Movement generation for robot arms

Kinematics and Attractor Dynamics for manipulators

robotic arms

they aren’t vehicles

movement generation for vehicles

- the floor is a 2D environment
- vehicle treated as point
- task: reach goal
- task: avoid obstacles
- not much more vehicles can do

arms: what changes?

- where we move: environment: 3D
- what we do: more tasks are possible at the same time or in sequence: e.g. manipulation
- an interesting point on the arm is the **end-effector**
- what we move: chain-of-links or segments geometry (**kinematic chain**)
- but moving a link can affect other links. complication.

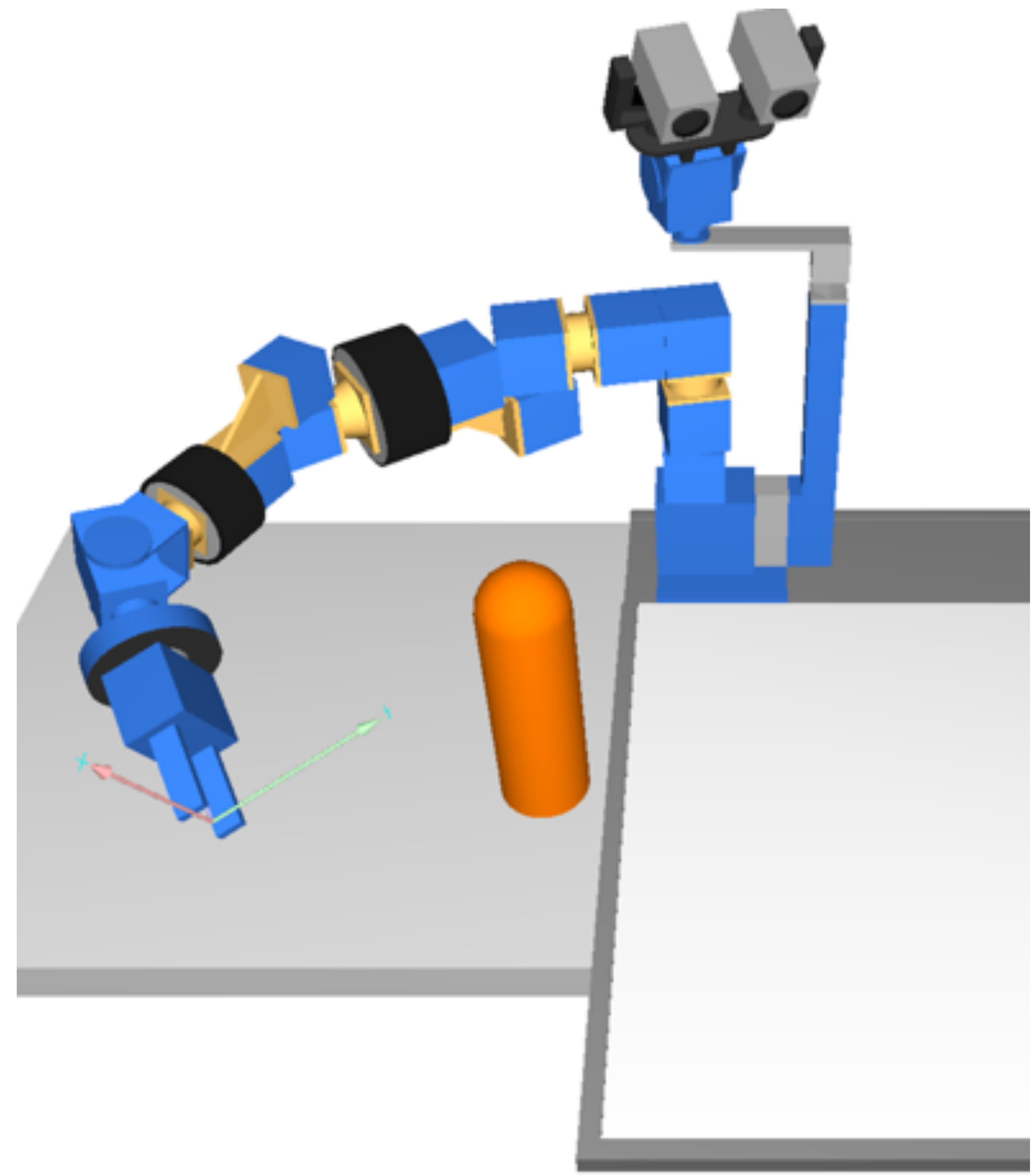
arms: what changes?

- different tasks active at different times: system needs to combine tasks that switch on/off all the time

- does Attractor Dynamics approach scale-up? what happens when multiple tasks are active at the same time? does it work? why?

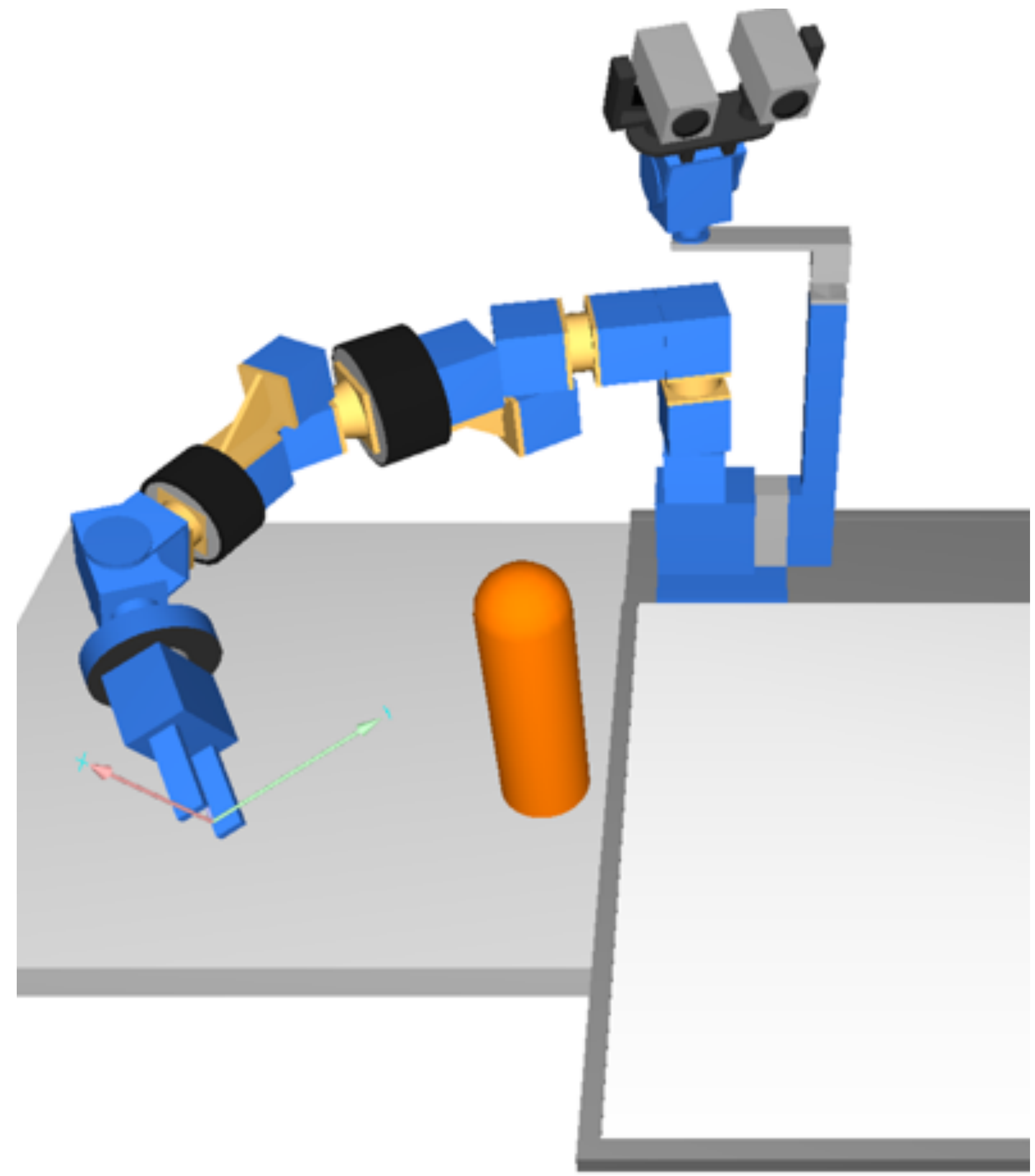
rigid bodies

- cannot treat robot as single point in space, anymore
- connected links
- orientation and translation for each link: two times 3 dimensions
- we need a way to relate our task to the links translation and orientation
- note: not always require specific orientation and specific translation for link at the same time

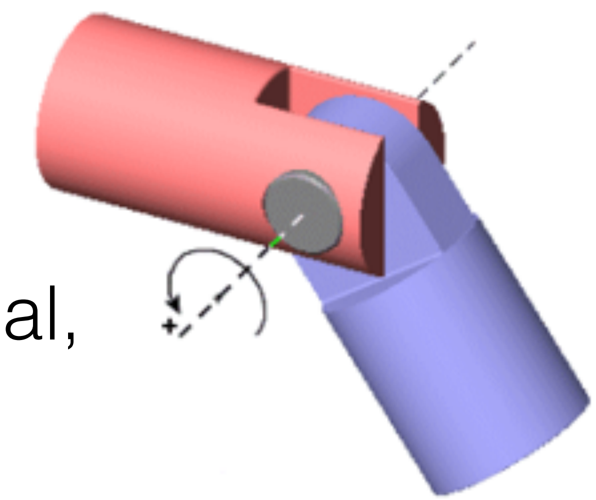


kinematics and kinetics

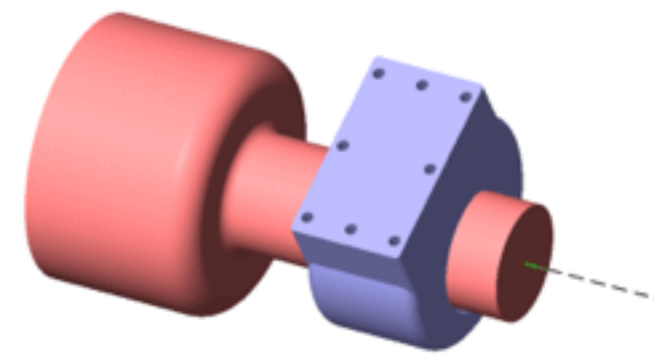
- **kinematics**: movement *without* forces
- **kinetics**: (dynamics, not in the mathematical sense) movement *with* forces
- important acting forces: gravitation, interaction of links
- we push kinetics out to low-level controller. modern robots know their own dynamics.



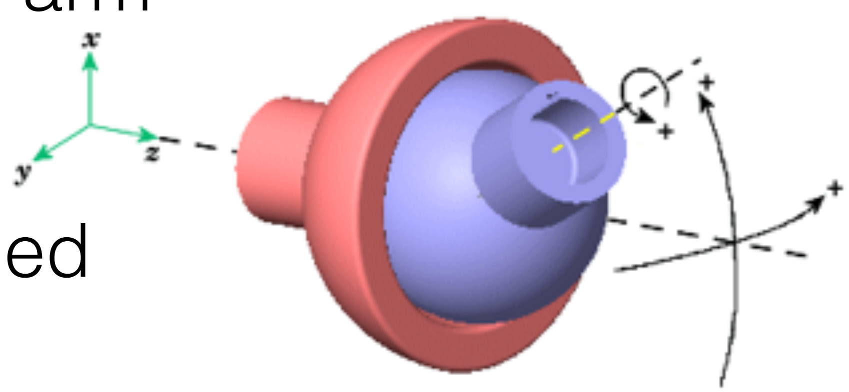
how does the arm move?



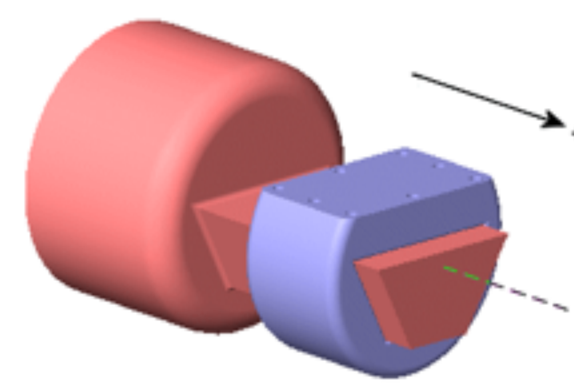
- joints: revolute, spherical, cylindrical, prismatic



- how many DoF and what kind of joints does the human arm have?



- typically position controlled servo-motors



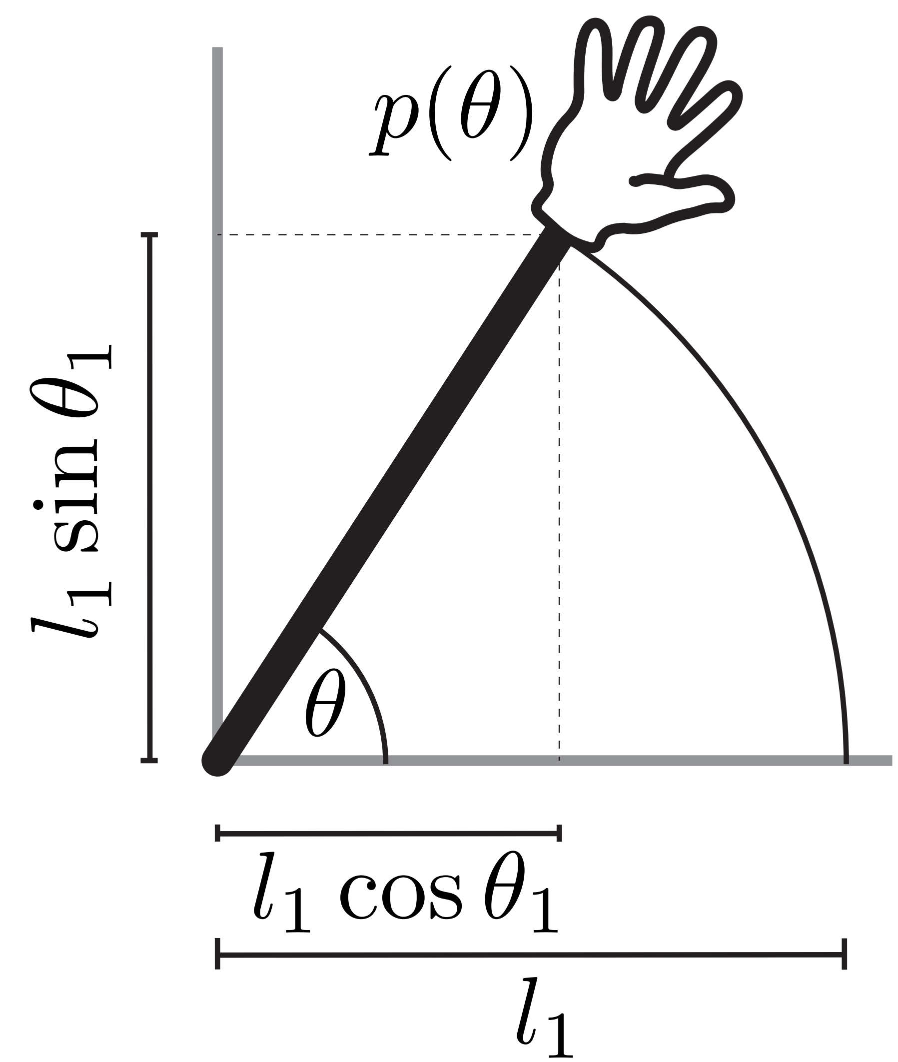
formal constraints

- **workspace**: either the environment or sometimes space of reachable positions p or x (vectors) of the end-effector. Euclidian.
- **configuration space**: space of all possible (here:) joint positions θ (vectors). Also **joint space**.
- task **constraints**: equations (equalities or inequalities) that need to be successfully satisfied for the task. can be vector-valued.
- **holonomic** constraints: expressible purely via configuration (and time). Reduces dimension of workspace.
- **non-holohomic** constraints: Velocity-based constraints. Introduces path-dependency. Typically vehicles are non-holonomic robots (can't move side-ways).
- **Degrees of Freedom**: dimensionality of configuration space.
- **Redundancy**: Compare DoF and dimensions of task contraints. More DoF than necessary? Infinite solutions to constraints possible.

