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A dynamical systems approach to task-level system integration used to plan and control autonomous vehicle motion

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Abstract

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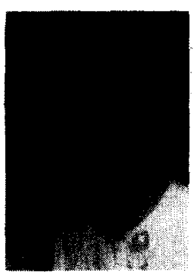
Autonomous systems with multiple sensory and effector modules face the problem of coordinating these components while fulfilling tasks such as moving towards a goal and avoiding sensed obstacles. We propose a set of organizational principles for dealing with this problem. The ideas are (a) to plan in terms of task-related variables that abstract from effector degrees of freedom and peripheral sensor coordinates but succinctly capture behavioral constraints; (b) to generate time courses of behavior through a dynamical system of the planning variables. Task constraints, such as targets to be reached, obstables to be avoided, etc. are expressed as parts of the planning dynamics in a principled fashion invoking concepts of the qualitative theory of dynamical systems. System integration is possible in the sense that all information provided by the various sensory modules and all information required by the various effector modules becomes part of the planning dynamics. Compression of such behavioral information is achieved in a second layer in which the relative strengths of different contributions to the planning dynamics are governed by competitive dynamics that separate convergent information, which is integrated by selecting a representative, from non-redundant information, which is kept invariant. The capability of the system to perform stable planning, make planning decisions, and integrate redundant as well as complementary information is demonstrated by software simulations. These include the simulation of control errors on both the effector and the sensor side.

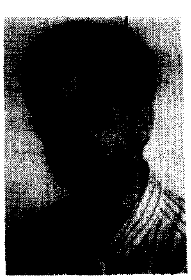
1. Introduction

In spite of much work on autonomous mobile robots (cf. [10,19] for a survey) the fundamental problem of integrating the sensory, effector, control, and reasoning modules remains a major challenge. Recently, the fact that biological movement systems demonstrate solutions to the problem has begun to make an impact on thinking about new approaches. Specifically, a broad spectrum of theoretical work on biological action-perception systems has led to a number of important notions that we briefly summarize: (a) Experimental work both in kinesiology (e.g., [30]) and neurophysiology (e.g., [13,7]) has revealed that reaching movements in monkeys and humans are planned by the central nervous system in task-related coordinate systems rather than peripheral effector coordinates (e.g., joint angle forces). In theoretical accounts of human movement abstract, function-specific control variables have been introduced as the central level of trajectory planning (e.g., [12,25]). (b) It has been shown that movement planning and movement control are closely intertwined in the central nervous system (e.g., [4,15,18]). (c) The notion of sensory-motor schemas (e.g., [2]) posits that the action-perception loop is to some extent functionally modular. A related idea is the concept of behavioral information ([29,27]) which suggests to analyze action-perception strictly in terms of behavioral requirements (this idea will be important below). (d) It has been shown that the coordination activity of the nervous system is governed by dynamic laws of relational variables [28].

At the same time two important new ideas have arisen in robotics and action planning. First, the approach of behavior-based robotics (cf. [6] for review) seeks to avoid overloaded central representations and to avoid the notorious difficulties of Artificial Intelligence solutions to the action planning problem [6]. In a limited sense this approach may also be viewed as guided by analogies with biological systems (see [1] for such discussion). Second, a new method has been developed for the path planning problem based on moving a system in adequately designed potential fields (e.g., [19,16,17,20,3,9]). Essentially, in the local versions of this approach, path planning is



Gregor Schöner (* 1958) studied mathematics and physics at the Universität des Saarlandes, Germany, obtaining a diploma in physics in 1982 with a thesis on theoretical solid state physics. His doctoral work in theoretical physics (1983–1985) at the Universität Stuttgart, Germany, was concerned with stochastic dynamics in self-organizing systems. A short stay at the Haskins Laboratories, New Haven, CT, USA in 1986 solidified a move into theoretical biology begun already in Stuttgart. This new line of research was deepened while working at the Center for Complex Systems, FAU, Boca Raton, Florida, first as a Research Associate, later as Research Assistant Professor, from 1986 to 1989. Borrowing theoretical concepts from the theory of nonlinear and stochastic dynamical systems, research on the coordination of biological movement, action-perception patterns, learning of coordination patterns and perceptual organization was performed in close alliance of theory and experiment. Since moving to the Institut für Neuroinformatik, Bochum, Germany, in 1989, the research interests in theoretical biology have expanded further to include modelling of temporal order in neurophysiology. A new direction is the application of ideas from theoretical biology to develop a dynamic approach to the design of perceiving and acting autonomous devices.



Michael Dose born in Düsseldorf, Germany. He studied computer science and physics at the University of Dortmund, Germany, where he received his diploma degree in computer science in 1989. Since then he has been working as a research assistant at the Institut für Neuroinformatik in Bochum, Germany, persuing doctoral studies in computer science and neurocomputing. He is a member of the German Computer Society (GI). His research interests include robotics, mobile robots, architectures for autonomous systems and theory of biological movement.

performed by a dynamical system, albeit not always engineered in a very direct way.

In this communication we propose to combine elements of the last two new engineering approaches with theoretical ideas flowing from theories of biological motion. Specifically, we propose a general strategy for dynamic task-level planning detailed in Section 2. The essential ideas are: (a) to introduce planning variables that capture the planned behavior of the system at the level on which tasks are defined, so that behavioral constraints can be directly and clearly expressed in terms of these variables; and (b) to achieve planning by generating a time course of the planning variables from a planning dynamics, a set of equations of motion of the planning variables. Different behavioral constraints contribute to the planning dynamics in a principled fashion. A second layer of dynamics is defined for the relative strenghts of different contributions to the planning dynamics. This layer, modelled on biological movement memory [26], leads through competition to sparse representation of sensed behavioral requirements. The benefits of such an approach include: (a) closed-loop stability of the action-perception behavior; (b) capability to make decisions based on sensory information; (c) accessibility to analysis by reducing the amount of interaction among different components of the dynamics.

After laying out the general strategy (Section 2), we present a concrete module for 2D path planning in Section 3 and demonstrate the properties of the planning dynamics in this example in Section 4.

2. Dynamic approach to system integration

2.1. Planning dynamics

Planning variables: A task-related and behavior-oriented level of description is defined by introducing variables, x, that express the desired or planned behavior of the system. In terms of these *planning variables* behavioral constraints on the system must be directly and clearly expressable, for instance, as points or parametrized sets in the planning space spanned by these variables. Transformation of sensory module information into the planning variables and from there into effector level coordinates must be possible, of course.

Planning dynamics: Planning is achieved by generating a time course of the planning variables from a planning dynamics, that is, a set of equations of motion of the planning variables. To be specific, we assume this dynamics is first order (i.e., that the planning variables have been chosen such that this is the case), formally:

$$\dot{x} = f(x). \qquad (1)$$

The vector-field $f(x)$ must be determined such as to capture task constraints as component forces that define attractors or repellors of the dynamical system. To design such vector-fields we make use of the language of the qualitative theory of dynamical systems (see [5] for elementary introduction, [14,22] for more advanced treatment). The following principles define the strategy with which the planning dynamics are determined.

