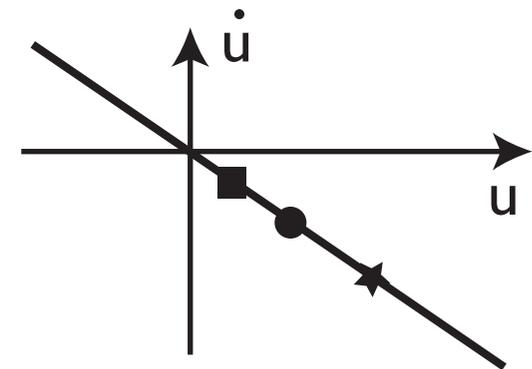
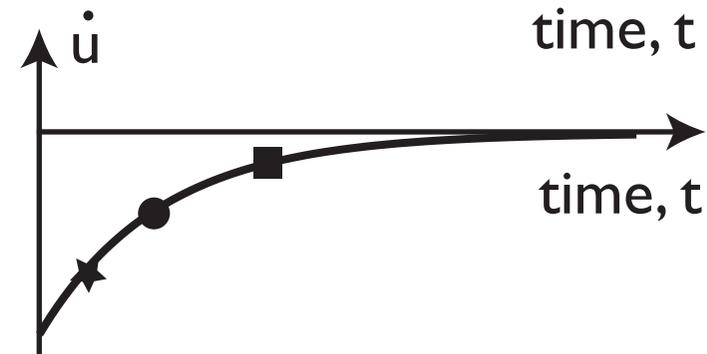
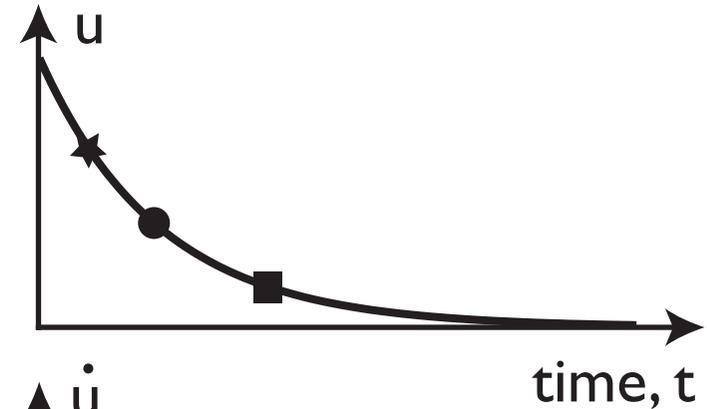


Summary: main conceptual points

Gregor Schöner, INI, RUB

Dynamical systems

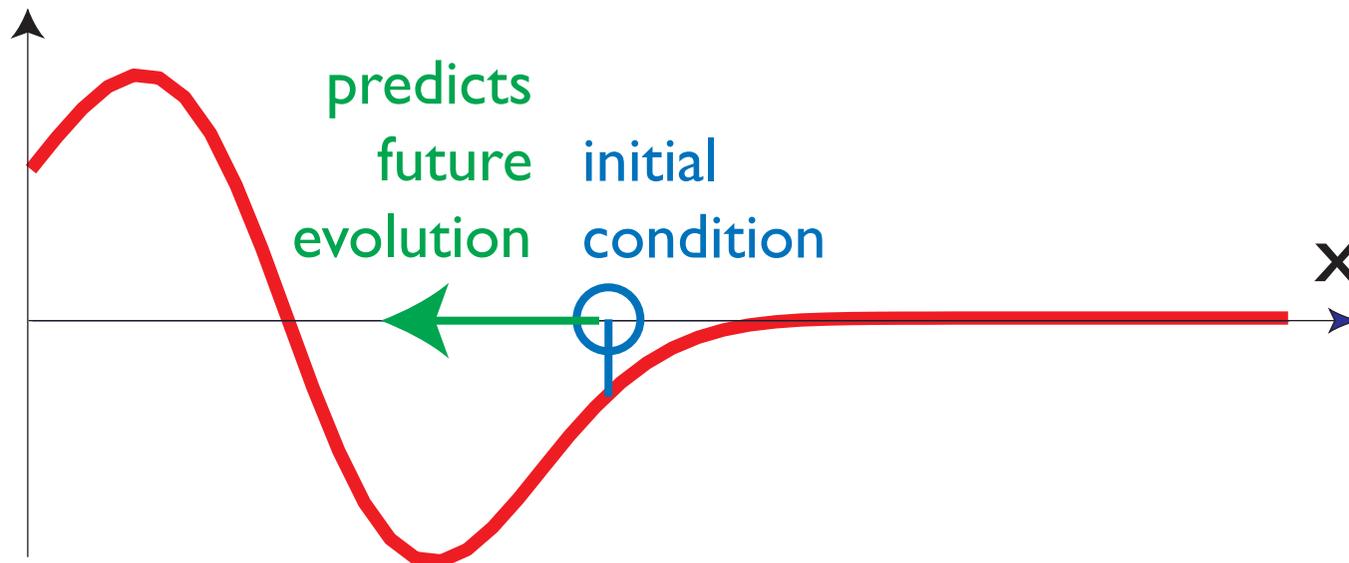
■ functional link between state and its rate of change



Dynamical system

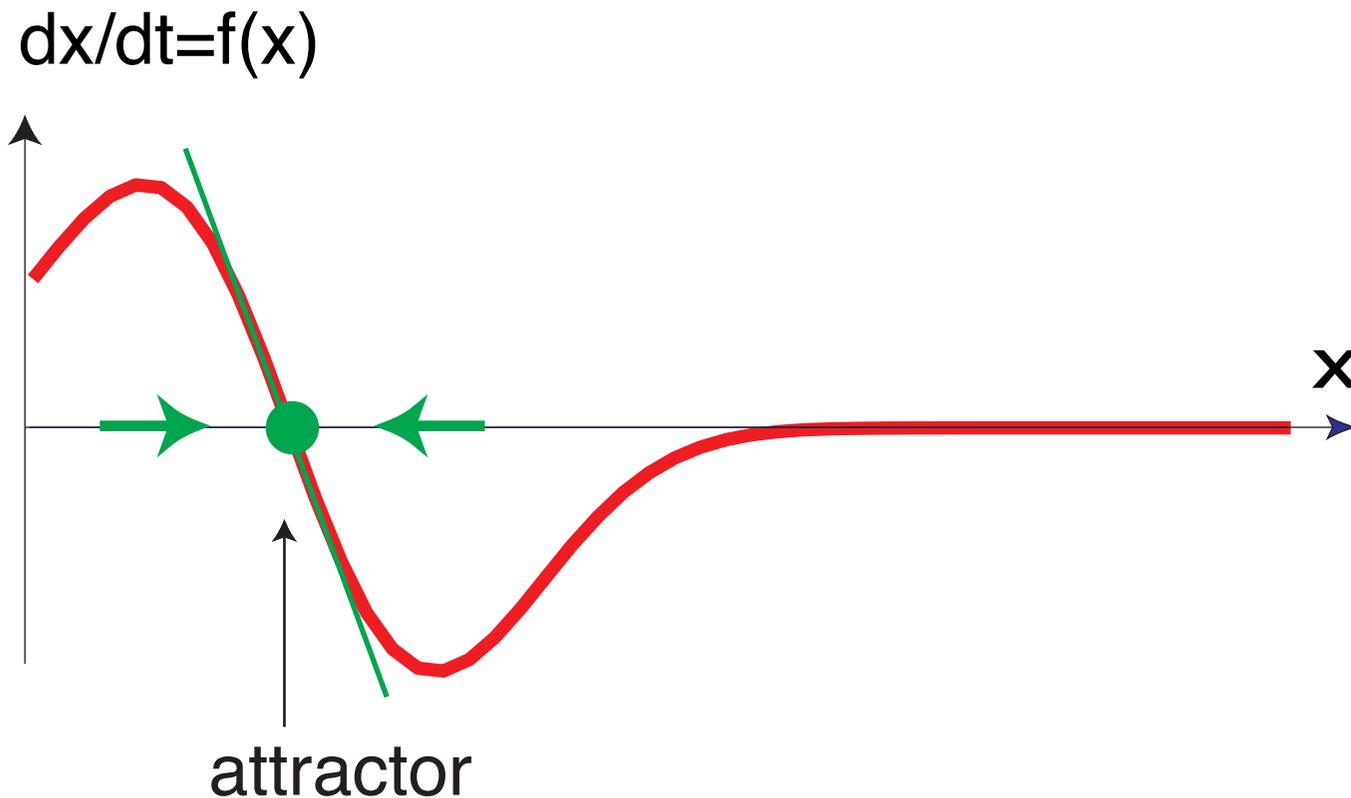
- present determines the future

$$dx/dt=f(x)$$



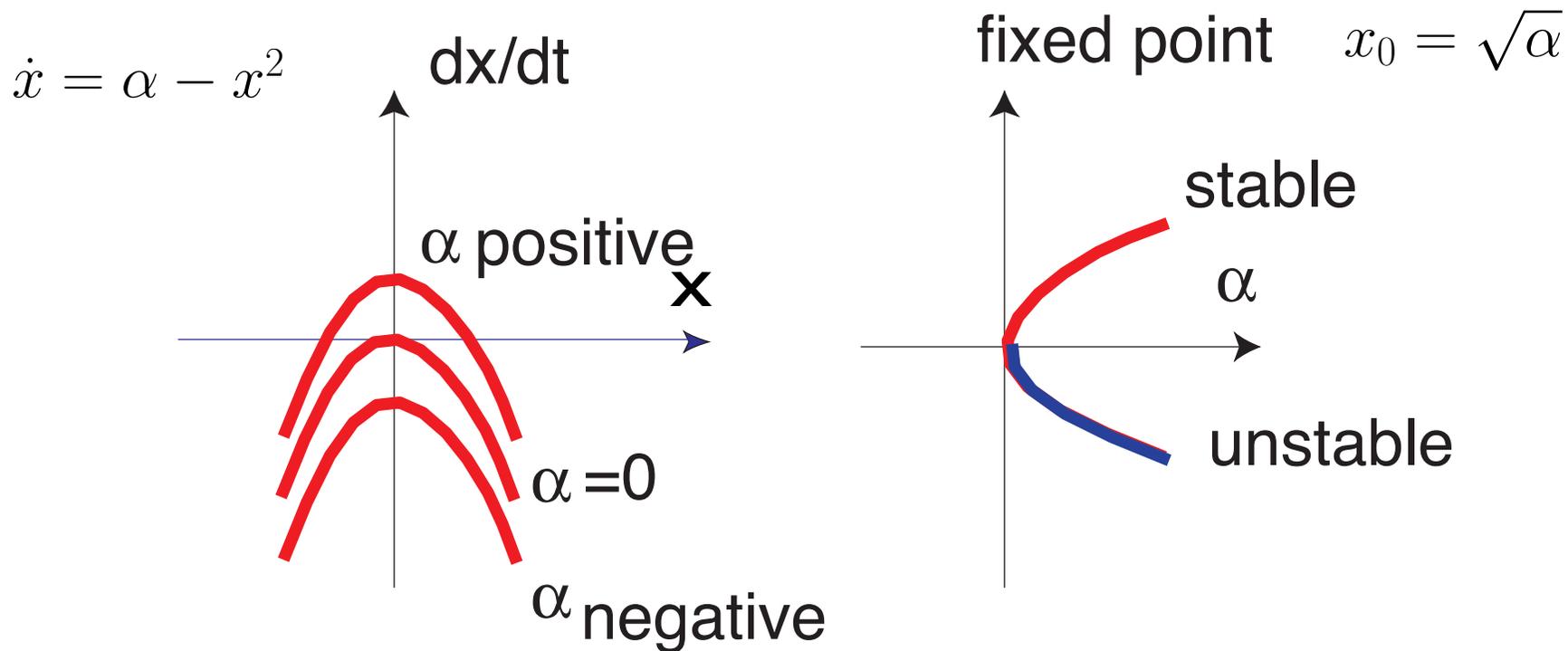
Dynamical systems

- **fixed point** = constant solution
- neighboring initial conditions converge = **attractor**



Bifurcations are instabilities

- In families of dynamical systems, which depend (smoothly) on parameters, the solutions change qualitatively at bifurcations
- at which fixed points change stability



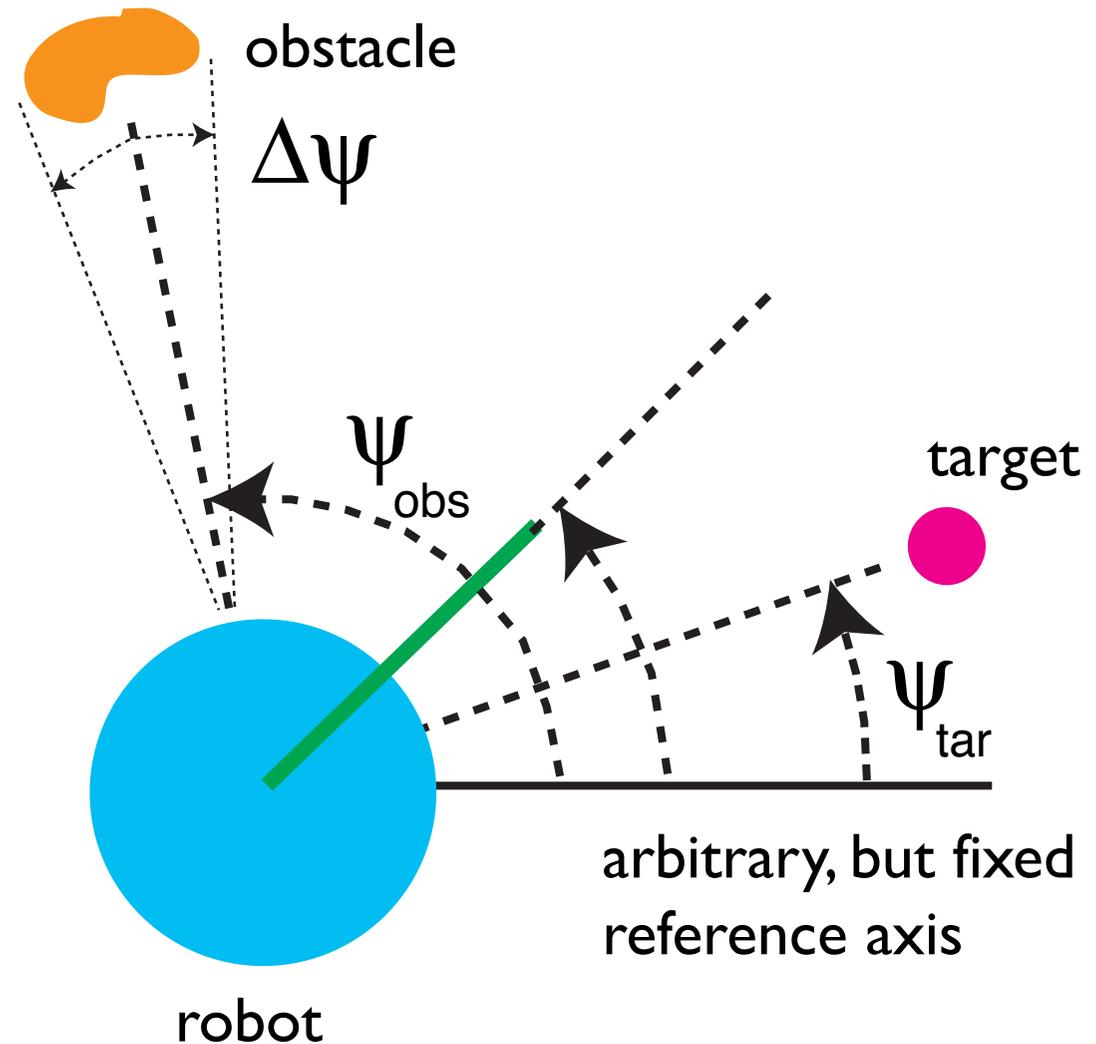
Basic ideas of attractor dynamics approach

- behavioral variables
- time courses from dynamical system:
attractors
- tracking attractors
- bifurcations for flexibility

