 + \theta_2) + l_3 \sin(\theta_1 + \theta_2 + \theta_3) \]
Redundant kinematics

=> (continuously) many inverse solutions…
Redundant kinematics

Use pseudo-inverses that minimize a functional (e.g., total joint velocity or total momentum)

\[
\begin{align*}
\dot{x} &= J(\theta) \dot{\theta} \\
\dot{\theta} &= J^+(\theta) \dot{x} \\
J^+(\theta) &= J^T (J J^T)^{-1}
\end{align*}
\]
or use extra degrees of freedom for additional tasks

[lossifidis, Schöner, ICRA 2004]
what is a DoF?

- variable that can be independently varied
- e.g. joint angles

muscles/muscle groups

- but: assess to which extent they can be activated independently...
- .. mode picture

Degree of freedom problem in human movement

\[ x = l_1 \cos(\theta_1) + l_2 \cos(\theta_1 + \theta_2) + l_3 \cos(\theta_1 + \theta_2 + \theta_3) \]

\[ y = l_2 \sin(\theta_1) + l_2 \sin(\theta_1 + \theta_2) + l_3 \sin(\theta_1 + \theta_2 + \theta_3) \]
for most tasks, there are many more degrees of freedom than task constraints…

e.g., 10 joints in the upper arm including scapular joints to control hand position and orientation (3 to 5 or 6 DoF)

but typically more: involve upper trunk movements

or even make a step to move

many muscles per joint (e.g. about 750 muscles in the human body vs. about 50 DoF)
Degree of freedom problem in human movement

- Nikolai Bernstein… 1930’s… in the Soviet Union
- "how to harness the many DoF to achieve the task"
Bernstein’s workers

highly skilled workers wielding a hammer to hit a nail... => hammer trajectory in space less variable than body configuration

as detected in superposing spatial trajectories of lights on hammer vs. on body.

but: camera frame anchored to nail/space, while initial body configuration varied
Bernstein’s workers

- Was the hammer position in space less variable than the joint configuration?
  - That is, does the task structure variance?
  - So that the solution to the degree of freedom problem lies in the variance/stability of the joint configuration?

- But: does this make any sense?
  - Different reference frames for body vs. task
  - Different units in the task vs joint space
Classical synergy concept

- The task-level motor commands ‘x’ activate synergies = groups of DoF through a forward neural network
Classical synergy concept

Motor command varies in time or across tasks => covariation of these muscle activations / DoF movements

leads to co-variation here

variation here
Classical synergy research strategy

- identify distinct synergies with the hope of finding a limited set => “the” synergies that explain multi-degree of freedom movement

- combine the time series of muscles/DoF under different conditions (sometimes including repetitions of movements) into one big data set and look for structure (e.g. principal components)

- if a small number of PC’s is sufficient to account for most of the variance, conclude that few synergies at work
Synergy: experimental use

E.g, Safavynia, Ting, 2012:

- SF Muscle Synergies
  - \( N_{\text{syn-S}} \)
  - \( W_1 \)
  - \( W_2 \)
  - \( W_3 \)
  - \( W_4 \)
  - \( W_5 \)
  - \( W_6 \)

- TF Muscle Synergies
  - \( N_{\text{syn-T}} \)
  - \( C_1 \)
  - \( C_2 \)
  - \( C_3 \)
  - \( C_4 \)
  - \( C_5 \)
  - \( C_6 \)

Varying Recruitment (C)

Fixed Weightings (W)

EMG

- Flexors
- Biarticular
- Extensors

J Neurophysiol • www.jn.org 2008, 2009. CoM kinematics are task-level variables that must transform based on feedback control of center of mass (CoM) motion (Lockhart and Ting 2007; Welch and Ting 2008, 2009). Moreover, the model can explain temporal patterns of muscle activity that vary with perturbation. In locomotion, a few temporal patterns can be recruited across step cycles to reproduce electromyographic (EMG) patterns. However, it may not be possible to combine is desired throughout a task in a feedback or feedforward manner. Here the nervous system organizes muscle activity spatially. The nervous system can variably recruit SF muscle synergies when a specific muscle combination is desired throughout a task in a feedback or feedforward manner.

Recent evidence suggests that low-dimensional temporal patterns of muscle activity in postural perturbations may be used to recruit SF muscle synergies. For example, fixed-duration temporal pulses are sufficient to elicit a range of postural configurations (Chvatal et al. 2011; McKay et al. 2005; Ivanenko et al. 2004). However, it may not be possible to vary across directions and trials is chosen to reproduce EMG activity sequences to recruit muscles during a task, consistent with feedforward system can variably recruit SF muscle synergies when a specific muscle recruitment throughout anterior-posterior (A-P) perturbations. We model based on CoM kinematics to reconstruct muscle synergy structure and recruitment in 10-ms bins out perturbation responses. To test this hypothesis, we examined the ability of the feedback model to reconstruct muscle synergy activity and EMG reconstructions. We then compared SF muscle synergy combinations of muscle activity and CoM kinematics compared with the initial postural response. We explicitly compared SF with TF muscle synergies on their ability to reconstruct EMG.
Classical synergy: critique of method

... no invariant set of synergies has emerged

confounds time, movement conditions, and trials

PCs are informative primarily about the geometry of the end-effector path.
and its variation with task

[Steele, Tresch, Perreault: J Neurophysiol 2015]
Classical synergy: critique of concept

- The variance across repetitions for a given task at a given point in time = signature of stability
- That variance is structured in the OPPOSITE way than predicted!
Classical synergy: critique of concept

motor commands

DoF/muscles

random variation here

leads to co-variation here

random variance here: uncorrelated
The many DoF are coordinated such that changes that affect the task-relevant dimensions are resisted against more than changes that do not affect task relevant dimension leading to compensation.