(1) Task constraints are parametrized in the planning variable space, i.e., they are expressed as points or sets in that space. These points or sets are referred to below as *behavioral information* [29].
(2) Behavioral information defines contributions, f_{info}, to the planning dynamics:
(a) Desired behaviors are modelled as attractors of the dynamics.
(b) Behaviors that must be avoided are modelled as repellors of the dynamics.
(3) Each such contribution to the planning dynamics is characterized by its range, d (measured, for instance, in terms of exponential decay of the strength of the corresponding contribution to the vector field).
(4) The contributions of various task constraints to the total vector-field are treated additively [1]:

$$\dot{x} = \sum_{y} f_{\text{info}}(x, y). \qquad (2)$$

Here various behavioral constraints are parametrized by vectors y.
(5) The design of the functional form of the various items of behavioral information is constrained by the following observation: The extent to which different contributions to the planning dynamics cooperate or compete is determined by the functional dependence or independence of the corresponding force functions. Linearly dependent functions lead through superposition to averaging among the corresponding constraints. Linearly independent functions allow for expression of constraints that are incompatible, contradictory, or independently valid. In that case the solutions of the planning dynamics may differ qualitatively (in number or nature) from the solutions designed into the individual contribution and thus may reflect decisions. Contributions may have the same functional form but centered around different values of the behavioral constraint, y. In this case the amount of overlap of the support areas determines the extent to which the contributions cooperate (averaging) or compete (multi-stability).
(6) The relative strength, w, of each contribution, $f_{\text{info}}(x, y)$, to the planning dynamics is assessed in terms of the time scale, $\tau_{rel} = |1/f_{\text{info}}(x=y, y)|$ of the dynamics of the individual contributions in the vicinity of its invariant solution.
(7) The planning dynamics must be augmented by stochastic forcing functions for conceptual reasons: Because some constraints are modelled as repellors, escape from such unstable invariant solutions must be garanteed.

Note that when the sensing–planning–action loop is closed, the parametrized task constraints may depend implicitly on time and explicitly on the planning variables, e.g., $y = y[x(t)]$. For instance, the behavioral requirements imposed by obstacles depend on the position of a robot vehicle, which in turn depends on the previously planned path, $x(t)$. As a result, the analysis of the planning dynamics underlying the definition of the various forces remains applicable in the closed loop situation only in an approximate sense. The range of validity of this approximation can be estimated on the basis of linear stability analysis of the invariant solutions. For given invariant solution, x_0, the

[1] This is essentially a recipe of how to define the individual contributions rather than a constraint: The individual contributions are defined such as to generate the desired phase portrait, characterized by invariant solutions and their stabilities, in the absence of all other contributions.

condition reads:

$$\left|\frac{\partial f_{\text{info}}}{\partial y}\right|\left|\frac{\partial y}{\partial x}\right|_{x=x_0} \ll \left|\frac{\partial f}{\partial x}\right|_{x=x_0}, \tag{3}$$

in other words, the rate of change of the vectorfield due to the change in behavioral information, y, must be much less than the rate of change due to the evolution of the planning variables.

Clearly, the identification of adequate planning variables is intimately linked with the identification of relevant behavioral constraints and their dynamic modelling. The two ideas sketched here therefore form a self-consistent loop rather than two subsequent steps in the design of a planning dynamics.

2.2. *Competitive dynamics of constraint representations*

Two types of problems remain unsolved by the planning dynamics as laid out above: (a) If redundant sensory information leads to multiple representation of the same underlying behavioral constraint then the strengths of the corresponding contributions to the planning dynamics add up. This would link multisensory convergence automatically to enhanced behavioral significance. For example, an obstacle avoidance system would perform more extensive evasive action if an obstacle had been sensed multiply. Some form of normalization is necessary to truly achieve system integration. (b) Local dynamic path planning systems suffer from spurious solutions if environments become too cluttered. In biology, the problem of avoiding collisions and the problem of finding a path in a constricted space are not necessarily linked. Mathematically, the problem of spurious solutions can be eliminated rigorously only at the cost of sacrificing the strict locality of the approach and by enduring considerable computations expense [9,23].

We propose to deal with both problems by introducing dynamic representations of behavioral constraints in terms of the relative strength factors, w. The dynamics of these relative strenghts must be competitive for those relative strengths representing redundant behavioral information. Such convergent information acts on the planning dynamics only through the selected representative. Only behavioral information of

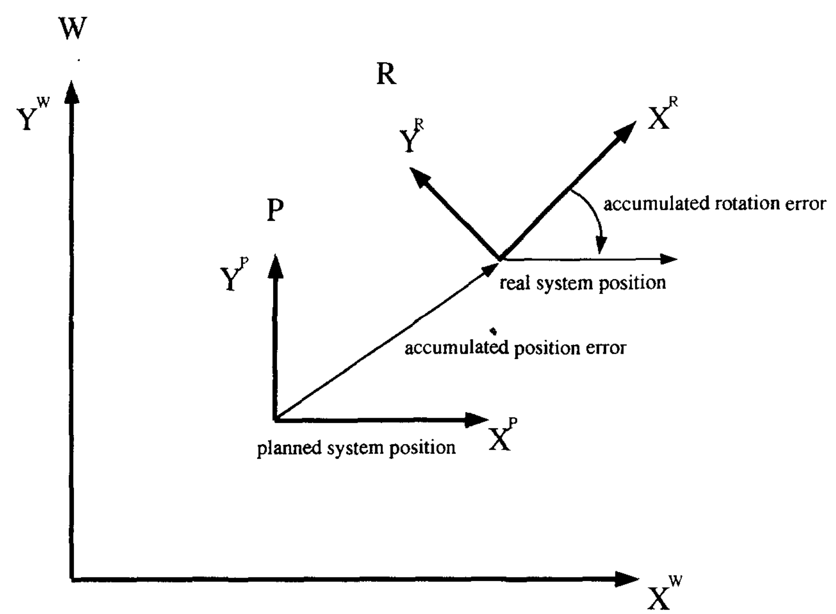


Fig. 1. The world coordinate system, (x^W, y^W), and the planning coordinate system, (x^P, y^P), are aligned, although this relative orientation need not be calibrated because only relative angles such as $\phi - \psi_{\text{object}}$ matter. The R-coordinate system is centered at the real position of the system and is rotated against the W-system. The shift from P-system to R-system as well as the rotation simulate effector errors which accumulate over time. The planning dynamics is stable against such errors.

the same type competes, e.g. behavioral information dealing with different obstacles or behavioral information dealing with multiple targets. Behavioral constraints that are independently valid and non-redundant do not compete. Mathematical forms of competitive dynamics have been discussed in the literature (e.g., [8,11]). Design of such highly non-linear dynamics becomes easier if parametrizations are chosen that are meaningful with respect to the invariant solutions. The time scale of the competitive dynamics must be chosen to be faster than the planning dynamics, so that sensory fusion occurs at a higher rate than planning decisions. It is then assured that the qualitative analysis of the planning dynamics remains valid as the relative strength parameters become dependent on time and on the planning state.

