Degree of Freedom problem, synergies, and the uncontrolled manifold

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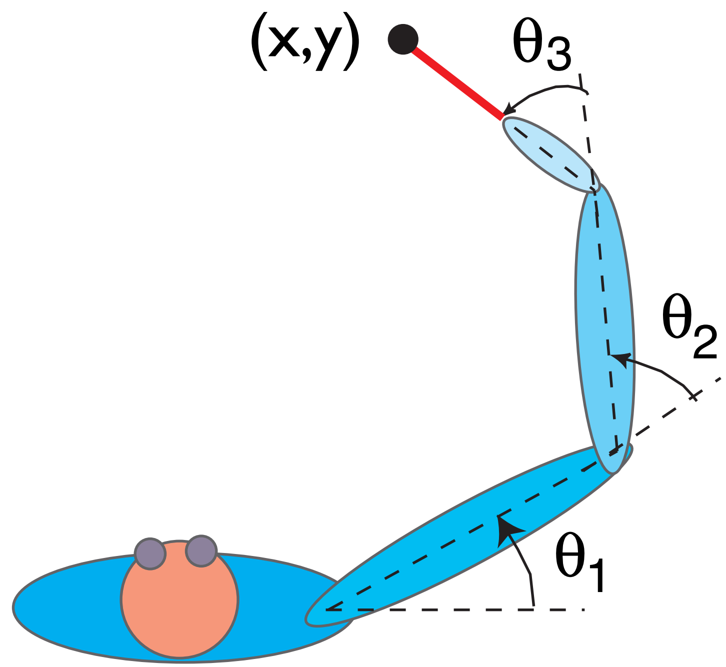
Degree of freedom problem

- what is a DoF?

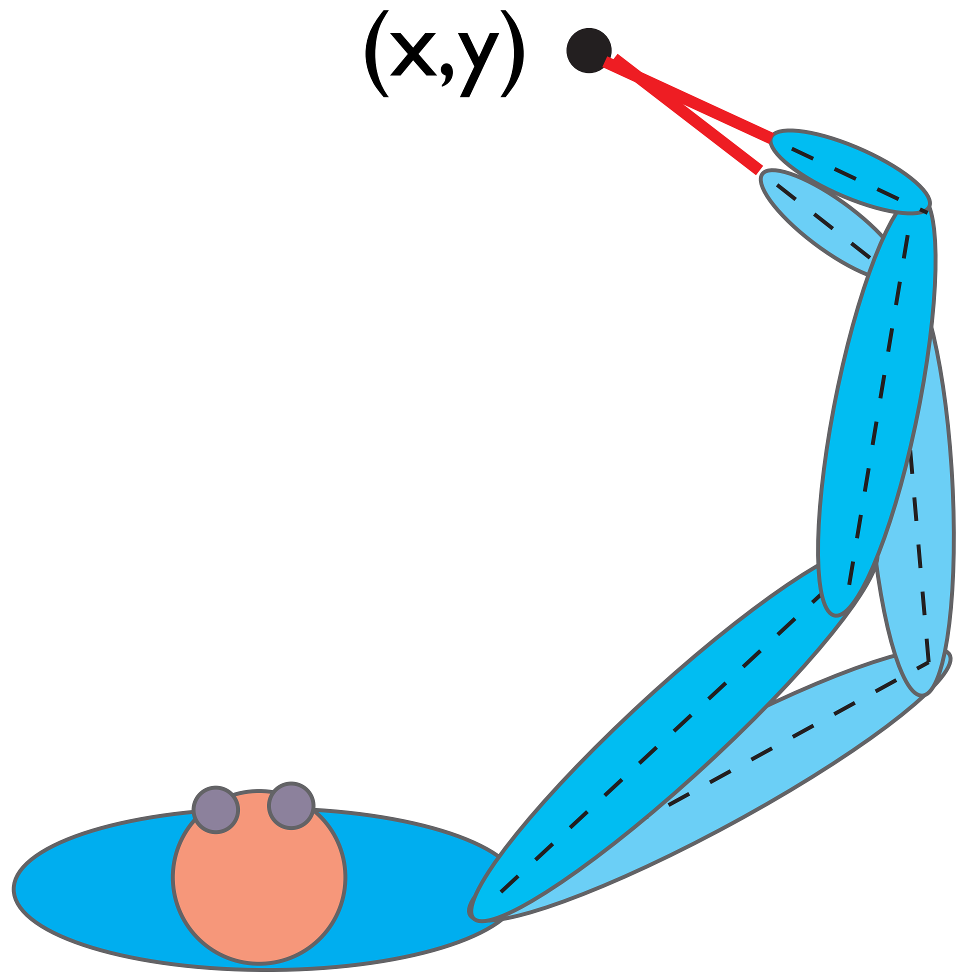


$x = l_1 \cos(\theta_1) + l_2 \cos(\theta_1+\theta_2) + l_3 \cos(\theta_1+\theta_2+\theta_3)$

$y = l_2 \sin(\theta_1) + l_2 \sin(\theta_1+\theta_2) + l_3 \sin(\theta_1+\theta_2+\theta_3)$

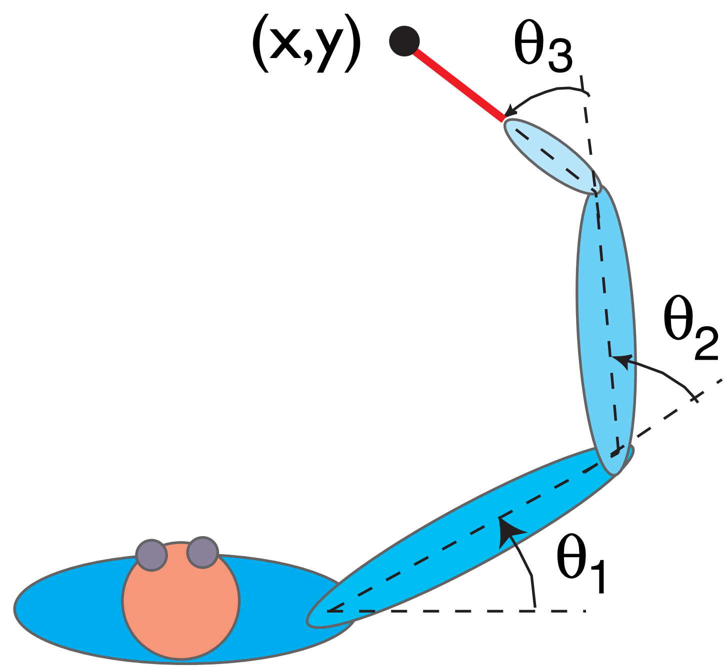
The degree of freedom problem

- The strongest form of the DoF problem: redundant effector systems
- redundance being defined relative to a task



Redundancy formalized

- task variable (x,y)
- elemental variables joint angles theta1, theta2, theta 3



$$x = l_1 \cos(\theta_1) + l_2 \cos(\theta_1+\theta_2) + l_3 \cos(\theta_1+\theta_2+\theta_3)$$
$$y = l_2 \sin(\theta_1) + l_2 \sin(\theta_1+\theta_2) + l_3 \sin(\theta_1+\theta_2+\theta_3)$$

Degree of freedom problem

- is it relevant?
- yes… in many tasks redundancy occurs
  - e.g., 3D positioning in point vs. >10 joints in an arm
- second level of redundancy:
  - many muscles per joint (e.g. about 750 muscles in the human body vs. about 50 Dof)