Kinematics

where is the hand?

- **forward kinematics**
- example: single revolute joint
- $p(\theta_1) = \begin{pmatrix} l_1 \cos\theta_1 \\ l_1 \sin\theta_1 \end{pmatrix}$
- generally: p is a function of θ

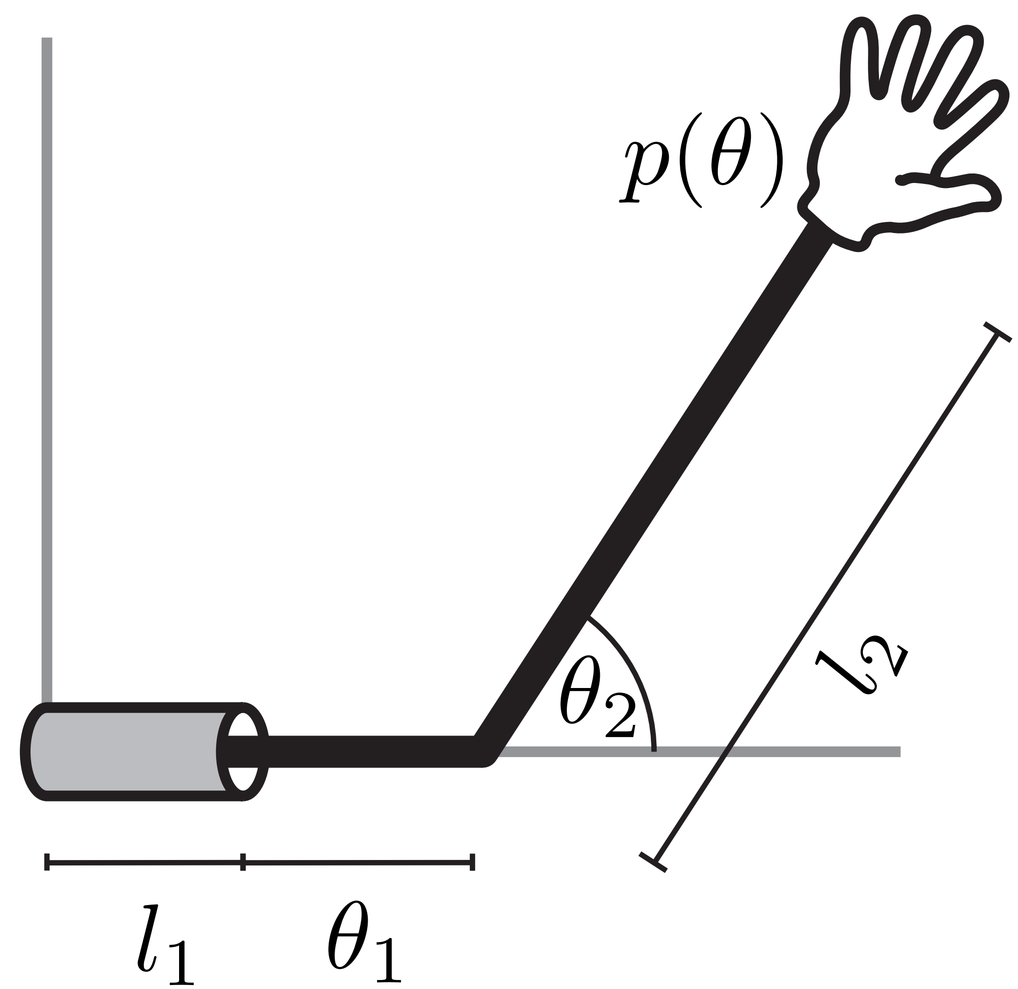


where is the hand?

- forward kinematics
- example: revolute joint and prismatic joint

$$p(\theta) = p(\theta_1, \theta_2)$$

$$= \begin{pmatrix} l_1 + \theta_1 + l_2 \cos\theta_2 \\ l_2 \sin\theta_2 \end{pmatrix}$$

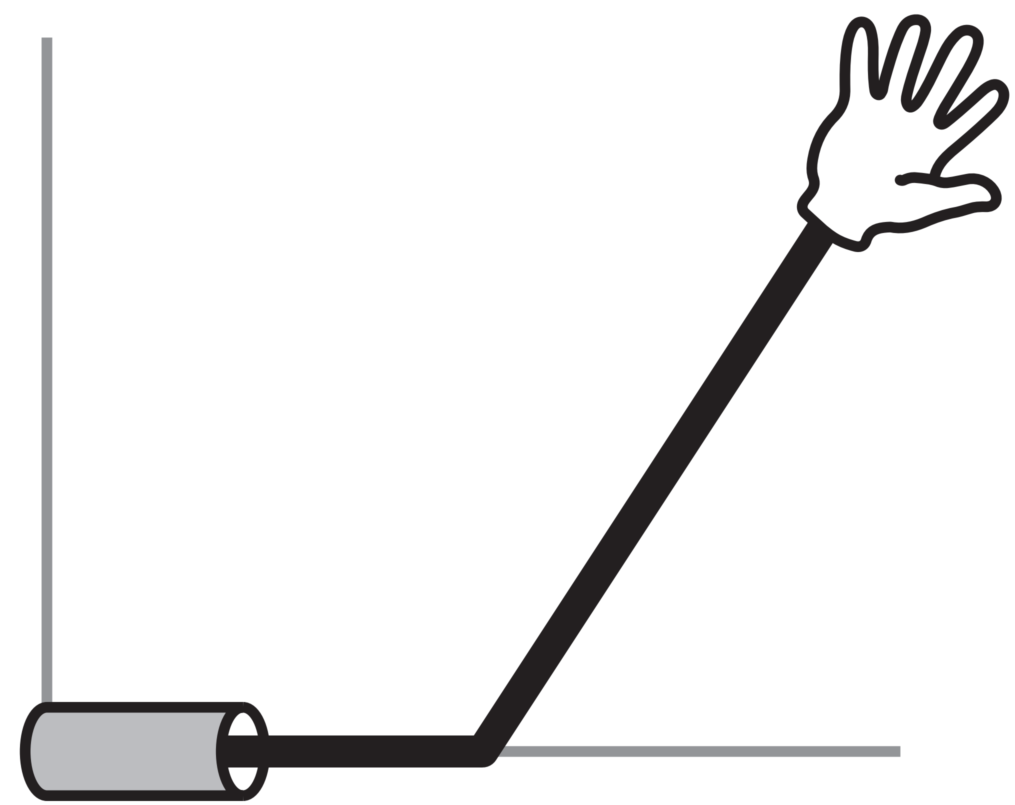


what happens if I move a joint?

- differential (forward) kinematics $\dot{p} = J\dot{\theta}$
- (kinematic) Jacobian matrix J

$$J = \begin{pmatrix} \frac{\partial p_1(\theta)}{\partial \theta_1} & \frac{\partial p_1(\theta)}{\partial \theta_2} \\ \frac{\partial p_2(\theta)}{\partial \theta_1} & \frac{\partial p_2(\theta)}{\partial \theta_2} \end{pmatrix}$$

$$= \begin{pmatrix} 1 & -l_2 \sin\theta_2 \\ 0 & l_2 \cos\theta_2 \end{pmatrix}$$

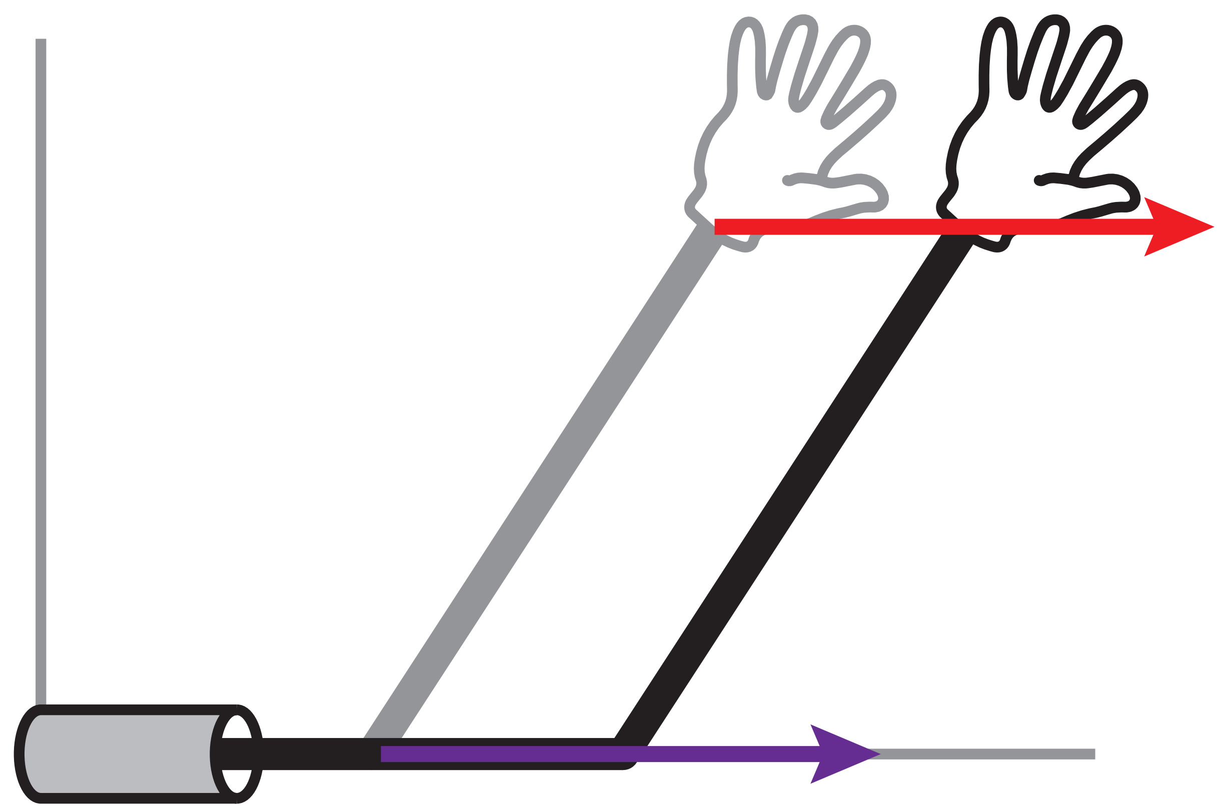


what happens if I move a joint?

(the first joint is the prismatic ‘slider’ joint)

$$J = \begin{pmatrix} \frac{\partial p_1(\theta)}{\partial \theta_1} & \frac{\partial p_1(\theta)}{\partial \theta_2} \\ \frac{\partial p_2(\theta)}{\partial \theta_1} & \frac{\partial p_2(\theta)}{\partial \theta_2} \end{pmatrix}$$

$$= \begin{pmatrix} 1 & -l_2 \sin\theta_2 \\ 0 & l_2 \cos\theta_2 \end{pmatrix}$$

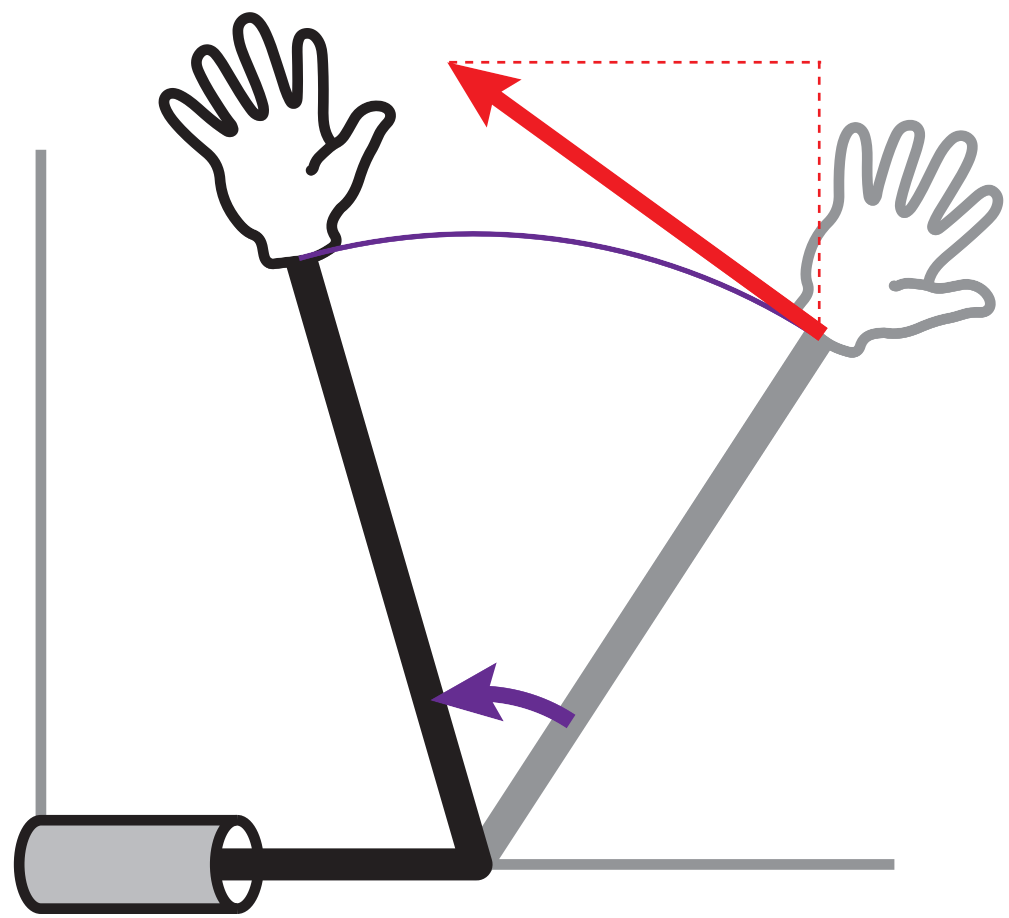


$$\dot{p} = J\dot{\theta}$$

what happens if I move a joint?

$$J = \begin{pmatrix} \frac{\partial p_1(\theta)}{\partial \theta_1} & \frac{\partial p_1(\theta)}{\partial \theta_2} \\ \frac{\partial p_2(\theta)}{\partial \theta_1} & \frac{\partial p_2(\theta)}{\partial \theta_2} \end{pmatrix}$$

$$= \begin{pmatrix} 1 & -l_2 \sin \theta_2 \\ 0 & l_2 \cos \theta_2 \end{pmatrix}$$



$$\dot{p} = J\dot{\theta}$$

what happens if I move a joint?