3. Two-dimensional path planning

Consider a mobile system that (a) may move in the plane, (b) senses obstacles and estimates their parameters direction, distance, and linear size, and (c) is provided from higher system modules with either represented or sensed target coordinates. Both obstacles and target may be moving.

3.1. The planning variables

As a conceptual device we introduce a world coordinate system (upper index W) in the plane, described mathematically by cartesian coordinates, (x^W, y^W), with fixed origin (cf. *Fig. 1*). Because relevant objects may often rest with respect to each other, object parameters usually vary slowly in the world coordinate system. This enables design of the planning dynamics in which these object parameters can be treated as adiabatic variables neglecting their dependence on time and planning state. Furthermore, the world coordinate system may serve to integrate the obstacle avoidance module into a behavioral hierarchy (cf. Discussion). In the presence of sensory information the world coordinate system has no operational meaning, however, because planning takes place in a second coordinate system (upper index P) centered in the mobile robot. Angles are measured from the x^P-axis, which is assumed parallel to the x^W-axis. Because only difference between angles enter into the actual planning dynamics calibration of the x^P-axis is not necessary.

In closed loop implementations, sensed objects are represented by their distance from the system, r_{obj}, and the angle under which they are seen from the x^P-axis.

The movement path is given by the heading direction $\phi(t)$ measured from the x^W-axis and the orbital velocity, $v(t)$ in the W-system. The heading direction essentially captures the planned path. Behavioral constraints like moving towards targets and avoiding obstacles can be expressed in terms of heading direction. Specically, the direction (from a reference axis, here x^P-axis) in which targets or obstacles are sensed specify, respectively, desired or undesired heading directions. The velocity may be used to implement additional task constraints like stopping in front of targets, going slowly through bends, etc. These task are dealt with only cursorily in this communication: we set velocity to a constant and turn vehicle motion off once we have reached the target to within a criterion distance. Note that in a sense the two-dimensional path planning task has been reduced to a one-dimensional dynamic problem.

3.2. Targets

The most basic task from which we construct all other desired behaviors is to move in a given direction, ψ. The corresponding contribution to the planning dynamics of $\phi(t)$ is extremely simple. A single fixed point attractor for ϕ at $\phi = \psi$ defined by

$$\dot{\phi} = a \sin(\phi - \psi) + \text{noise}, \tag{4}$$

where a determines the time scale of the planning dynamics: $\tau_\phi = 1/a$. (Note that due to the angular character of ϕ of the dynamics must be 2π-periodic. For escaping instable fixpoints we add a gaussian white noise term. This equation is essentially the circular analog of linear dynamics having a single point attractor.) Only the difference $\phi(t) - \psi$ enters in the dynamics, rendering performance independent of calibration of the planning coordinate system.

With this basic module, other behaviors can be constructed. For example, moving toward a sensed target takes just this form where ψ is chosen as the angle, ψ_{target}, under which the target is seen relative to the x^P-axis. Note that as the system moves with finite velocity, the parameter ψ_{target} becomes a function of time and a functional of $\phi(t)$. The planning dynamics must be sufficiently fast for a given vehicle velocity (choice of τ_ϕ) so that ψ_{target} varies much more slowly than $\phi(t)$. As another task consider moving toward and along a straight line path in W-coordinates. This task can be decomposed into (a) moving parallel to the straight line path, (b) moving toward the straight line path. These targets are modelled by two functions of the form of Eq. (4) with ψ oriented parallel and towards the line path, respectively. Superposition of these target functions defines the dynamics, a case of achieving averaging through linearly dependent contributions to the planning dynamics. Other interesting tasks include movement toward one of multiple targets (multiply functions of the form of Eq. (4) with range limiting factor leading to linear independence for targets sufficiently separated) and movement along arbitrary, parametrized paths (same decomposition as above but locally).

3.3. Obstacles

Sensory system components are assumed to provide the obstacle parameters: (a) ψ_{obst}: angle from the x^P-axis at which the obstacle center is seen (although only $\psi_{\text{obst}} - \phi$ matters); (b) τ_{obst}: distance from the vehicle to the obstacle; and (c) R_{obst}: size of the obstacle as described by a radius. From this we determine the angle, $2\Delta\psi_{\text{obs}} = 2\arcsin(R_{\text{obst}}/r_{\text{obst}})$, subtended by the obstacle.

Obstacles specify the behavior of avoiding to head toward the obstacle, specifically, to avoid heading anywhere within the interval of directions, $[\psi_{\text{obst}} - \Delta\psi_{\text{obst}}, \psi_{\text{obst}} + \Delta\psi_{\text{obst}}]$. Dynamically, this constraint is expressed by defining a force-field contribution to the ϕ-dynamics that erects a repellor at ψ_{obst}. The repelling force must extend to the limits of the interval. A refinement is to specify the repelling direction interval with an adquately scaled vehicle size, d_{vehicle}, added to the obstacle's size, so that $\Delta\psi_{\text{total}} = \arcsin[(R_{\text{obst}} + d_{\text{vehicle}})/r_{\text{obst}}]$. We distinguish the following factors of this dynamic contribution: (a) The proper repelling force $f_{\text{obst}}(\phi, \psi_{\text{obst}}, r_{\text{obst}}, \Delta\psi_{\text{total}})$; (b) a factor range $_{\text{angular}}(\phi, \psi_{\text{obst}}, \Delta\psi_{\text{total}})$ defining an angular range such that outside this range, parametrized by δ as $[\psi_{\text{obst}} - \Delta\psi_{\text{total}} - \delta, \psi_{\text{obst}} + \Delta\psi_{\text{total}} + \delta]$, the repelling force is negligibly small; (c) a factor, $\text{range}_{\text{spatial}}(r_{\text{obst}}, R_{\text{obst}})$ limiting the spatial range, parametrized by d_{obst}, over which the force has appreciable strength; (d) the normalized relative strength factor, $|w(t)| \in [0, 1]$ resulting from competitive dynamics (see below). The mathematical details are given in Appendix A.

The contributions of different sensed obstacles have isomorphic functional form, but each function is centered around the corresponding direction to the obstacle, ψ_i. Because the contributions have finite range in ϕ-space, the amount of overlap of the respective ranges determines the extent to which multiple obstacle forces cooperate or act independently. For example, for closely matching ranges, different contributions superpose such that an obstacle in an averaged direction is avoided. When the ranges overlap little, the different contributions are linearly independent and the individual direction in which obstacles are seen are avoided with the possibility of moving into directions between these forbidden zones. The cross-over from one regime to the other is mediated by instabilities enabling decision making (cf. Section 4).

