Summary: main conceptual points

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Dynamical systems

Functional link between state and its rate of change
Dynamical system

The present determines the future

\[ \frac{dx}{dt} = f(x) \]

The diagram illustrates the concept of a dynamical system, with the initial condition predicting the future evolution of the system.
Dynamical systems

- **fixed point** = constant solution
- neighboring initial conditions converge = **attractor**

\[
\frac{dx}{dt} = f(x)
\]

Diagram:
- Fixed point
- Attractor
Bifurcations are instabilities

- In families of dynamical systems, which depend (smoothly) on parameters, the solutions change qualitatively at bifurcations at which fixed points change stability.

\[ \dot{x} = \alpha - x^2 \]

- A positive \( x_0 = \sqrt{\alpha} \) is stable.
- A negative \( \alpha \) is unstable.
Basic ideas of attractor dynamics approach

- behavioral variables
- time courses from dynamical system: attractors
- tracking attractors
- bifurcations for flexibility
vehicle moving in 2D: heading direction

constraints: obstacle avoidance and target acquisition

Behavioral variables: example

robot

$\Delta \psi$

$\Psi_{\text{obs}}$

$\Psi_{\text{tar}}$

arbitrary, but fixed reference axis

obstacle

target
Behavioral dynamics: example

behavioral constraint: target acquisition

vehicle

target

\psi

\frac{d\phi}{dt}

attractor

\psi_{tar}
behavioral constraint: obstacle avoidance

Behavioral dynamics: example

robot

obstacle

arbitrary, but fixed reference axis

$\Delta \psi$

$d\phi/dt$

repellor

$\psi_{obs}$
Behavioral dynamics

Each contribution is a “force-let” with:
- Specified value
- Strength
- Range

\[ d\phi/dt \sim \text{strength} \]

\[ \psi_{\text{tar}} \]

Specified value

~Strength

Range
Behavioral dynamics: bifurcations

constraints not in conflict
Behavioral dynamics

constraints in conflict

obstacle

target

obstacle

dφ/dt

ϕ
transition from “constraints not in conflict” to “constraints in conflict” is a bifurcation
In a stable state at all times

Vehicle

Target

Obstacle

Heading direction

$d\phi/dt$

$\phi$
Obstacle avoidance: sub-symbolic

- obstacles need not be segmented
- do not care if obstacles are one or multiple: avoid them anyway…

\[
\begin{align*}
\Delta \psi &= \text{obstacle} \\
\theta_{\text{obs}} &= \psi_{\text{obs}} \\
\phi &= \text{repellor}
\end{align*}
\]
more than one sensor in this low-level implementation will lead to a sensible avoidance behavior: The extended obstacle "bleeds through" to other sensors with different distance values. Two virtual obstacles are detected at directions ψ1 and ψ2 under different distance values.

On the top: with respect to Figure 3. is a decreasing function of the distance, the net contribution is zero. The contributions from all seven sensors are summed: the corresponding forcelet inside an attractive region and σo is the sensed distance. The angle subtended by half the vehicle's width is the maximum repulsion strength of this contribution in fact represents exactly one obstacle, we are inclined to ask whether extended obstacles that can appear on obstacle dynamics. The resultant repeller is at ψ6.

On the bottom: three repulsive forcelets are erected at these directions. In this figure, sensors 5 and 6 specify a frontal direction, specifies an obstacle at direction ϕ3. Two repulsive forcelets centered at these directions are erected. In that figure, distances are 40, 30 and 40 cm respectively. On the spot Figures 4.

[from: Bicho, Jokeit, Schöner]
Bifurcations
2nd order attractor dynamics to explain human navigation

\[ \ddot{\phi} = -b\dot{\phi} - k_g(\phi - \psi_g)(e^{-c_1d_g} + c_2) + k_o(\phi - \psi_o)(e^{-c_3|\phi - \psi_o|})(e^{-c_4d_o}) \]

inertial term

damping term

attractor goal heading

repellor obstacle heading

[Fajen Warren…]
model-experiment match: goal

experiment

model
model-experiment match: obstacle

Experiment

![Graphs showing model-experiment match for obstacle experiments](image1)

Model

![Graphs showing model-experiment match for obstacle models](image2)
Alternative 2nd order approach

\[ \dot{\omega} = (\alpha + \frac{1}{2}\pi)c_{obs}F_{obs} + \alpha \omega - \gamma \omega^3 \]

(a) dynamics of turning rate  
(b) dynamics of turning rate  
(c) dynamics of turning rate  
(d) dynamics of turning rate

[Bicho, Schöner, 97]
Timing in nervous systems

- Absolute timing
- Coordination: relative timing
- External mechanical contribution to timing
- External perceptual contribution to timing
- Biomechanical contribution to timing
Relative vs. absolute timing