Bernstein problem

- Nikolai Bernstein… 1930’s… in the Soviet Union
- “how to harness the many DoF to achieve the task”

Bernstein’s workers

- highly skilled workers wielding a hammer to hit a nail… => hammer trajectory in space less variable than body configuration
  - as detected in superposing spatial trajectories of lights on hammer vs. on body..
- but:
  - camera frame anchored to nail/space, while initial body configuration varied

Bernstein’s workers

- was the hammer position in space less variable than the joint configuration?
  - that is, does the task structure variance?
  - so that the solution to the degree of freedom problem lies in the variance/stabilty of the joint configuration?
- but: does this make any sense?
  - different reference frames for body vs. task
  - different units in the task vs joint space

The concept of a synergy: classical

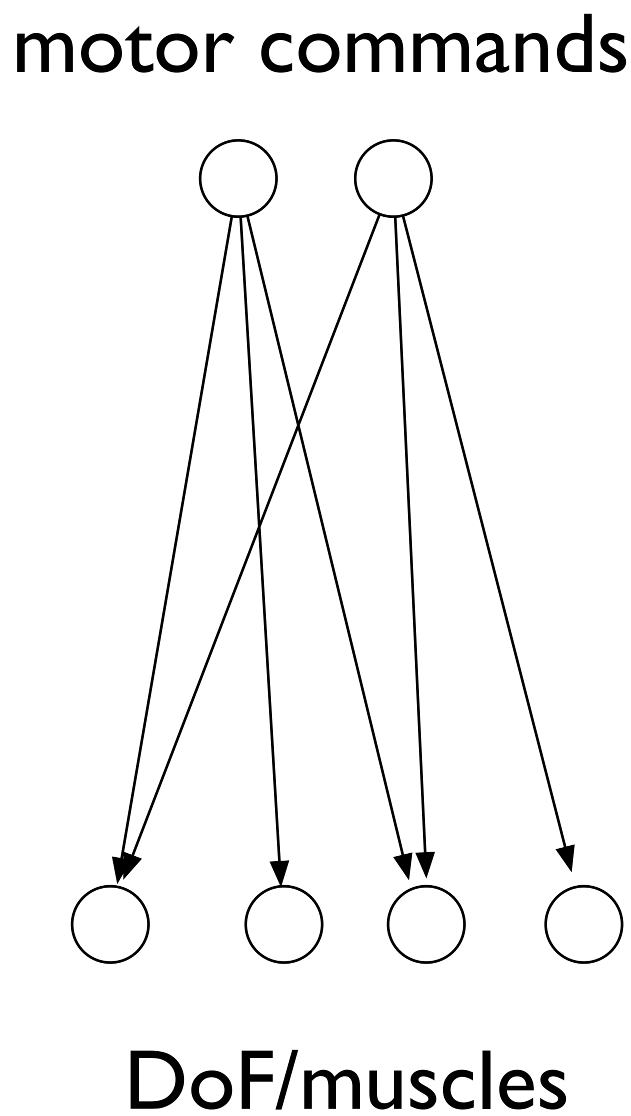
- multiple degrees of freedom/muscles are co-activated in a characteristic pattern
  - leading to covariation of these DoF/muscle activations in time
  - leading to covariation of these DoF/muscle activation when movement parameters are varied
  - “sharing” aspect of synergies

Classical synergy: data analysis

- Throw data from time series under different conditions (sometimes including repetitions of movements) into one big data set and look for principal components
- if a small number of PC's is sufficient to account for most of the variance, conclude that few synergies at at work
- or establish co-variation among DoF/ muscles directly by testing for significant correlation

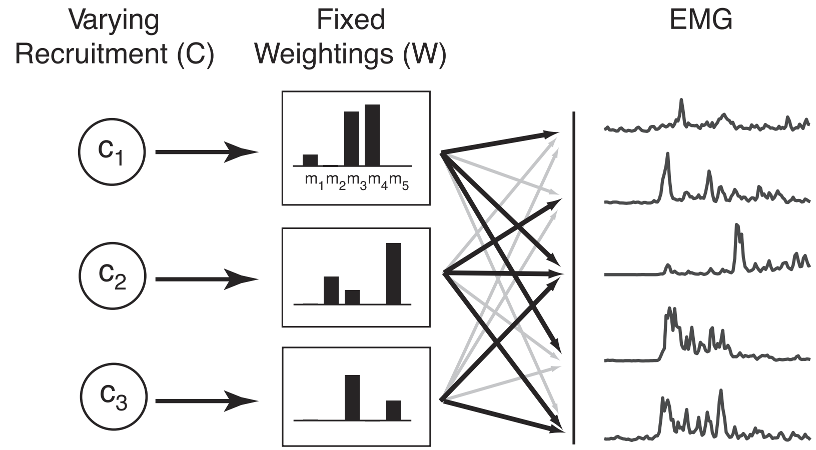
Classical synergy: accounts

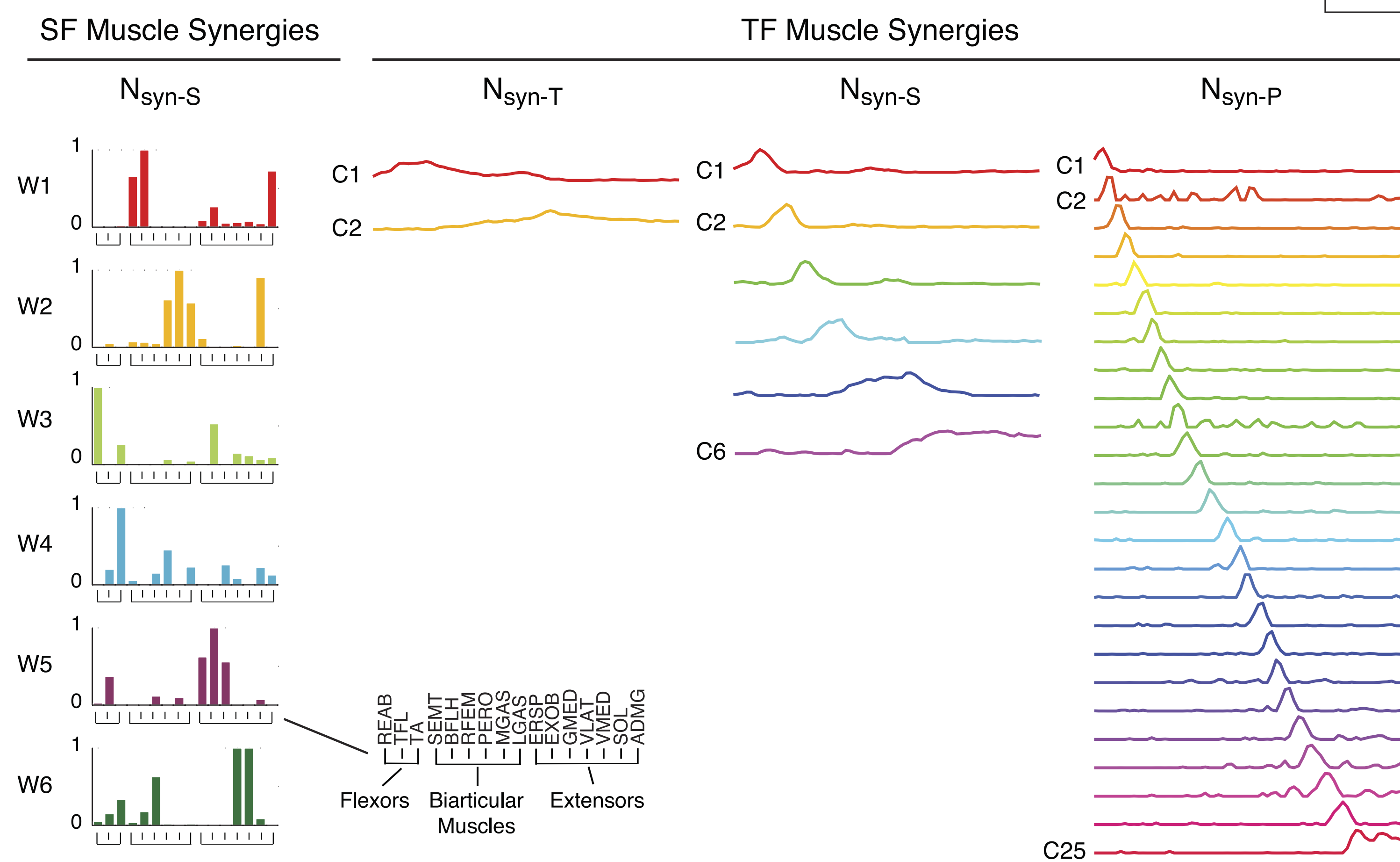
- A small number of descending commands specify the synergies
- these are distributed to a large number of DoFs/muscles by a forward neural network
  - the forward connectivity pattern defines the synergy
  - induces the co-variation among the DoF when the command varies.