The complete ϕ-dynamics read:

$$\dot{\phi} = f_{\text{target}} + \sum_i |w_i| \text{range}_{\text{spatial}}(r_i, R_i) \times \text{range}_{\text{angular}}(\phi, \psi_i, \Delta\psi_{(\text{total},i)}) \times f_{\text{obst}}(\phi, \psi_i, r_i, \Delta\psi_{(\text{total},i)}), \quad (5)$$

where f_{target} is one of the target forces discussed earlier.

3.4. Competitive dynamics

The ϕ-dynamics generates reasonable paths on its own and endows the system with the desired closed-loop stability (see Section 4). However, as discussed earlier (cf. Section 2.2) problems (a) with multiple estimates of the same obstacle (lack of normalization) and (b) with spurious solutions in clustered environments still remain unresolved. We propose to deal with both problems by weeding out the obstacle representation through competitive dynamics of the relative strength factors, $w_i(t)$. The idea is that obstacles with sufficient overlap compete leading to activation of one representative, which is chosen in the most conservative manner, that is, as the one closest to the vehicle. Obstacles with little or no overlap do not compete and remain individually active with full strength. The overlap function need not be literally the geometric overlap of obstacles, but may take into account vehicle size, required safety margins, etc.

Among a number of functional forms for competitive dynamics that have been proposed in the literature (e.g., [8,11]) we have chosen the following form ([26], for details see Appendix B):

$$\dot{w}_i = \alpha(i)\left(w_i - w_i^3\right) - \sum_{j \neq i} \gamma(i, j) w_j^2 w_i + \text{noise}. \tag{6}$$

This form has a number of technical advantages: (a) A potential exists guaranteeing relaxation to stationary state and prohibiting oscillatory and other nontrivial invariant solutions. (b) A number of attractor states can be determined analytically and their stability can be analyzed explicitly. This makes the 'mathematical engineering' problem much easier to handle. (c) The coefficients of the dynamics have immediate meaning in terms of the nature and stability of the various attractor solutions: (1) The function $\alpha(i)$ determines the competitive advantage of the i-th contribution: essentially, among competing solutions the one, i_{win}, with the largest positive factor $\alpha(i_{win})$ is turned on ($w_{i_{win}} \to \pm 1$) while the competing contributions are turned off ($w_j \to 0$) (for a more precise formulation see Appendix B). (2) The function $\gamma(i, j)$ determines the degree to which contributions i and j compete: For $\gamma(i, j) \sim \alpha(i)$ the contribution j competes with the contributions i, while for $0 < \gamma(i, j) \ll \alpha(i)$, the relative strength dynamics of contribution i is independent of that of contribution j.

The generation of competition through dynamics (rather than by means of symbolic computation) leads to continous behavior. For example, if the optimal representative of redundant information is not unique, multistable dynamics result in which hysteresis or history effects prevent the system from oscillating among possible options. Symbolic computation is more difficult to integrate into stable control behavior.

4. Results

We first focus on the ϕ-dynamics, keeping the relative strengths w_i fixed, to demonstrate the properties of stability and flexibility. Subsequently, the working in unison of the two layers of dynamics is examined. We simulated the planning dynamics in software by numerically solving the corresponding differential equations.

4.1. Stability of the planning dynamics

The proposed planning dynamics is stable if operated in closed loop, i.e., when obstacle and target parameters are derived at each point in time from sensory input and movement commands take effect in the physical world (instantiated and situated system). To simulate this situation we store objects in world coordinates and determine the two planning parameters distance and angle by transforming into the P-system and using polar coordinates. Control errors on the effector side are introduced by allowing that at each point in time the system position may be translated and rotated against the planned position (cf. *Fig. 1*). These errors are assumed to be gaussian random variables with constant mean and variance per unit time (leading to linear drift of the mean and gaussian white noise for the random error in the continuous time limit). To keep track of the accumulating error we introduce a coordinate system (upper index R, cf. *Fig. 1*) centered at the real system position and rotated to represent real heading direction of the system. A second error with constant mean and variance is added to the object parameters distance and angle to simulate sensor errors, but this error is not accumulated over time. Note that the object parameters are determined in the R-system but are then fed without further transformation into the planning dynamics which operate at the level of the P-system. *Fig. 2* illustrates that the system, symbolized by a crossed circle, stably achieves its tasks in spite of accumulating and random effector and sensor error. For instance, in the simulated closed-loop operation an externally induced sudden shift of the vehicle position in the plane would be compensated successfully (although, if large enough, it may lead to a different path being followed toward the target). This form of independent in closed-loop of a calibra-

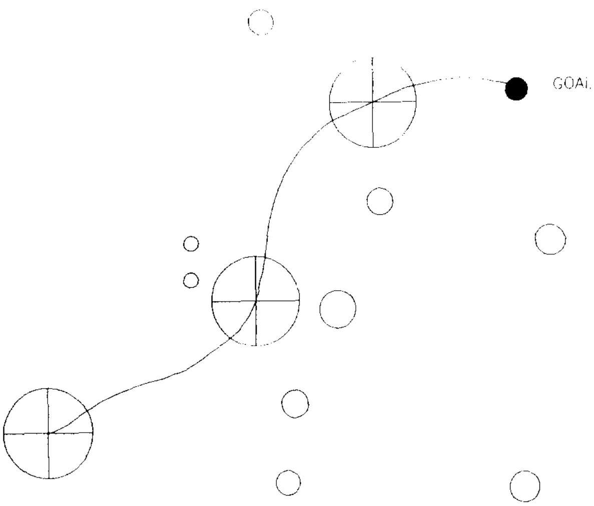


Fig. 2. Simulation of the planning dynamics: a target position is achieved while avoiding sensed obstacles in spite of accumulating effector and random sensor errors. The vehicle is symbolized by a crossed circle, the size of which reflects the vehicle size, d_{vehicle}. The obstacles are open circles, the target is the filled circle. The figure shows the real trajectory and the vehicle at different times.

tion of the planning coordinate system is illustrated in *Fig. 3*.

Stability and independence of calibration of the command level is important if the obstacle avoidance module is to be incorporated into a behavioral hierarchy. It assures that the behavioral level of this module (e.g., avoiding sensed obstacles) works stably in relation to the information provided by the sensory systems on the same level independently of the quality of task constraints provided by higher modules (e.g., world coordinate system). Stability is achieved because behavioral information (e.g., ψ_{obst}) is weakly time and state-dependent through choice of coordinate system.