Behavioral variables: example

2

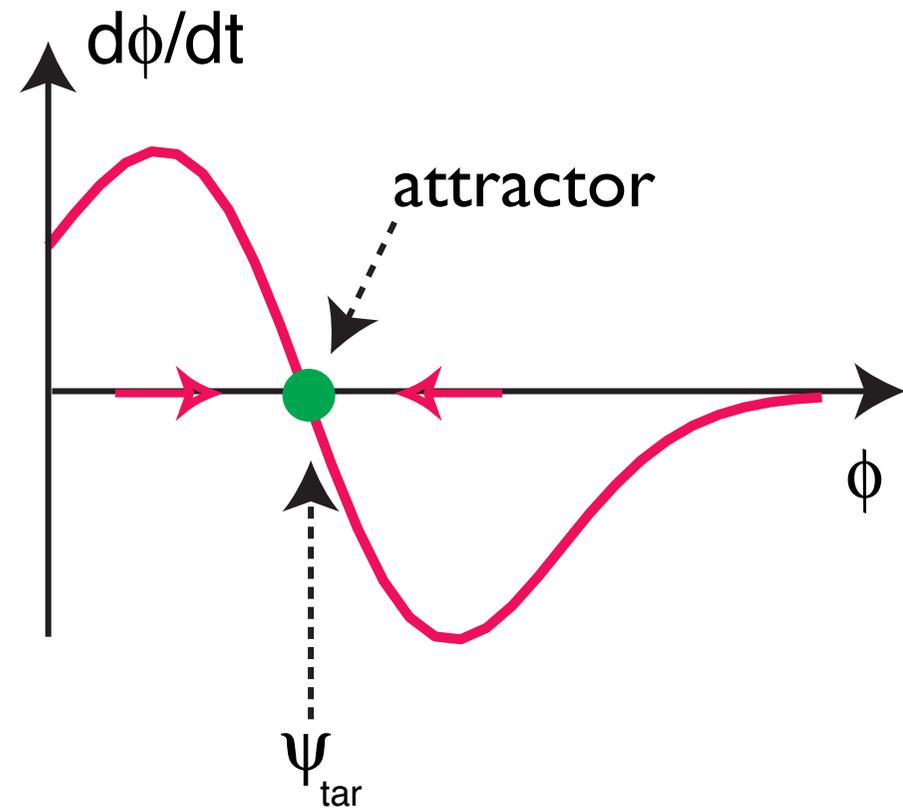
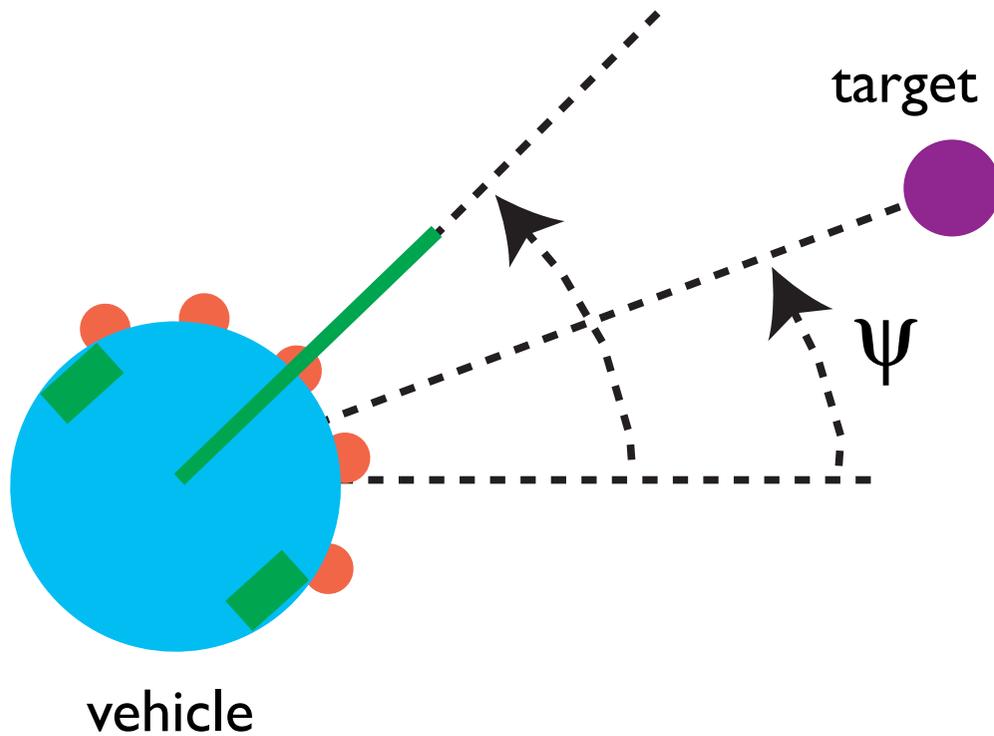
- vehicle moving in 2D: heading direction
- constraints: obstacle avoidance and target acquisition



Behavioral dynamics: example

2

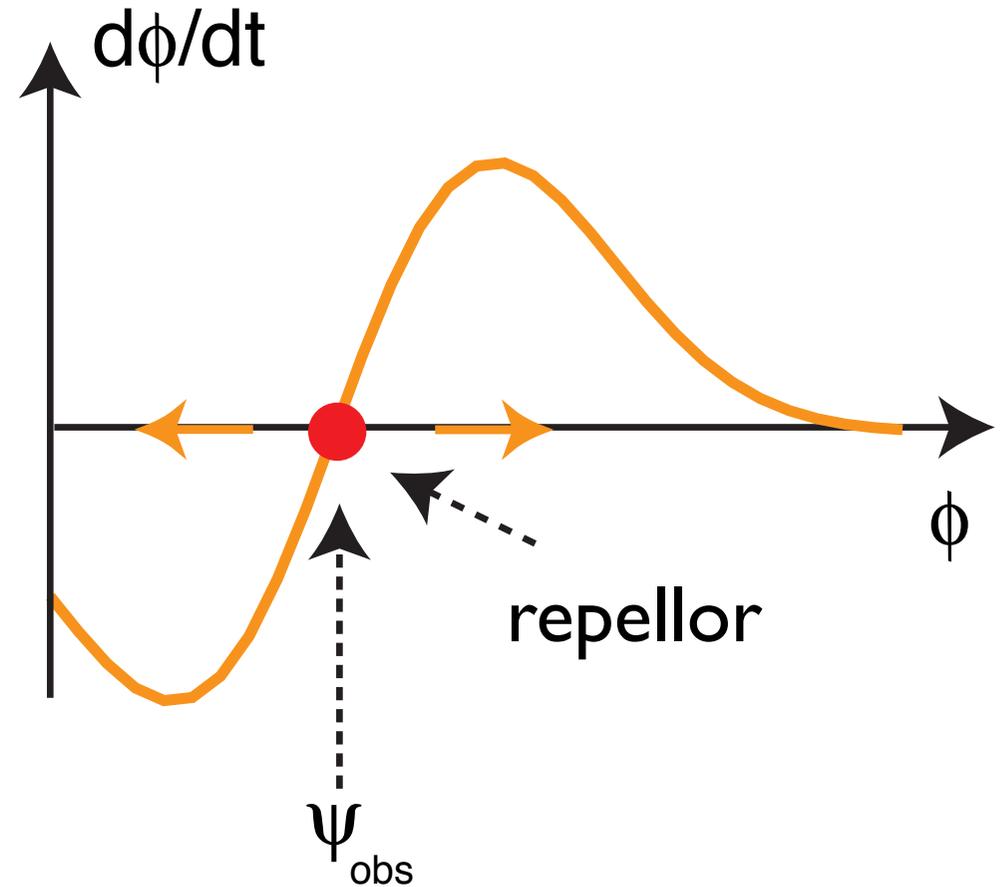
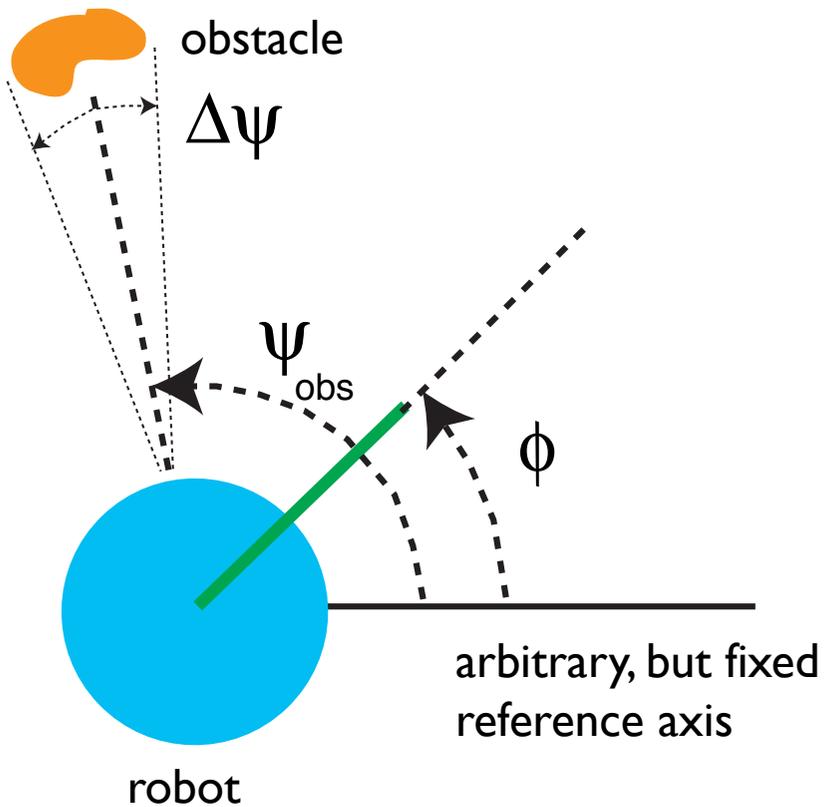
- behavioral constraint: target acquisition



Behavioral dynamics: example

2

■ behavioral constraint: obstacle avoidance



Behavioral dynamics

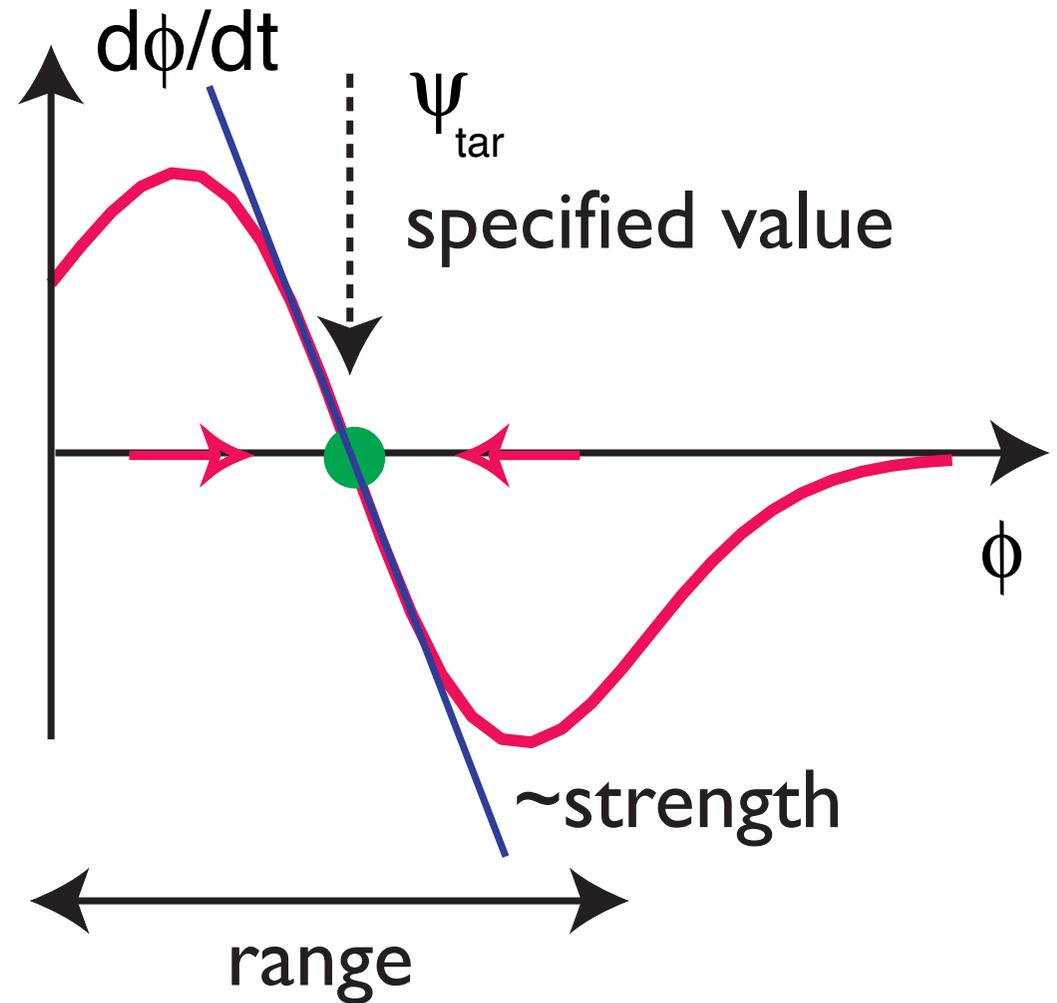
2

■ each contribution is a “force-let” with

■ specified value

■ strength

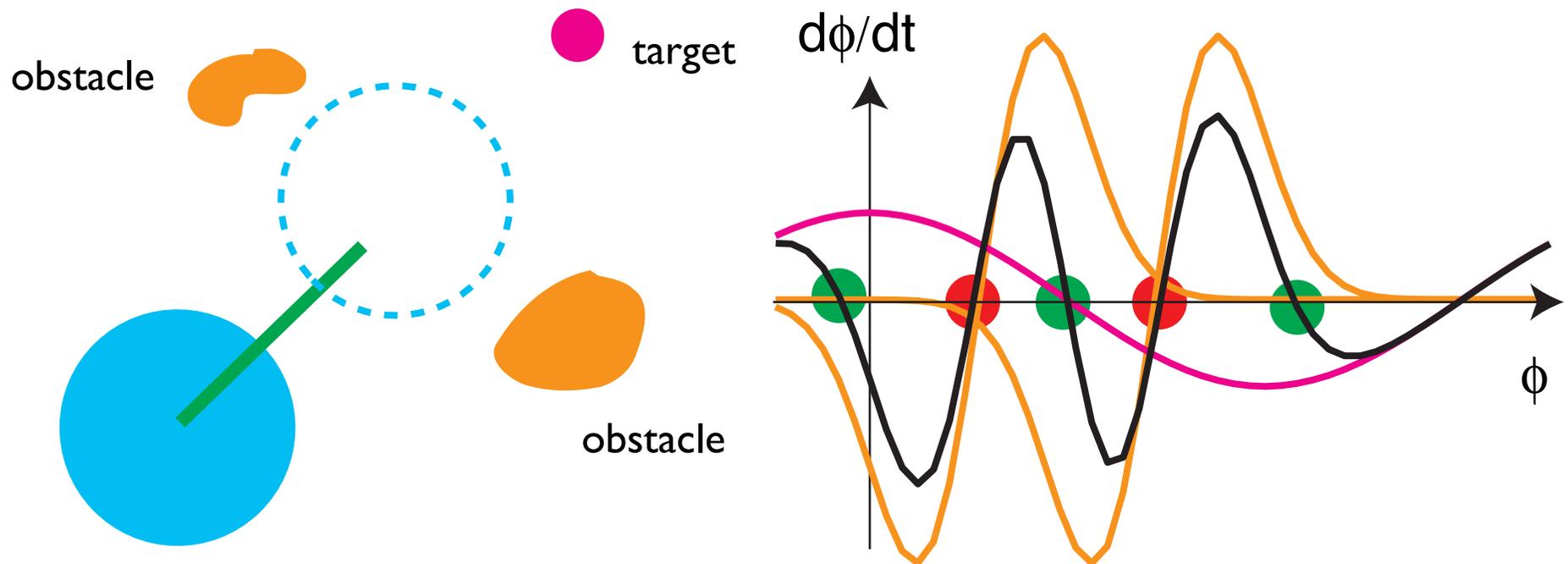
■ range



Behavioral dynamics: bifurcations

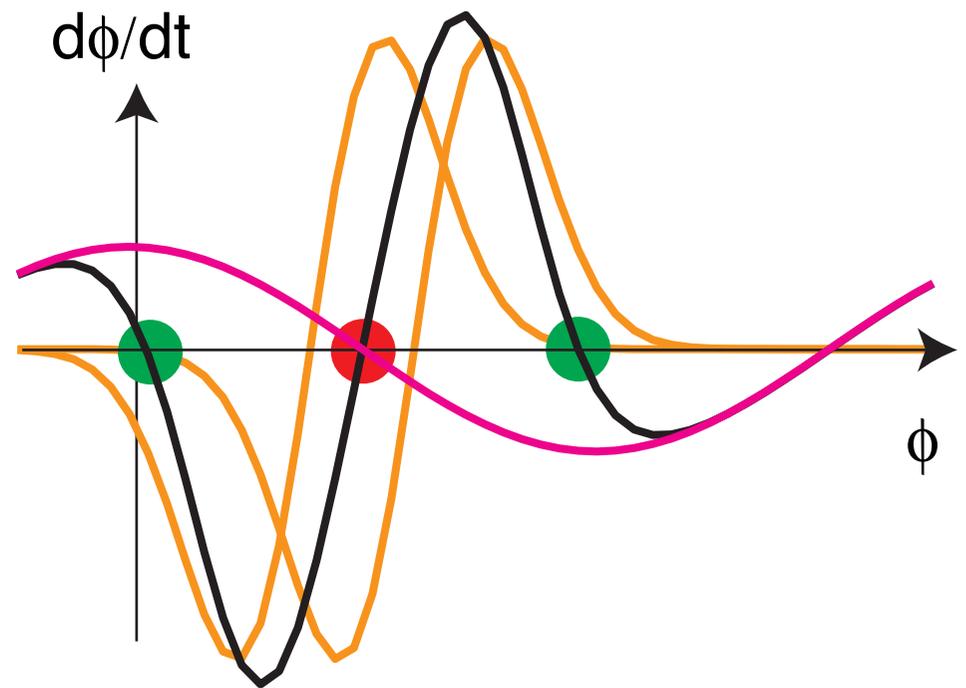
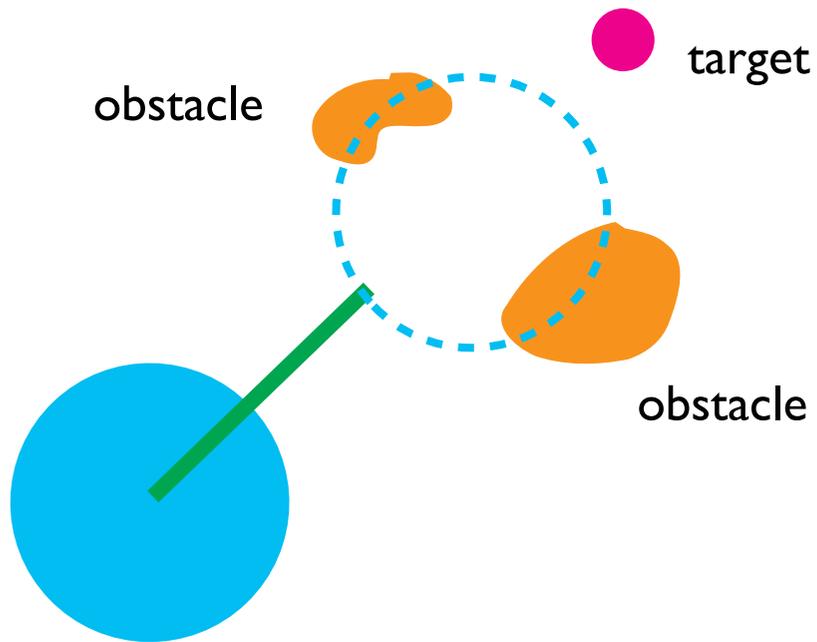
2