Synergy: experimental use

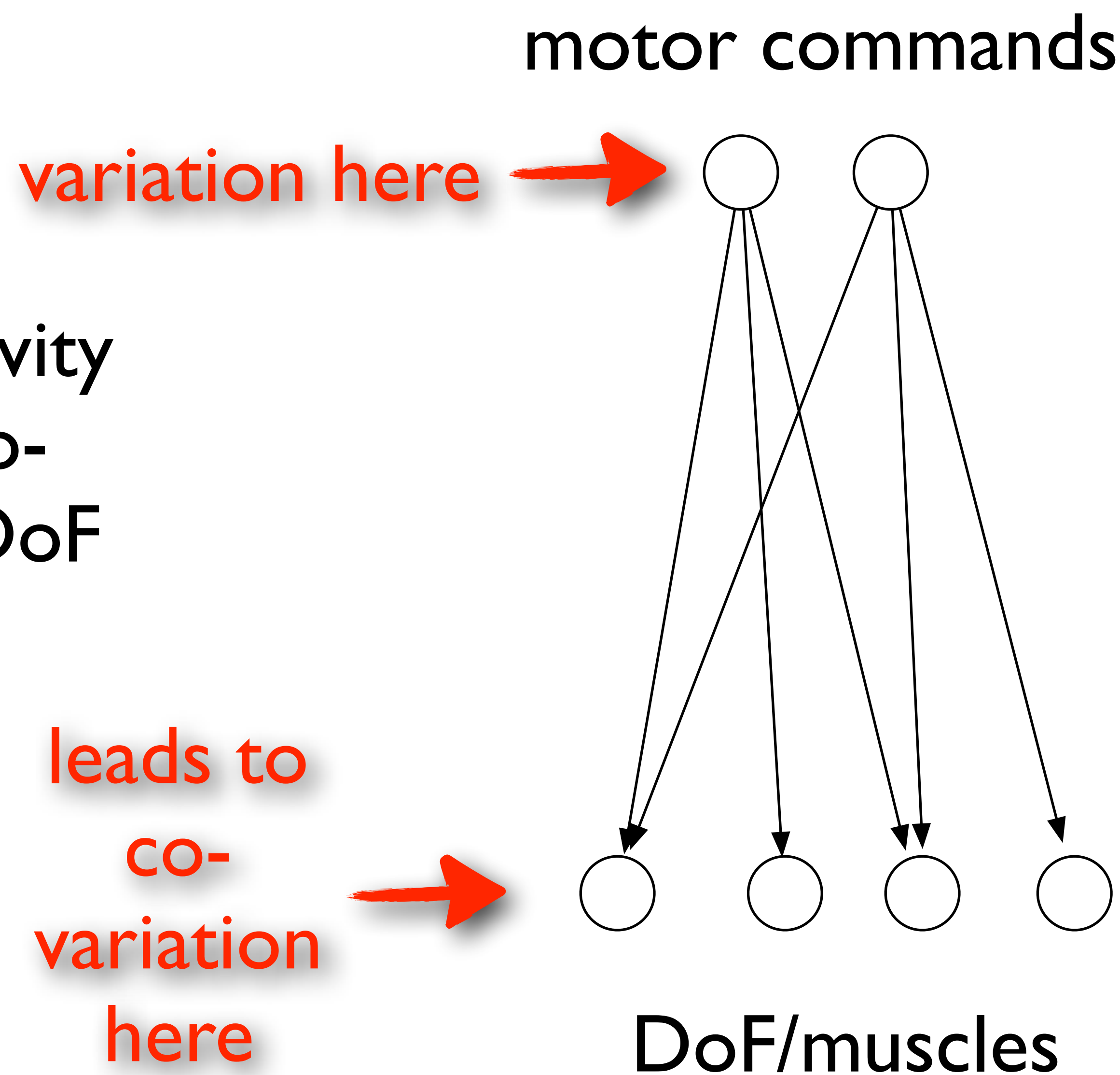
- E.g, Safavynia, Ting, 2012:





Classical synergy: accounts

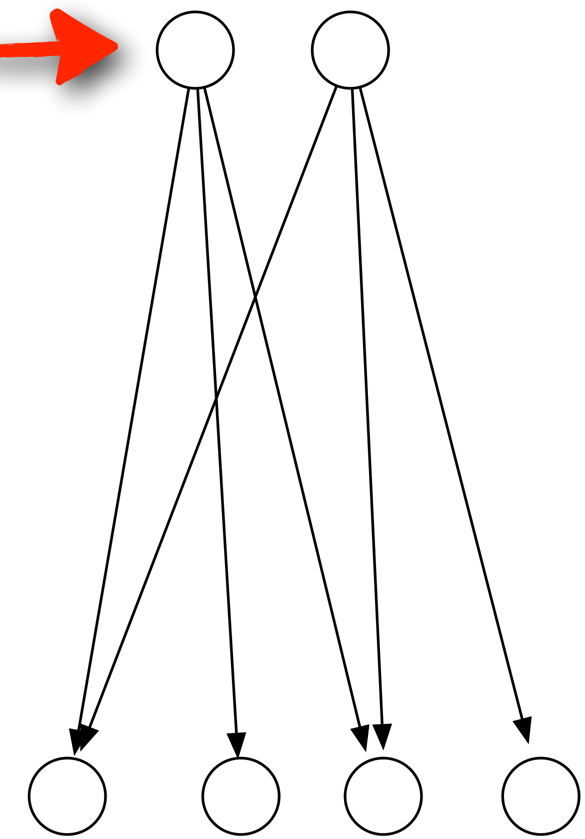
- The forward connectivity pattern induces the co-variation among the DoF when the command varies.