The dynamic architecture accomodates moving obstacles as demonstrated in *Fig. 4*. Clearly, as long as the change in obstacle parameters is slow in comparison to the relaxation time of the planning dynamics, no conceptual difference exists between stationary and moving obstacle avoidance. The range of available velocities limits, of course, the obstacle speeds that can be handled successfully by the system. Furthermore, without extra effort we may also let two autonomous systems, governed each by a dynamical planning architecture, avoid collision with each other: The

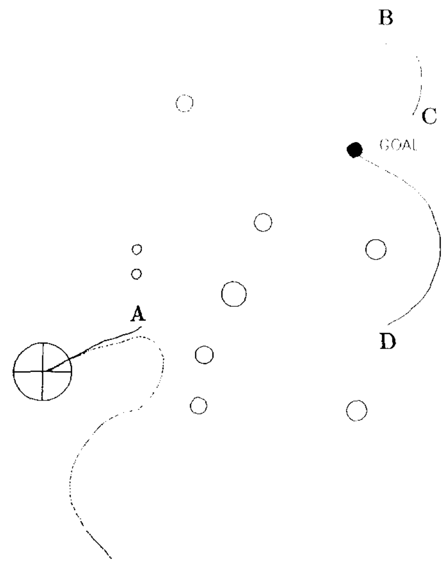


Fig. 3. Independence of the obstacle avoidance module from calibration of the planning coordinate system: Initially the vehicle moves toward a target position. At position A the vehicle is abruptly moved to point B while keeping its heading direction. The planning dynamics changes the heading direction adequately. At point C another abrupt shift puts the system to point D. At the same time the heading direction is abruptly rotated by π. The vehicle proceeds towards the goal avoiding successfully a sensed obstacle. We display both the real trajectory (solid line) and the 'planned' trajectory (dotted line). Note that due to the external perturbations, the planning coordinate system is completely decalibrated both translationally and rotationally. This does not affect the simulated closed-loop performance. Note, however, that we assume that the sensory modules solve the segregation problem of detecting which is target and which is obstacle.

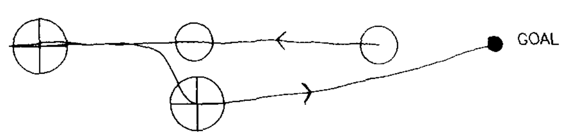


Fig. 4. Dynamic obstacle avoidance: While the vehicle moves to the goal positions on the right side of the figure, a dynamic obstacle, depicted by an open circle, moves to the left. The intermediate positions of obstacle and vehicle are simultaneous.

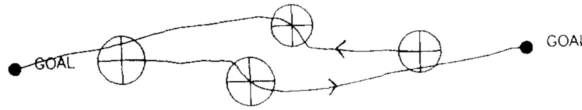


Fig. 5. Collision avoidance of two vehicles: Assuming that the two vehicles sense each other as dynamic obstacles collision avoidance results from dynamical obstacle avoidance. The depicted intermediate positions are simultaneous.

two systems need only detect each other as obstacles. This is demonstrated in the simulation shown in *Fig. 5*.

4.2. Flexibility through instabilities

The contributions of the different obstacles are linearly independent if the shared support is zero, that is, if the intervals $[\psi_i - \Delta\psi_{i,\text{total}} - \delta, \psi_i + \Delta\psi_{i,\text{total}} + \delta]$ and $[\psi_j - \Delta\psi_{j,\text{total}} - \delta, \psi_j + \Delta\psi_{j,\text{total}} + \delta]$ are disjoint. These contributions become increasingly dependent, as their overlap increases. Therefore, the degree of angular overlap between obstacles determines to which extent the corresponding behavioral reqirements are averaged or act independently. At the crossover between these two limit cases instabilities may occur. This is illustrated in *Fig. 6*: When two obstacles are sufficiently far from each other, the corresponding vector-field contributions share little support and are thus linearly independent. The summed force-field has repellors corresponding separately to each requirement with an attractor in between. If we decrease the distance between the two obstacles, their shared support area decreases. At a critical distance value, the attractor