- differential (or instantaneous) kinematics provide a relationship between velocities

$$\dot{p} = J\dot{\theta} \qquad J = \frac{\partial p(\theta)}{\partial \theta}$$

- note: J changes when θ changes

- what happens when J is singular? kinematic **singularity**. rank changes

- since J changes, these singularities can appear and disappear (at certain configurations) while moving

- **nullspace** of J: space of all $\dot{\theta}$ that project to a $\dot{p}$ of 0.

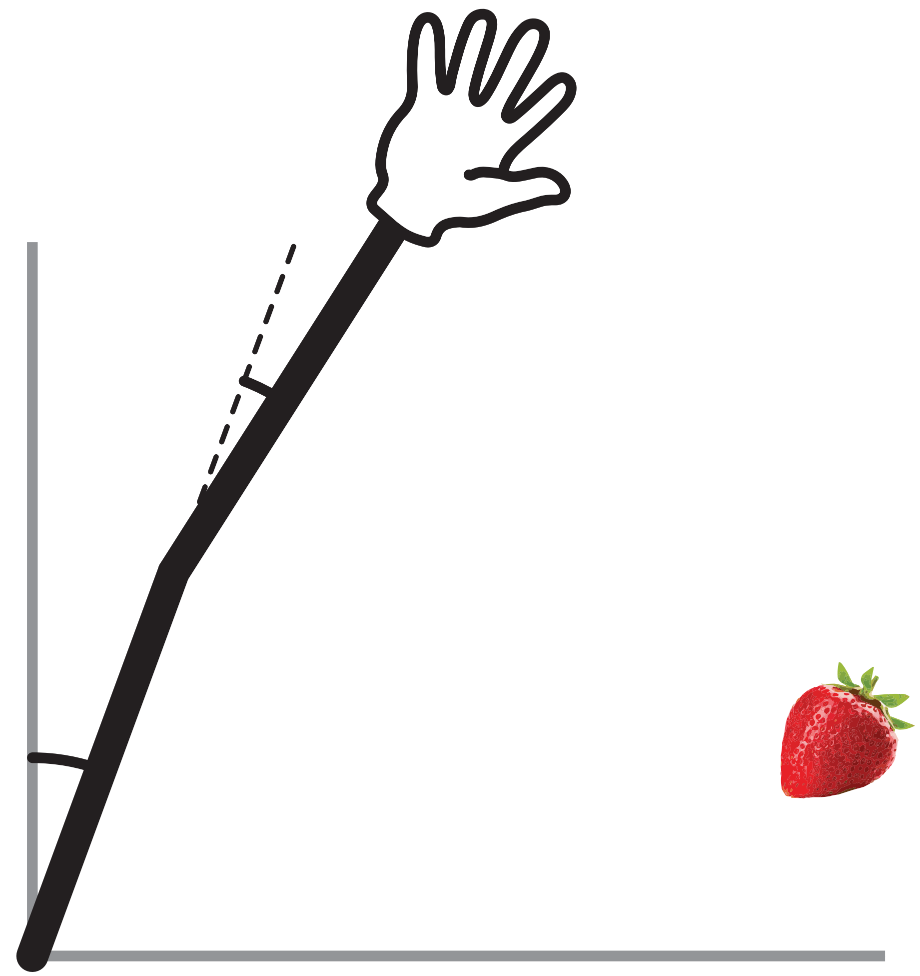
how do I get the hand to where I want it?

- we now need to look at the inverse problem: what joints do I need to set to what values to reach a certain point in workspace?

- **closed form** solution (**inverse** of the forward kinematics)

- the forward kinematics $p(\theta)$ can in general not be analytically inverted

- geometrical construction. depends on geometry of robot!



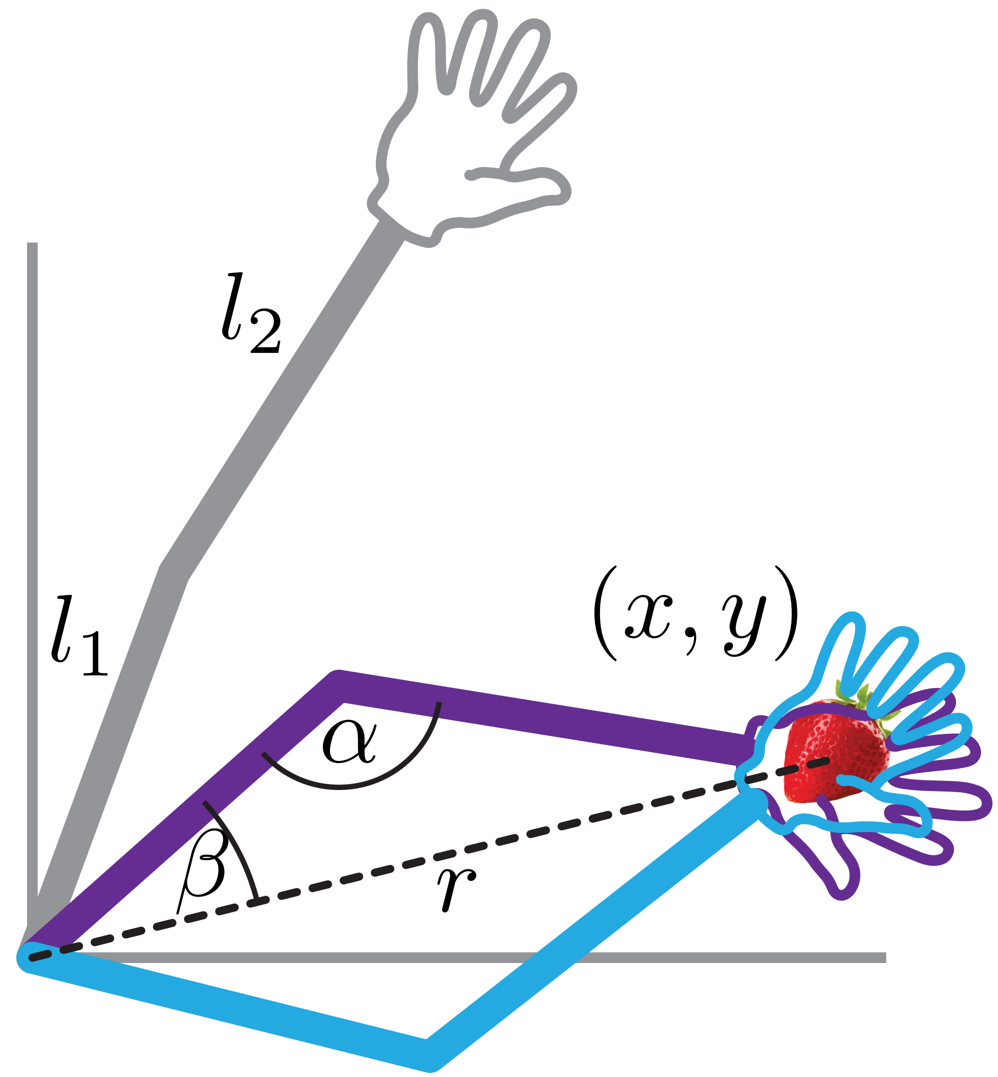
how do I get the hand to where I want it?

$$\theta_1 = \arctan_2(y, x) \pm \beta$$

$$\theta_2 = \pi \pm \alpha$$

$$\alpha = \cos^{-1}\left(\frac{l_1^2 + l_2^2 - r^2}{2l_1 l_2}\right)$$

$$\beta = \cos^{-1}\left(\frac{r^2 + l_1^2 - l_2^2}{2l_1 l_2}\right)$$



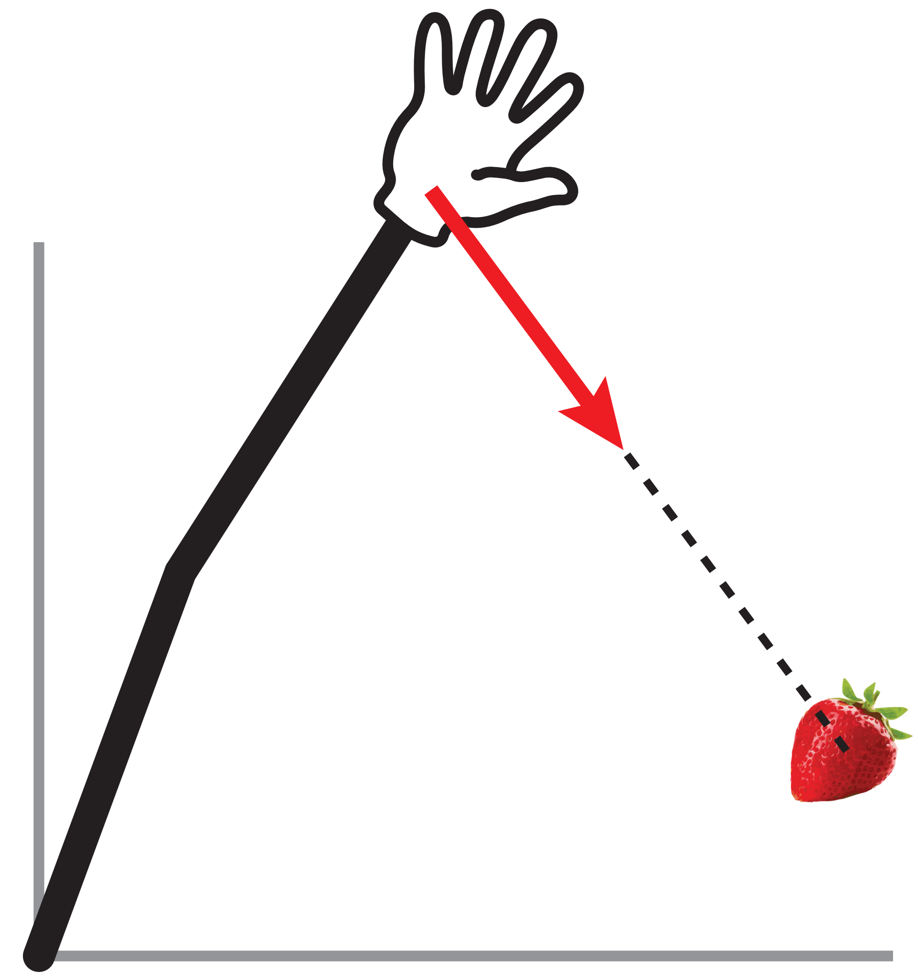
how do I get the hand to where I want it?

- the differential kinematics may be simpler to invert?

$$\dot{p} = \frac{dp}{dt} = \frac{dp}{d\theta}\frac{d\theta}{dt} = J\dot{\theta}$$

- … if J is invertible.
- is J singular?
- is J even quadratic?
- iff invertible: $\dot{\theta} = J^{-1}\dot{p}$

  we can calculate a commanded joint velocity
- integrate $\dot{\theta}$ to θ to send commands



inverse of differential kinematics

- if J is not invertible we can use the **Moore-Penrose pseudoinverse** J^+ instead of J^{-1}

- defined as:

$$J^+ = (JJ^T)^{-1}J^T$$

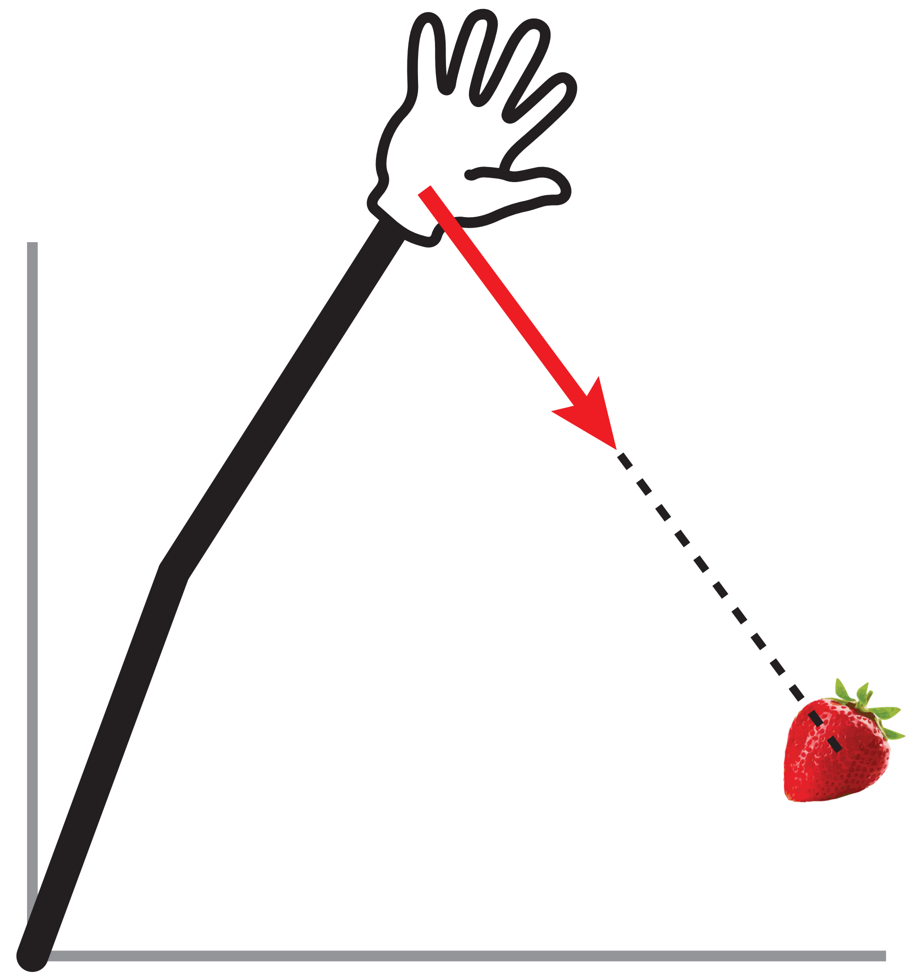
- to invert the equation:

$$\dot{p} = J\dot{\theta}$$

- (apply from the left) to get:

$$\dot{\theta} = J^+\dot{p}$$

- this is a commonly used generalized matrix inverse



inverse of differential kinematics

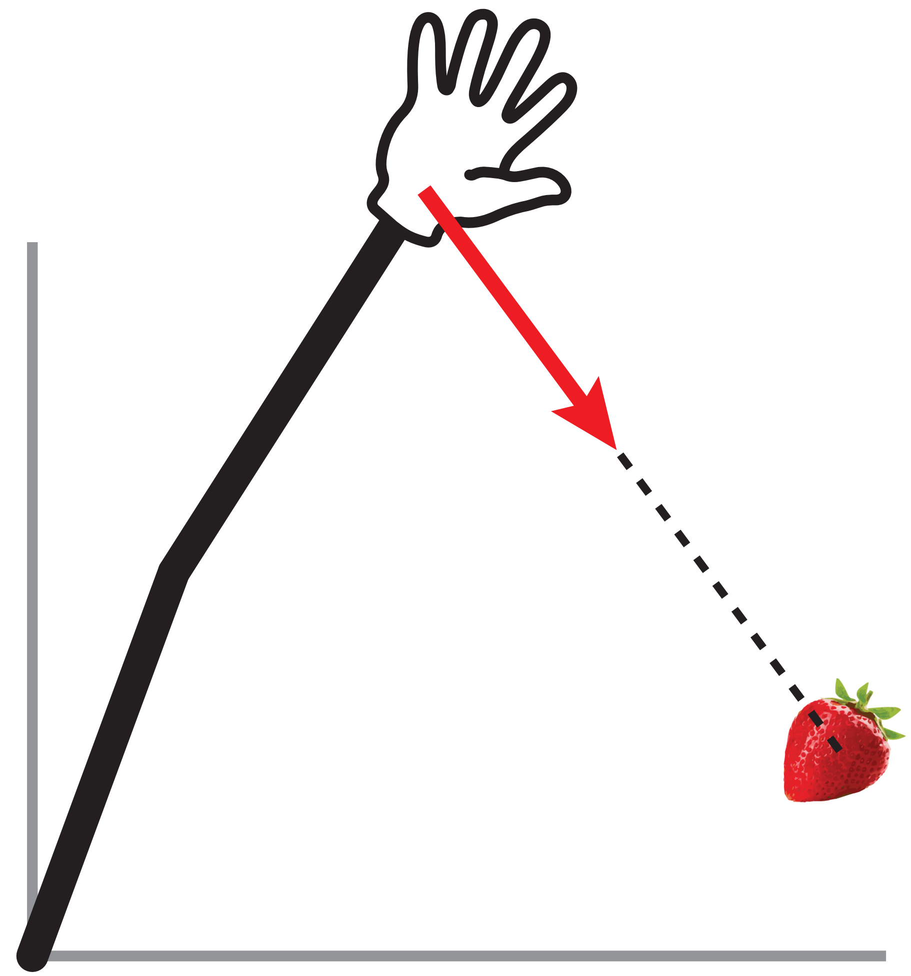
- J^+ in $\dot{\theta} = J^+ \dot{p}$ has useful properties:

- if there are many solutions (typically yes) it minimizes:

$$|\dot{\theta}|$$

- net effect: this prefers lower speeds over all joints than one joint with high speed. Good!

- if there is no solution it finds the solution with the smallest error



inverse of differential kinematics

- not implemented in this manner, though:

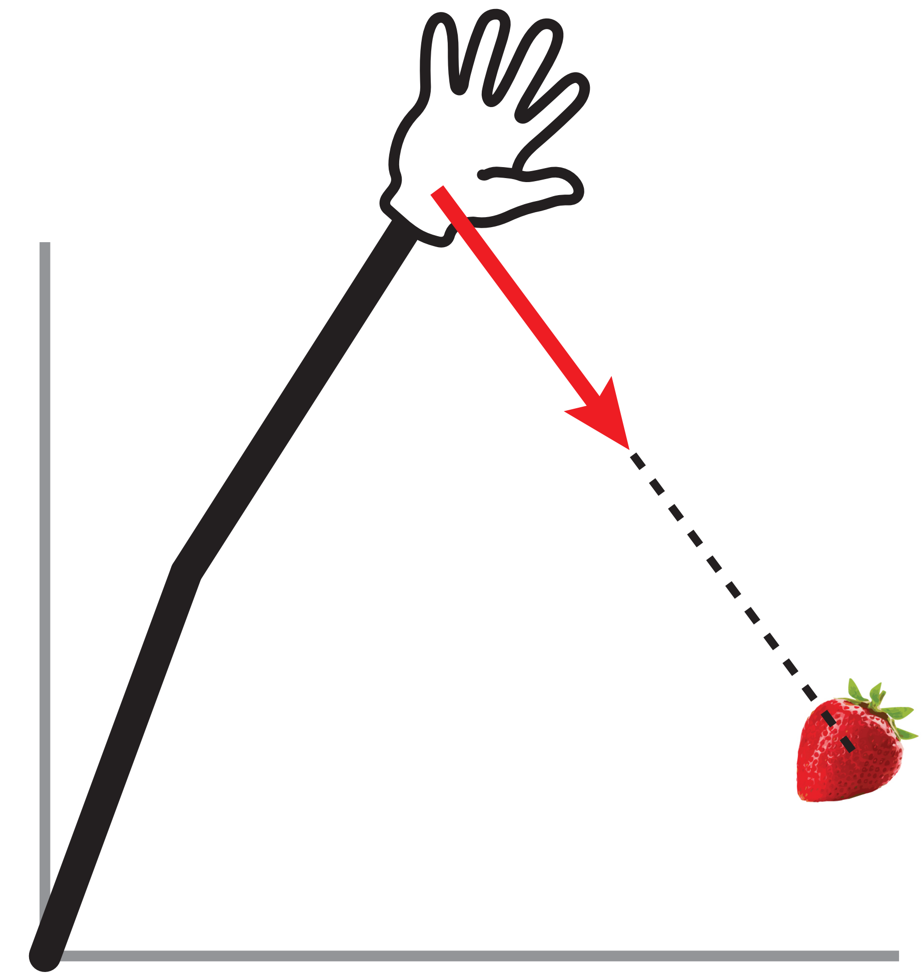
$$J^+ = (JJ^T)^{-1}J^T$$

- but as singular value decomposition

$$J = U\Sigma V^T$$

$$J^+ = V\Sigma^+ U^T$$

- where U and D are specific orthogonal matrices and $\sum$ is a diag matrix of ‘singular values’ - all based on the Eigenvalues and Eigenvectors of JJ^T and J^TJ



more on the Moore Penrose Pseudo-Inverse

- if Σ is a matrix of 'singular values' σ in the diagonal entries
- then Σ^+ is the transposed matrix of the reciprocal values:

$$\frac{1}{\sigma}$$

- PROBLEM: the singular values become very small when we get close to singularities (directions in which we cannot move)
- workaround: use the **damped pseudo-inverse** with the corresponding entries:

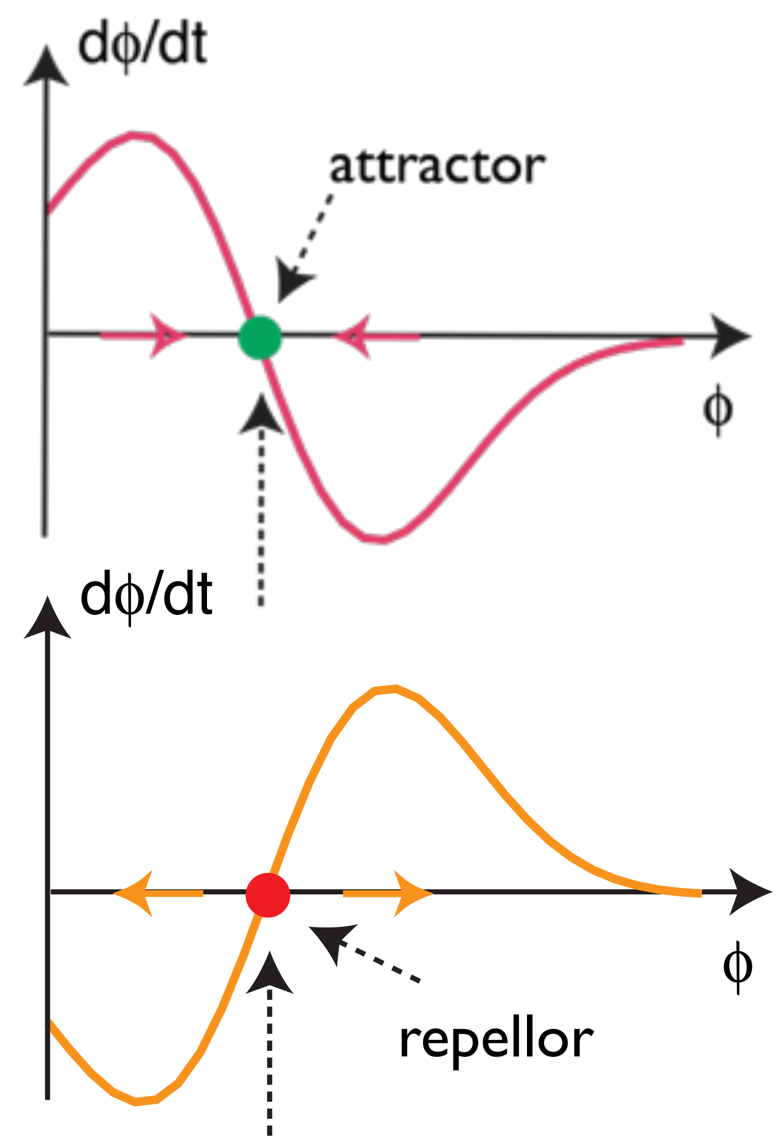
$$\frac{1}{\sigma + \epsilon}$$

Attractor Dynamics for robot arms

recap: tasks in Attractor Dynamics

- task as differential equation
- task is adhered-to if system is in a fixed-point
- move quickly into attractor state
- in reality: near attractor suffices
- avoidance: repellors
- task akin "**forcelet**"

$$\dot{\phi} = f(\phi)$$
$$\dot{\phi} = 0$$



generating complex movements

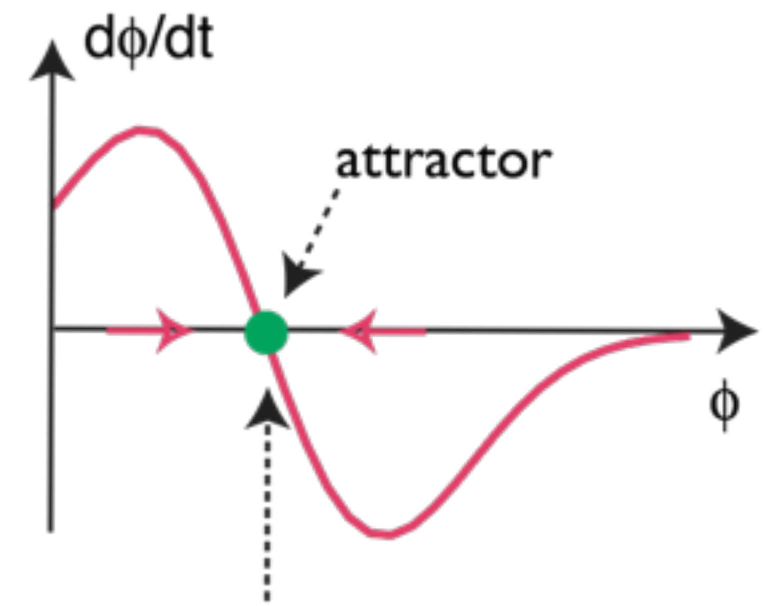
different tasks

- reach bottle
- grasp bottle
- pour the drink
- put bottle on table
- avoid obstacles

- hand position, bottle position
- hand orientation, hand opening, hand closing
- bottle orientation, glass position, glass filling
- obstacle positions, if any

different tasks

- different variables "ϕ" are relevant for different tasks
- a task can be expressed as constraint on that variable (stable fixed-point in a dynamical system)
- but how do ϕ and $\dot{\phi}$ relate to θ and $\dot{\theta}$ in joints?
- task defines submanifold on configuration space
- different tasks live on different sub- manifolds of configuration space. how can this work?