Classical synergy: variance

motor commands

variance here



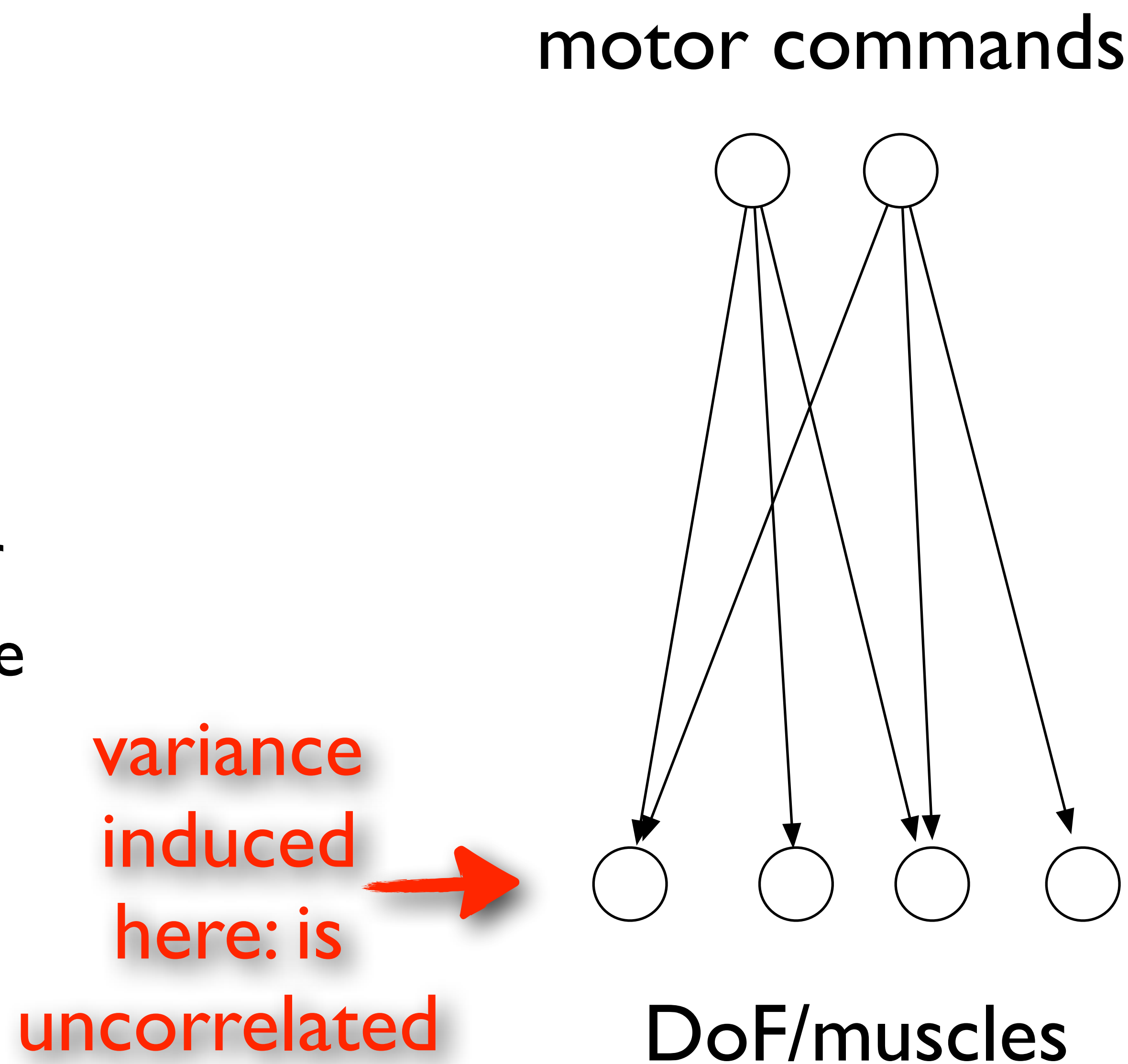
leads to covariance here

DoF/muscles

- Variance that arises from stochastic motor commands induces covariance at DoFs
- = opposite of the UCM ("anti-correlation" or "compensation")
- => tension between classical synergy and UCM

Classical synergy: variance

- variance induced at the level of the individual DoF/muscles is uncorrelated
- = the null hypothesis of UCM: the same variance in all direction
- => tension between classical synergy and UCM

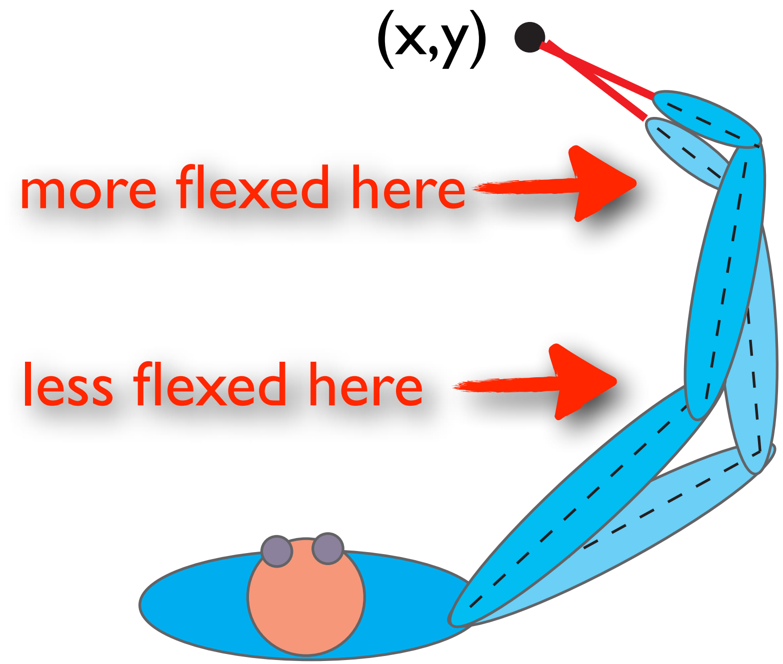


Classical synergy: DoF problem

- classical synergy concept is thought of as a solution to the DoF problem: shared input

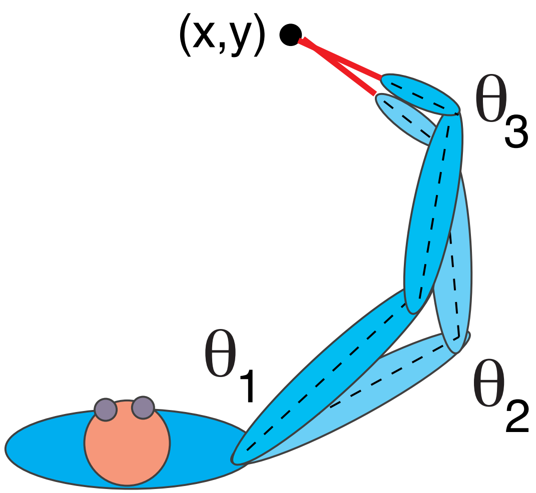
uncontrolled manifold (UCM)