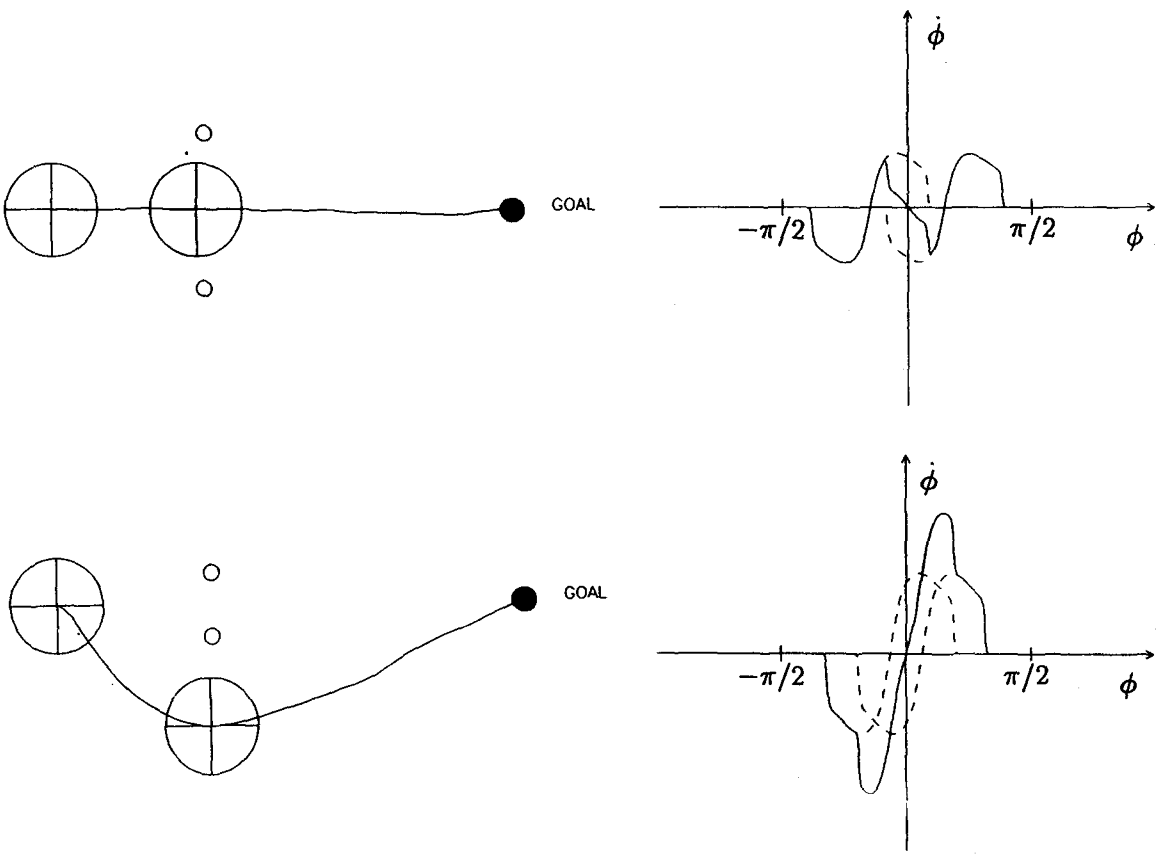


Fig. 6. Decisions in the planning dynamics: Two situations are shown, with obstacles separated by more than a vehicle size (top) and less than a vehicle size (bottom). In the left column the realized paths are plotted: The vehicle passes between the obstacles to reach the target through the most direct path if possible. For obstacles that are too closely spaced the path changes qualitatively and now circumnavigates both obstacles on one side. In the right columnn the corresponding initial ϕ-dynamics are plotted. The contributions to the individual obstacles are shown (dashed lines) as well as their superposition (solid line). On top, the obstacles are sufficiently separated (in angle) so that the individual contributions share only little support and the two separate repellors are erected. Between the two repellors an attractor exists allowing for passage between the obstacles. In the bottom panel the obstacles are sufficiently close to each other (in angle) to share a fair amount of support: the two repellors combine into one intermediate repellor. This qualitative change of the dynamics as a function of the distance of the two obstacles is a bifurcation.

between the two repellors becomes unstable and the summed vector-field has a single repellor, now reflecting the requirements imposed by both obstacles in an averaged fashion. Behaviorally, this bifurcation leads to a qualitative change in the planned path. For large distance between obstacles, three paths are possible, depending on initial conditions: passing the two obstacles on the right or on the left, or else, passing in between the two obstacles. For smaller distances, the path in between the obstacles is not available anymore. The range of initial conditions from which this path is chosen shrinks to zero at the critical distance. Note that the critical distance depends on the vehicle size and is determined – in the functional description chosen here – exactly by the distance below which the vehicle fails to pass physically between the two obstacles.

This instability reflects both an active form of system integration and an essential piece of flexiblity of the path planning system: System integration is achieved in the sense that two (and analogously, multiple) obstacles that are sufficiently close to each other are fused into just one obstacle in terms of the actual behavior. Flexibility is achieved in the sense that as the sensed environment changes the system may change its planning solution continuously, but also discontinuously. This is illustrated in *Fig. 7* where an additional obstacle leads to the selection of a qualitatively different corresponding path. Again, instabilities are at the origin of such flexibility: if the additional constraint is gradually increased in strength from zero, a bifurcation in the path planning dynamics occurs. Near the decision regime multistable regimes usually exist (except in special symmetric situations), in which several paths are possible. An advantage of the dynamic approach to path planning as compared to symbolic decision making is that *hystersis* leads in such situation to the maintenance of stability within the decision zone: The system will tend to persist in one solution to the path planning problem until another solution is more stable than the realized one.

The problems dynamic path planners have in cluttered environments (cf. Introduction, [9]) can be discussed in terms of instabilities. In potential field formalisms the problems appear as stable stationary positions of the vehicle other than the target. In the present approach the problem shows

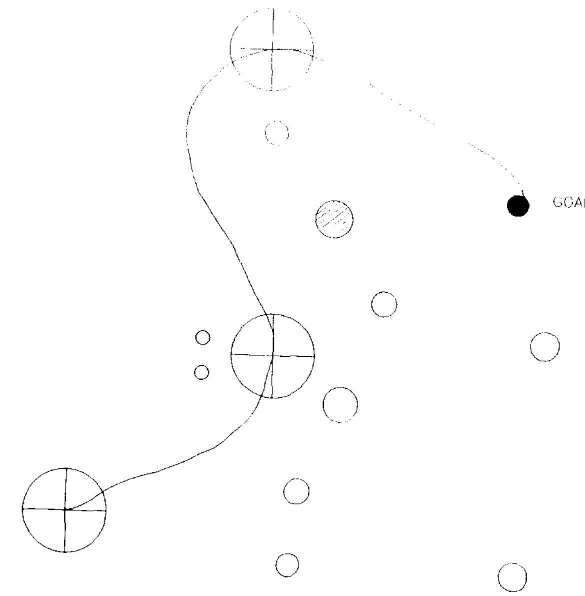


Fig. 7. Flexibility: Here an obstacle has been added to the configuration of *Fig. 2* (hatched circle). The resulting path takes a different route. If in a Gedankenexperiment we increase the strength of the added obstacle from 0 to a finite value then the abrupt change of the path also represents a bifurcation of the planning dynamics.

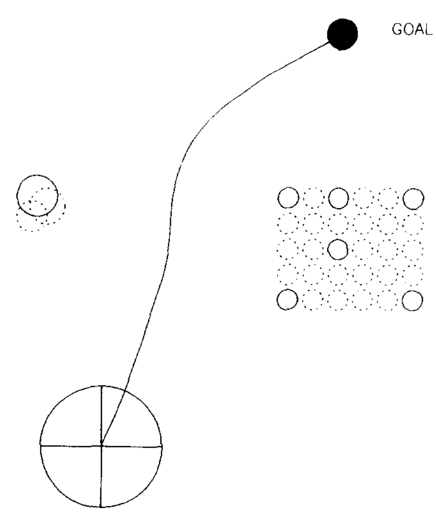


Fig. 8. The reduction of the obstacle contributions through competitive dynamics of relative strengths is illustrated in a standard situation. Obstacles with relative strength near one are plotted as solid circles, obstacles with near zero strength as dashed circles. Note that the active representation may change as the vehicle moves. Rasters of obstacle contributions as shown on the right may arise from visual obstacle detection procedures such as the inverse perspective method [21]. The representation at the final state is shown.

up in the form of attractors in the ϕ-dynamics which allow the system to pass between obstacles even if the passage way is narrower than the vehicle size (plus safety margin). For two individual obstacles the instability discussed above prevents such moves. Essentially, the attractor in the heading direction between the two obstacle collides with two repellors (one stemming from each obstacle) and is turned unstable in a pitchfork bifurcation. By analyzing this bifurcation it is possible to design at which critical distance between two obstacles this instability is to occur based on the spatial constraints of obstacle size, safety margin, etc. If force functions of more than two obstacles overlap, this analysis becomes quite difficult (cf. top panel of *Fig. 9*). Essentially, the bifurcation analysis is strictly local in nature and this locality is destroyed as additional obstacles must be taken into account. The same mechanism underlies the problem of spurious minima in potential field approaches, in which velocity and heading direction are coupled, however, leading to the vehicle stopping rather than passing through narrow passages. Ways of dealing with this problem must deal with nonlocality. One way to do this to design the vector-field of the dynamics based on non-local information, e.g., by solving a Laplace equation into which all obstacles and targets enter as boundary conditions [9]. In our approach a dynamics of representation weeds out the set of represented obstacles and thus tends to reduce cluttering.