■ constraints not in conflict



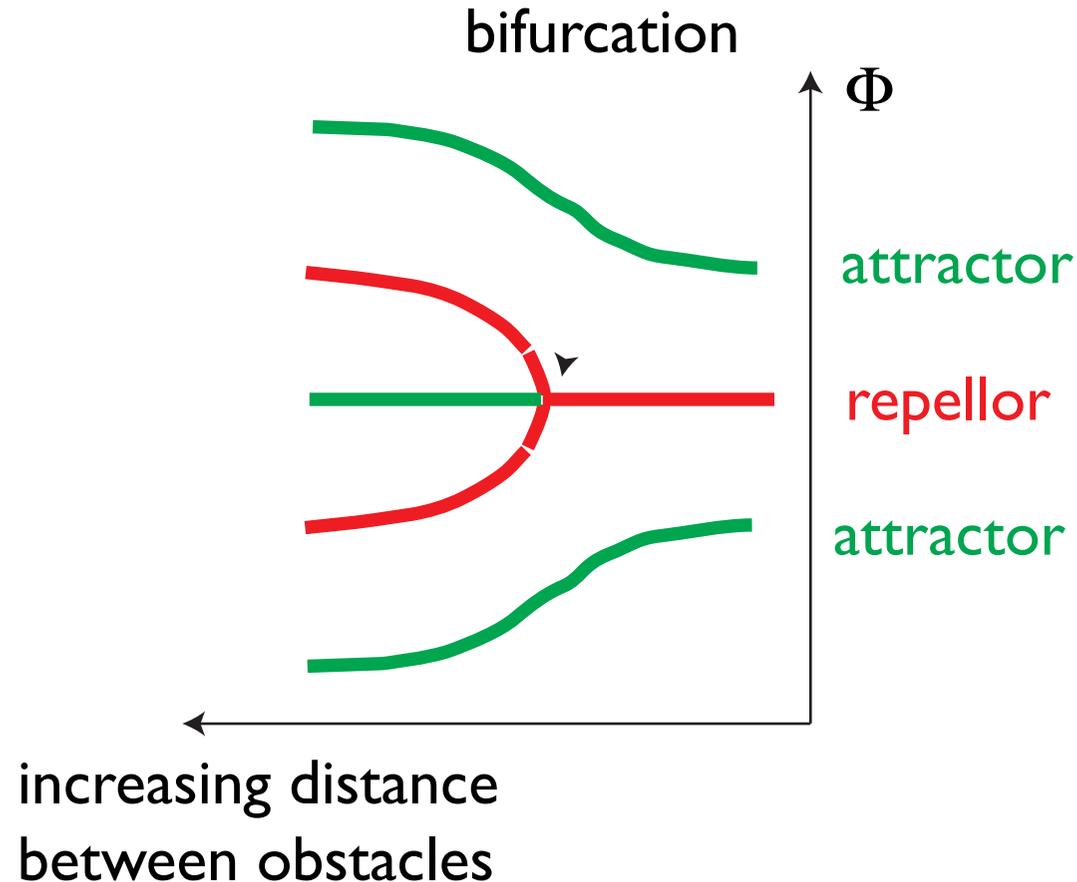
Behavioral dynamics

■ constraints in conflict



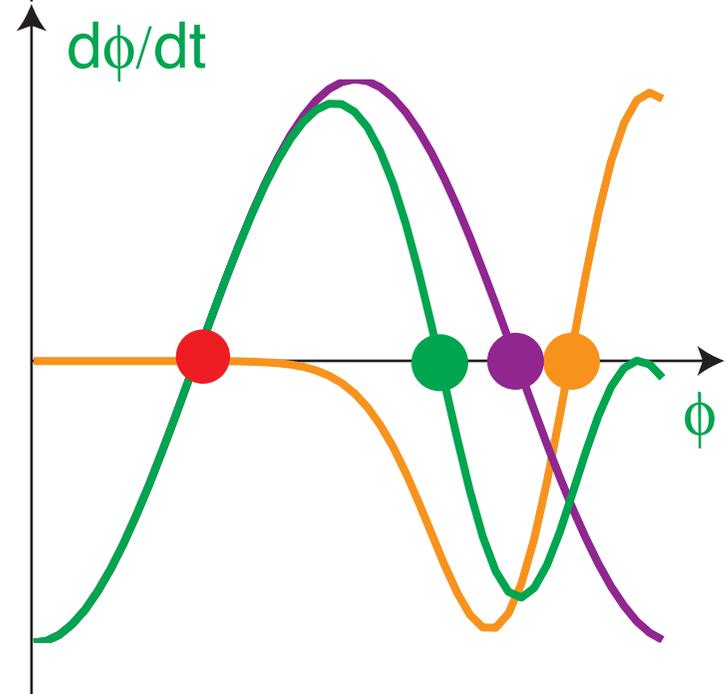
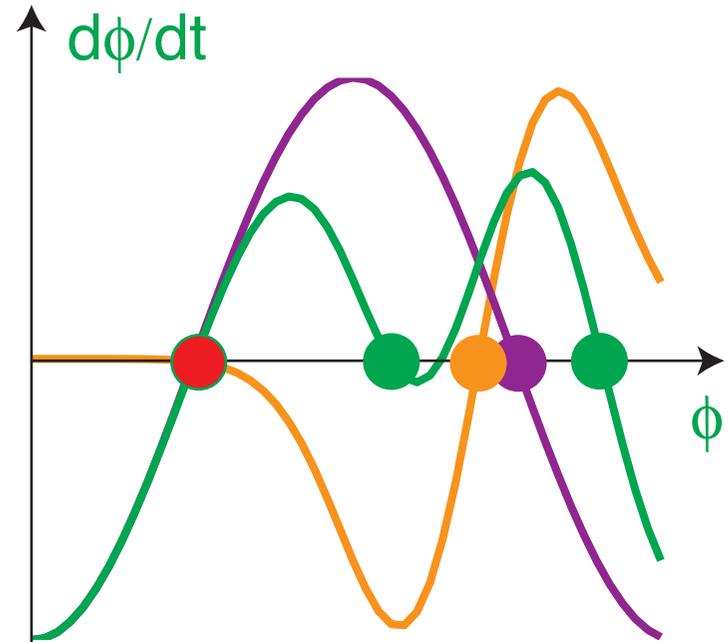
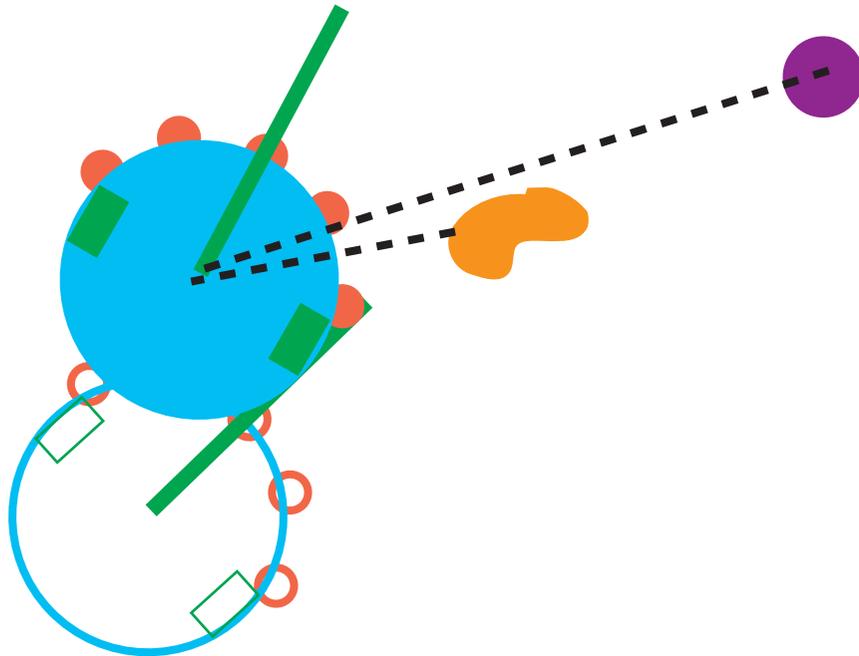
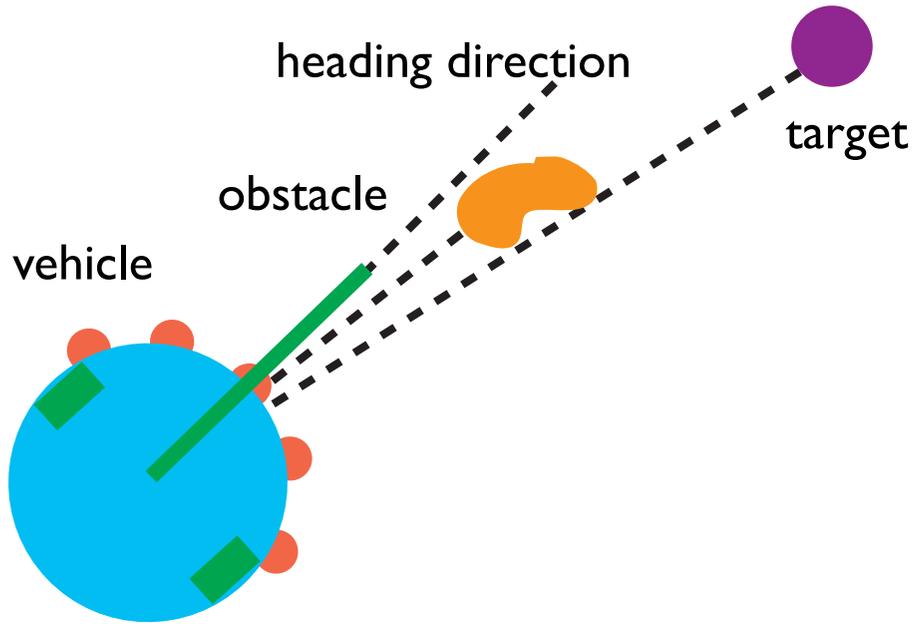
Behavioral dynamics

- transition from “constraints not in conflict” to “constraints in conflict” is a bifurcation



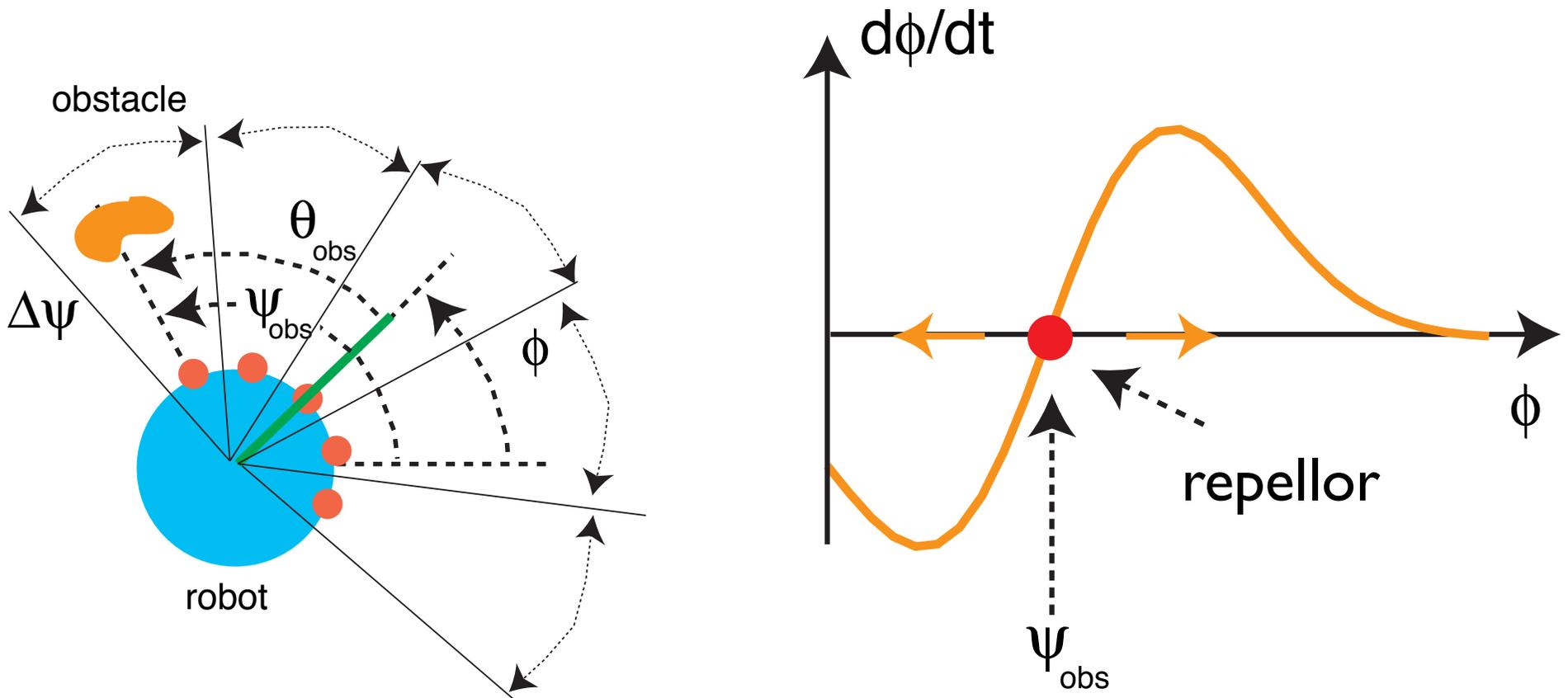
In a stable state at all times

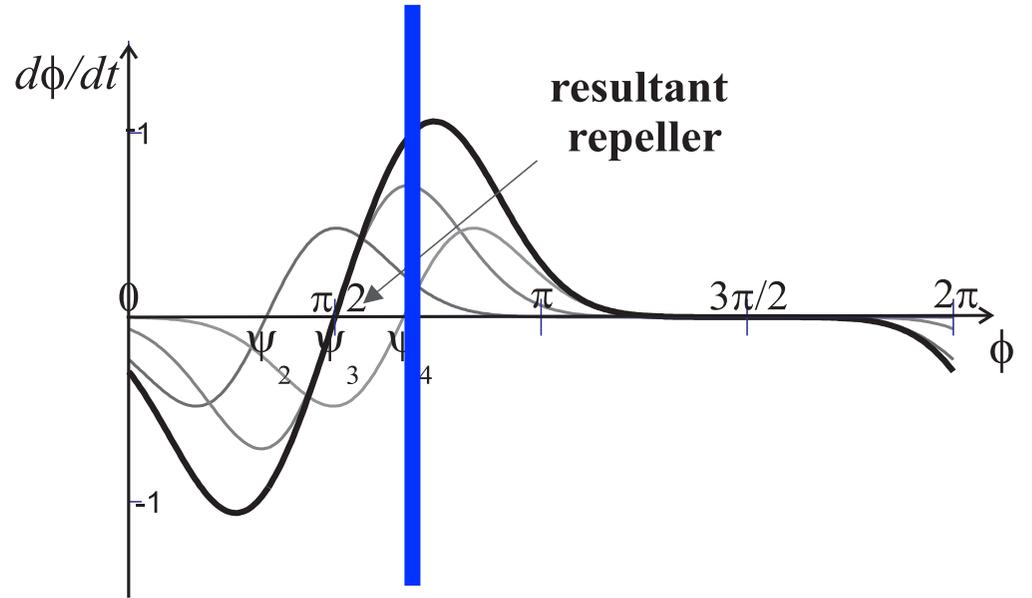
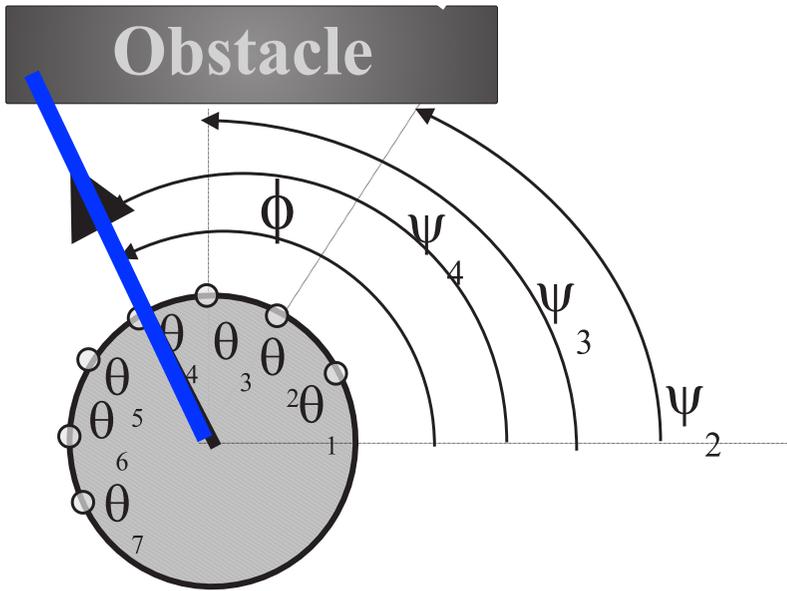
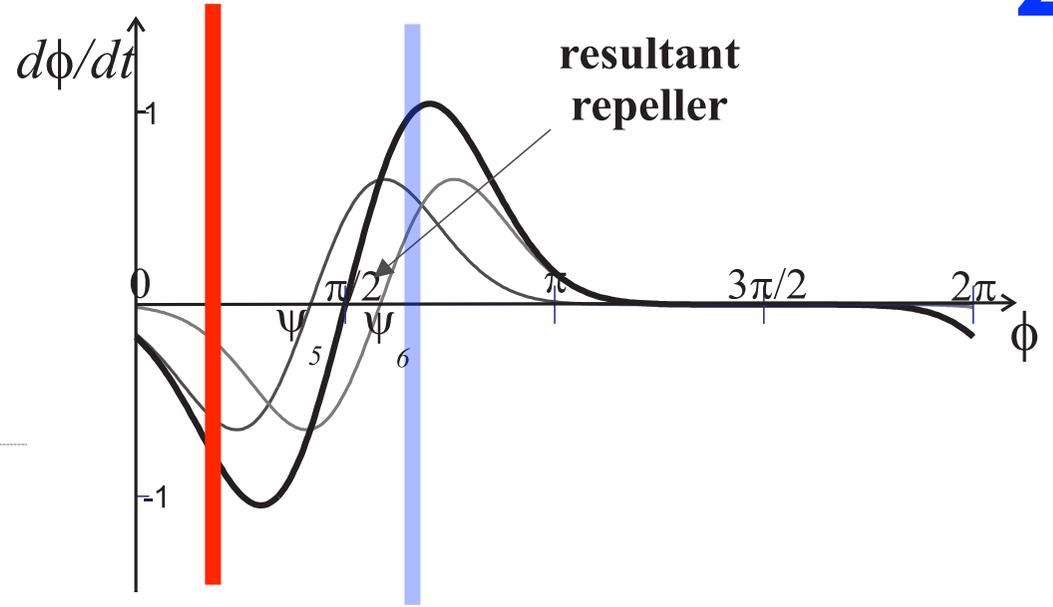
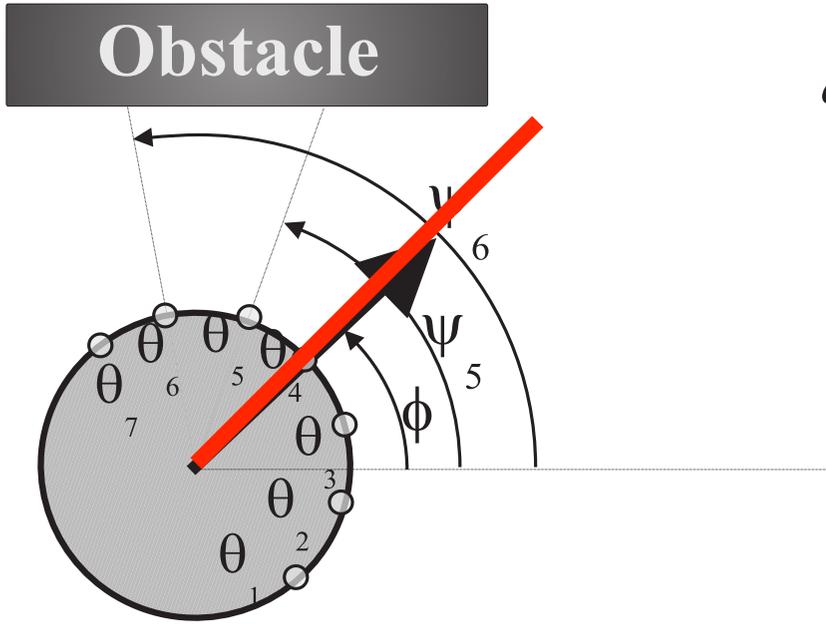
2