[Scholz, Schöner, EBR 126:289 (99)]
align trials in time

hypothesis about task variable

compute null-space (tangent to the UCM)

predict more variance within null space than perpendicular to it
supplement hypothesis testing by checking for correlation (Hermann, Sternad...)

look for increase in variance of task variable when correlation within data is destroyed
Example 1: pointing with 10 DoF arm at targets in 3D

task variable: hand movement direction in space

UCM
orthog UCM
Example 2: shooting with 7 DoF arm at targets in 3D

[from Scholz, Schöner, Latash: EBR 135:382 (2000)]
Example 2: shooting with 7 DoF arm at targets in 3D

[from Scholz, Schöner, Latash: EBR 135:382 (2000)]
Synergy: critique of concept

Motor commands

DoF/muscles

Variation here

Variance induced here: is uncorrelated

Leads to co-variation here
UCM synergy: decoupling

motor commands

insert a perturbation here

compensatory change here

arm in space
i.e. along the task-equivalent manifold, but not in other joint space that do not a...that joint angle variance increases in those directions of A distinct feature is that for most joints, variance in-
be due to time-normalization as discussed previously. ing that some of the mid-movement variability might variance (for movement-extent, see Figure 5), suggest-
with a distinctive peak of variance in mid-movement. Many joint variance profiles are roughly bell shaped, participants. Some features emerge as common traits.

Fig. 8 (Movement 6).

Fig. 7

Approach to the UCM clavicular

Variance (m

Sterno-

0.02

0.04

3.1 Experimental protocol

relations, we set particular terms to zero to probe their role.

noise sources were selectively set to zero to demonstrate magnitude of the resultant variance. In some simulations, adjusted by hand to achieve appropriate orders of mag-

The only parameters adjusted to fit data in this pa-

configuration evolve in the presence of noise according

initial end- vector is subjected to noise (see Equation 3). (2) (11)

Time (s)

End-vector position and joint configuration across

References in joint trajectory variability across

was considered, but found to have minimal influence on

state-dependence of noise sources (Harris and Wolpert,

and

where

ψ

⇣

⇣

⌘

⌘

0.5 rad

·

·

⌘

⌘

0.4

0.3

0.2

0.1

End-vector variance obtained from simulation when

End-vector variance,

End-e
t

²

(figures 3).

Time (s)

Extents

Subject 1

Subject 2

Subject 3

Model

Variance (rad²)

Normalized Time

Normalized Time

Normalized Time

Normalized Time

sterno-clavicular

shoulder

elbow

wrist

Martin, Reimann, Schöner, 2018]
Val` ere Martin et al.

Subject 1 Subject 2 Subject 3 Model

Movement 1 Movement 2 Movement 3 Movement 4

Variance/DoF (rad$^2$)

<table>
<thead>
<tr>
<th>Subject 1</th>
<th>Subject 2</th>
<th>Subject 3</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>V$</td>
<td></td>
<td>$</td>
<td>V$_\perp$</td>
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<tr>
<td>Movement 1</td>
<td>Movement 2</td>
<td>Movement 3</td>
<td>Movement 4</td>
</tr>
</tbody>
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Below we will use this analysis to study the causes of the structure of variance in the model.

4.5 What causes the UCM effect?

The model enables us to identify possible causes of the UCM structure of variance by varying model compo-
model

biomechanical dynamics

\[ M(\theta) \cdot \ddot{\theta} + H(\theta, \dot{\theta}) = T_m \]

muscle models

\[ T_i = K_l \cdot \left( (e^{K_{nl}(\theta_i - \lambda_i^p)} + 1) - (e^{-K_{nl}(\theta_i - \lambda_i^m)} - 1) \right) \]

\[ + \mu_{bl} \cdot \text{asinh}(\dot{\theta}_i - \dot{\lambda}_i) + \mu_{rl} \cdot \dot{\theta}_i. \]
neural dynamics of lambda

\[ \ddot{\lambda} = (J^+ E) \cdot \left( \begin{array}{c} -\beta_v J \cdot \dot{\lambda} + \beta_v u - J \cdot \dot{\lambda} \\ -\beta_{s1} E^T \cdot (\lambda - \theta_d) - \beta_{s2} E^T \cdot (\dot{\lambda} - \dot{\theta}_d) - E^T \cdot \dot{\lambda} \end{array} \right) \]

\[ \dot{v} = -\beta_v (v - u(t)), \]

\[ v(t) = J[\lambda(t)] \cdot \dot{\lambda}(t), \]