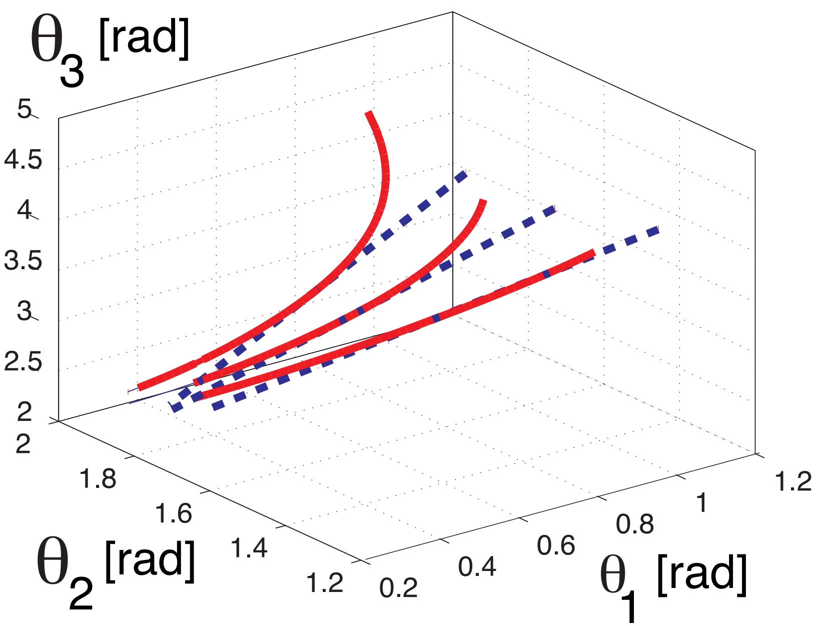
- the many DoF are coordinated such that variance that affects a smaller number of task variables is smaller than variance that does not affect a task variable
  - leading to compensation among DoF (or "anti-correlation")



UCM synergy: data analysis

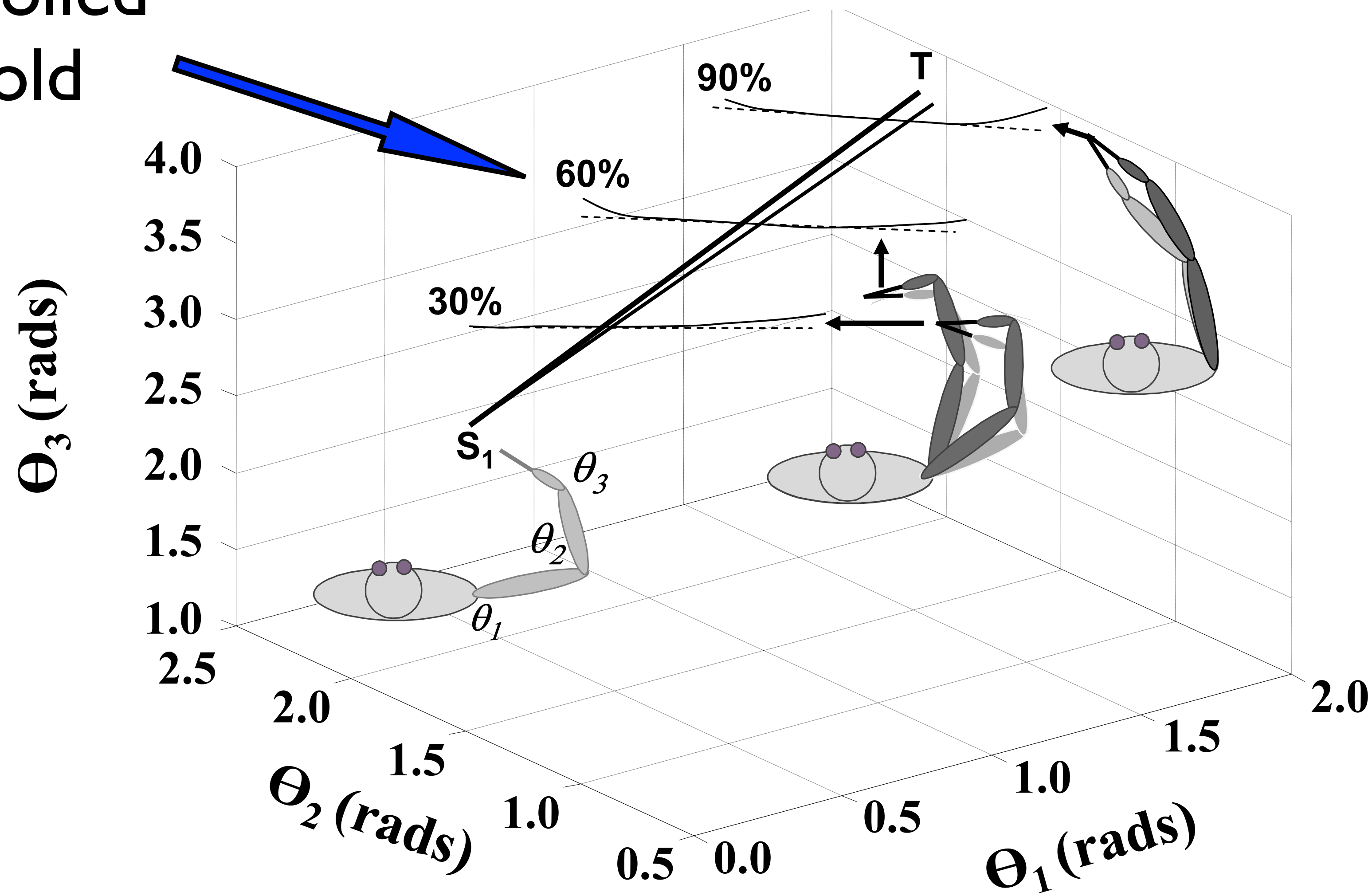
- hypothesis testing
  - align trials in time, computer variance at each time slices
  - formulate hypothesis about task variable
  - compute null-space (tangent to the “uncontrolled manifold”)
  - predict there is more variance within null space than perpendicular to it





uncontrolled manifold hypothesis

uncontrolled manifold



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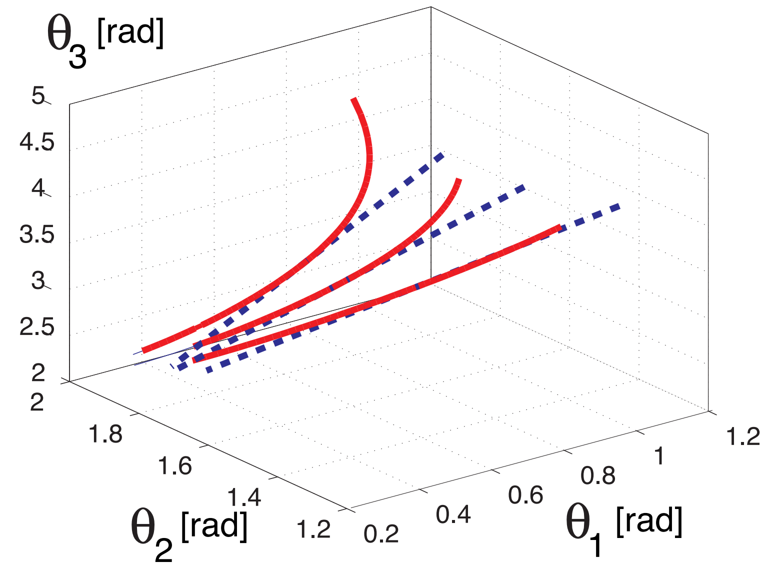