Obstacle avoidance: sub-symbolic 4

- obstacles need not be segmented
- do not care if obstacles are one or multiple: avoid them anyway...

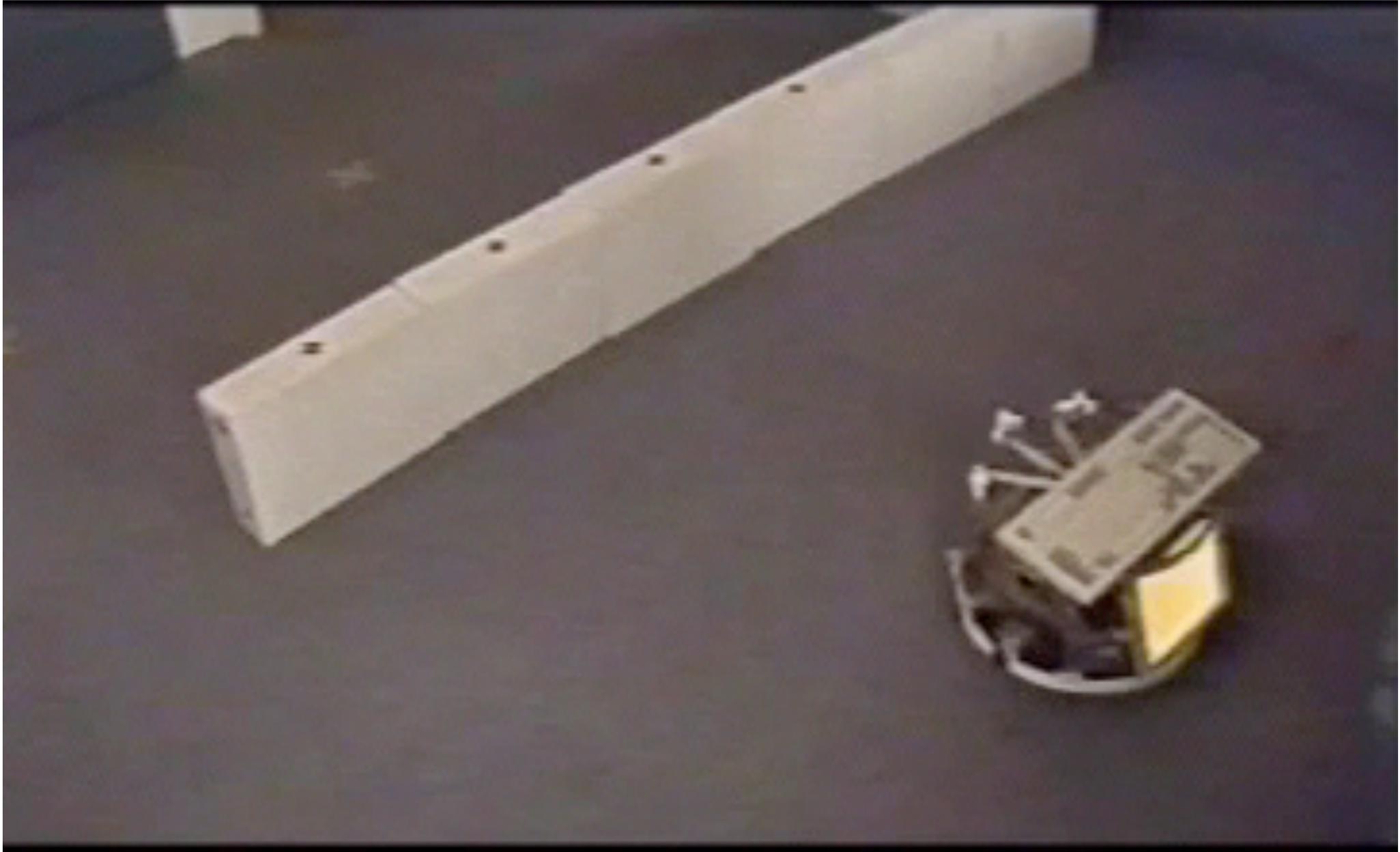




■ => dynamics invariant!

Bifurcations

4



2nd order attractor dynamics to explain human navigation

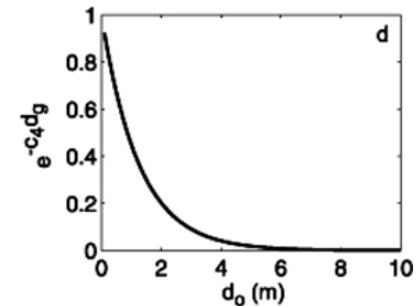
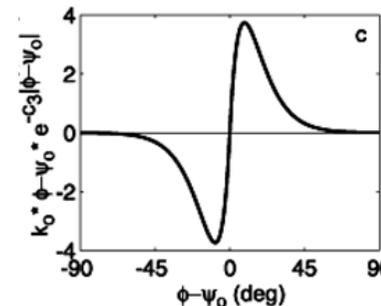
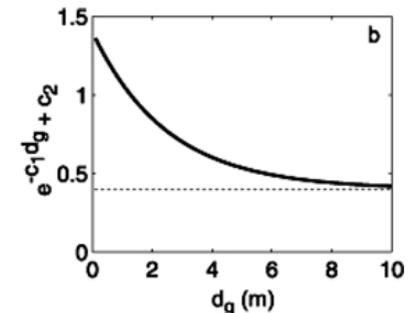
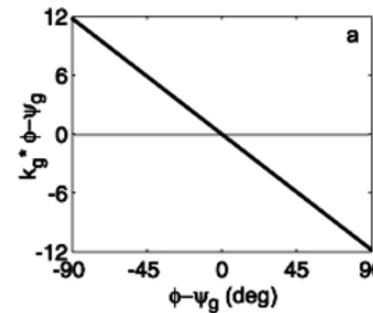
inertial term

damping term

attractor goal heading

$$\ddot{\phi} = -b\dot{\phi} - k_g(\phi - \psi_g)(e^{-c_1 d_g} + c_2) + k_o(\phi - \psi_o)(e^{-c_3 |\phi - \psi_o|})(e^{-c_4 d_o})$$

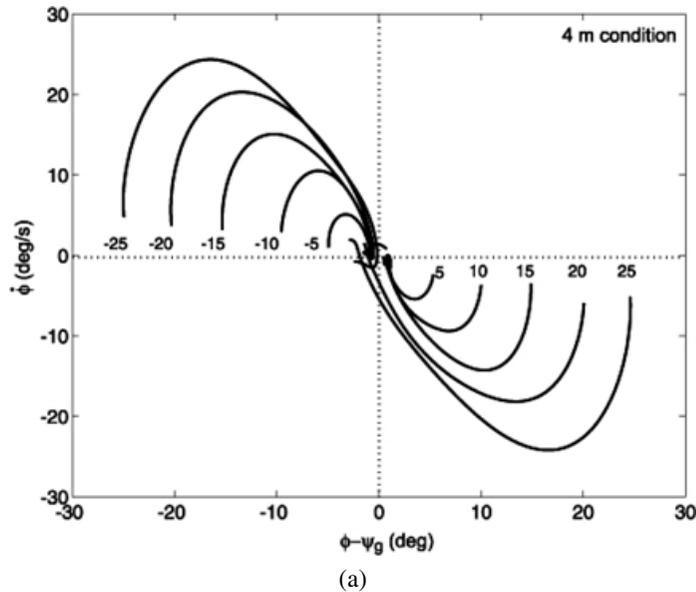
repellor obstacle heading



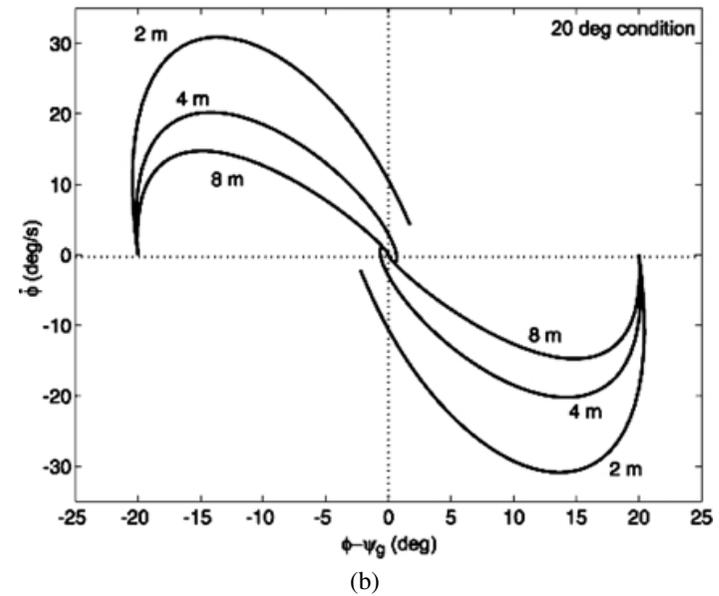
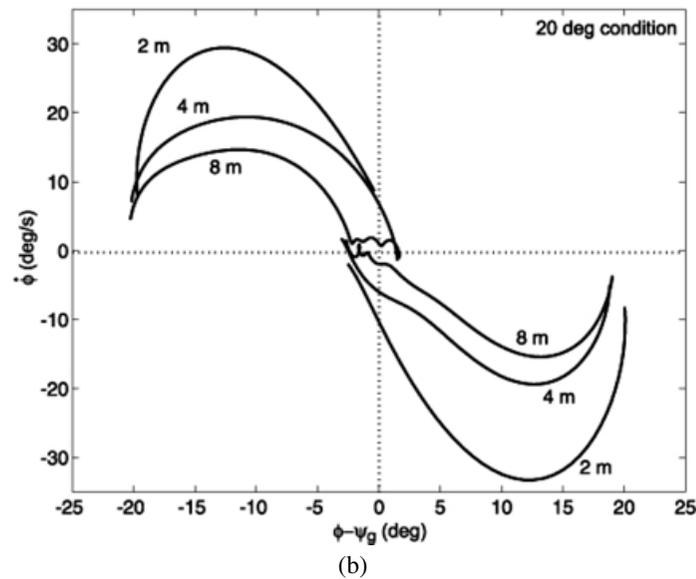
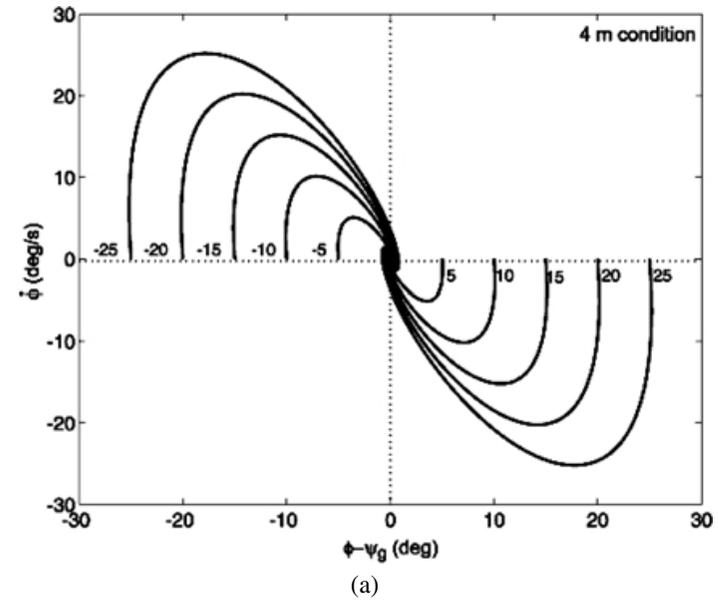
[Fajen Warren...]

model-experiment match: goal

experiment



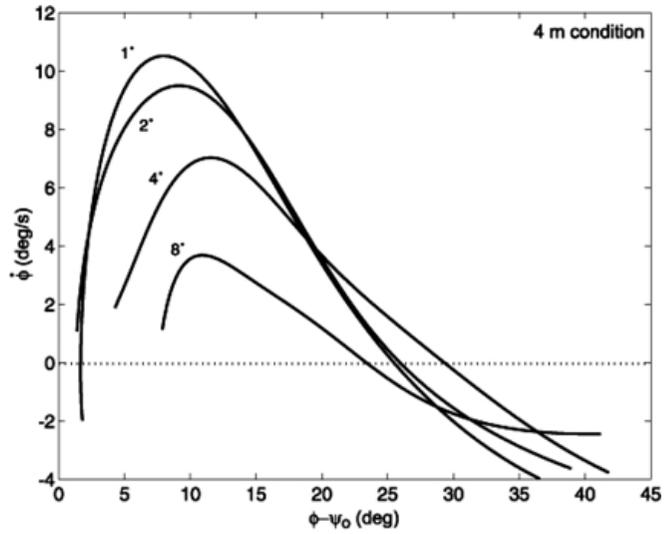
model



model-experiment match: obstacle

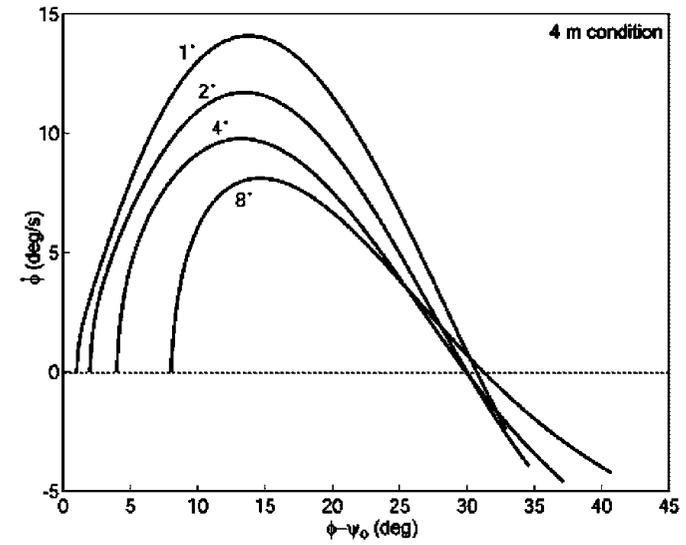
5

experiment

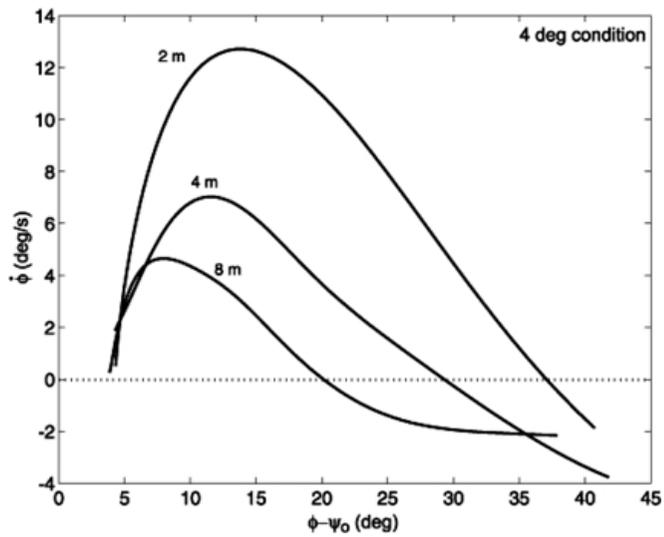


(a)