back-coupling

timing signal
This hypothesis says that when the estimated real joint configuration, $\theta_d$, deviates from the virtual joint configuration, $\lambda$, then lead to an update of the virtual joint configuration within the null-space of the Jacobian (brought about by projecting the difference onto the basis vectors of the null-space, $E^T$). The same kind of mechanism may occur at the level of joint velocities (second term). The real joint configuration must be sensed and estimated, leading to processing delays (index $d$; see appendix D for details). This form of back-coupling of the real into the virtual joint configuration dynamics implies both stabilization of the joint configuration within the uncontrolled manifold (through the terms dependent on $\lambda$ and $\dot{\lambda}$) and driving virtual self-motion (when the terms $(\lambda - \theta_d)$ and $(\dot{\lambda} - \dot{\theta}_d)$ are different from zero). The projection of the back-coupling term onto the null-space ensures that the dynamics within the space of self-motion depends on only the components of $\lambda$ and $\dot{\lambda}$ within that subspace, so that the range-space and null-space remain decoupled.

That this neuronal dynamics is a closed description in the space of the virtual joint configuration $\lambda$ and velocity $\dot{\lambda}$ is seen by replacing all references to the end-effector velocity, $v$, and the self-motion velocity, $s$, by virtual joint velocities using equations 2.5 and 2.6:

$$\ddot{\lambda} = (J^+ E) \cdot \begin{pmatrix} -\beta_v J \cdot \dot{\lambda} + \beta_v u \\ 0 \end{pmatrix}$$

To implement the model, the matrices $J(\lambda)$, $E(\lambda)$, $\dot{J}(\lambda)$, and $\dot{E}^T(\lambda)$ are computed analytically.

2.7 Muscle-Joint Model. The virtual joint configuration $\lambda$ and velocity $\dot{\lambda}$ drive the muscle joint systems. These are modeled by reducing a detailed, nonlinear muscle model (Gribble et al., 1998) to its essentials, limiting the number of parameters. First, we fuse all muscles acting onto a given joint into an effective muscle joint model that covers both agonist and antagonist activity. As a result, the descending commands are condensed into the virtual joint angle, $\lambda(t)$, and virtual joint velocity, $\dot{\lambda}(t)$. The state-dependent generation of muscle torques at a given joint, $\mathbf{T}_i(\lambda, \dot{\lambda}, \theta, \dot{\theta})$, can then be characterized by a single function, $\mathbf{T}_i(\lambda, \dot{\lambda}, \theta, \dot{\theta})$, where $\theta(t)$ and $\dot{\theta}(t)$ are the real joint angle and velocity. At rest and in the absence of external forces, the muscle joint system is at equilibrium at $\mathbf{T} = 0$ and $\theta = \lambda$. Depending on the time course of the virtual joint trajectory, $\lambda(t)$ and the biomechanics of the arm, the realized joint trajectory may deviate significantly from the virtual trajectory. This is why taking into account the nonlinear dependence of muscle force generation on muscle state is important (Gribble et al., 1998).
where does this come from?

start with pseudo-inverse of: \( v = J\dot{\lambda} \)

\[
\dot{\lambda} = J^+ v \\
\ddot{\lambda} = J^+ \dot{v} \quad [ + J^+ v \approx 0 ]
\]

a neuron, \( n \), encoding rate of change of \( \lambda \):

\[
\dot{n} = J^+ \dot{v} \quad \text{<= insert timing signal} \quad \dot{v} = -v + u \\
\dot{n} = J^+ (-v + u) \quad \text{<= insert} \quad v = J\dot{\lambda} \\
\dot{n} = J^+ (-J\dot{\lambda} + u) \quad \text{<= replace} \quad n = \dot{\lambda} \\
\dot{n} = J^+ (-Jn + u) \\
\dot{n} = -J^+ Jn + J^+ u
\]
where does this come from?

\[
\begin{align*}
\dot{n} &= -J^+ J n + J^+ u \\
\dot{n} &= -n + n - J^+ J n + J^+ u \\
\dot{n} &= -n + (1 - J^+ J) n + J^+ u
\end{align*}
\]
\[
\dot{n} = -n + (1 - J^+ J)n + J^+ u
\]
how does this do the UCM effect?

\[ \dot{n} = -n + (1 - J^+ J)n + J^+ u \]

within the range-space

\[ \dot{n} = -n + J^+ u \]

=> stability within the range-space
how does this do the UCM effect?

\[ \dot{n} = -n + (1 - J^+ J)n + J^+ u \]

projection onto null-space

feed-forward from timing command

within the null-space

\[ \dot{n} = -n + n + 0 \]

\[ \dot{n} = 0 \]

=> no stability within the null-space
Conclusion

- The problem of inverse kinematics is part of the broader “degree of freedom problem”

- Neither robots nor human movement systems can use a simple 1:1 optimal solution, but must allow self-motion to avoid drifts into singular configurations.

- Humans have considerable self-motion and stabilize movement much less within the UCM (self-motion) space than orthogonal to it.

- Beyond the feed-forward few-to-many mappings, this involves compensatory coupling among motor commands.