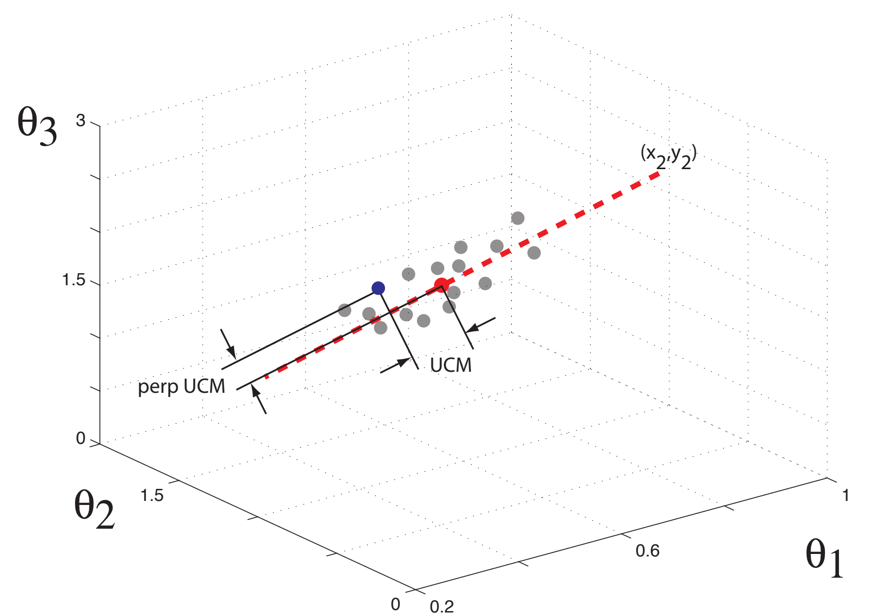

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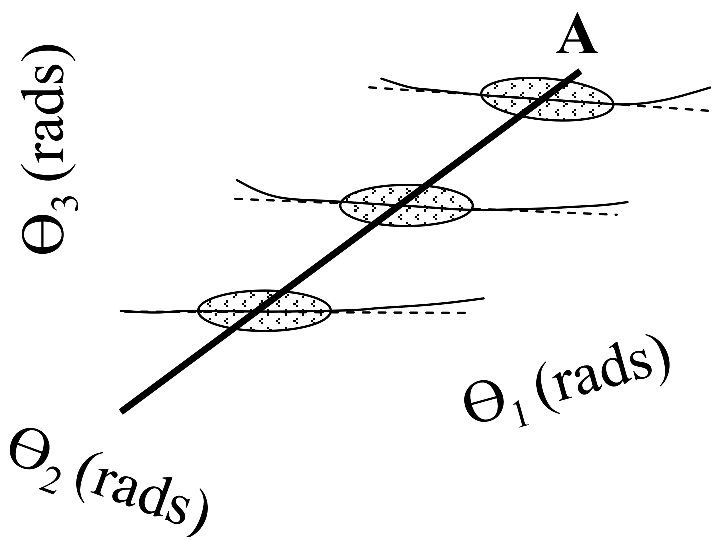
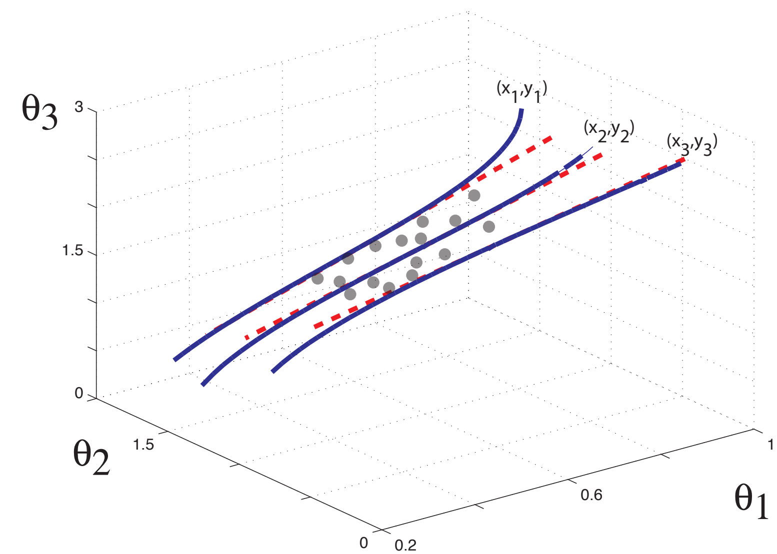
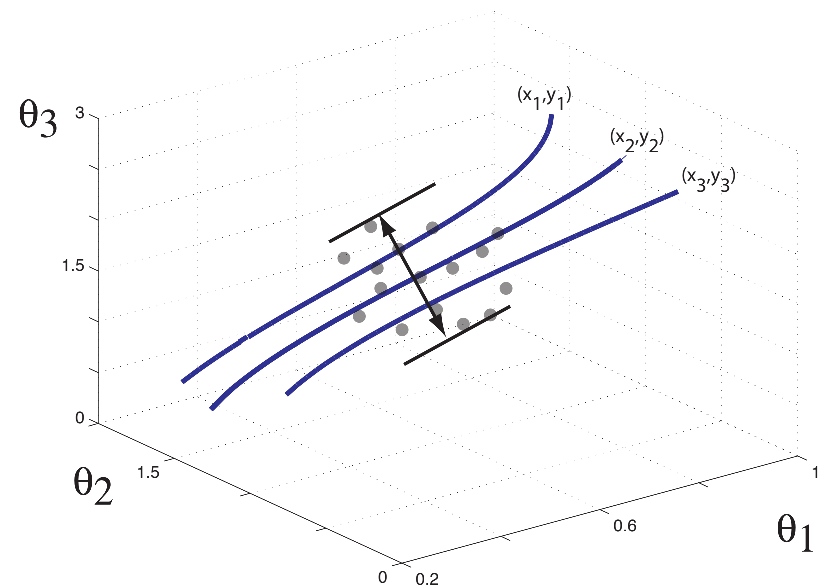