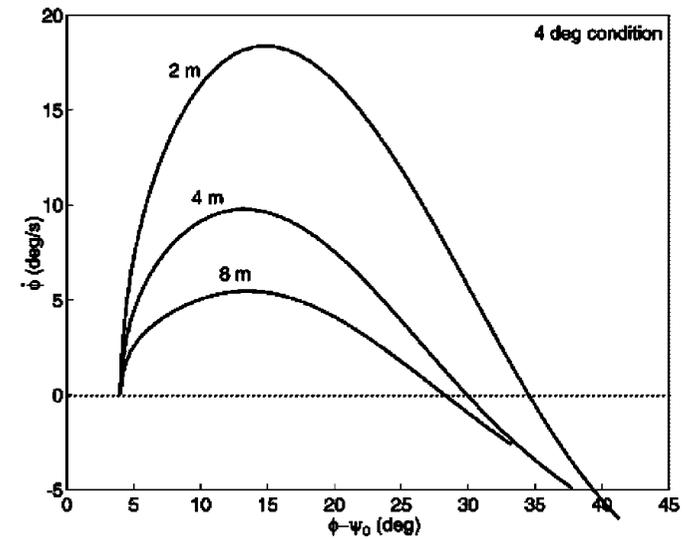
model



(a)



(b)

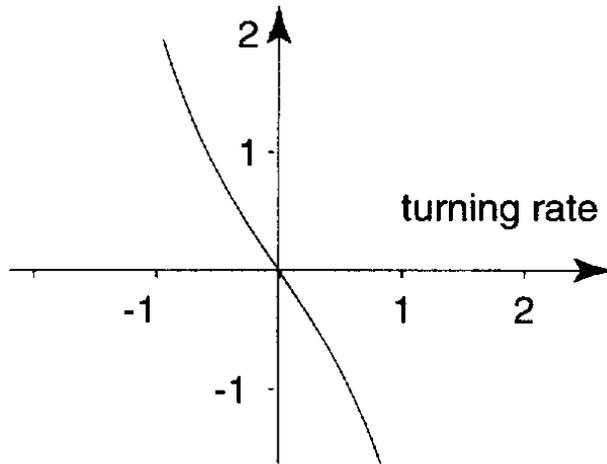


(b)

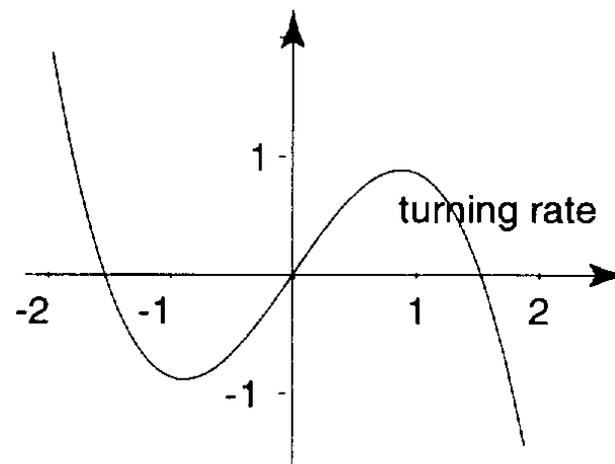
Alternative 2nd order approach

$$\dot{\omega} = (\alpha + \frac{1}{2}\pi)c_{\text{obs}}F_{\text{obs}} + \alpha\omega - \gamma\omega^3$$

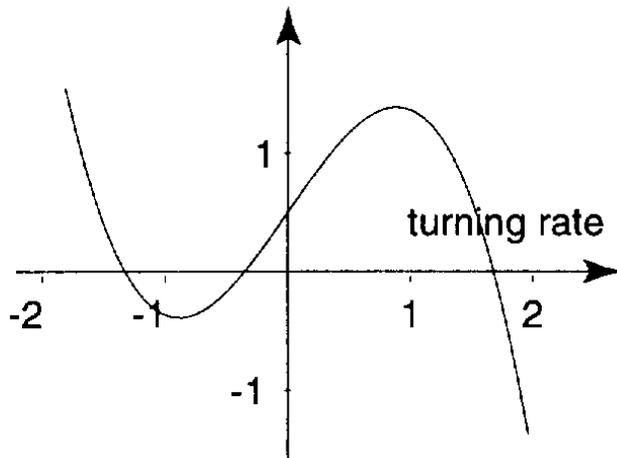
(a) dynamics of turning rate



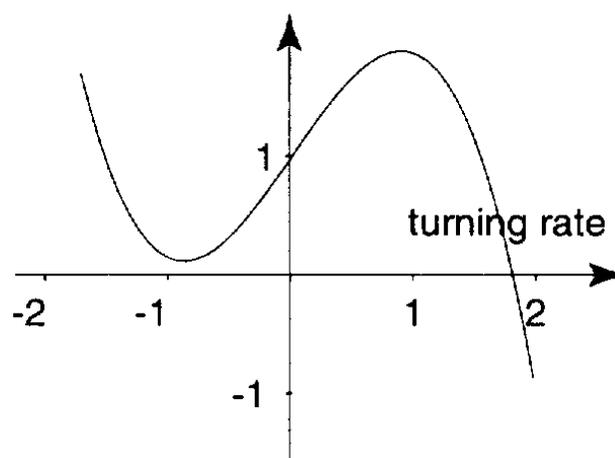
(b) dynamics of turning rate



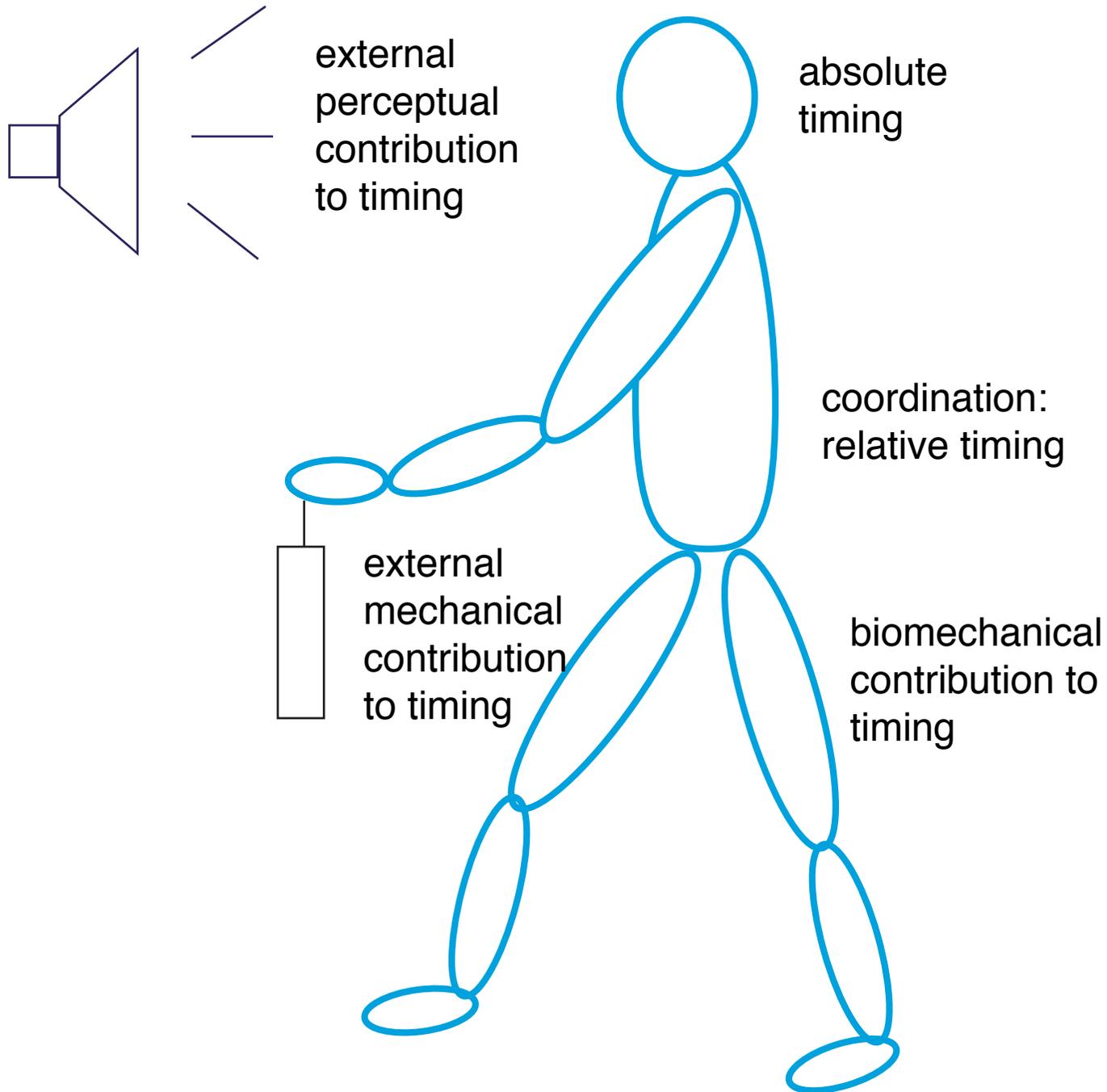
(c) dynamics of turning rate



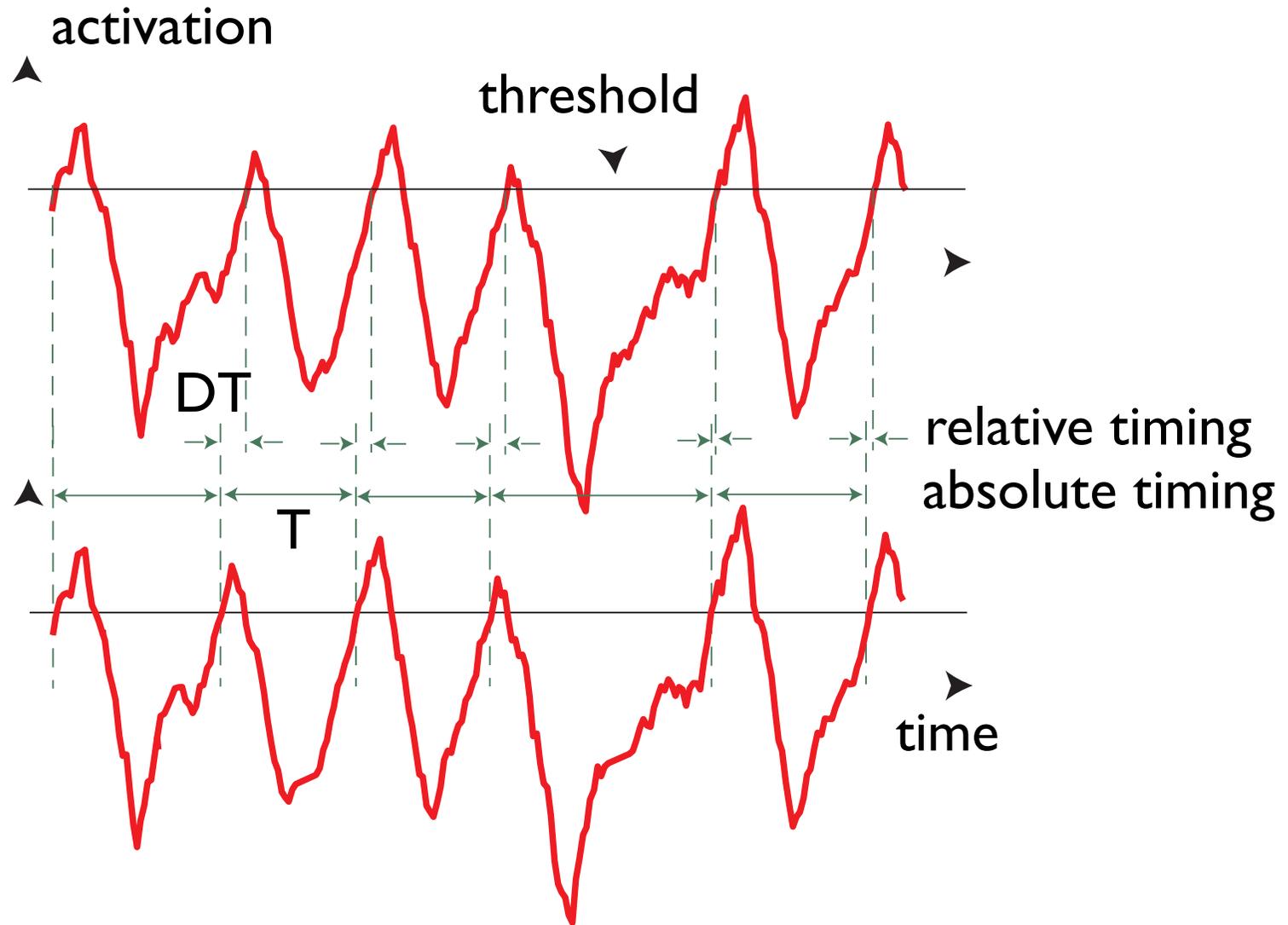
(d) dynamics of turning rate



Timing in nervous systems



Relative vs. absolute timing



relative phase= DT/T

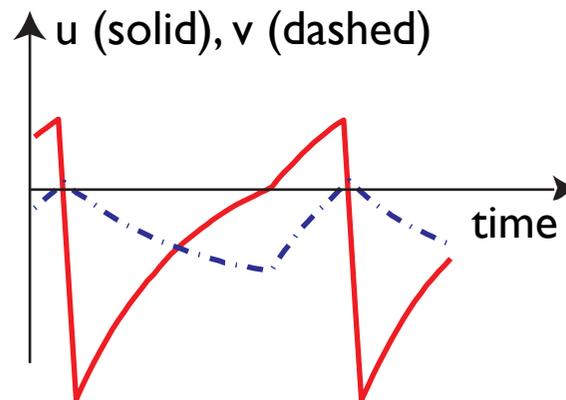
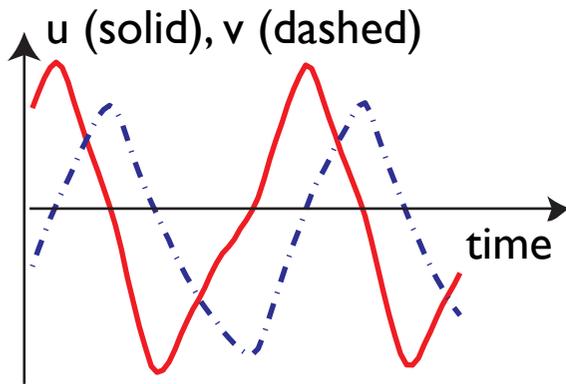
Neural oscillator

7

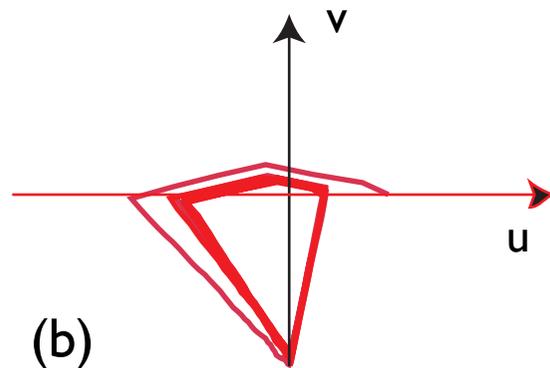
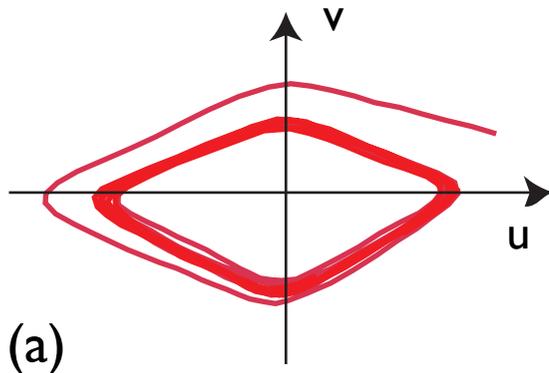
■ relaxation oscillator

$$\tau \dot{u} = -u + h_u + w_{uu}f(u) - w_{uv}f(v)$$

$$\tau \dot{v} = -v + h_v + w_{vu}f(u),$$

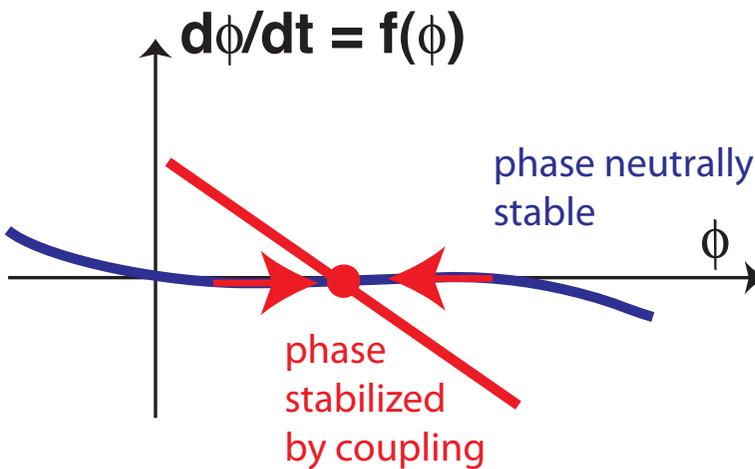
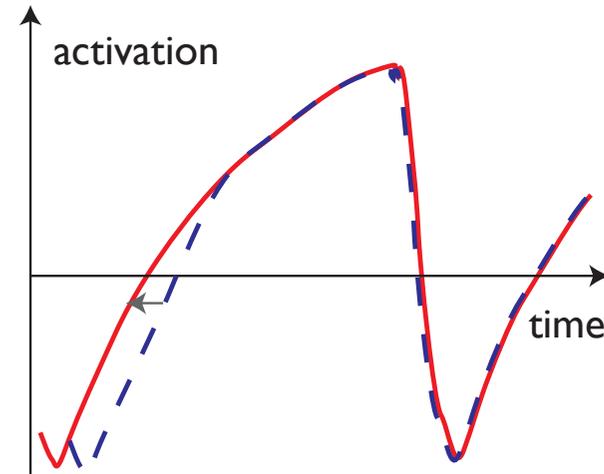