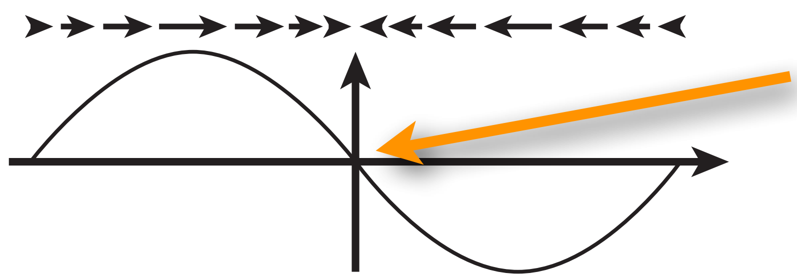


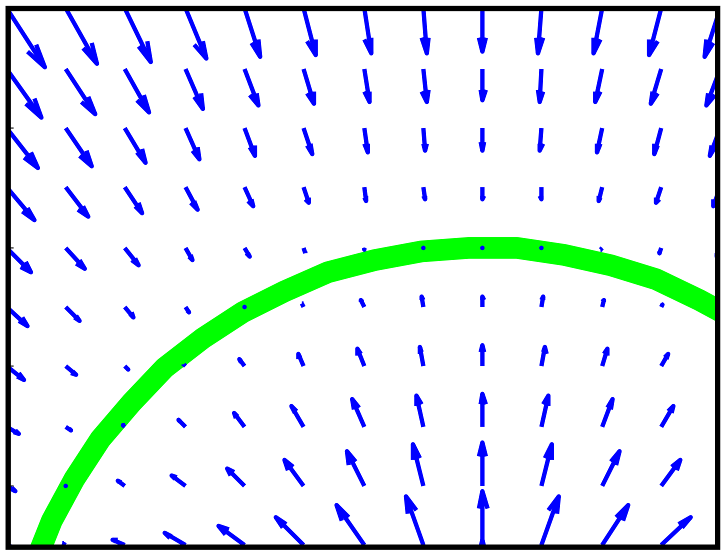
$$\dot{\phi} = f(\phi)$$

$$\phi = ?$$

independent stabilization

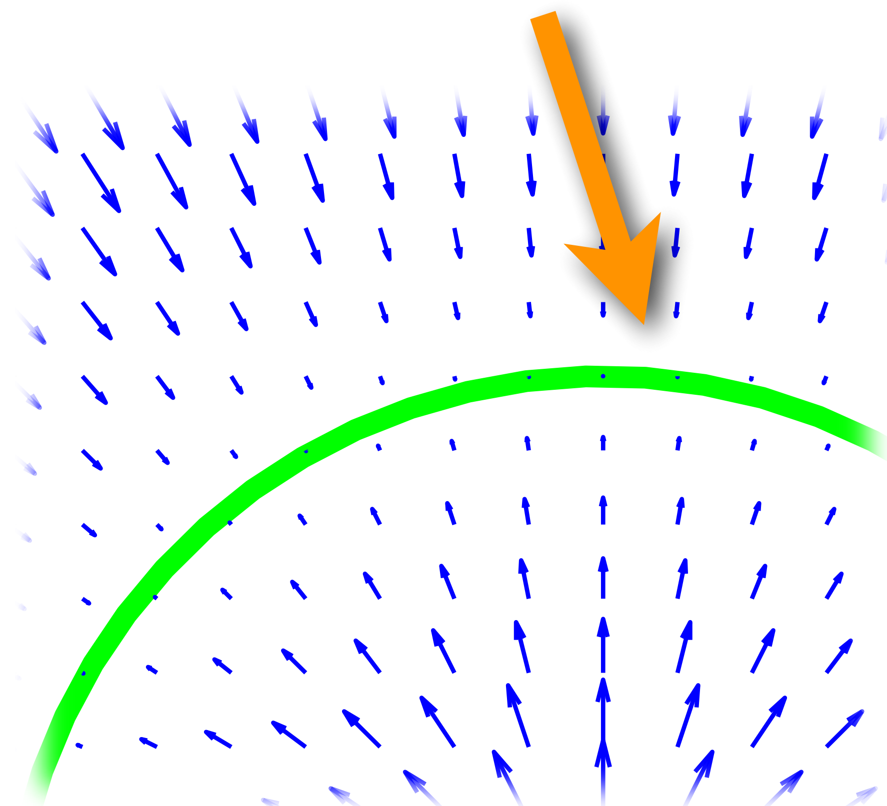
- independent forcelets
- each a (possibly different) relevant variable
- constraints expressed as attractors/repellors in dynamical system over that relevant variable only
- find joint space changes that realize this task (independently)





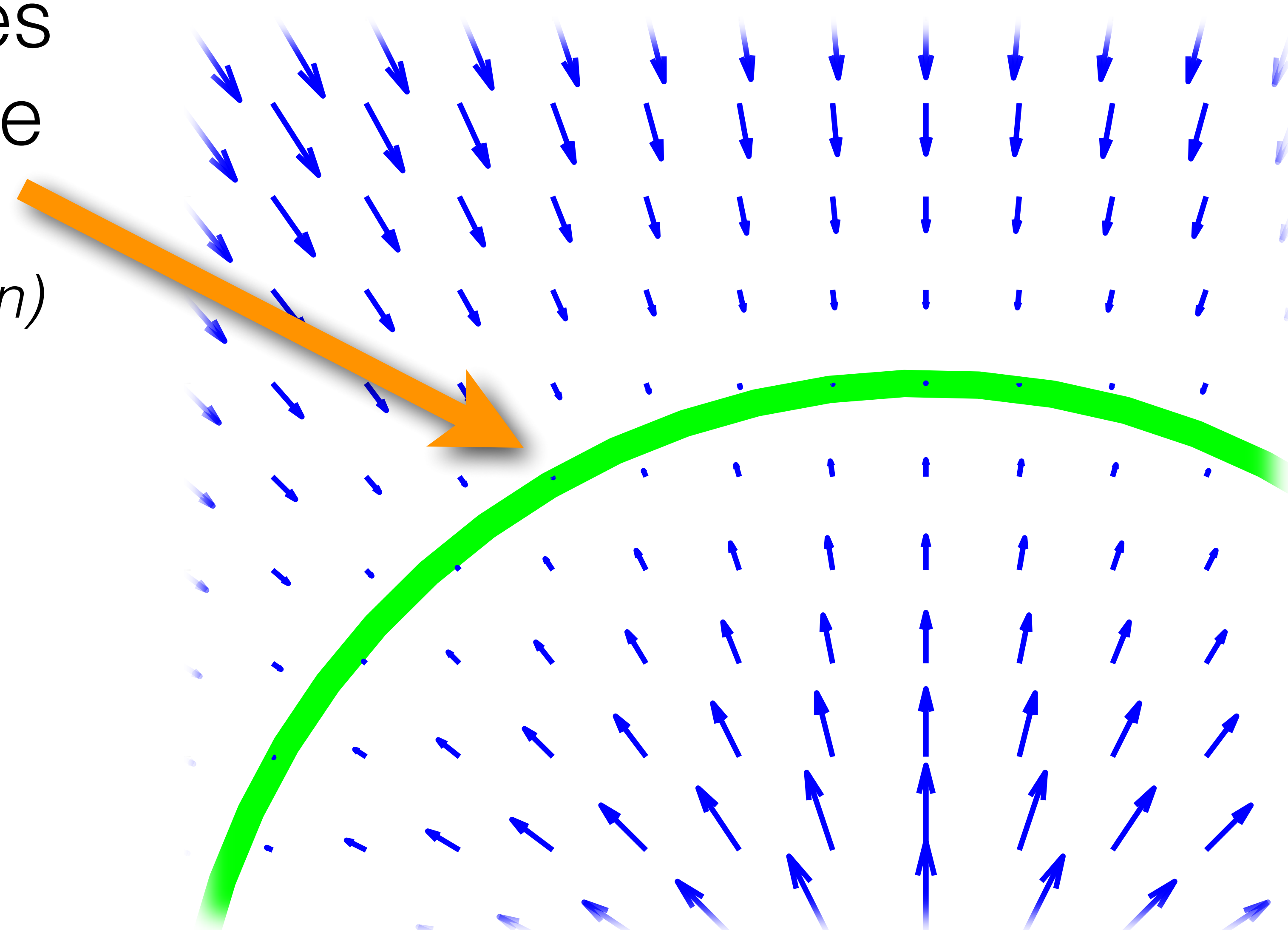
reintegration of independent tasks

- superposition of independent forcelets
- now new vector field realizes compromise of tasks



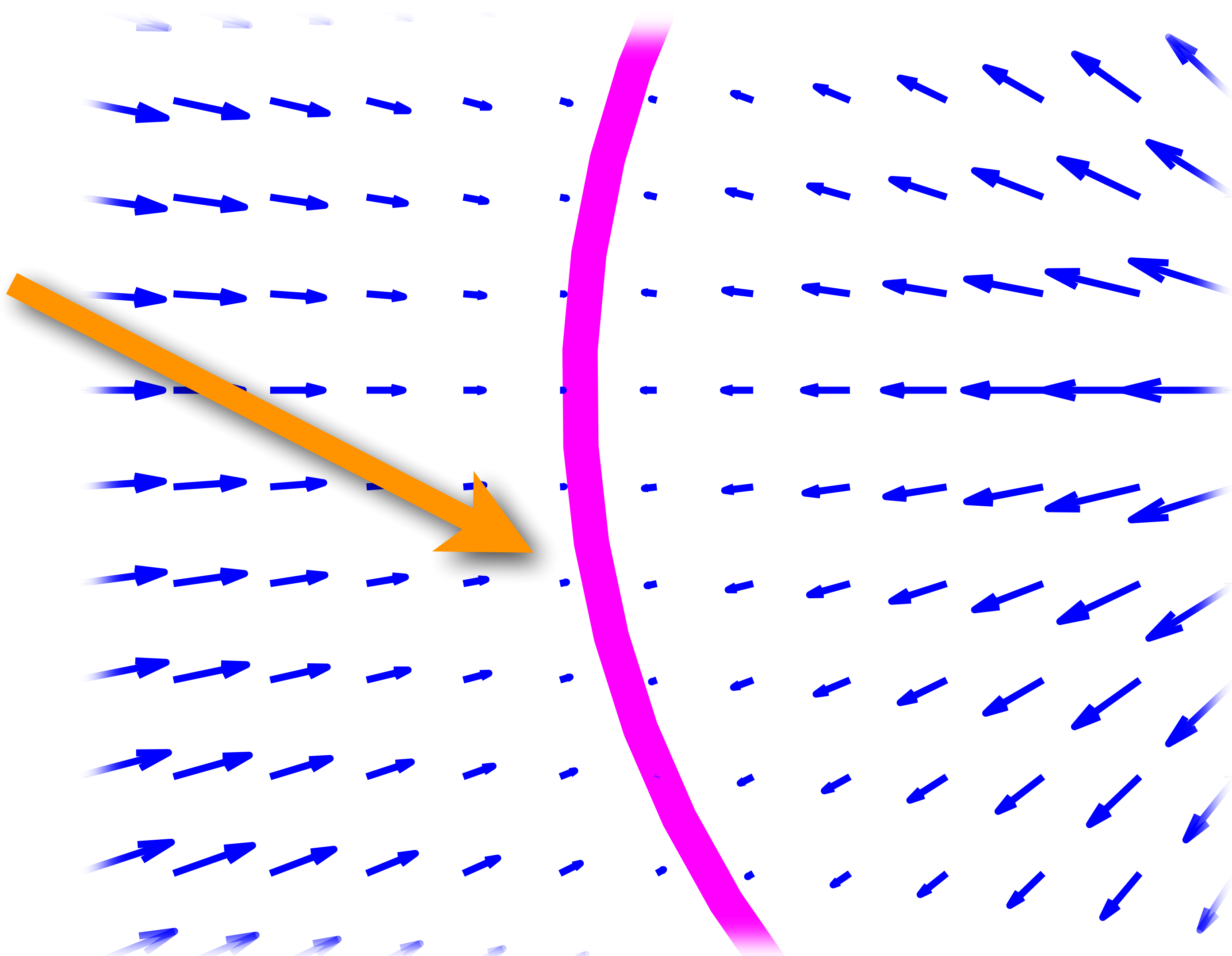
reintegration of independent tasks

joint angles
that realize
task 1
(hand position)