Relative phase = \frac{\text{DT}}{\text{T}}
Neural oscillator

**relaxation oscillator**

\[ \tau \dot{u} = -u + h_u + w_{uu}f(u) - w_{uv}f(v) \]
\[ \tau \dot{v} = -v + h_v + w_{vu}f(u), \]

To see this, imagine a periodic time course of activation (Fig. 5). All levels of activation (except at the turning points) are then passed through in two directions, once at increasing and once at decreasing activation. Thus, such activation values do not uniquely specify the future. A second variable, here called “inhibition,” is needed, to disambiguate the future: each activation level is passed through once at a smaller and once at a larger level of this second variable. Thus, clocks cannot be built as dynamical systems in terms of activation alone!

Stable periodic solutions, to which the system is attracted from nearby states are called limit cycle attractors. An example of a dynamical system supporting limit cycle attractors of an activation–inhibition pair of variables is

\[ \tau \dot{u} = -u + h_u + w_{uu}f(u) - w_{uv}f(v) \]
\[ \tau \dot{v} = -v + h_v + w_{vu}f(u), \]

equations first analyzed by Amari (1977). The first two terms of each equation describe two linear uncoupled dynamical systems, each with a stable fixed point at the resting levels of activation, \( h_u \), and of inhibition, \( h_v \). A sigmoid function, \( f(u) = \frac{1}{1 + \exp(-\beta u)} \), makes the system nonlinear in terms of “self-excitation” (\( w_{uu} \)) and of coupling between activation and inhibition variables (\( w_{vu}, w_{uv} \)). For appropriate choices of these parameters, a limit cycle attractor emerges (Fig. 6). The stability of the periodic solution manifests itself by attraction of neighboring states toward the limit cycle. The activation-based stochastic timer model emerges as the limit case, in which the vector field is structured such that a period of graded activation growth is followed by a more rapid phase of activation decay (Fig. 6b). In fact, abstractly speaking, any clock is a limit cycle attractor of a dynamical system (see, e.g., Andronov, Vitt, & Khaikin, [Amari 77])

[Amari 77]
Coordination from coupling

- Coordination = stable relative timing emerges from coupling of neural oscillators

Coordination from coupling

**3.2. Dynamic Timing Models**

Coupling is the central concept for understanding relative timing within dynamic timing models. Mathematically, two dynamic timers, \((u_1, v_1)\) and \((u_2, v_2)\), are mutually coupled if the dynamic variables of one timer contribute to the dynamic equations of the second and vice versa. For the Amari oscillator model presented earlier [Eqs. (6) and (7)], for instance, a simple form of mutual coupling is generated by the terms carrying the coefficient, \(c\), in these equations:

\[
\tau \dot{u}_1 = -u_1 + h_u + w_{uu}f(u_1) - w_{uv}f(v_1)
\]

\[
\tau \dot{v}_1 = -v_1 + h_v + w_{vu}f(u_1) + cf(u_2)
\]

\[
\tau \dot{u}_2 = -u_2 + h_u + w_{uu}f(u_2) - w_{uv}f(v_2)
\]

\[
\tau \dot{v}_2 = -v_2 + h_v + w_{vu}f(u_2) + cf(u_1)
\]

These are only two out of a great variety of possible coupling terms. They generically generate phase locking, so that the two oscillators adopt identical frequencies and align matching parts of their activation trajectory (Fig. 11). This relative time order is stable; that is, when the two oscillators start out with differently aligned trajectories or are perturbed away from the stable alignment, then the dynamics drives the timers back to the stable timing relationship.

A characterization of relative timing independently of the underlying activation states is possible through the concept of relative phase. Its empirical definition is based on reference events (here the moments in time when activation pierces a threshold leading to a motor event such as a tap). The latency between matching events of two activation functions divided by the current cycle time of either of the activation functions is the relative phase, \(\phi/\Delta T/T\) (Fig. 9). (Relative phase may be normalized)

Instabilities of relative timing

A. TIME SERIES

EXT
FLEX

Position of Right Index Finger
Position of Left Index Finger

B. CYCLE ESTIMATE OF RELATIVE PHASE

C. INDIVIDUAL SAMPLE ESTIMATE OF RELATIVE PHASE

Schöner, Kelso (Science, 1988)
Instabilities of relative timing

Schöner, Kelso (Science, 1988)
Dynamics Movement Primitives
Spaces for robotic motion planning

kinematic model
\[ x = f(\theta) \]
\[ \dot{x} = J(\theta)\dot{\theta} \]

inverse kinematic model
\[ \theta = f^{-1}(x) \]
\[ \dot{\theta} = J^{-1}(\theta)\dot{x} \]

