Neural Dynamics

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Activation

how to represent the inner state of the Central Nervous System?

=> activation concept
Activation

- neural state variables
  - membrane potential of neurons?
  - spiking rate?
  - ... population activation...
Activation

- Activation as a real number, abstracting from biophysical details
- Low levels of activation: not transmitted to other systems (e.g., to motor systems)
- High levels of activation: transmitted to other systems
- As described by sigmoidal threshold function
- Zero activation defined as threshold of that function
Activation

- compare to connectionist notion of activation:
  - same idea, but tied to individual neurons

- compare to abstract activation of production systems (ACT-R, SOAR)
  - quite different... really a function that measures how far a module is from emitting its output...
Activation dynamics

- activation variables $u(t)$ as time continuous functions...

\[ \tau \frac{\dot{u}(t)}{dt} = f(u) \]

- what function $f$?
Activation dynamics

\[ \tau \dot{u} = \xi_t \]

- start with \( f=0 \)

\[ \tau \dot{u} = \xi_t \]

probability distribution of perturbations
Activation dynamics

\[ \tau \dot{u} = -u + h + \xi_t. \]

- need stabilization
In a dynamical system, the present predicts the future: given the initial level of activation $u(0)$, the activation at time $t$: $u(t)$ is uniquely determined.

$$\frac{du(t)}{dt} = \dot{u}(t) = -u(t) + h \quad (h < 0)$$
mental simulation

=> dynamical systems tutorial Mathis Richter
Neural dynamics

- Stationary state = fixed point = constant solution
- Stable fixed point: nearby solutions converge to the fixed point = attractor

\[ \frac{du(t)}{dt} = \dot{u}(t) = -u(t) + h \quad (h < 0) \]
Neural dynamics

Attractor structures ensemble of solutions = flow

\[ \tau \dot{u}(t) = -u(t) + h \]

resting level

time, t

resting level

vector-field

du/dt = f(u)
Neuronal dynamics

- Inputs = contributions to the rate of change
  - Positive: excitatory
  - Negative: inhibitory
- Shifts the attractor
- Activation tracks this shift (stability)

\[ \tau \dot{u}(t) = -u(t) + h + \text{inputs}(t) \]
=> simulation
tutorial on numerics

dynamical system
continuous time

differential
quotient
approximates the
derivative in
discrete time

Euler iteration
equation in
discrete time

\[ \dot{u} = f(u). \]

\[ \dot{u}(t_i) \approx \frac{u(t_i) - u(t_{i-1})}{\Delta t} \]

\[ u(t_i) = u(t_{i-1}) + \Delta t f(u(t_{i-1})). \]
Matlab code
Neuronal dynamics with self-excitation

\[ \tau \dot{u}(t) = -u(t) + h + S(t) + c\sigma(u(t)) \]
Neuronal dynamics with self-excitation

\[ \tau \dot{u}(t) = -u(t) + h + S(t) + c\sigma(u(t)) \]

\[ u(0) = u_0 \]

- \[ \Rightarrow \] nonlinear dynamics!
Neuronal dynamics with self-excitation

\[ \tau \dot{u}(t) = -u(t) + h + S(t) + c\sigma(u(t)) \]
Neuronal dynamics with self-excitation

- at intermediate stimulus strength: bistable
- “on” vs “off” state

\[ \tau \dot{u}(t) = -u(t) + h + S(t) + c\sigma(u(t)) \]
Neuronal dynamics with self-excitation

- Increasing input strength $\Rightarrow$ detection instability
Neuronal dynamics with self-excitation

- decreasing input strength  => reverse detection instability

\[
\frac{du}{dt} \quad \text{resting level, } h
\]

stable

unstable

stimulus strength

fixed point

stimulus strength
Neuronal dynamics with self-excitation

- The detection and the reverse detection instability create discrete events out of input that changes continuously in time.
=> simulation
Neuronal dynamics with competition

\[ \tau \dot{u}_1(t) = -u_1(t) + h - \sigma(u_2(t)) + S_1 \]
\[ \tau \dot{u}_2(t) = -u_2(t) + h - \sigma(u_1(t)) + S_2 \]
the rate of change of activation at one site depends on the level of activation at the other site

mutual inhibition

\[ \tau \dot{u}_1(t) = -u_1(t) + h - \sigma(u_2(t)) + S_1 \]
\[ \tau \dot{u}_2(t) = -u_2(t) + h - \sigma(u_1(t)) + S_2 \]

sigmoidal nonlinearity
Neuronal dynamics with competition

- to visualize, assume that $u_2$ has been activated by input to positive level
- $\Rightarrow$ then $u_1$ is suppressed
why would $u_2$ be positive before $u_1$ is? E.g., it grew faster than $u_1$ because its inputs are stronger(inputs match better)

$\Rightarrow$ input advantage translates into time advantage which translates into competitive advantage
Neuronal dynamics with competition

vector-field in the absence of input

ID cut through vector-field

\[ \frac{du}{dt} = f(u) \]
Neuronal dynamics with competition

vector-field (without interaction) when both neurons receive input

\[ \frac{du}{dt} = f(u) \]

stimulus determined state

ID cut through vector-field

activated level
only activated neurons participate in interaction!

sigmoidal nonlinearity
Neuronal dynamics with competition

Vector-field of mutual inhibition

Site 1 inhibits site 2

Site 2 inhibits site 1

Interaction combined
Neuronal dynamics with competition

vector-field with strong mutual inhibition: bistable
Neuronal dynamics with competition

before input is presented

after input is presented
Neuronal dynamics with competition

\[ \Rightarrow \text{biased competition} \]

stronger input to site 1:
attractor with activated $u_1$ stronger,
attractor with activated $u_2$ weaker, may become unstable

\[
\begin{align*}
\text{input} & \quad \text{interaction} & \quad \text{total} \\
\end{align*}
\]
Neuronal dynamics with competition

=> biased competition

before input is presented

after input is presented
=> simulation
=> hands-on exercise NOW
in the robotics lab..
where do activation variables come from?

=> DFT lecture