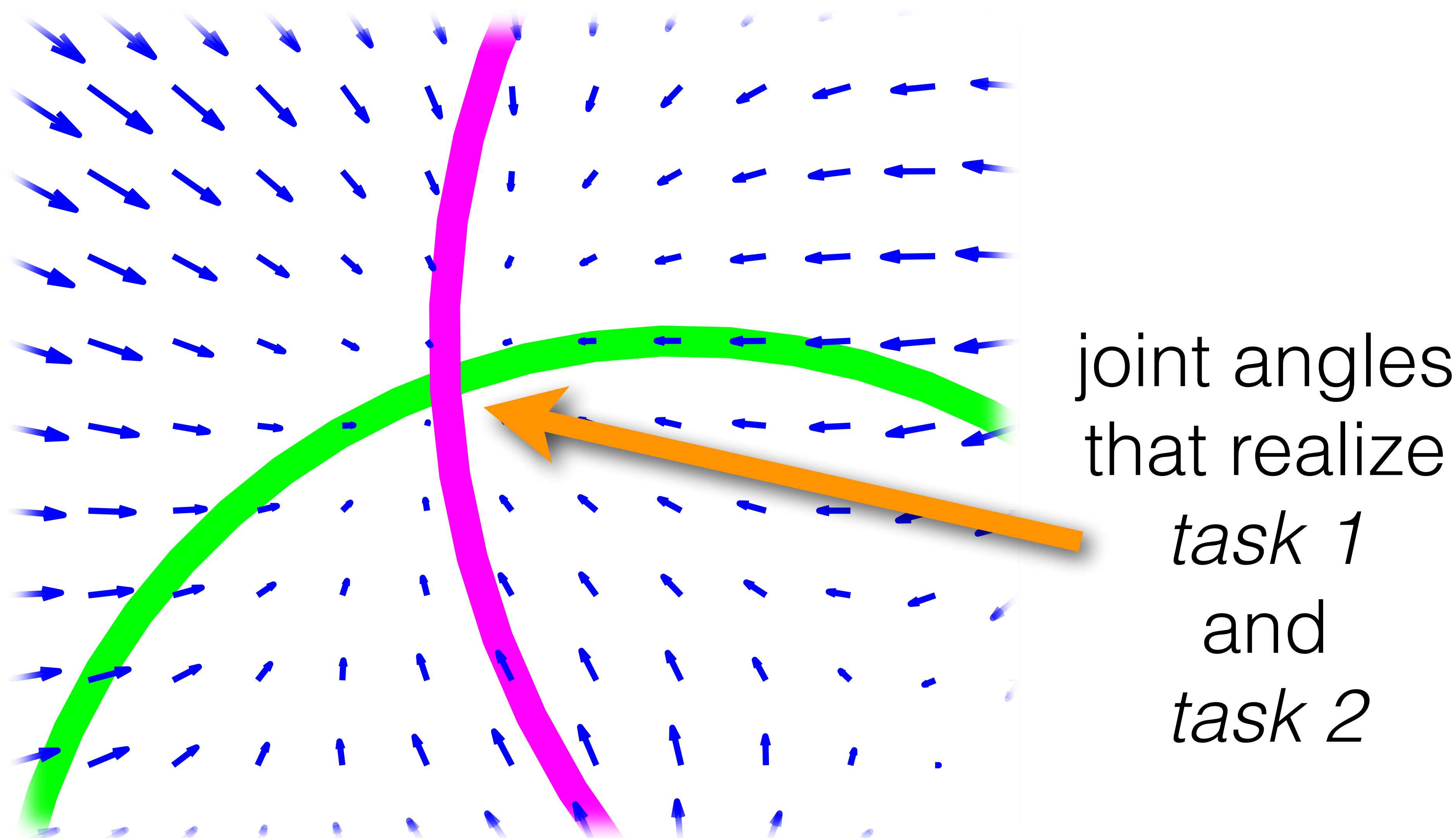


reintegration of independent tasks

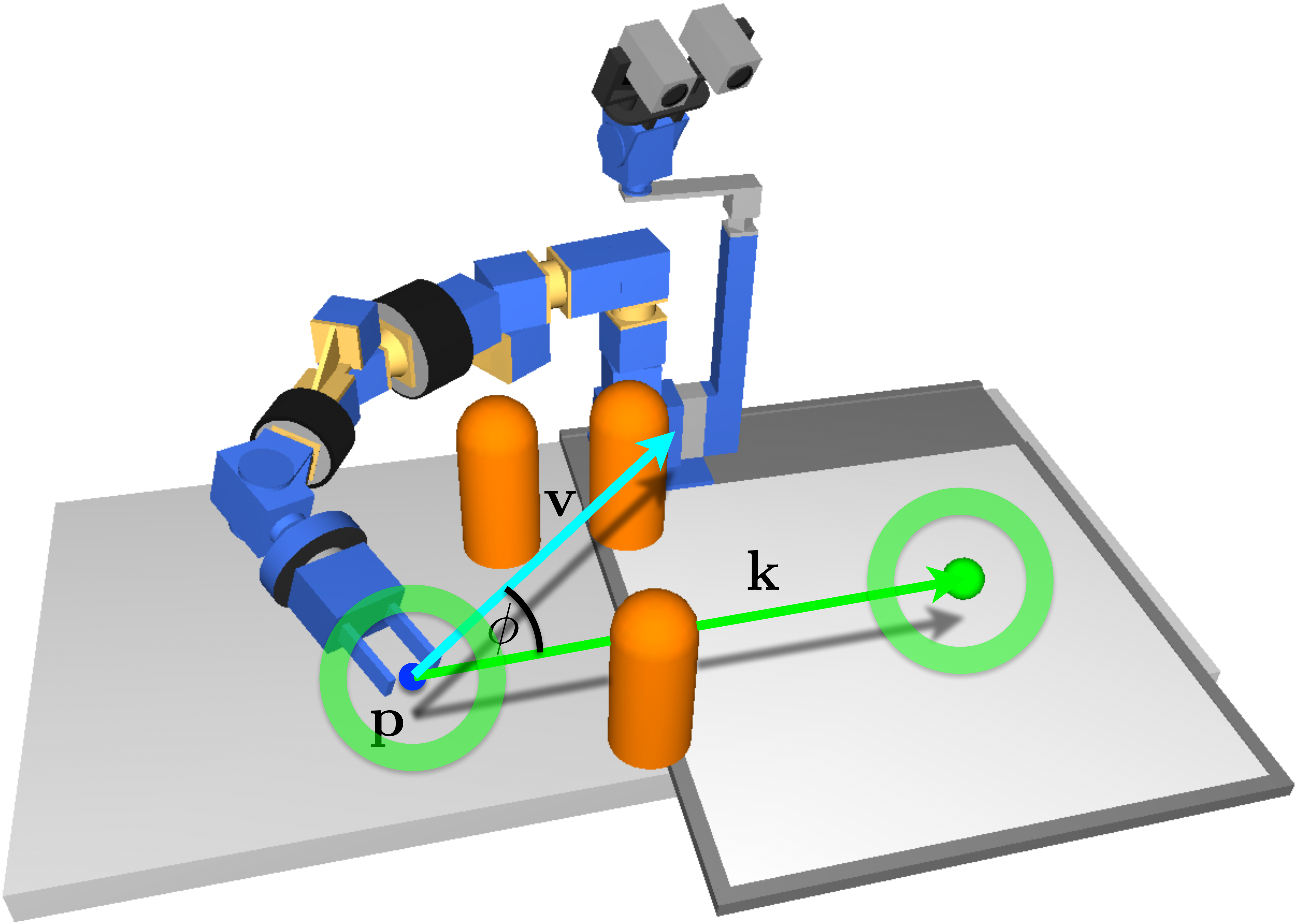
joint angles
that realize
task 2
(hand orientation)



reintegration of independent tasks



reaching



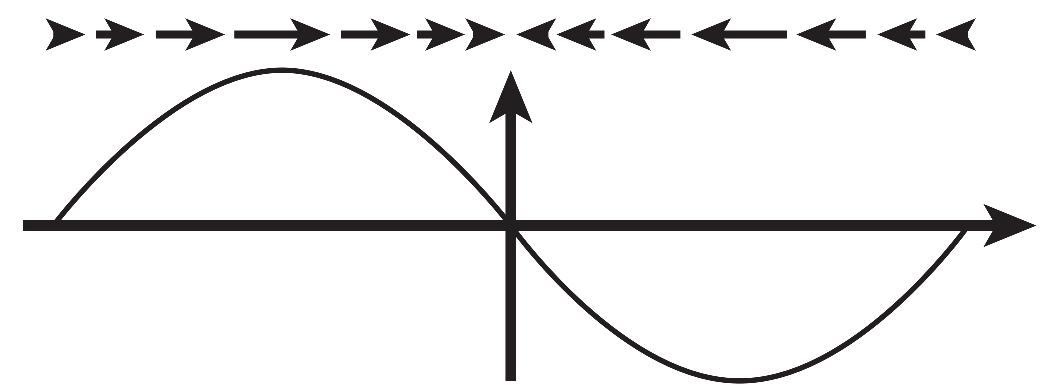
reaching

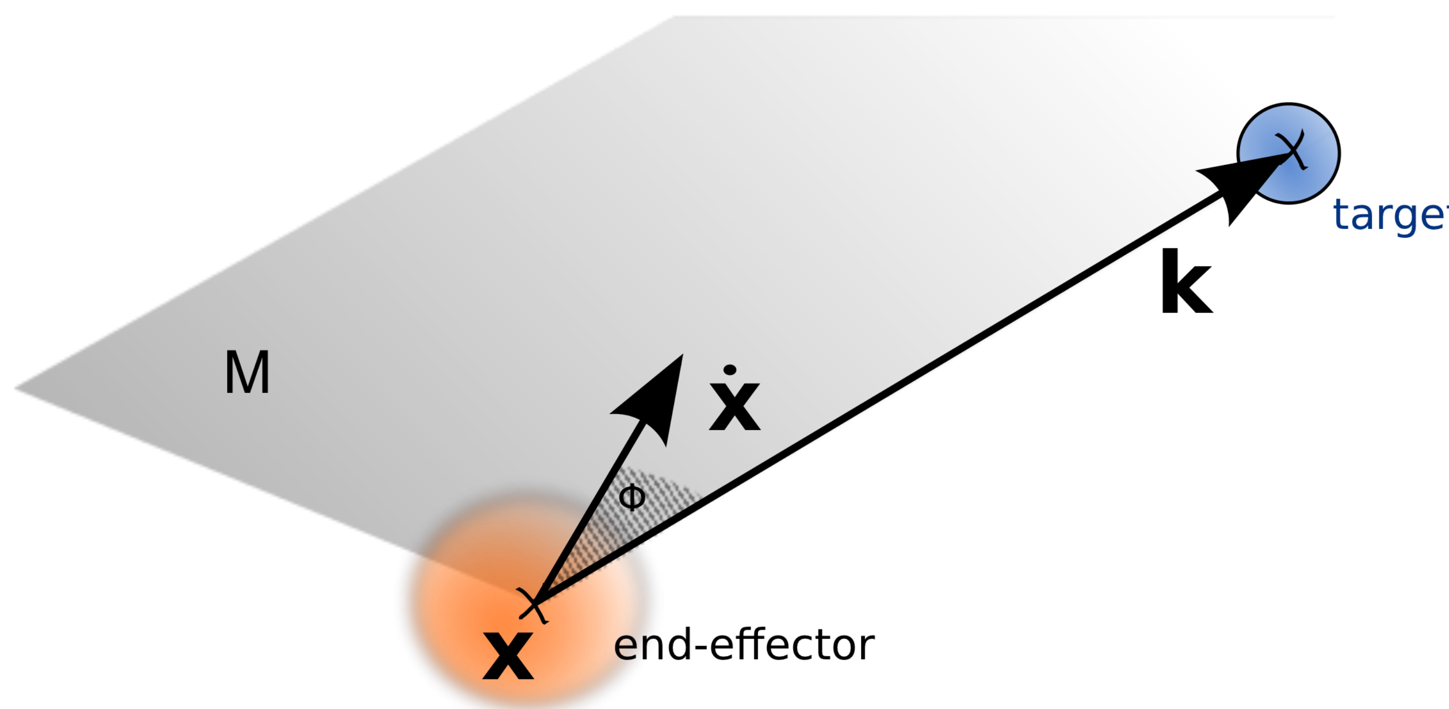
- deviation angle dynamics, analogously to heading angle dynamics (define a plane M)

$$\dot{\phi} = f_{dir} = -\alpha_\phi \sin \phi$$

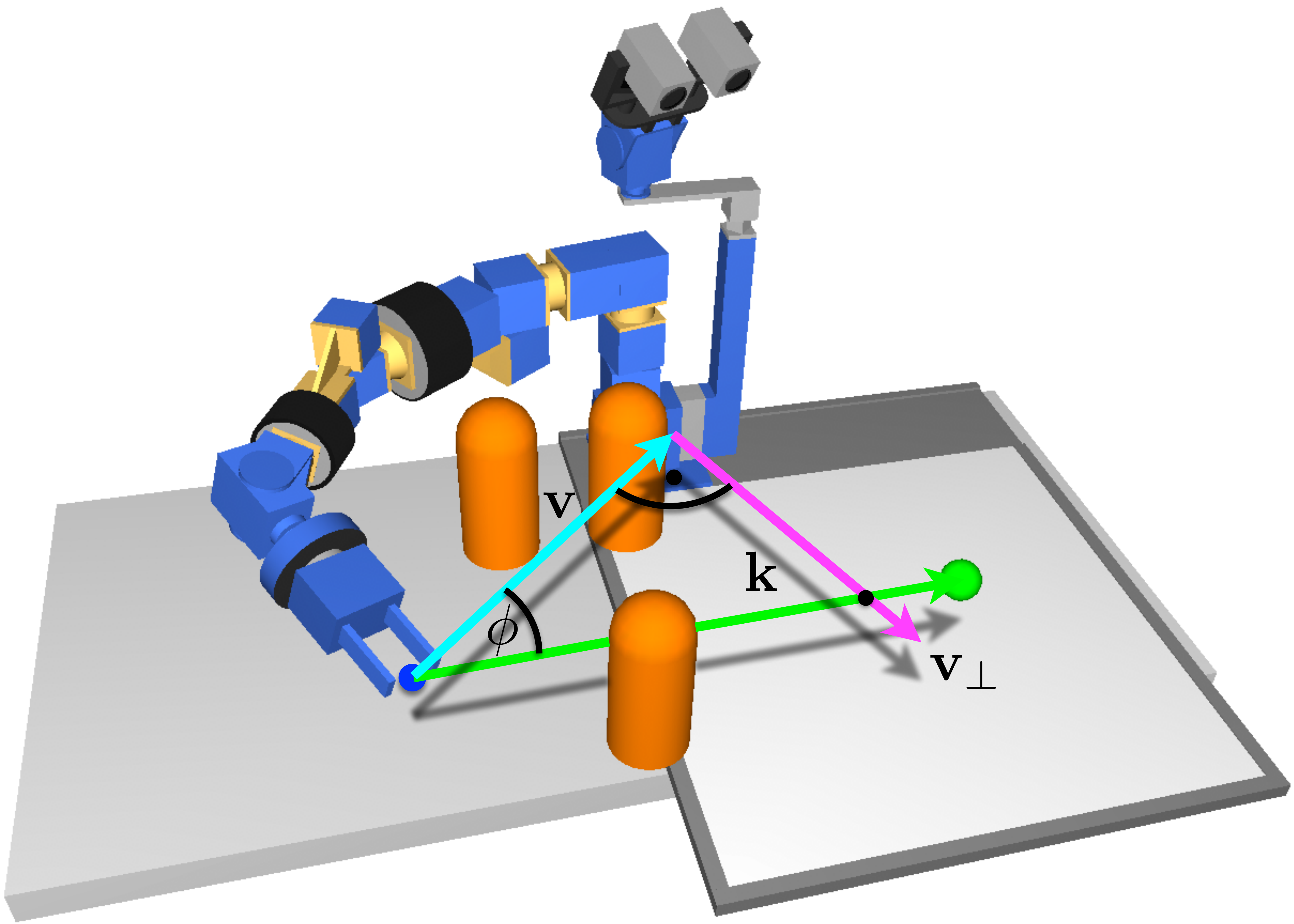
- angle: $\phi = \measuredangle(\dot{x}, k)$

- insert a step: In workspace, what vector would realize the change?

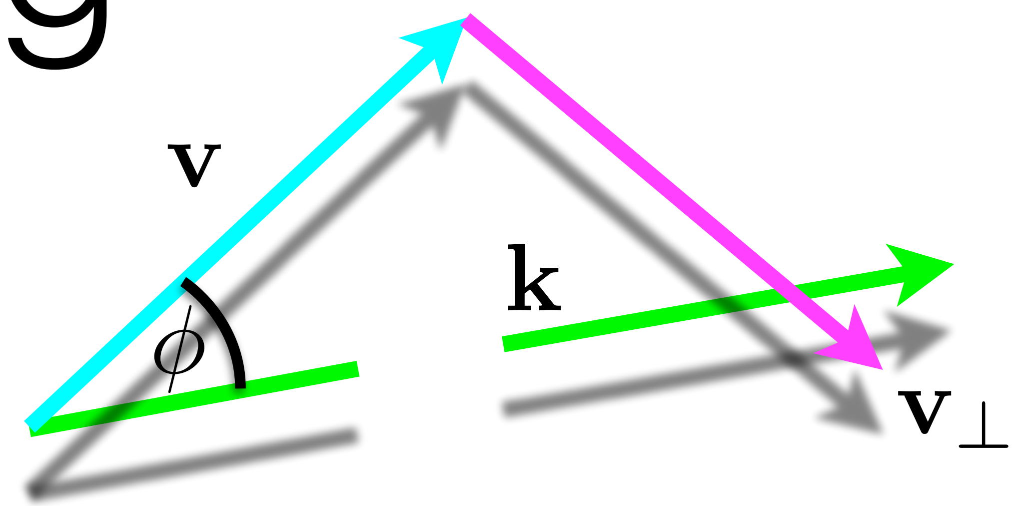




reaching



reaching



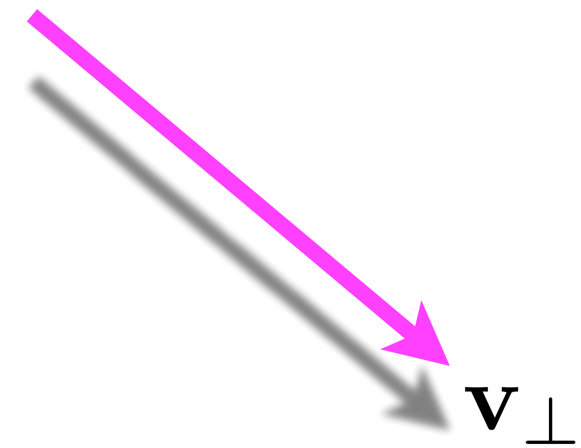
- from geometry we can find:

$$\mathbf{v}_\perp = \left(\mathbf{k} - \frac{\langle \mathbf{k}, \mathbf{v} \rangle}{\langle \mathbf{v}, \mathbf{v} \rangle}\mathbf{v}\right) \frac{|\mathbf{v}|}{|\mathbf{k} - \frac{\langle \mathbf{k}, \mathbf{v} \rangle}{\langle \mathbf{v}, \mathbf{v} \rangle}\mathbf{v}|}$$