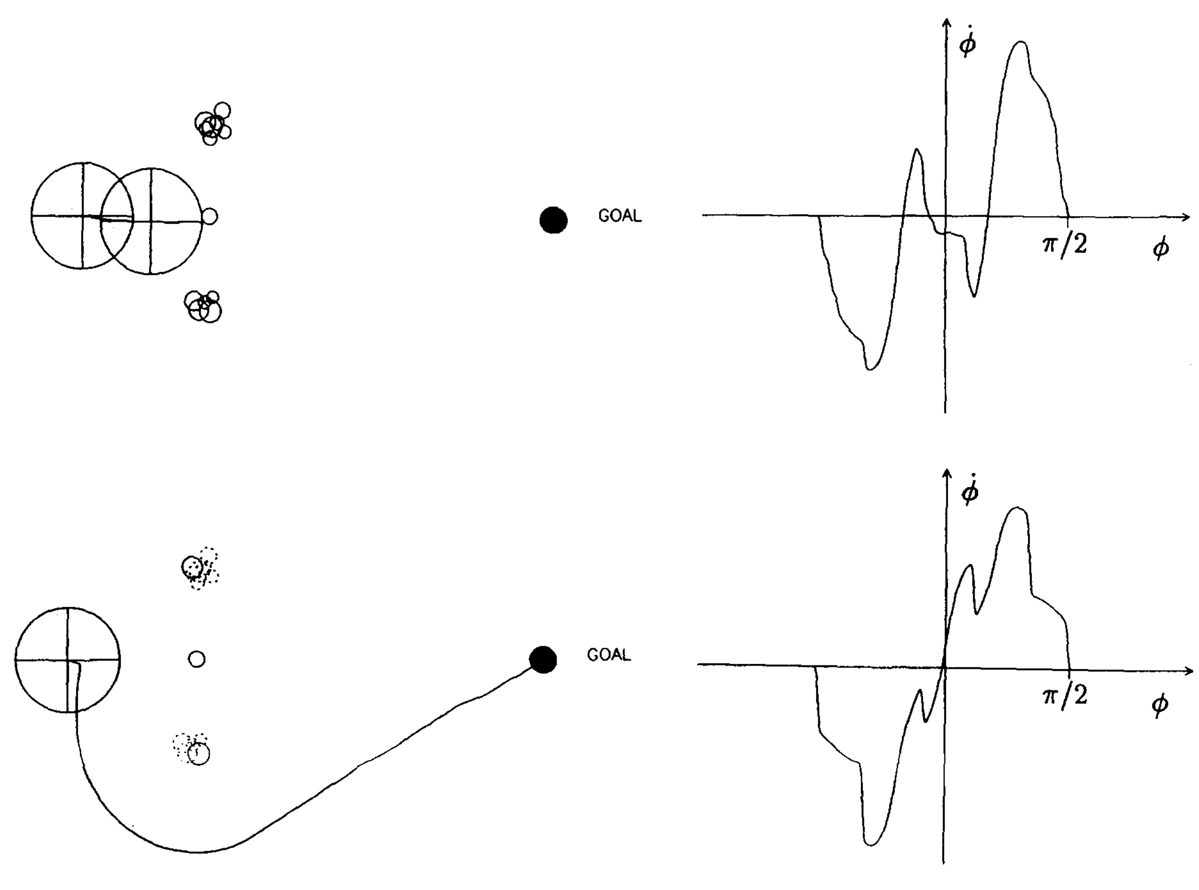


Fig. 9. Alleviating the problems of path planning in cluttered environments through competitive dynamics: The three clusters of obstacles shown in the upper left panel lead to a spurious attractor in the ϕ-dynamics as illustrated in the upper right panel. The corresponding path leads to collision with the central obstacle (because velocity is constant in our simulations). The spurious attractor is due to the non-local effects of multiple contributions. When the competitive dynamics or relative strengths is implemented (bottom panels), the clusters are weeded out and three normalized representative contributions are selected (solid circles) while most obstacle forces are turned off (dotted circles). The spurious attractor disappears and no danger of collision exits.

4.3. Reducing representations through competitive dynamics

Analytic analysis (see Appendix B) for two overlapping obstacle contributions shows that two cases can be distinguished: (a) For large overlap ($\gamma(i, j) > \alpha(i)$) the contributions compete and lead to stable activation of the contribution i closer to the system: $w_i = 1$, $w_j = 0$. (b) If the system is very close to the obstacles ($\gamma(i, j) < \alpha(i)$), then both contributions are activated but with a reduced weight leading to averaging among the obstacles:

$$w_i = \sqrt{\frac{\alpha(i)\alpha(j) - \alpha(j)\gamma(i, j)}{\alpha(i)\alpha(j) - \gamma(i, j)\gamma(j, i)}}\,, \tag{7}$$

and analogously for w_j. For very small overlap ($\gamma(i, j) \ll \alpha(i)$) the contributions have relative strength near one: $w_i \approx w_j \approx 1$. Essentially this means that competition is induced whenever the overlap of obstacle contributions measured on an adequate scale is larger than the distance between obstacles and system. *Fig. 8* illustrates how the dynamics of the representation reduces obstacle information in this case: Competition leads to selection of a few representatives for a group of overlapping obstacles. Note that the representation may change as the vehicle moves. The competitive advantage is determined by the distance of the system from the obstacle creating the tendency to select the closest sensed obstacle as a representative. This mechanism is useful as the vehicle moves along a wall sampled as a row of sensed obstacles as shown in *Fig. 10*: At all times a suitable obstacle close to the vehicle is activated. The bottom panel of *Fig. 9* explains how

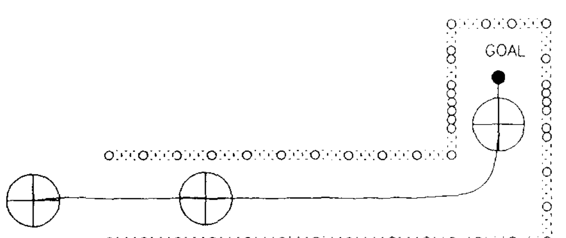


Fig. 10. The system moves smoothly along a wall sampled as a string of obstacles. Note the weeding out of the representation. The relative strengths of different contributions are indicated using the same convention as in *Fig. 8*. The situation depicted is the one corresponding to the last intermediate position before reaching the goal.

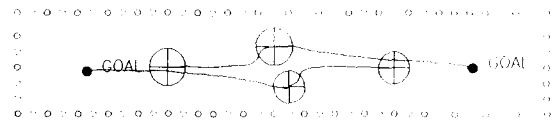


Fig. 11. This final figure combines all features: two vehicles avoid each other as well as a set of obstacles that samples walls. The dynamics of the obstacle relative strength reduces the wall representation.

the competitive dynamics of the relative strengths alleviates the problems with cluttered environments.