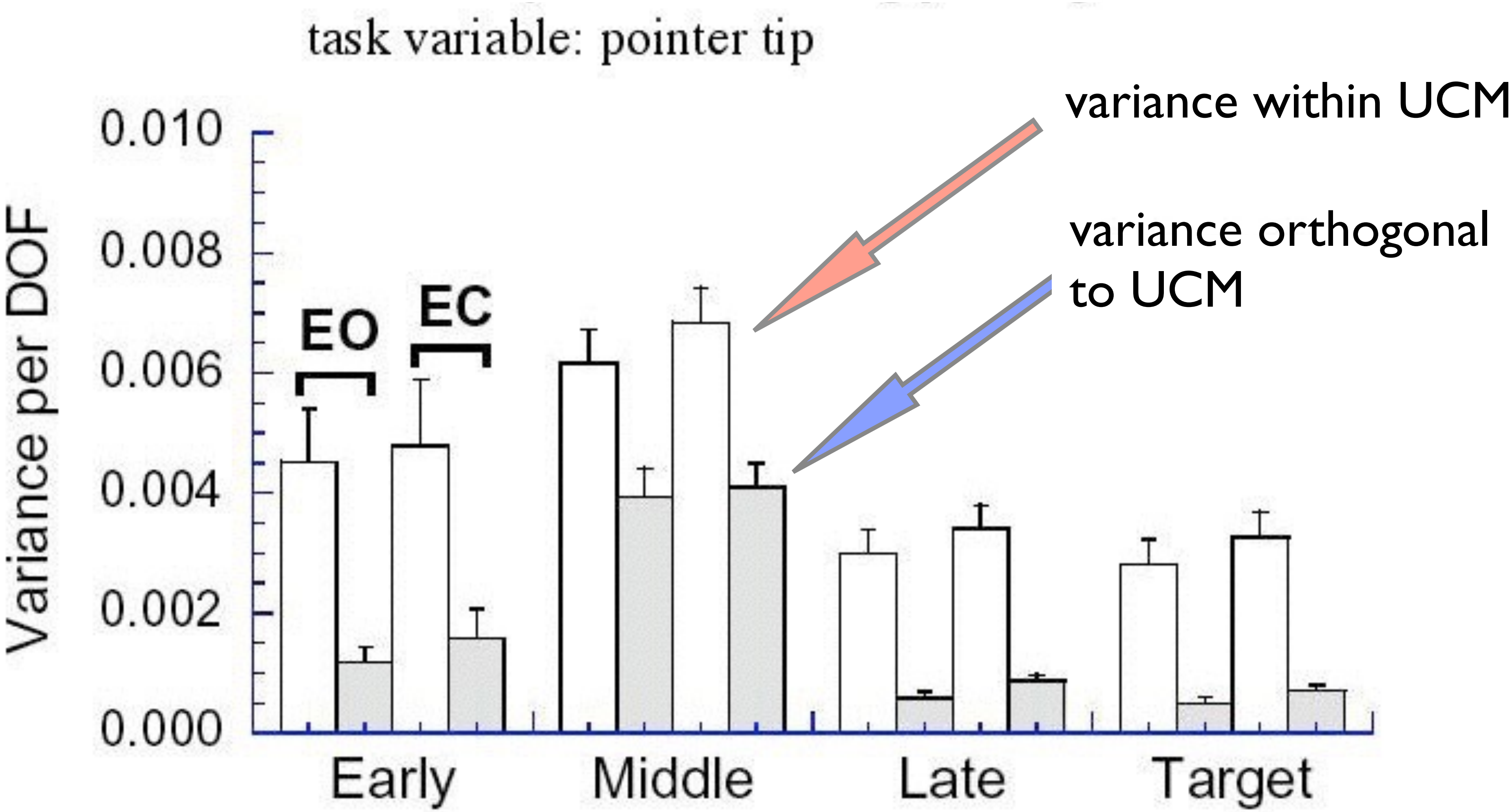
UCM synergy: data analysis

- supplement hypothesis testing by checking for correlation (Hermann, Sternad...)
  - look for increase in variance of task variable when correlation within data is destroyed





Example 1: pointing with 10 DoF arm at targets in 3D

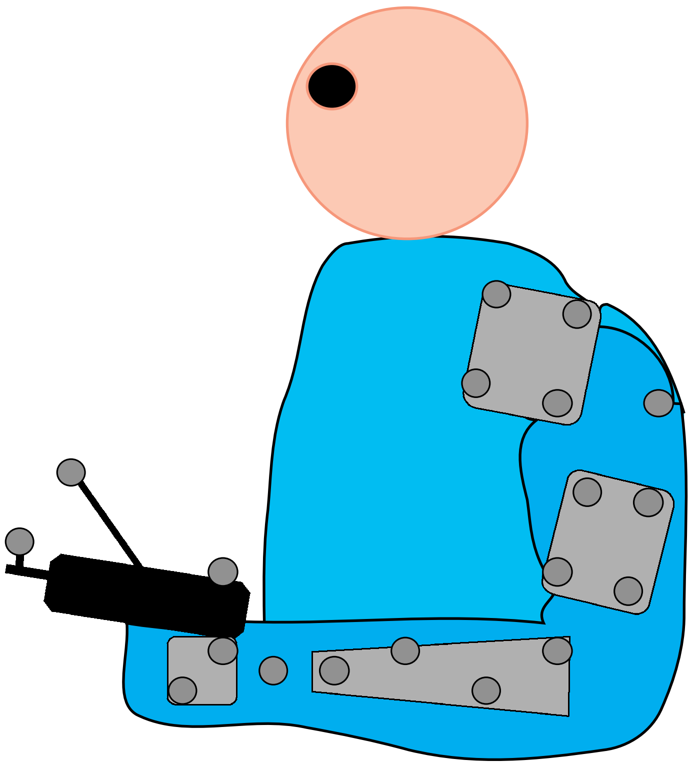


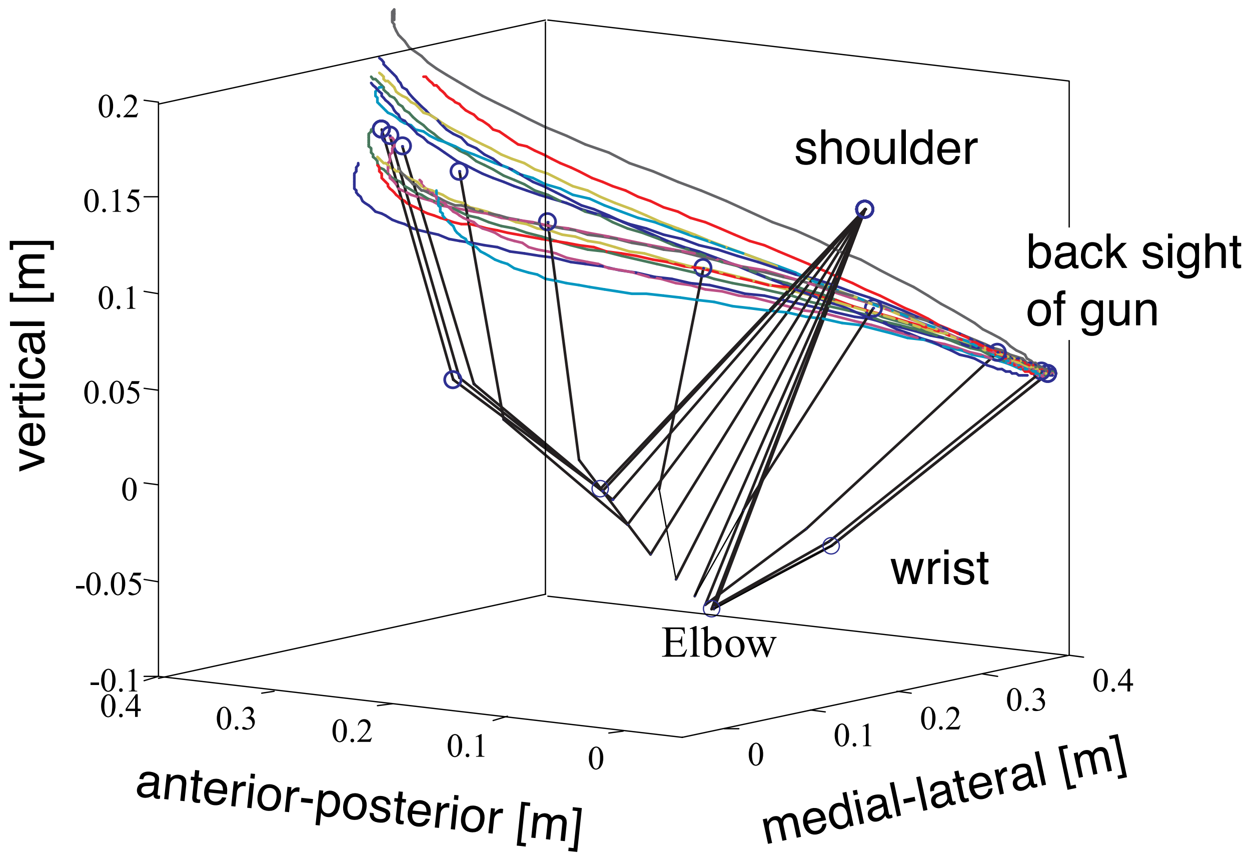
[from: Tseng, Scholz, Schöner: Motor Control (2002)]

task specificity of the structure of the joint variance

- is joint variance always structured by the end-effector spatial position?
- no: depends on task