- transformation of forcelet into workspace:

$$\mathbf{f}_{dir} = f_{dir} \cdot \mathbf{v}_\perp = -\alpha_\phi \sin\phi \cdot \mathbf{v}_\perp$$

reaching



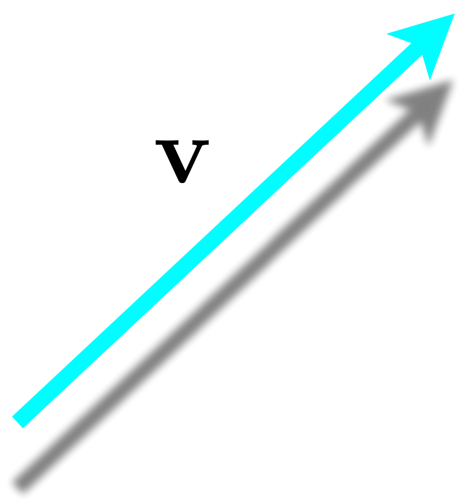
- transformation from workspace into joint-space:
- per inverse differential kinematics:

$$J^+ = J^T \left(JJ^T\right)^{-1}$$

$$\mathbf{F}_{dir} = J^+ \cdot \mathbf{f}_{dir} = -\alpha_\phi \sin\phi \cdot J^+ \cdot \mathbf{v}_\perp$$

- we now have a “forcelet” in joint space

speed



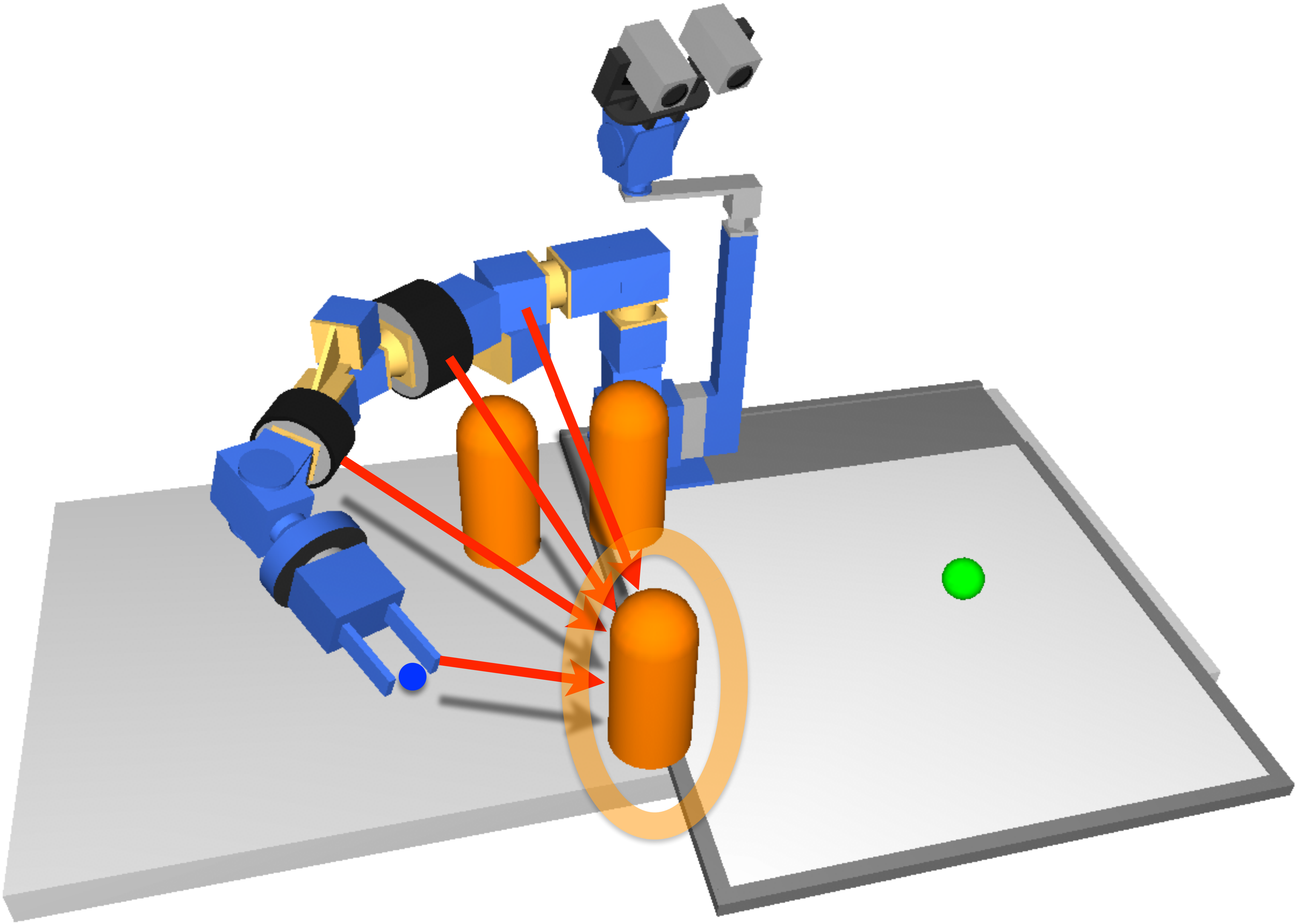
- analogous to the vehicle scenario, speed treated as independent task:
- $v = |\mathbf{v}|$
- select a desired speed: v_{des}
- $\hat{\mathbf{v}} = \dfrac{\mathbf{v}}{|\mathbf{v}|}$

$$f_{vel} = -\alpha_{vel}(v - v_{des})$$

$$\mathbf{f}_{vel} = f_{vel} \cdot \hat{\mathbf{v}}$$

$$\mathbf{F}_{vel} = J^{+} \cdot \mathbf{f}_{vel}$$

obstacle avoidance



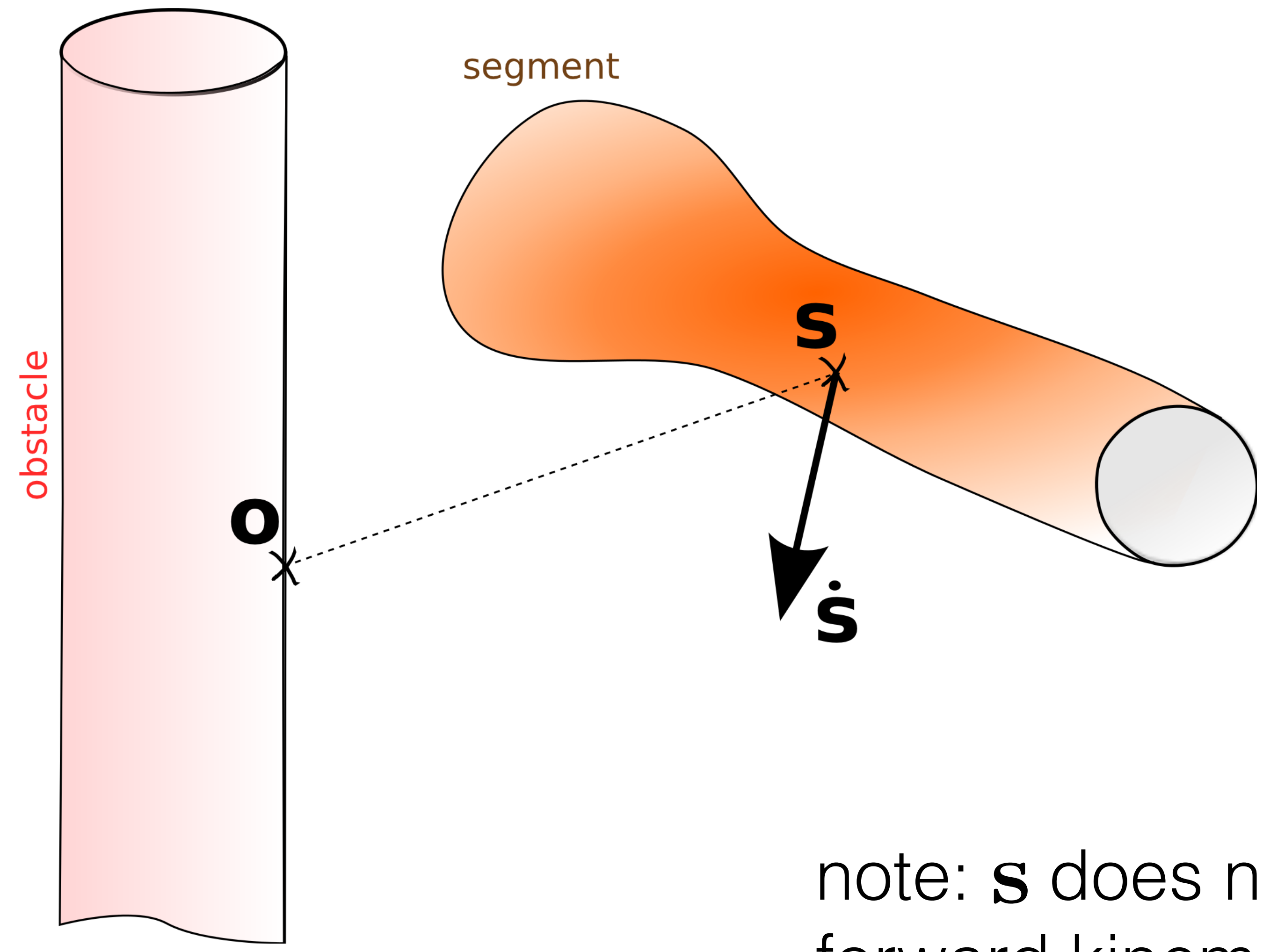
obstacle avoidance

- finding a instantaneous joint change that enacts the required (instantaneous) task change: find direction that moves the relevant task variable *into* its *attractor*

- other take on it: find direction that moves the relevant task variable *away* from its repellor

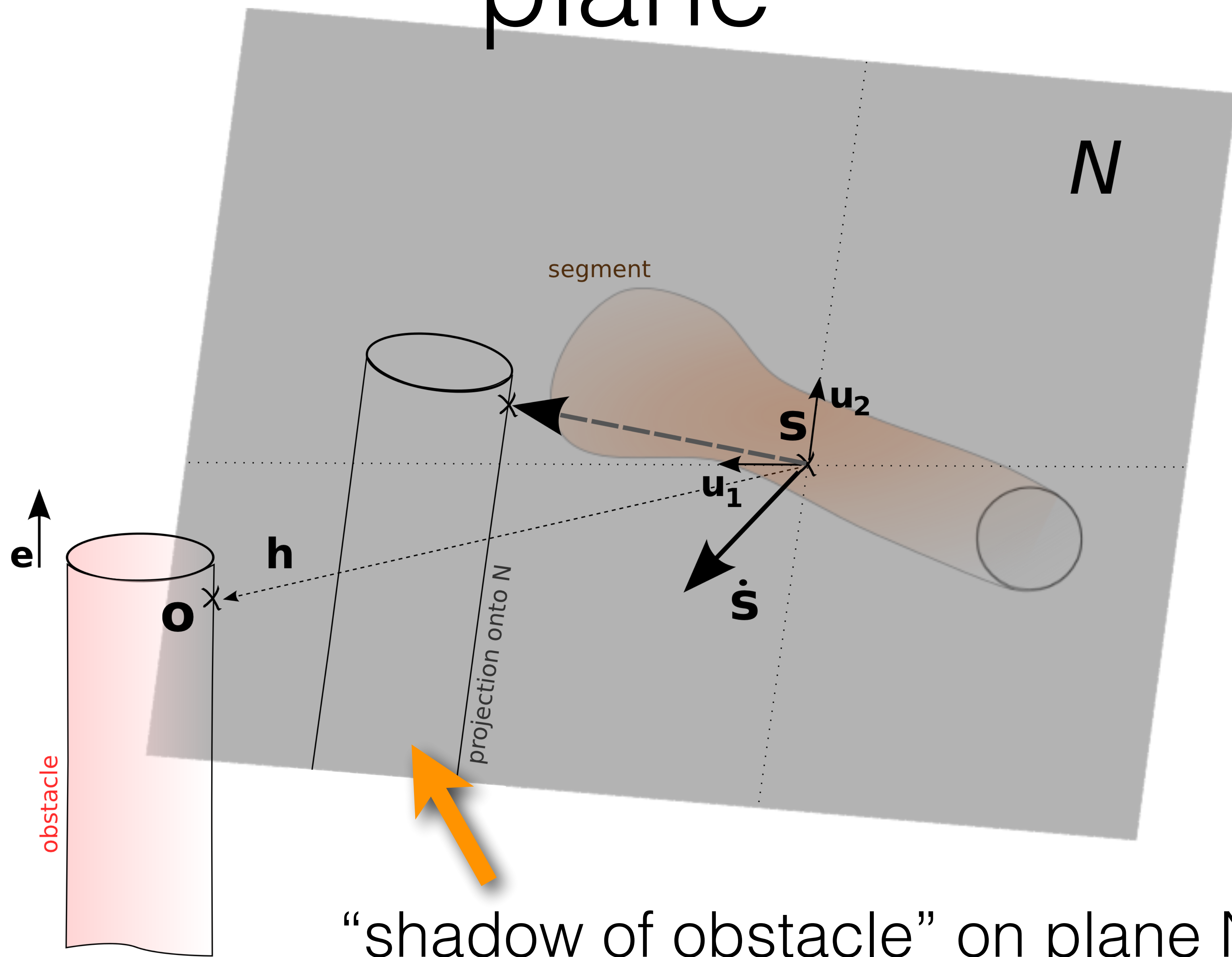
- problem: *all links* must be able to avoid. but moving proximal links also moves distal ones (kinematic chain) for *every link*:

- find closest points $\mathbf{o}$ on obstacle and $\mathbf{s}$ on link
- in what direction does link point $\mathbf{s}$ currently move?
- in what direction should it move?