5. Discussion

In this report we have presented a general strategy to integrate sensory and effector systems in a task-related manner through defining planning dynamics on the level of behavior-based variables. All behavioral constraints are represented as forces acting on the planning dynamics. The relative strengths of different contributions are governed by a competitive dynamics which selects representatives of redundant information. By numerically solving a model system describing path planning in 2D including simulation of sensor and effector error we have demonstrated (1) the stability of the planning dynamics in closed-loop operation; (2) the flexibility of the path planning dynamics including capability to change paths abruptly as the sensed environment changes continuously; (3) the capability of the relative strength dynamics to reduce sensory information in a behaviorally relevant way. The strengths of our specific solution to the path planning problem are (a) straightforward design of behavioral and task constraints into dynamics of the planning variables; (b) accessibility of the dynamics to analytic treatment granting control of instabilities; and (c) reduction of a 2D planning problem to a one-dimensional dynamic problem. *Fig. 11* illustrates some of the features of our method: avoidance of dynamical obstacles, collision avoidance of two vehicles, moving along walls, reducing representations.

From the present work generalization is necessary and possible in at least two directions. First, the action-perception loop implemented here is essentially a module for obstacle avoidance in 2D. We have not exploited so far the fact that the design carefully takes into account organizational principles of behavior-based robotics. In a next step, the existing module can be incorporated into a hierarchy of action-perception loops. The next higher level, for instance, could be concerned with determining target positions based on its own sensory information and parametrically modulated by target properties (e.g., sensory parameters: 'look for red ball' etc.). The output of this module would not act directly on the effectors but serve to provide the target parameters to the present module. This level would essentially solve the segregation problem hinted at in *Fig. 3*.

A second generalization is to use the approach in more abstract planning tasks. In this paper, one-dimensional planning dynamics were considered. In principle, however, formulations in more than one dimension are possible as well, the only cost being the increased analytical difficulty. On the other hand, formulations in two dimensions may be particularly advantageous. In polar coordinates, one dimension captures the direction of change, such as the angular variable used in this article. The second dimension, a radial variable, is free to control the rate of change and thus to provide for rate invariance. Higher-dimensional problems (e.g., a robot arm) can be mapped onto two dimensions by exploiting topographic representations (e.g., [24]). In this way, more complex trajectory planning in general control situations can be achieved with essentially the same strategy as laid out in this article.

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Appendix

A. Obstacle forces: Mathematical details

The repelling function:

$$f_{\mathrm{obst}}(\phi, \psi_{\mathrm{obst}}, r_{\mathrm{obst}}, \Delta\psi_{\mathrm{total}}) = (\phi - \psi_{\mathrm{obst}}) \frac{1}{\Delta\psi_{\mathrm{total}}} \exp\left[1 - \left|\frac{\phi - \psi_{\mathrm{obst}}}{\Delta\psi_{\mathrm{total}}}\right|\right]. \tag{8}$$

The angular range:

$$\mathrm{range}_{\mathrm{angular}}(\phi, \psi_{\mathrm{obst}}, \Delta\psi_{\mathrm{total}}) = \tfrac{1}{2}\left[\tanh(h_1\{\cos(\phi - \psi_{\mathrm{obst}}) - \cos(2\Delta\psi_{\mathrm{total}} + \delta)\}) + 1\right], \tag{9}$$

where h_1 determines the force cut-off's steepness. (In the simulations we always chose $h_1 = 4./(\cos(2\Delta\psi_{\mathrm{total}}) - \cos(2\Delta\psi_{\mathrm{total}} + \delta))$.)

The spatial range:

$$\mathrm{range}_{\mathrm{spatial}}(r_{\mathrm{obst}}, R_{\mathrm{obst}}) = \exp\left[-\frac{r_{\mathrm{obst}} - R_{\mathrm{obst}} - d_{\mathrm{vehicle}}}{d_{\mathrm{obst}}}\right], \tag{10}$$

where we scale according to the distance from the vehicle to the outer boundary of the obstacle.

B. Competitive dynamics: Mathematical details

In the dynamics Eq. (6) the relative strengths are normalized to $[-1, 1]$ with a redundant sign, so that the actual strength to be used is the modulus, $|w_i| \in [0, 1]$. The competitive advantage of contribution i:

$$\alpha(i) = 1 + \exp\left(-\frac{r_i - R_i - d_{\mathrm{vehicle}}}{d_\alpha}\right) \in]1, 2]. \tag{11}$$

The overlap of contributions i and j:

$$\gamma(i, j) = \tfrac{1}{2} t_h f\left[1 - \tanh\left(2.5\left(\frac{d(i, j) - \max(R_i, R_j) - d_\gamma}{\min(R_i, R_j) + d_\gamma}\right)\right)\right] \in]0, t_h[, \tag{12}$$

where $d(i, j)$ is the distance between the centers of the two obstacles and $f = (R_j + d_\gamma)/(R_i + d_\gamma)$ for $R_i > R_j$ and $f = 1$ else. $\gamma(i, j)$ jointly with f defines an overlap measure of two obstacles that carefully deals with cases where one obstacle is contained in the other. t_h is a constant. Note that $\alpha(i) > 0$ and $\gamma(i, j) > 0$ at all times to assure boundedness.

We note the following analytical results in useful limit cases:

(1) The competitive states where all relative strengths are zero except one (index, say, $i_{\max}$) which is unity are stationary solutions. Such states are stable as long as $\alpha(j) < \gamma(j, i_{\max})$ for all $j \neq i_{\max}$. If all obstacles considered compete (finite

$\gamma(i, j)$), and we consider a situation where the system approaches such a group of obstacles, then initially all these competitive states are multistable until one state first violates this condition. The index j for which the condition is first violated (maximal $\alpha(j)$) defines a competitive state for $i_{\max}=j$ which is the only one that remains stable. Hence, the corresponding obstacle is turned on, all others are turned off. [For proof, first convince yourself that $w_{i_{\max}} = \pm 1$, $w_j = 0$ $(j \neq i)$ is a stationary solution of the dynamics and then perform linear stability theory.] Obstacles that have no or negligable overlap with all other obstacles are turned on at all times.

(2) For two obstacles all stationary states can be determined analytically and linear stability theory can be performed (although it is lengthy) We find:

(1) $w_i = w_j = 0$ is unstable for any $\alpha(i) > 0$, $\alpha(j) > 0$.
(2) $w_i = 0$; $w_j = 1$ is stable as long as $\alpha(i) < \gamma(i, j)$ (and analogously for the symmetric case).
(3) For $\alpha(i) > \gamma(i, j)$ and $\alpha(j) > \gamma(j, i)$ there is a noncompetitive solution

$$w_i = \sqrt{\frac{\alpha(i)\alpha(j) - \alpha(j)\gamma(i, j)}{\alpha(i)\alpha(j) - \gamma(i, j)\gamma(j, i)}}\,. \qquad (13)$$

These two conditions are necessary for the existence and sufficient for the stability for the solution. Note that this solution leads for $\gamma(i, j) \to 0$ and $\gamma(j, i) \to 0$ to the case of non-overlapping obstacles which do not compete at all.

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