Prof. Dr. G. Schöner, Institut für Neuroinformatik gregor.schoener@rub.de

Autonomous robotics

May 12, 2017

Exercise 3 Dynamical Systems

This exercise is due on the May 18, 2017.

1 Linear dynamics

Consider this simple linear dynamical system:

$$\dot{x} = -\alpha(x - x_0)$$

where x is the dynamical variable and $x_0 < 0$ and α are two parameters.

1. Compute the fixed point of this dynamics formally. A fixed point is a value, x_0, of the dynamical variable, x, at which the rate of change of the dynamical variable is zero.

2. Make a plot of the dynamics ($\dot{x}$ vs. x) for $\alpha > 0$ and $\alpha < 0$. Mark the fixed point of this dynamics and interpret graphically the parameter α. Based on the graph, discuss whether the fixed point is stable or not for the two signs of α. (The fixed point is stable if solutions starting nearby converge in time to the fixed point. The fixed point is unstable if solutions starting nearby may diverge from the fixed point).

3. Write down the general solution of this equation for $x_0 = 0$ and any initial condition $x(0)$. (If you don't know how to do this, look this up in any textbook on differential equations, e.g., the freely downloadable Scheinermann, E.R., Invitation to Dynamical Systems (at `https://github.com/scheinerman/InvitationToDynamicalSystems`), equations 2.3 there). Plot the time courses of the solution for $\alpha > 0$ for different initial conditions and discuss the asymptotic behavior for large times.

2 Nonlinear dynamics

Consider this dynamical system:

$$\dot{x} = f(x) = \alpha - x^2$$

where x is the dynamical variable and α is a parameter. (This equation is the normal form of the tangent bifurcation. Use any text book, including Scheinerman, for help and for background on this bifurcation. The lecture slides also treat this example.)

1. Compute the fixed points of this dynamics by solving $\dot{x} = 0$.
2. Determine the stability of the fixed points by computing the derivative of the dynamics, $f(x)$, at the fixed point and examining the sign as α is varied).
3. Make plots of the dynamics (drawing $\dot{x}$ against x) for $\alpha < 0$, $\alpha = 0$ and $\alpha > 0$. Draw the bifurcation diagram, that is, draw the fixed points as a function of the parameter, α, marking branches on which the fixed point is stable and branches on which it is unstable.