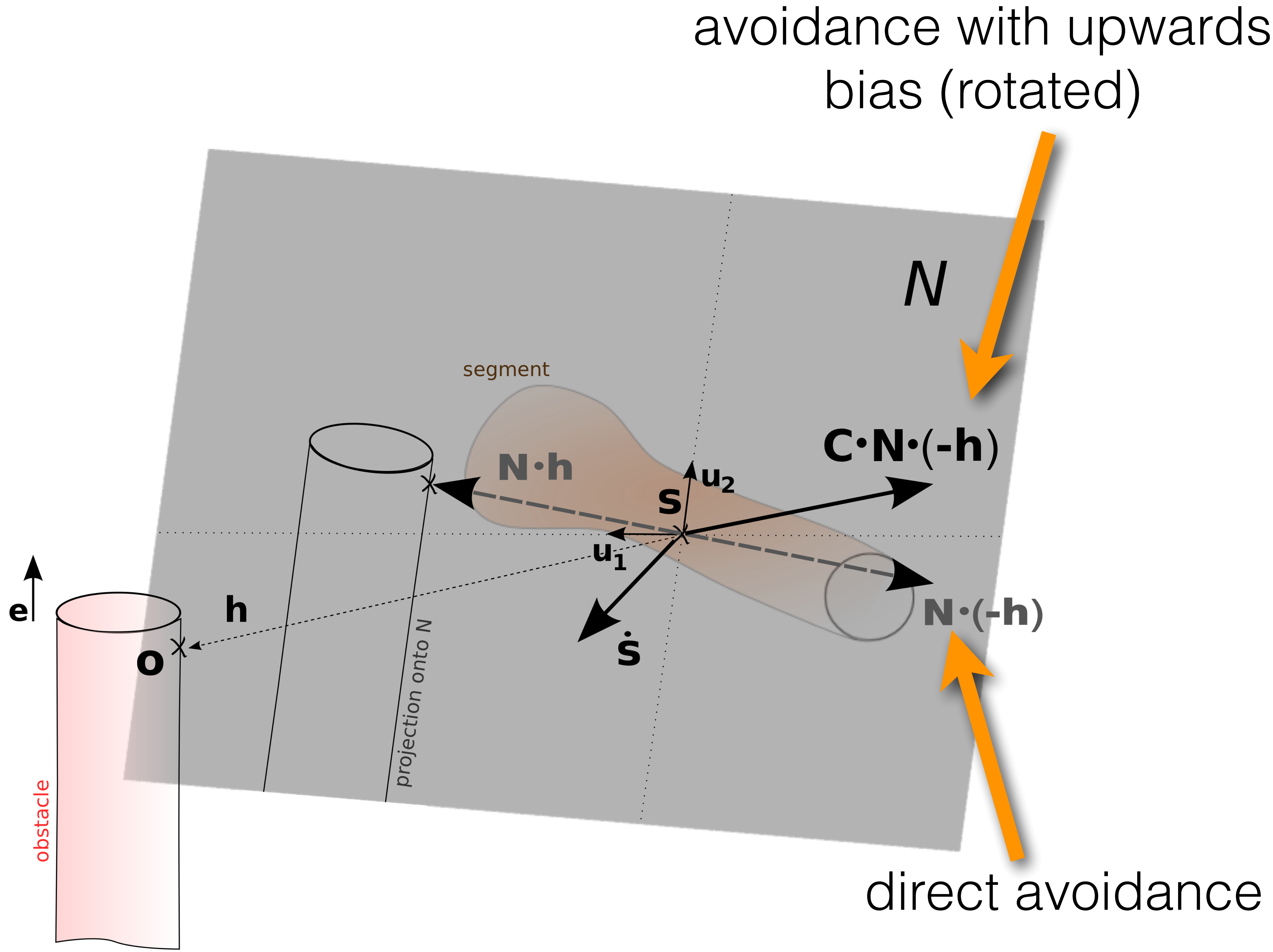


note: $\mathbf{s}$ does not have the same forward kinematics and not the same Jacobian as the end-effector!

construction on normal plane

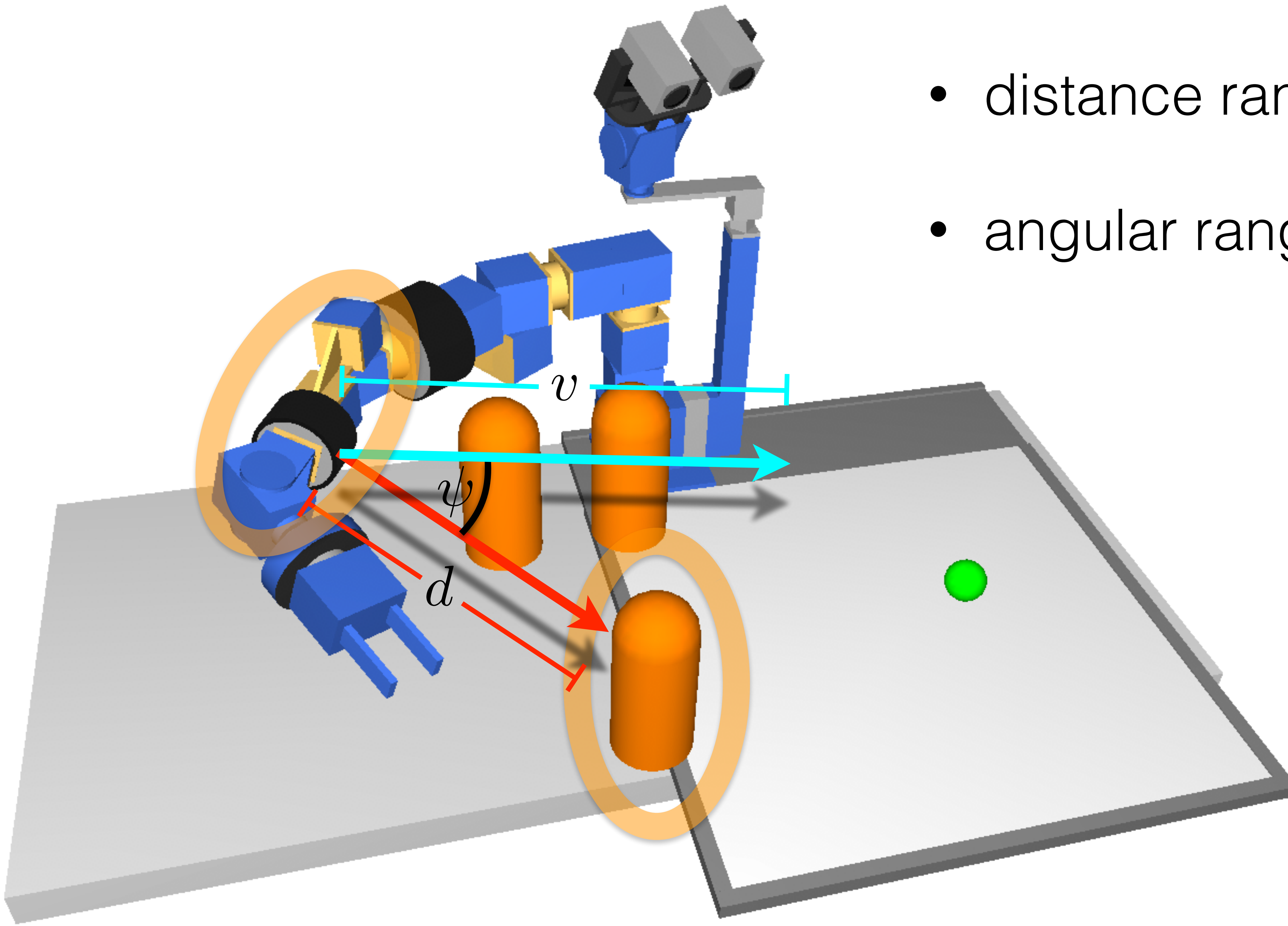


“shadow of obstacle” on plane N



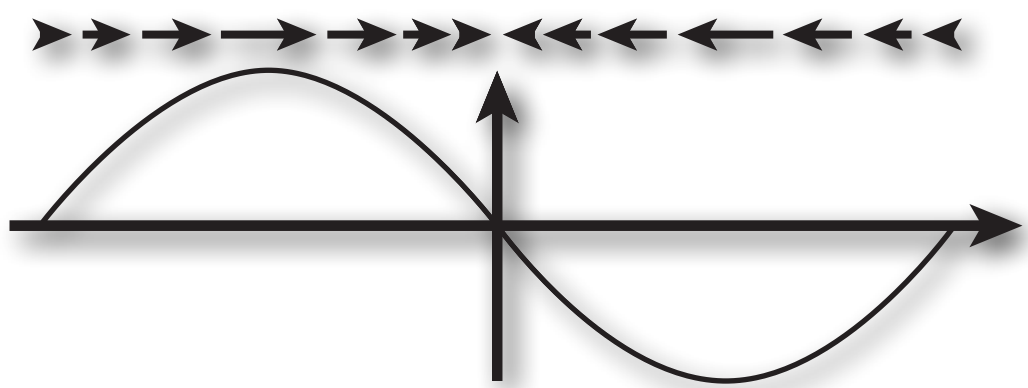
avoidance with upwards bias (rotated)
N
segment
C·N·(-h)
N·h
S
u2
u1
e
h
O
N·(-h)
Ṡ
projection onto N
obstacle
direct avoidance

other parameters

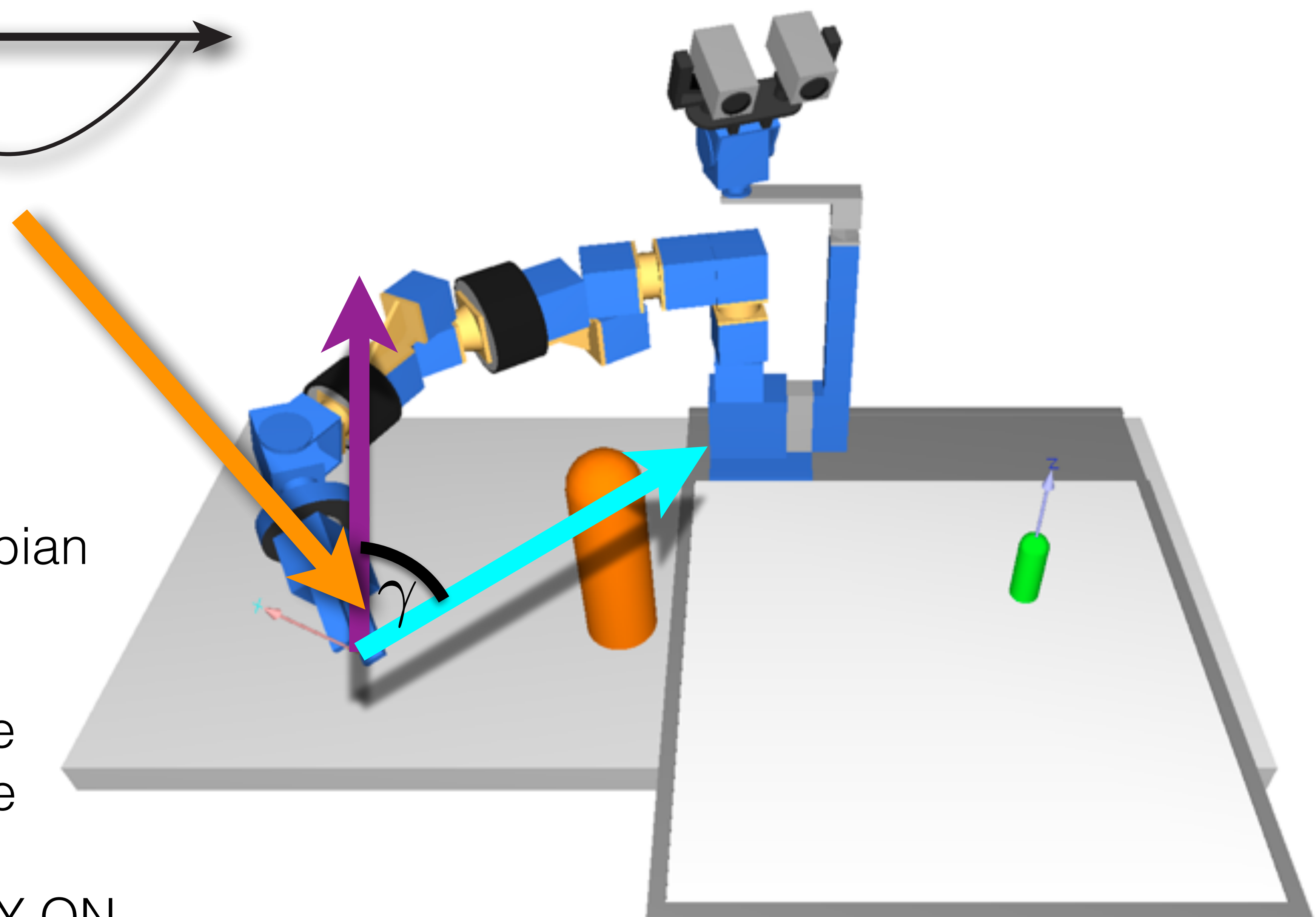


- distance range
- angular range

gripper orientation

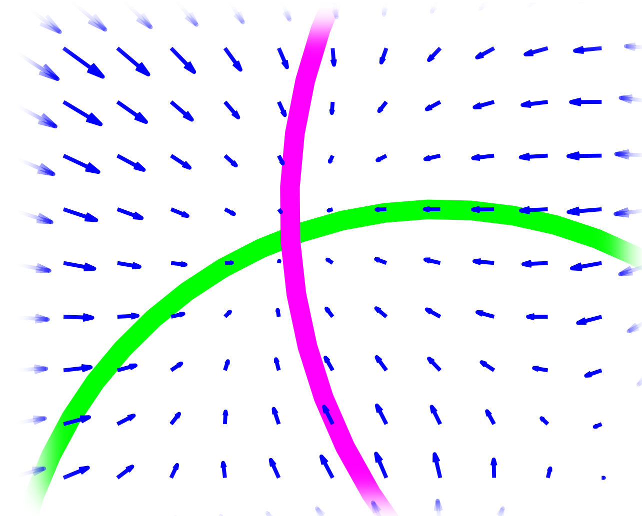


$$f_{ori} = -\alpha_\gamma \sin \gamma$$



- angle dynamics
- different geometrical construction and Jacobian but same principle
- requires one DoF of the system, thus preferable only enforce when necessary. NOT ALWAY ON

superposition of tasks



- finally, superpose all independently stabilizing vector fields:

$$\mathbf{F} = \mathbf{F}_{dir} + \mathbf{F}_{vel} + \sum_{obs,seg} \mathbf{F}_{obs}$$

- interpret the vector-field as acceleration command:

$$\ddot{\theta} = \mathbf{F}$$