Exercise 2, Oct 19, 2017

The exercises use the interactive MATLAB simulator `launcherTwoNeuronSimulator`. This program simulates two activation variables, informally called neurons, with external input, self-excitation, interaction, and noise, as defined by the equations

$$\begin{aligned}\tau_1\ \dot{u}_1(t) &= -u_1(t) + h_1 + s_1(t) + c_{11}\ g(u_1(t)) + c_{12}\ g(u_2(t)) + q_1\ \xi_1\\ \tau_2\ \dot{u}_2(t) &= -u_2(t) + h_2 + s_2(t) + c_{21}\ g(u_1(t)) + c_{22}\ g(u_2(t)) + q_2\ \xi_1\end{aligned}$$

where the sgimoid function, g, is given by

$$g(u) = \frac{1}{1 + \exp(-\beta u)}$$

1. Use the simulator to explore the dynamics of a single activation variable with variable input, as specified by

$$\tau_1 \dot{u}_1(t) = -u_1(t) + h_1 + s_1(t) + q_1\ \xi_1$$

   (a) Tracking: Explore how the activation variable tracks a shifting input. Use the s_1 slider to set the input parameter to different values and observe how the zero-crossing of the phase plot of u_1 is shifted around. Observe how the state variable tracks the input by relaxing to the new attractor, both in the trajectory plot and the phase plot.

   (b) Relaxation time: Note how the state changes faster initially when the distance to the new attractor is larger, but the overall shape of the relaxation curve is always the same. Compare relaxation times for different values of τ: Use the `Parameters` button to set τ_2 to a value that is significantly different from the value of τ_1 (to do this, select the corresponding node in the drop-down menu in the parameter panel, enter the desired value of τ, and click Apply). Use the same resting level and non-zero stimulus for u_1 and u_2, then Reset both activation variables to observe the differences in relaxation time. Do this for several different parameter settings.

   (c) Stability: Set the relaxation time parameters to very different values, for example, 10 and 1000. Add a small amount of noise to both systems and observe how the activation variable with higher relaxation time deviates significantly further from the resting level and takes longer to return to it eventually (use $h = 0$, no input, $q_1 = q_2$). How is this effect reflected in the two-dimensional combined trajectory plot?

2. Explore the dynamics of a single neuron with self-excitation, as specified by

$$\tau_1 \dot{u}_1(t) = -u_1(t) + h_1 + s_1(t) + c_{11}\ g(u(u_1(t)) + q_1\ \xi_1$$

   For this exercise, set the relaxation time parameters of both activation variables back to their initial values, $\tau_1 = \tau_2 = 20$, and set the resting levels back to $h_1 = h_2 = -5$. Start with a stimulus amplitude of zero.

(a) Detection: Increase self-excitation strength, c_{11}, of the activation variable to a medium value and note the nonlinearity emerging in the phase plot. Move the system through the detection instability by increasing the stimulus amplitude systematically. Move the system back through the reverse detection instability by decreasing the stimulus.

(b) Hysteresis: Modify the self-excitation and stimulus to put the system u_1 into the bistable regime, then copy the parameter values to u_2 in order to create two identical systems. Demonstrate the hysteresis effect of this system by temporarily varying the stimulus of one system. After resetting the stimulus to the old value, the activation variables of these two identical systems should relax to different attractors.

(c) Perturbations: Find parameter settings for a bistable system with moderate self-excitation, reset the system, and let it relax to the off-attractor. Subject the system to a random perturbation by temporarily adding a lot of noise to the system. Does the system stay in the off-state after the perturbation or switch to the on-state? Repeat this process several times and note the ratio of returns versus switches. How does this ratio change when you vary the self-excitation strength?