Attractor dynamics approach to behavior generation: dynamical systems tutorial

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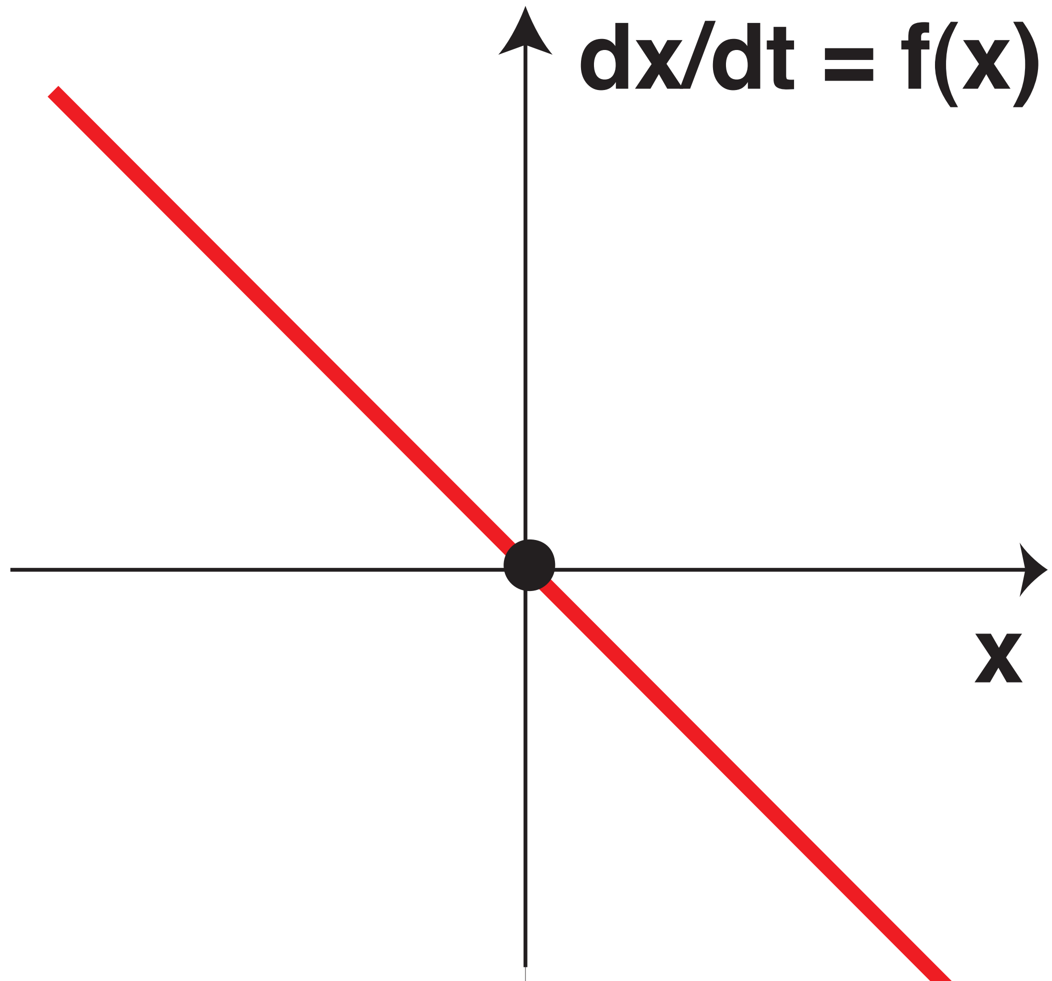
Dynamical systems: Tutorial

- the word "dynamics"
  - time-varying measures
  - range of a quantity
  - forces causing/accounting for movement => dynamical systems
- dynamical systems are the universal language of science
  - physics, engineering, chemistry, theoretical biology, economics, quantitative sociology, ...