[Amari 77]



Coordination from coupling

- coordination = stable relative timing emerges from coupling of neural oscillators



$$\tau \dot{u}_1 = -u_1 + h_u + w_{uu}f(u_1) - w_{uv}f(v_1)$$

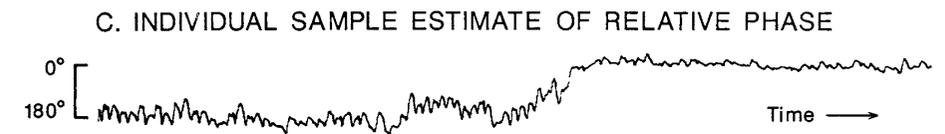
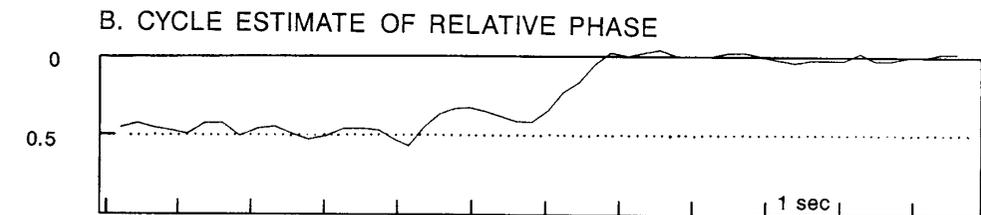
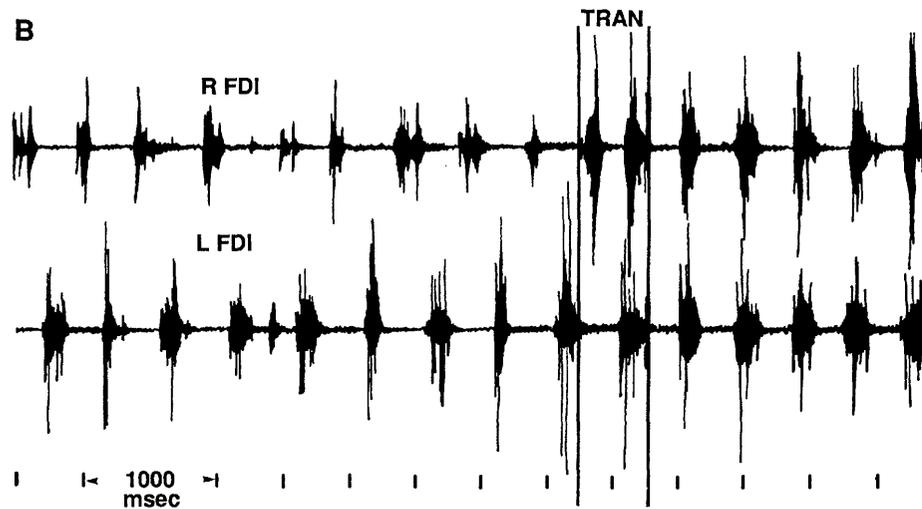
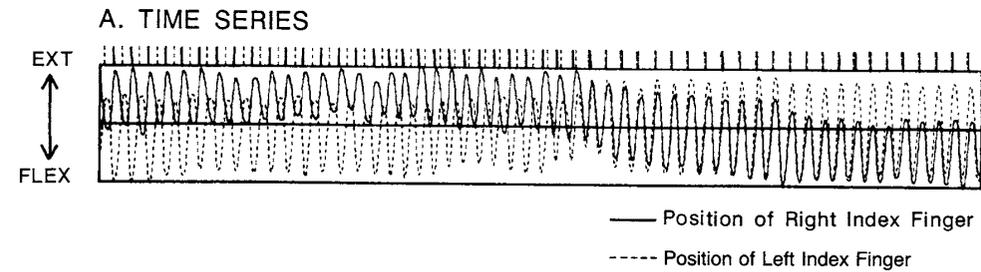
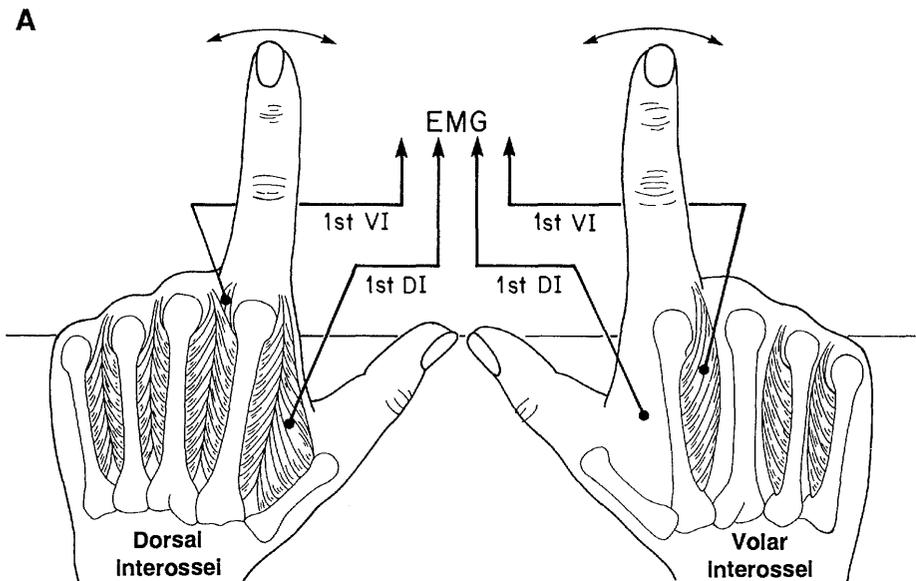
$$\tau \dot{v}_1 = -v_1 + h_v + w_{vu}f(u_1) + cf(u_2)$$

$$\tau \dot{u}_2 = -u_2 + h_u + w_{uu}f(u_2) - w_{uv}f(v_2)$$

$$\tau \dot{v}_2 = -v_2 + h_v + w_{vu}f(u_2) + cf(u_1)$$

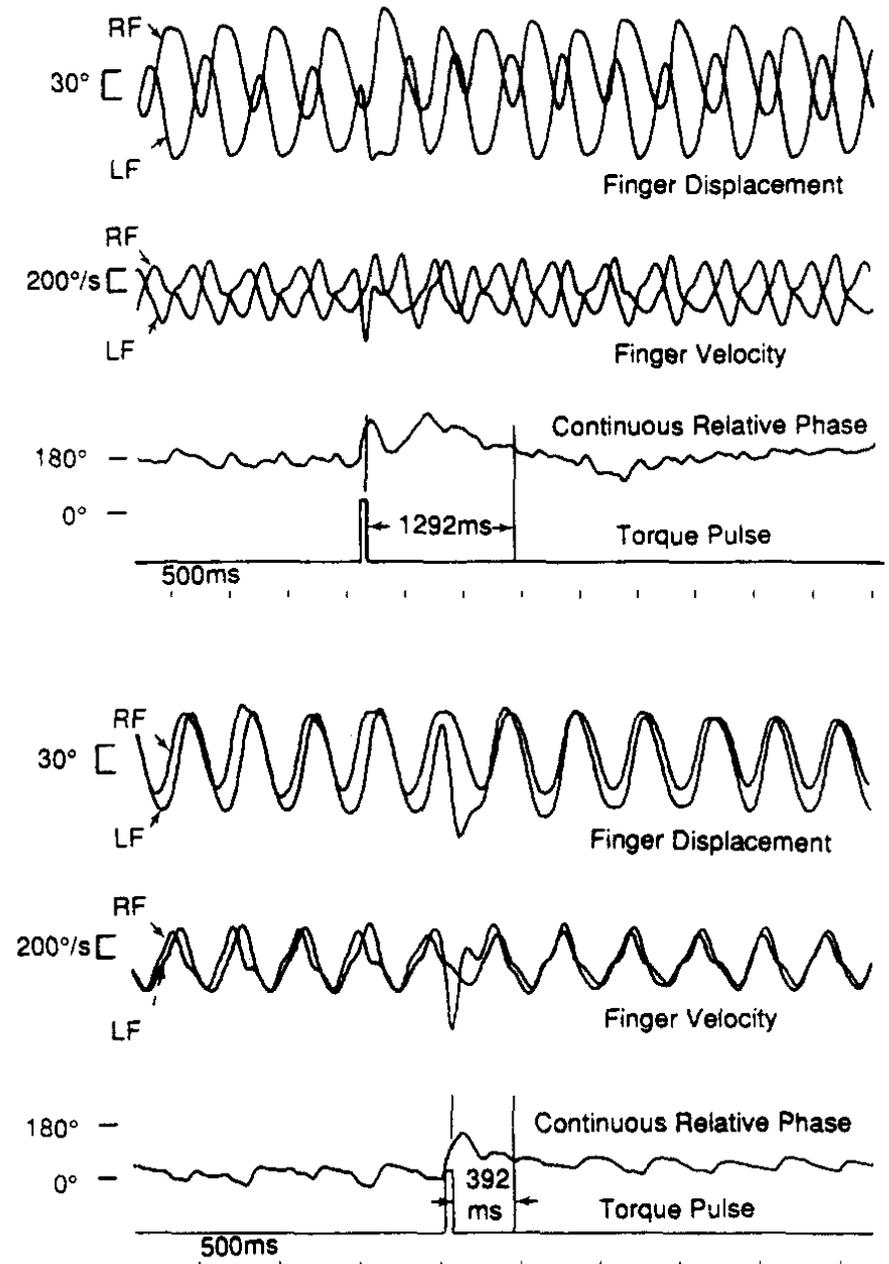
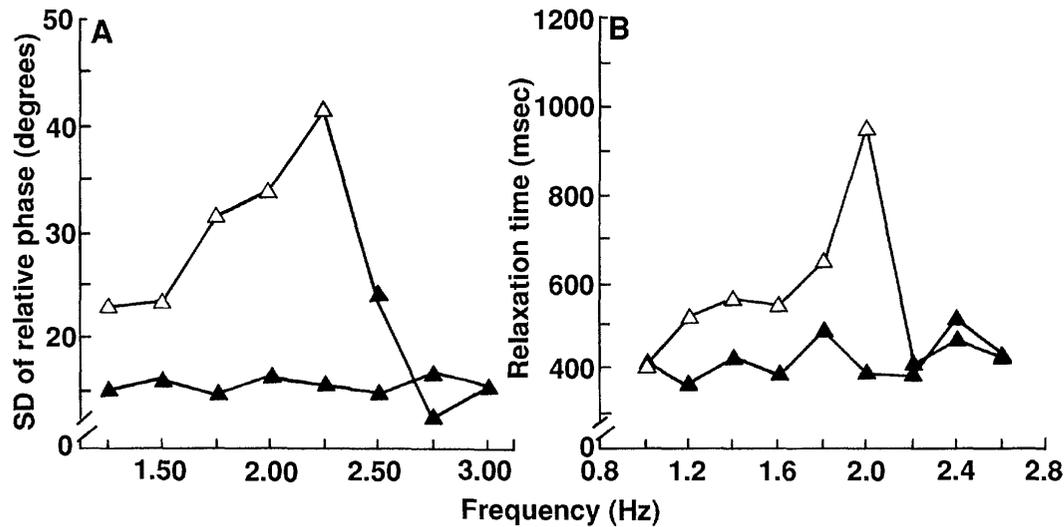
Instabilities of relative timing

7



Instabilities of relative timing

7



Schöner, Kelso (Science, 1988)

Dynamics Movement Primitives

8

Spaces for robotic motion planning 9

kinematic model

$$\mathbf{x} = \mathbf{f}(\theta)$$

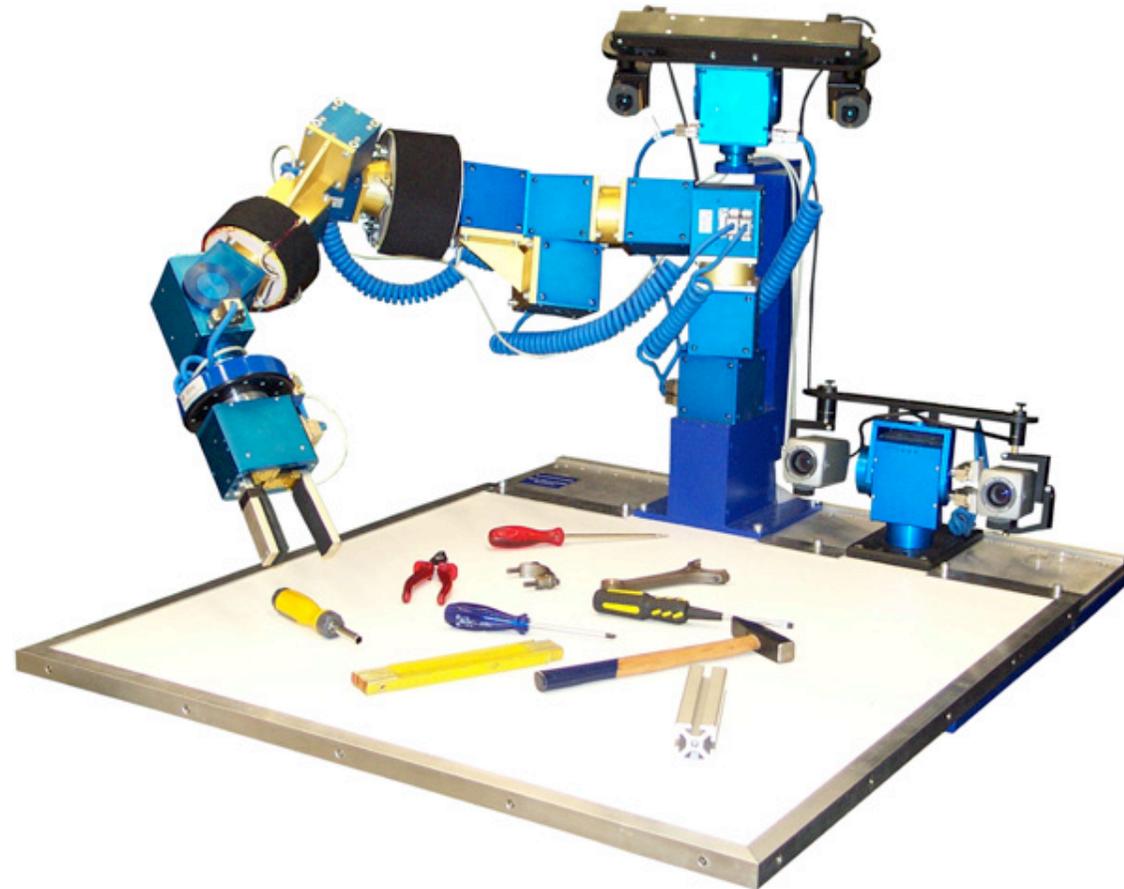
$$\dot{\mathbf{x}} = \mathbf{J}(\theta)\dot{\theta}$$

inverse kinematic model

$$\theta = \mathbf{f}^{-1}(\mathbf{x})$$

$$\dot{\theta} = \mathbf{J}^{-1}(\theta)\dot{\mathbf{x}}$$

- transform end-effector to configuration space through inverse kinematics
- problems of singularities and multiple “leafs” of inverse...



Degree of freedom problem in human movement

■ what is a DoF?

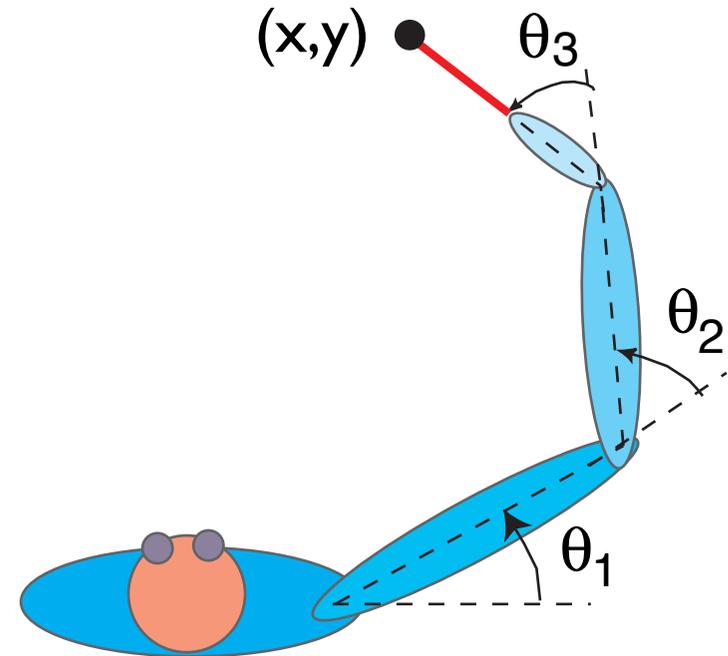
■ variable that can be independently varied

■ e.g. joint angles

■ muscles/muscle groups

■ but: assess to which extent they can be activated independently...

