Exercise 2 Obstacle avoidance

Read the short conference paper "Using Attractor Dynamics to Control Autonomous Vehicle Motion" by Estela Bicho, Pierre Mallet, and myself published in 1998 (available on the course web page). This paper covers pretty much the last lecture. Use this paper as a resource to answer the following questions.

The core idea of obstacle avoidance presented in this paper is that every sensor \( i = 1, \ldots, 7 \) contributes a repulsive force-let, \( f_{\text{obs},i}(\phi) \), to the dynamics of heading direction, \( \dot{\phi} \):

\[
\dot{\phi} = \sum_{i} f_{\text{obs},i}(\phi) + f_{\text{tar}}
\]

(Equations 4 and 7 in the paper). That force-let is defined as follows:

\[
f_{\text{obs},i}(\phi) = \lambda(d_i)(\phi - \psi_i) \exp \left[ -\frac{(\phi - \psi_i)^2}{2\sigma_i^2(d_i)} \right].
\]

It repels from the direction, \( \psi_i \), into which the sensor is pointing. See the paper for the formulas for \( \lambda(d_i) \) and \( \sigma(d_i) \).

1. Make a drawing of the robot and mark angles \( \phi, \theta_i, \) and \( \psi_i \) (for one sensor, \( i \), only). Write down the equation that links these three angles.

2. Plot the linear term in the force-let and describe the geometrical meaning of the two parameters, \( \psi_i \) and \( \lambda \).

3. Plot the Gaussian factor and describe the geometrical meaning of the two parameters, \( \psi_i \) and \( \sigma \).

4. Plot the product. Is the slope of the dynamics at \( \phi = \psi_i \) affected by the Gaussian? Why or why not?

5. Replace \( \psi_i \) by \( \theta_i \) and \( \phi \) in the force-let. What do you learn about the contribution of sensor number 4?

6. Plot the target force-let

\[
f_{\text{tar}} = -\lambda_{\text{tar}} \sin(\phi - \psi_{\text{tar}})
\]

7. Plot the sum of the target and one obstacle force-let in the case \( \psi_i = \psi_{\text{tar}} \). Do this plot for two cases: \( \lambda(d_i) >> \lambda_{\text{tar}} \) (obstacle close) and \( \lambda(d_i) < < \lambda_{\text{tar}} \) (obstacle far). Interpret the resulting dynamics.

8. Can you plot the critical case in which the dynamics transitions between these two limit cases? Discuss...