- transform end-effector to configuration space through inverse kinematics
- problems of singularities and multiple “leafs” of inverse...
what is a DoF?
- variable that can be independently varied
- e.g. joint angles

muscles/muscle groups
- but: assess to which extent they can be activated independently...
- .. mode picture

Degree of freedom problem in human movement

\[ x = l_1 \cos(\theta_1) + l_2 \cos(\theta_1 + \theta_2) + l_3 \cos(\theta_1 + \theta_2 + \theta_3) \]
\[ y = l_2 \sin(\theta_1) + l_2 \sin(\theta_1 + \theta_2) + l_3 \sin(\theta_1 + \theta_2 + \theta_3) \]
the many DoF are coordinated such that changes that affect the task-relevant dimensions are resisted against more than changes that do not affect task relevant dimension leading to compensation

[Scholz, Schöner, EBR 126:289 (99)]
UCM synergy: data analysis

- align trials in time
- hypothesis about task variable
- compute null-space (tangent to the UCM)
- predict more variance within null space than perpendicular to it
Example 1: pointing with 10 DoF arm at targets in 3D

Task variable: hand movement direction in space
Example 2: shooting with 7 DoF arm at targets in 3D

[from Scholz, Schöner, Latash: EBR 135:382 (2000)]
Example 2: shooting with 7 DoF arm at targets in 3D

[Graphs showing gun spatial position and gun orientation to target, with annotations indicating variance within UCM and variance perpendicular to UCM.]

[from Scholz, Schöner, Latash: EBR 135:382 (2000)]
UCM synergy: decoupling

motor commands

insert a perturbation here

compensatory change here

arm in space
Example 3: posture

- Inverted pendulum hypothesis predicts the opposite than UCM
- but: find signature of UCM synergy

![Graphs showing variance per DOF for different posture positions with UCM and ORT vectors.](image)
UCM synergy: from feedback

leads to change here

passes this to other DoF

insert a perturbation here
compensatory change here

\( \hat{c}(t) \hat{c}(t) \)

body in space

Reimann, Schöner, Biological Cybernetics 2017
Movement entails change of posture

- Muscle-joint systems have an equilibrium point during posture that is stable against transient perturbation.

- That equilibrium point is shifted during movement so that after the movement, the postural state exists around a new combination of muscle lengths/joint configurations.
Fig. 1. This figure shows the full movement generation architecture. Some details are hidden in connections for clarity’s sake, but are marked with text stating “including . . . ”. See text for more details.

B. Generation of virtual trajectory

The movement plan feeds into a two-layer DNF, consisting of $u_{\text{pex}}$ and $u_{\text{pin}}$ (see Figure 1, C),

$$
\dot{u}_{\text{pex}}(x, t) = u_{\text{pex}}(x, t) + h + s_{\text{pex}}(x, t)
$$

and

$$
\dot{u}_{\text{pin}}(x, t) = u_{\text{pin}}(x, t) + h + s_{\text{pin}}(x, t),
$$

with

$$s_{\text{pex}}(x, t) = s_{\text{pin}}(x, t) = s_{\text{pla}}(x, t) + c_{\text{mov}}(u_{\text{int}}(t)).$$

The two-layer structure of $u_{\text{pex}}$ and $u_{\text{pin}}$ serves as a neural oscillator. Transient activation is created in the excitatory layer, which the more slowly evolving inhibitory layer suppresses over time. This dynamics thus performs a one-shot active transient in response to input. The oscillation is parameterized by the movement plan $s_{\text{pla}}$ and is switched on by the activation of a neural node $u_{\text{int}}$, which expresses the intention to generate movement. Both layers use a semi-linear output function $\varphi(\cdot)$ instead of $\varphi(\cdot)$,

$$\varphi_{\text{pex}}(x, t) = \varphi_{\text{u}}(x, t)$$

for $u_{\text{pex}}(x, t) > 0$ else $0$.

This assures that no movement is created as long as $u_{\text{pex}}$ is below threshold. Note that $u_{\text{pex}}$ and $u_{\text{pin}}$ cover a larger spatial area than $u_{\text{tar}}$ and $u_{\text{ini}}$, as their coordinate system expresses relative distance to the end-effector. Consequently, if the end-effector is at the target, the target appears in the center of $u_{\text{pex}}$ and $u_{\text{pin}}$ with a distance of zero to the end-effector.

From the relative position of the target in $u_{\text{pex}}$, a velocity vector $v$ is extracted by integrating over the represented domain $X = \{(x_1, x_2) \in \mathbb{R}^2 : 50 \leq x_1, x_2 \leq 50\}$:

$$v(t) = \int_{X} \varphi_{\text{pex}}(u_{\text{pex}}(x, t)) \, dx_1 \, dx_2.$$
Architecture

![Diagram showing trajectories and velocity profiles](image)

[Zibner, Tekülve, Schöner, ICDL 2015]