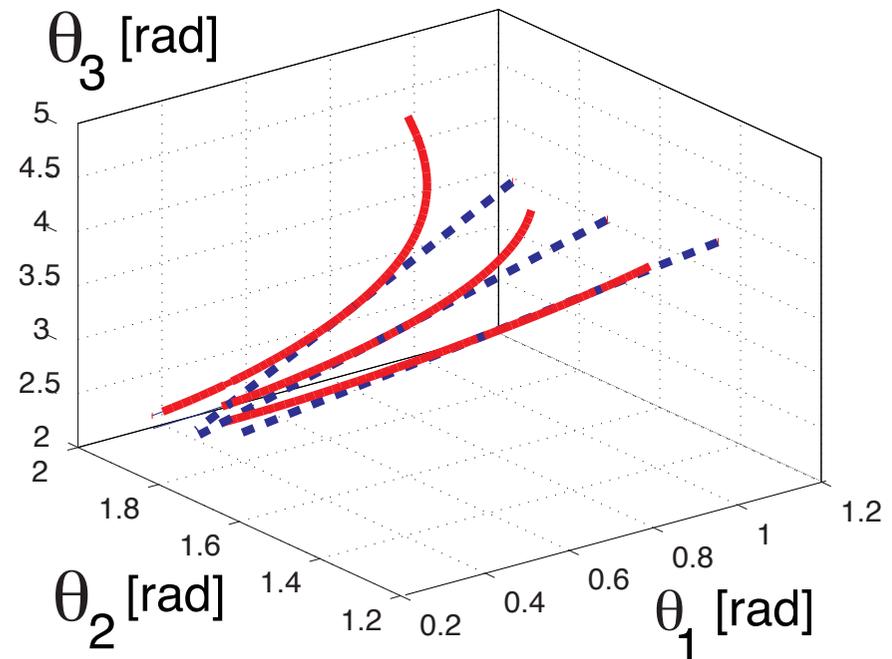
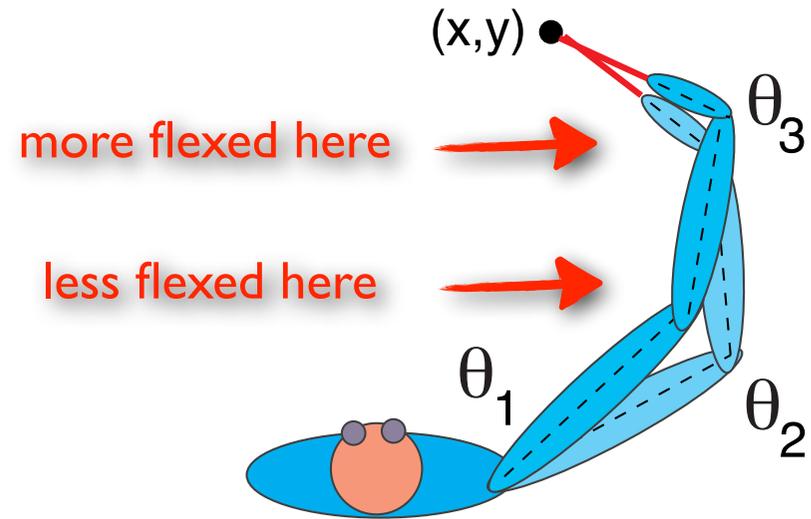
■ .. mode picture



$$x = l_1 \cos(\theta_1) + l_2 \cos(\theta_1 + \theta_2) + l_3 \cos(\theta_1 + \theta_2 + \theta_3)$$
$$y = l_1 \sin(\theta_1) + l_2 \sin(\theta_1 + \theta_2) + l_3 \sin(\theta_1 + \theta_2 + \theta_3)$$

Concept of the UnControlled Manifold 9

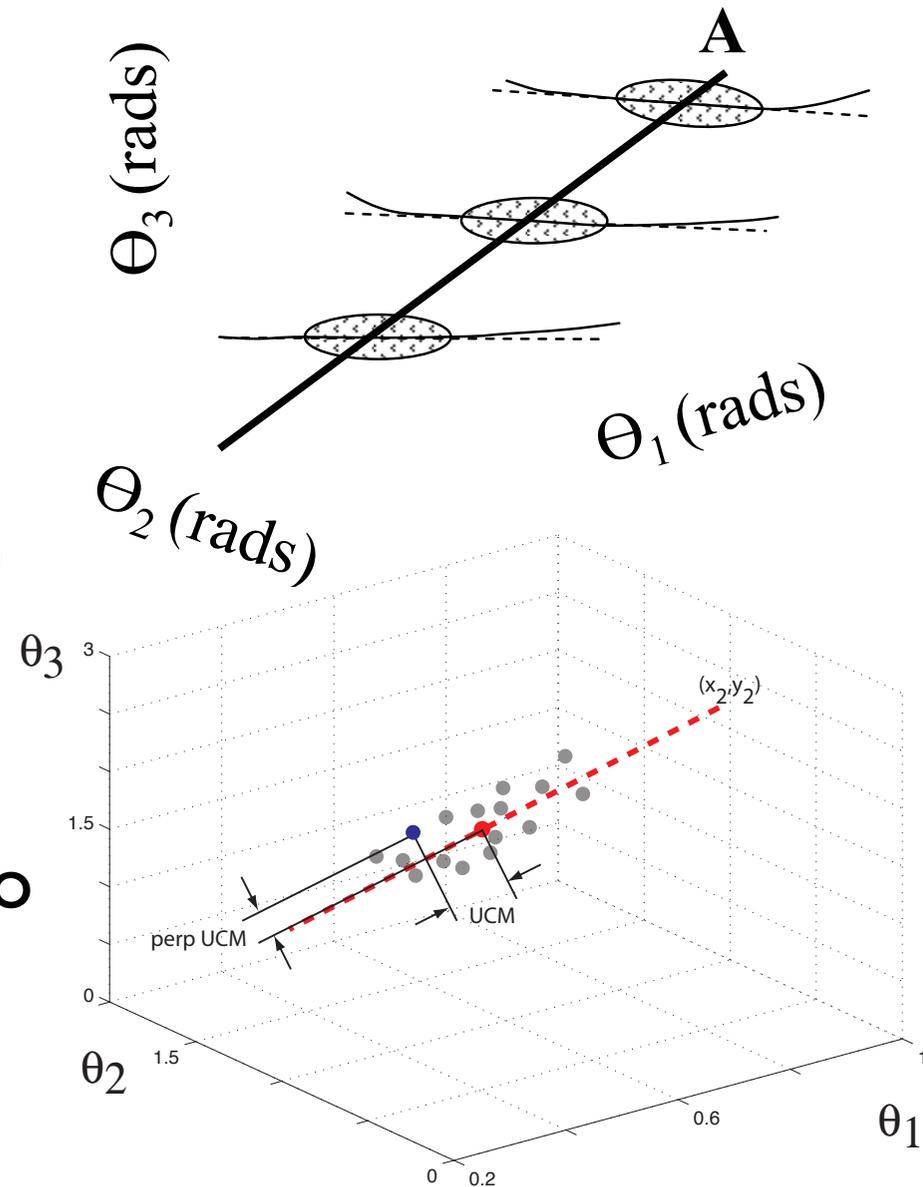
- the many DoF are coordinated such that changes that affect the task-relevant dimensions are resisted against more than changes that do not affect task relevant dimension
- leading to compensation



UCM synergy: data analysis

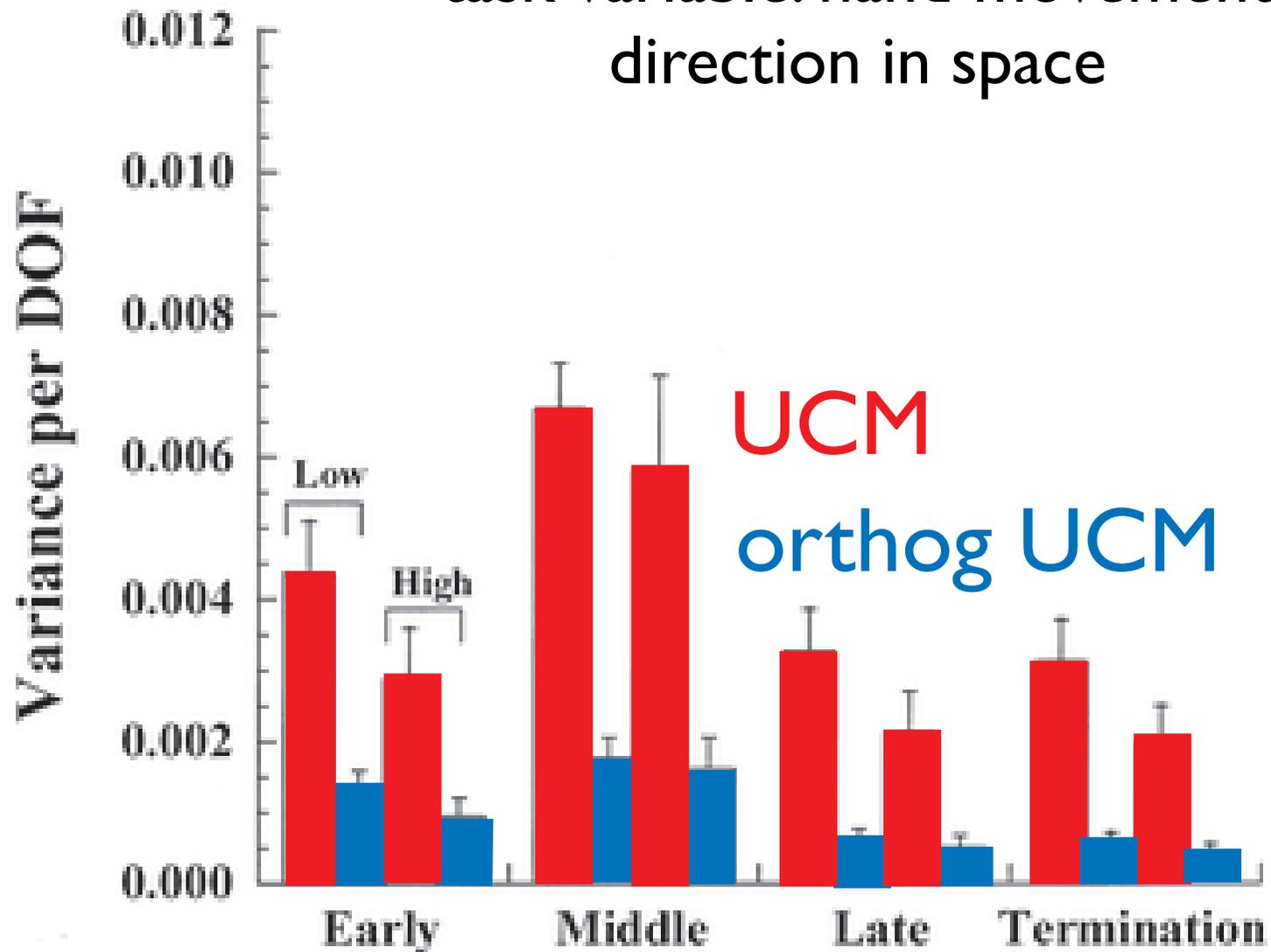
9

- align trials in time
- hypothesis about task variable
- compute null-space (tangent to the UCM)
- predict more variance within null space than perpendicular to it



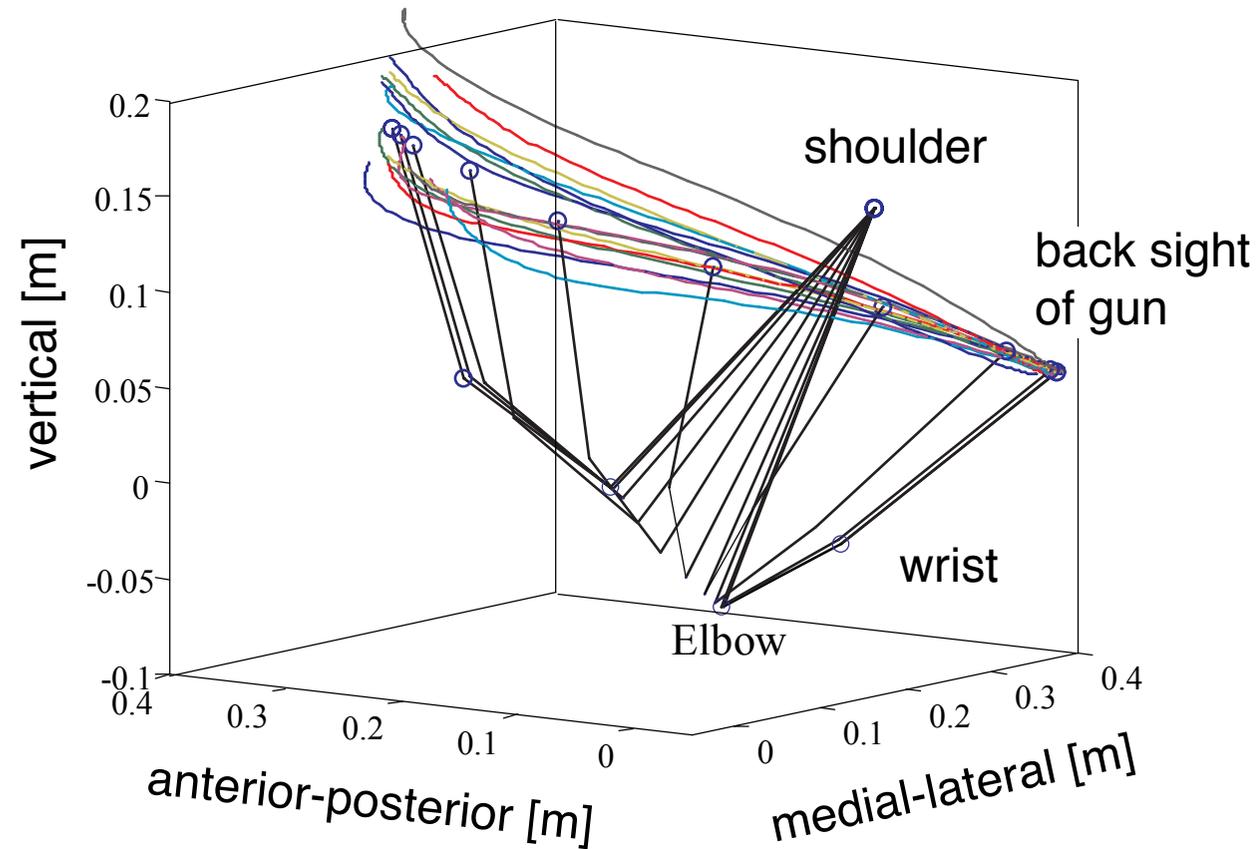
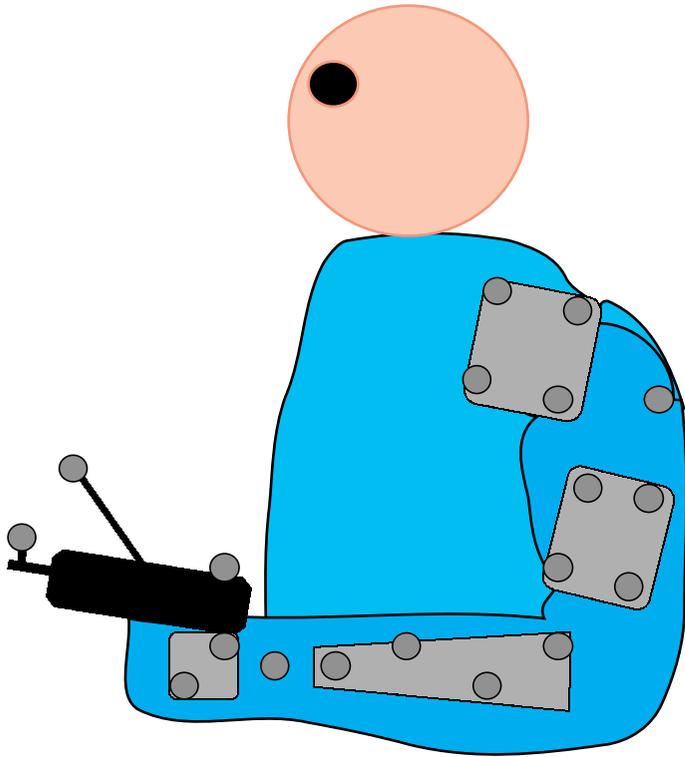
Example 1: pointing with 10 DoF arm at targets in 3D

task variable: hand movement direction in space



Example 2: shooting with 7 DoF arm at targets in 3D

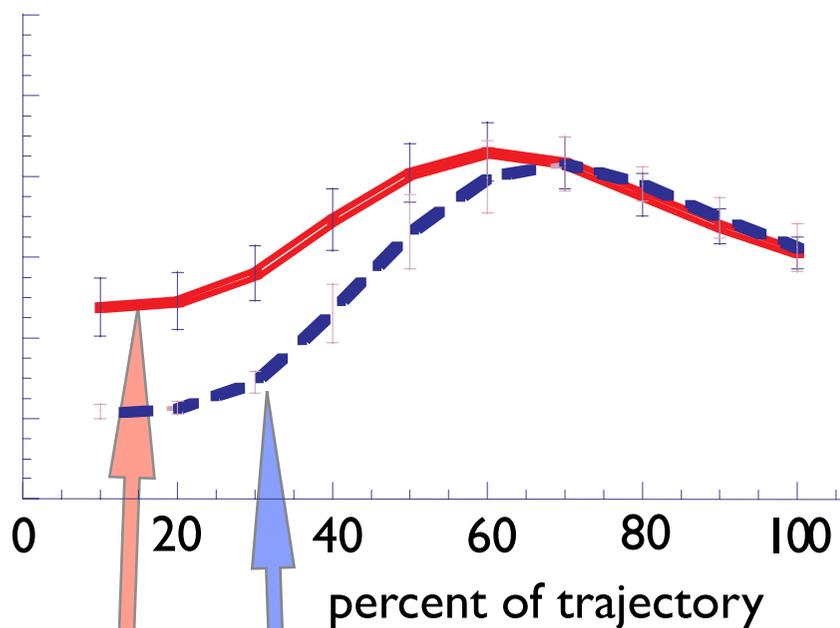
9



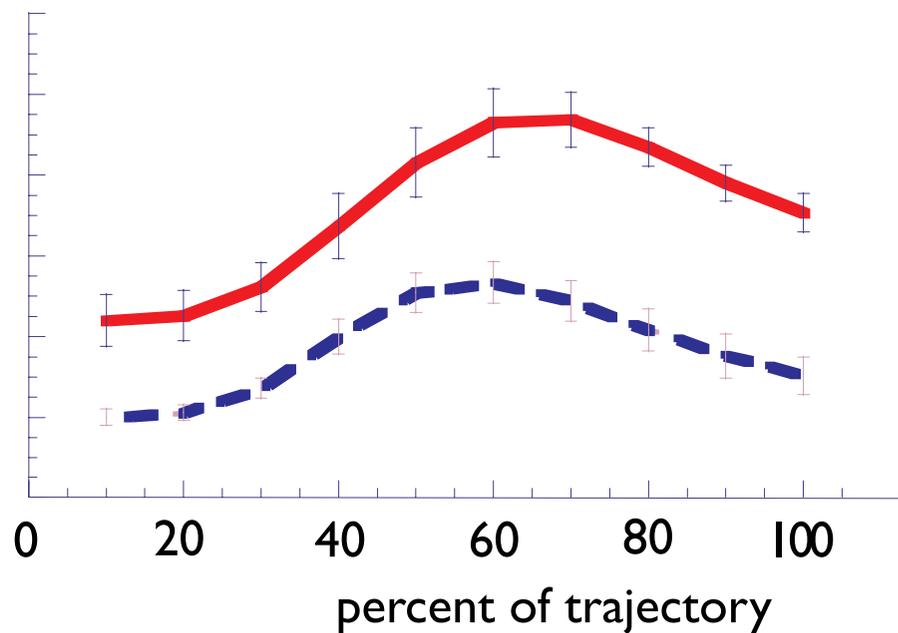
[from Scholz, Schöner, Latash: EBR 135:382 (2000)]

