

May 3, 2019

## Exercise 2 [triple points]

This is a longer and more complex exercise, that wins you triple bonus points according to the rules. Hand in your essay on Thursday, the 23th of May, 2019, which gives you 3 weeks to perform the work. The text can be written in English or German. Use complete sentences and organize your text into sections. This is important for the list of questions asked in points 2 and 3 below.

Read the paper (available from the course web page): A Dynamical Model of Visually-Guided Steering, Obstacle Avoidance, and Route Selection by B R Fajen, W H Warren, S. Temizer, L P Kaelbling *International Journal of Computer Vision* **54**(1/2/3), 1334, 2003

In a first reading, ignore problems you have comprehending, only look up words critical to your understanding, try to get a picture of the whole article first. Mark passages you don't understand (but don't work through them yet), underline points you do understand and find important. Try to gain an understanding of what the components of the paper are.

Two of the questions below ask you to do some simple math, looking things up things in books or web pages where you do not know the methods. An excellent resource is a book that is available online as a free download (thanks to the author's generosity): Edward R. Scheinerman's Invitation to Dynamical Systems. See here: <http://www.ams.jhu.edu/ers/books/invitation-to-dynamical-systems/>.

1. Write a short summary of what you think are the main goals of the paper. Do not simply copy the abstract, in fact, avoid copying text verbatim. You will do best to first write a draft of this summary and then come back to this summary later after working on the other questions. Then revise this text as your understanding of the paper improves.
2. To establish the link between this model and the model of Schöner and Dose (1992) study the two contributions to Equation 4 separately. First study:

$$\ddot{\phi} = -b\dot{\phi} - k_g(d)(\phi - \psi_g)$$

where I have absorbed the distance dependence terms into a function  $k_g(d)$ . Rewrite this equation to compare it to a damped harmonic oscillator. To do so, look up the topic "harmonic oscillator" in a text book or web resource (a math treatment is in Scheinermann, page from page 64 on, but look up a simpler text first). Then establish a mapping of the equation to the damped harmonic oscillator equation. Assume that the distance,  $d$ , remains constant (e.g., the robot

is rotating on the spot). By looking at the solutions of harmonic oscillators in the source you found, you can predict the solutions to this equation. Describe, what these solutions imply for how the robot's orientation in space evolves in time. Distinguish under-damped and overdamped regimes.

3. Second, study

$$\ddot{\phi} = -b\dot{\phi} + k_o(d)(\phi - \psi_0)$$

in which I have absorbed the distance dependent term into  $k_o$  and neglected the range-limiting exponential. Articulate, why this is no longer a damped harmonic oscillator. Expand this equation into two first order differential equations by introducing a second variable,  $\dot{\phi} = \omega$  (which gives you one of the two first order differential equations). Compute the fixed point of that system of differential equations by setting the left sides to zero. Determine two equations for the deviations  $(\epsilon, \delta)$  from the fixed point in  $(\phi, \omega)$  space. Compute the eigenvalues of the two by two matrix you obtain on the right hand side. If you don't know how to do that, look up the problem of eigenvalues on a two by two matrix in a text book or online (e.g., on page 50 of Scheinermann). Examine the sign of the eigenvalues and compare to the linear stability theorem reviewed in the tutorial lecture on dynamical systems. Does this match what you expect of an obstacle avoidance term?

4. The paper describes an alternative model, the "potential field" approach. To better understand this method, read Arkin: Motor Schema Based Mobile Robot Navigation. *International Journal of Robotics Research* 8:92-110 (1989), also available on the web page. This paper you need to read in less detail. Based on the two descriptions, deliver your own verbal account for how the potential field approach works. Summarize why the authors of the Fajen et al paper reject this as a model of human walking?
5. Write a short text about what you find the most important or most interesting point of the Fajen et al work. You may also make critical assessments, if you disliked some aspect of the paper.