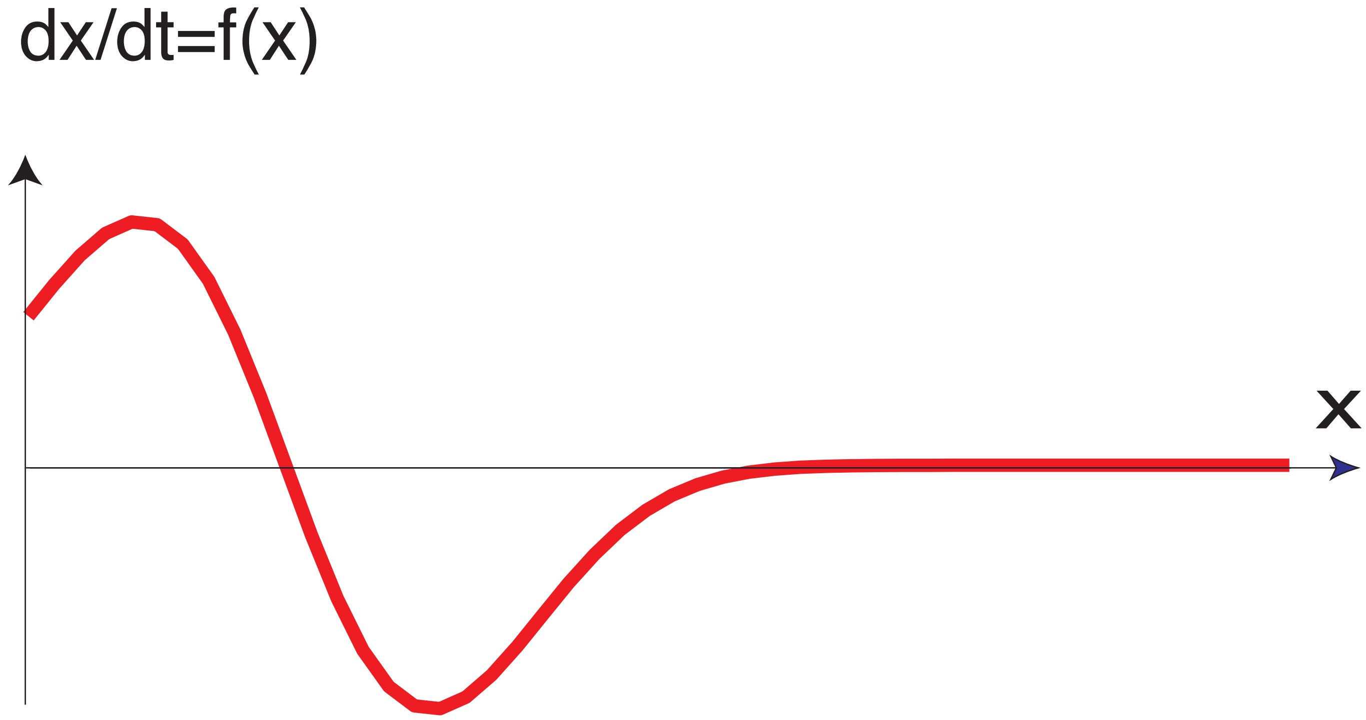
time-variation and rate of change

- variable x(t);
- rate of change dx/dt

dynamical system



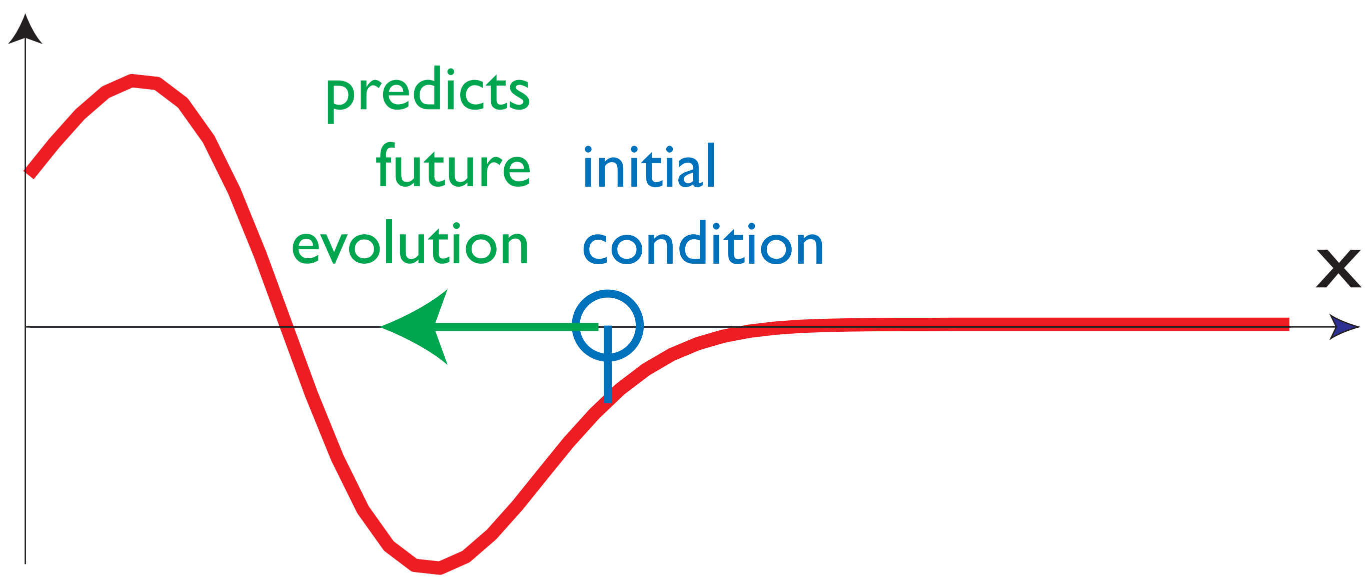
dynamical system



dynamical system

- present determines the future
  - given initial condition
  - predict evolution (or predict the past)

$dx/dt=f(x)$



dynamical systems

- x: spans the state space (or phase space)
- f(x): is the "dynamics" of x (or vector-field)
- x(t) is a solution of the dynamical systems to the initial condition x_0
  - if its rate of change = f(x)
  - and x(0)=x_0

Dynamical systems

- as differential equations: initial state determines the future

$$\dot{x} = f(x)$$

Dynamical systems

- a vector of initial states determines the future: systems of differential equations:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) \quad \text{where} \quad \mathbf{x} = (x_1, x_2, \ldots, x_n)$$

Dynamical systems

- continuously many variables x(y) determine the future = an initial function x(y) determines the future
  - partial differential equations
  - functional differential equations

$$\dot{x}(y,t) = f\left(x(y,y), \frac{\partial x(y,t)}{\partial y}, \ldots\right)$$

$$\dot{x}(y,t) = \int dy' g\left(x(y,t), x(y',t)\right)$$

Dynamical systems

- a piece of past trajectory determines the future
  - delay differential equations
  - functional differential equations

$$\dot{x}(t) = f(x(t-\tau))$$

$$\dot{x}(t) = \int^{t} dt' f(x(t'))$$

numerics

- sample time discretely
- compute solution by iterating through time

$$\dot{x} = f(x)$$

$$t_i = i * \Delta t; \qquad x_i = x(t_i)$$

$$\dot{x} = \frac{dx}{dt} \approx \frac{\Delta x}{\Delta t} = \frac{x_{i+1} - x_i}{\Delta t}$$

$$x_{i+1} = x_i + \Delta t * f(x_i)$$

[forward Euler]

linear dynamics

- => simulation