Example 2: shooting with 7 DoF arm at targets in 3D

gun spatial position



gun orientation to target



[from Scholz, Schöner, Latash: EBR 135:382 (2000)]

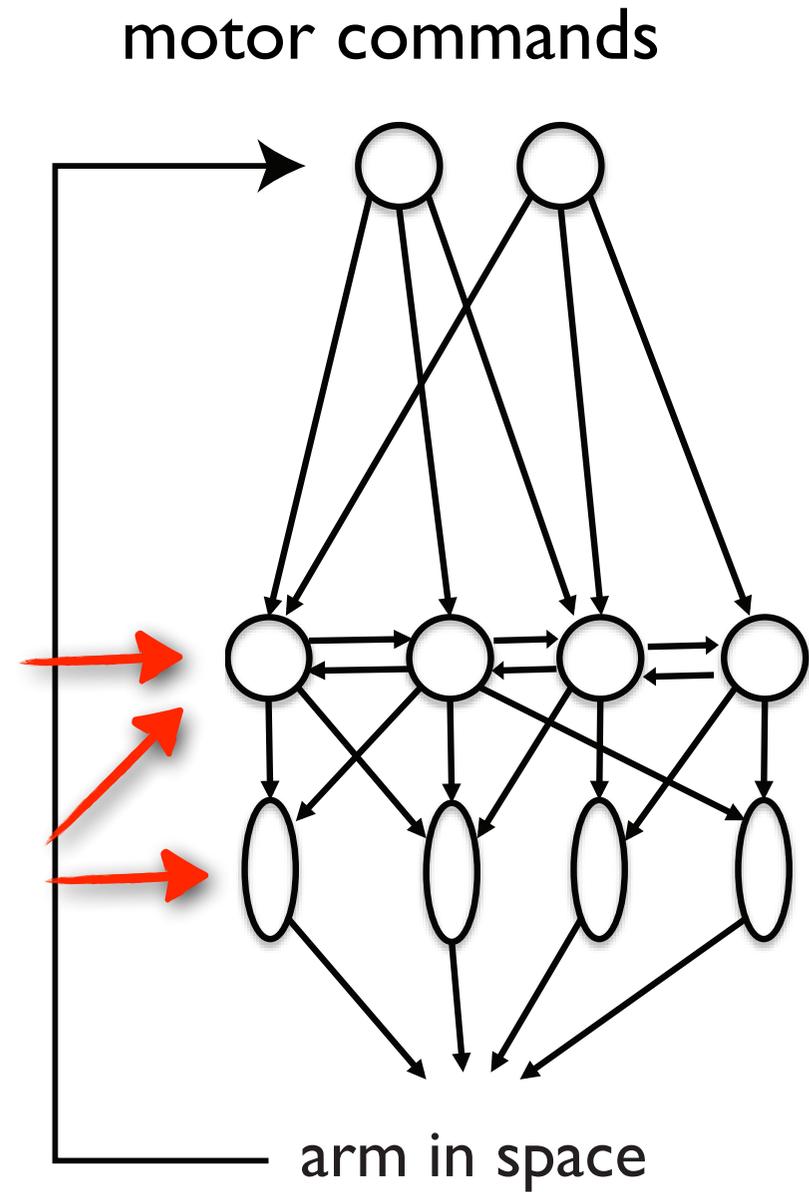
variance
within
UCM

variance
perpendicular
to UCM

UCM synergy: decoupling

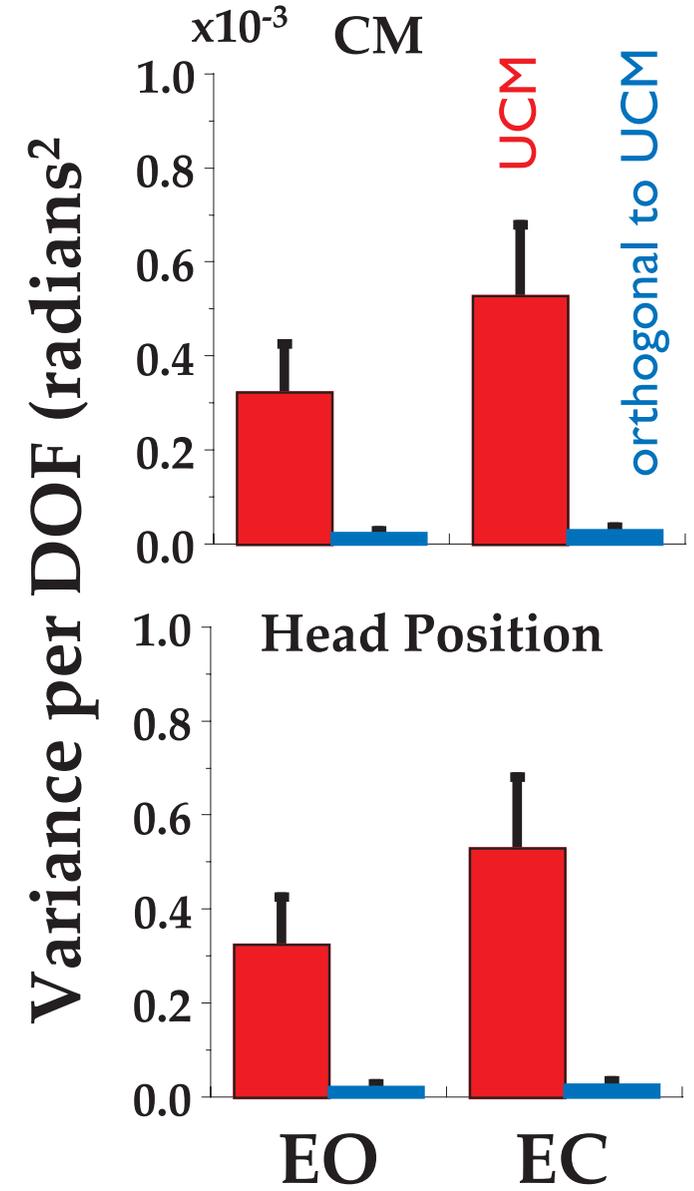
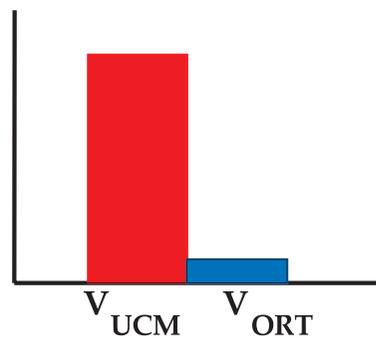
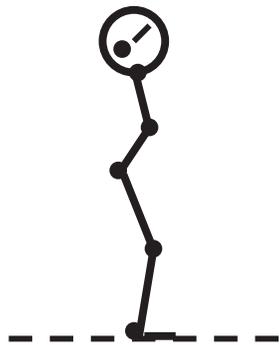
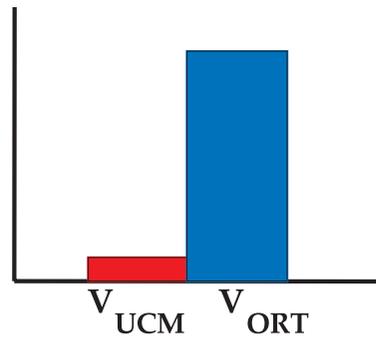
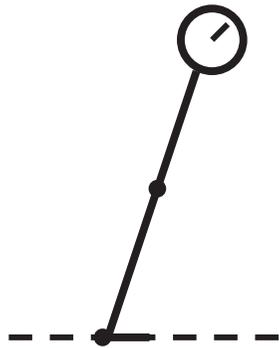
insert a perturbation here

compensatory change here



Example 3: posture

- Inverted pendulum hypothesis predicts the opposite than UCM
- but: find signature of UCM synergy



UCM synergy: from feedback

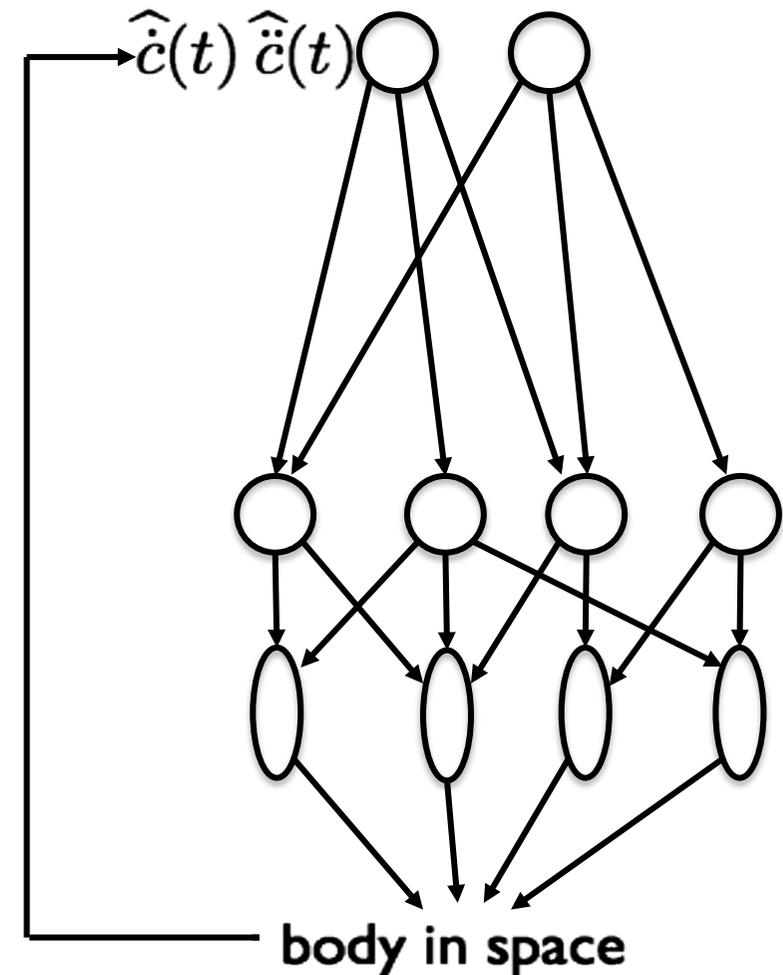
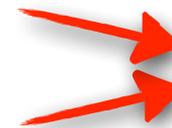
leads to change here



passes this to other DoF

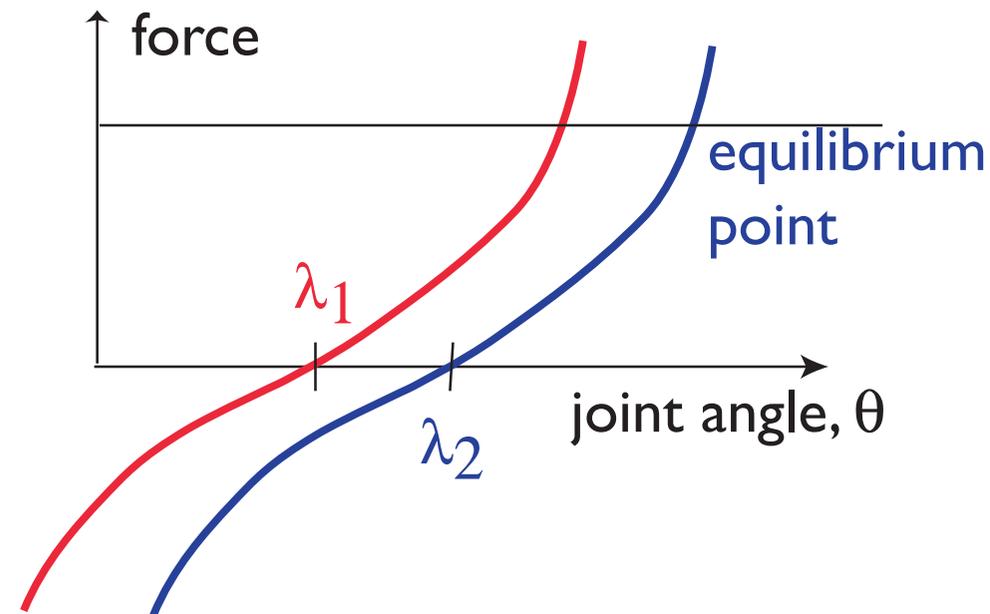


insert a perturbation here
compensatory change here

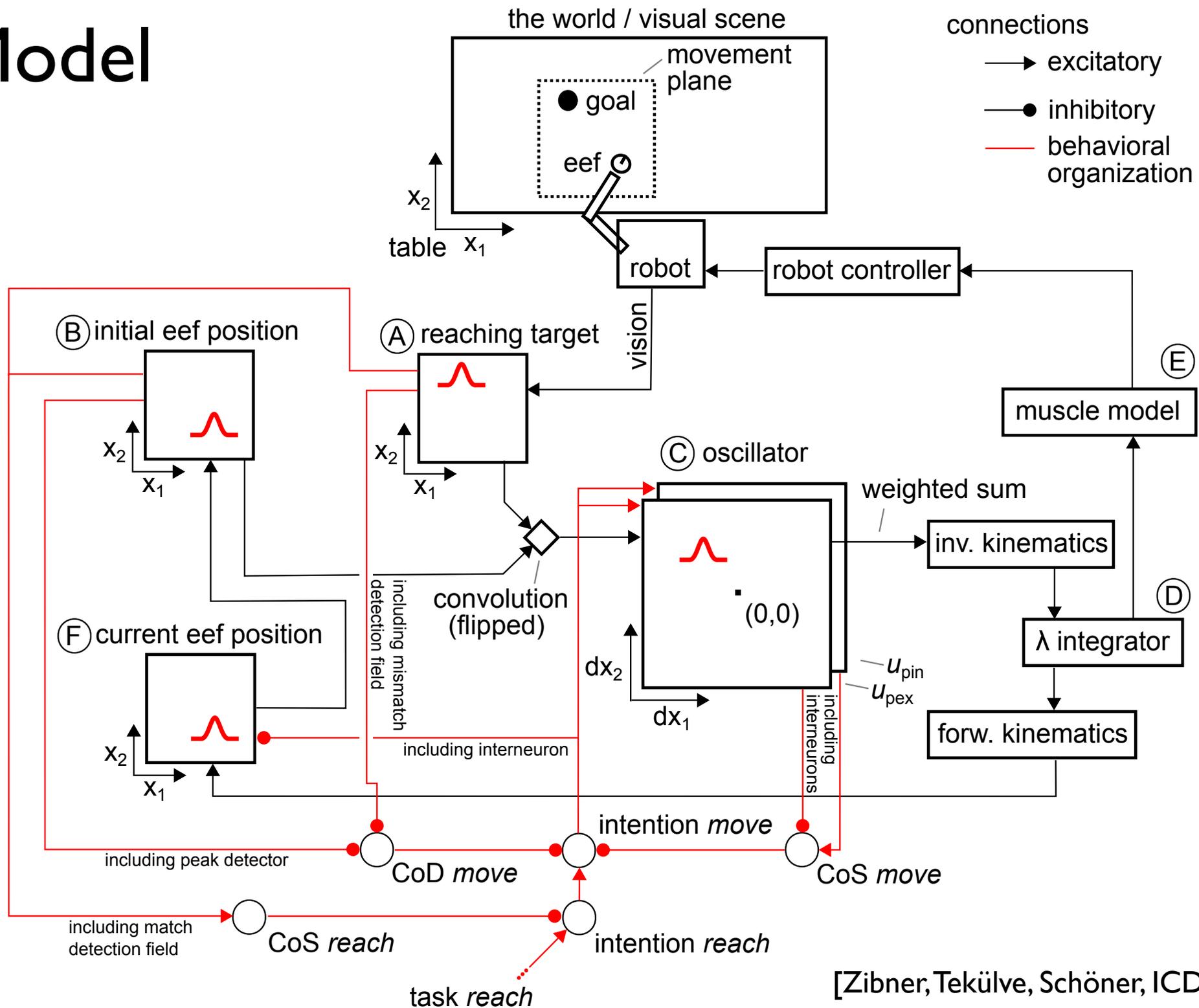


Movement entails change of posture

- muscle-joint systems have an equilibrium point during posture that is stable against transient perturbation
- that equilibrium point is shifted during movement so that after the movement, the postural state exists around a new combination of muscle lengths/joint configurations



Model



- connections
- excitatory
 - inhibitory
 - behavioral organization

Architecture