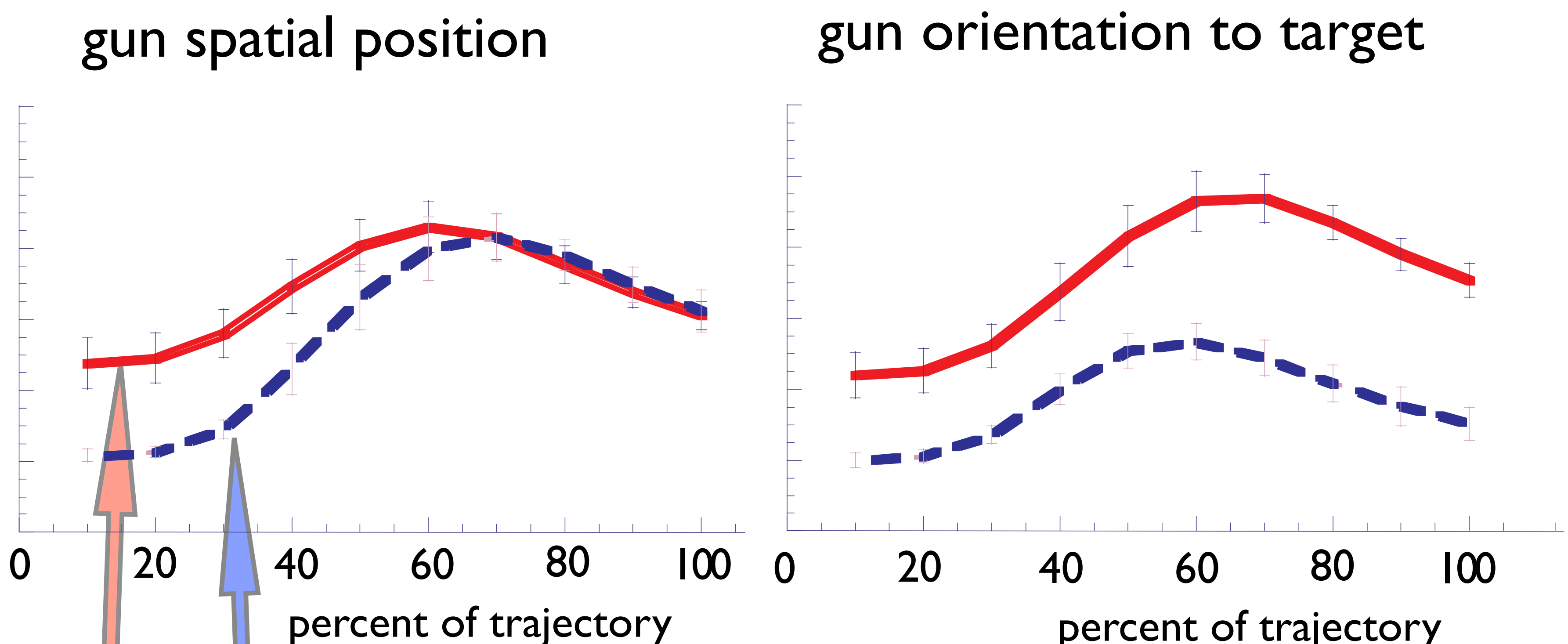
Example 2: shooting with 7 DoF arm at targets in 3D





[from Scholz, Schöner, Latash: EBR 135:382 (2000)]

Example 2: shooting with 7 DoF arm at targets in 3D



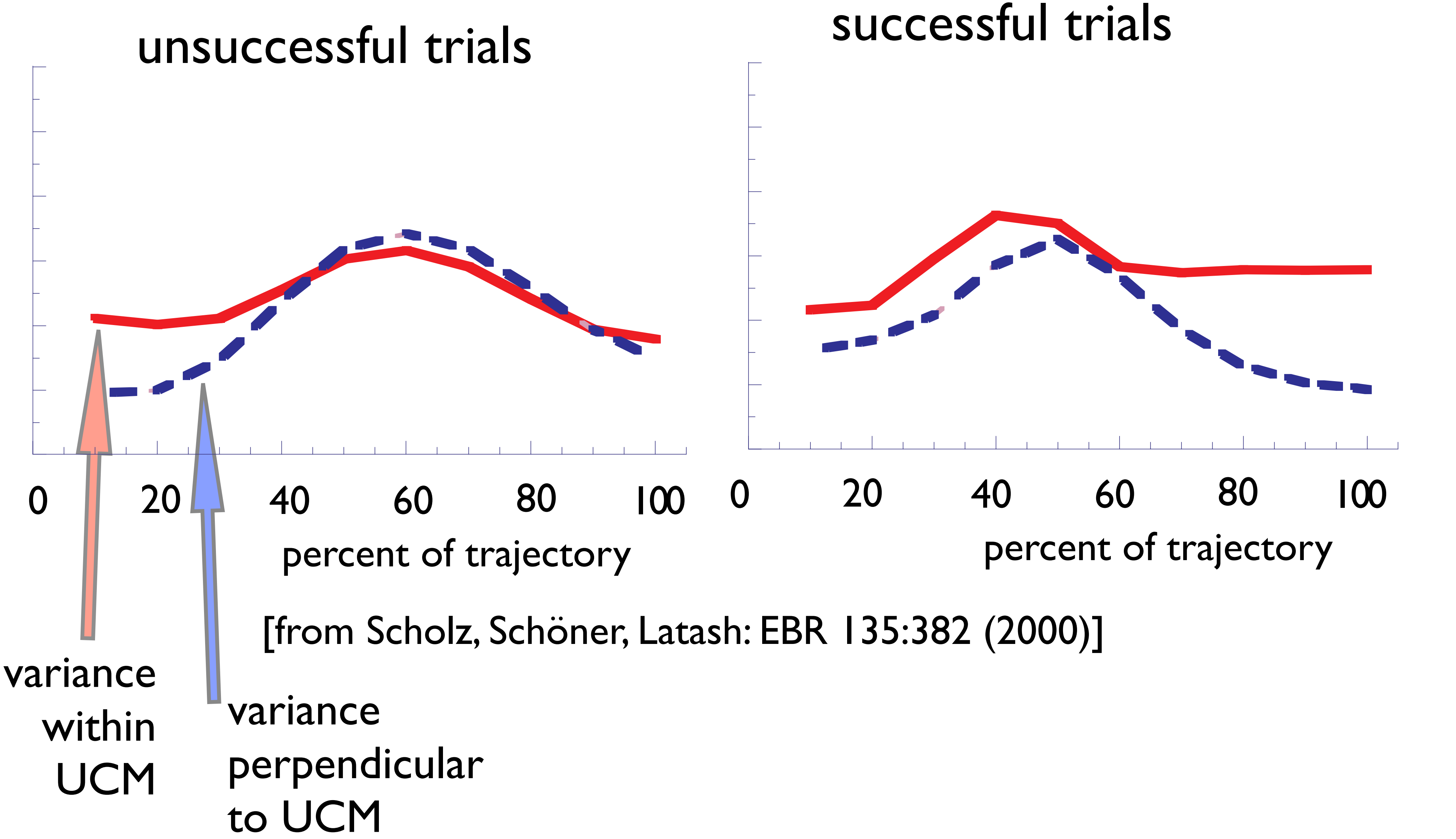
[from Scholz, Schöner, Latash: EBR 135:382 (2000)]

variance within UCM

variance perpendicular to UCM

Example 2: shooting with 7 DoF arm at targets in 3D

hypothesis: gun orientation, data from one participant

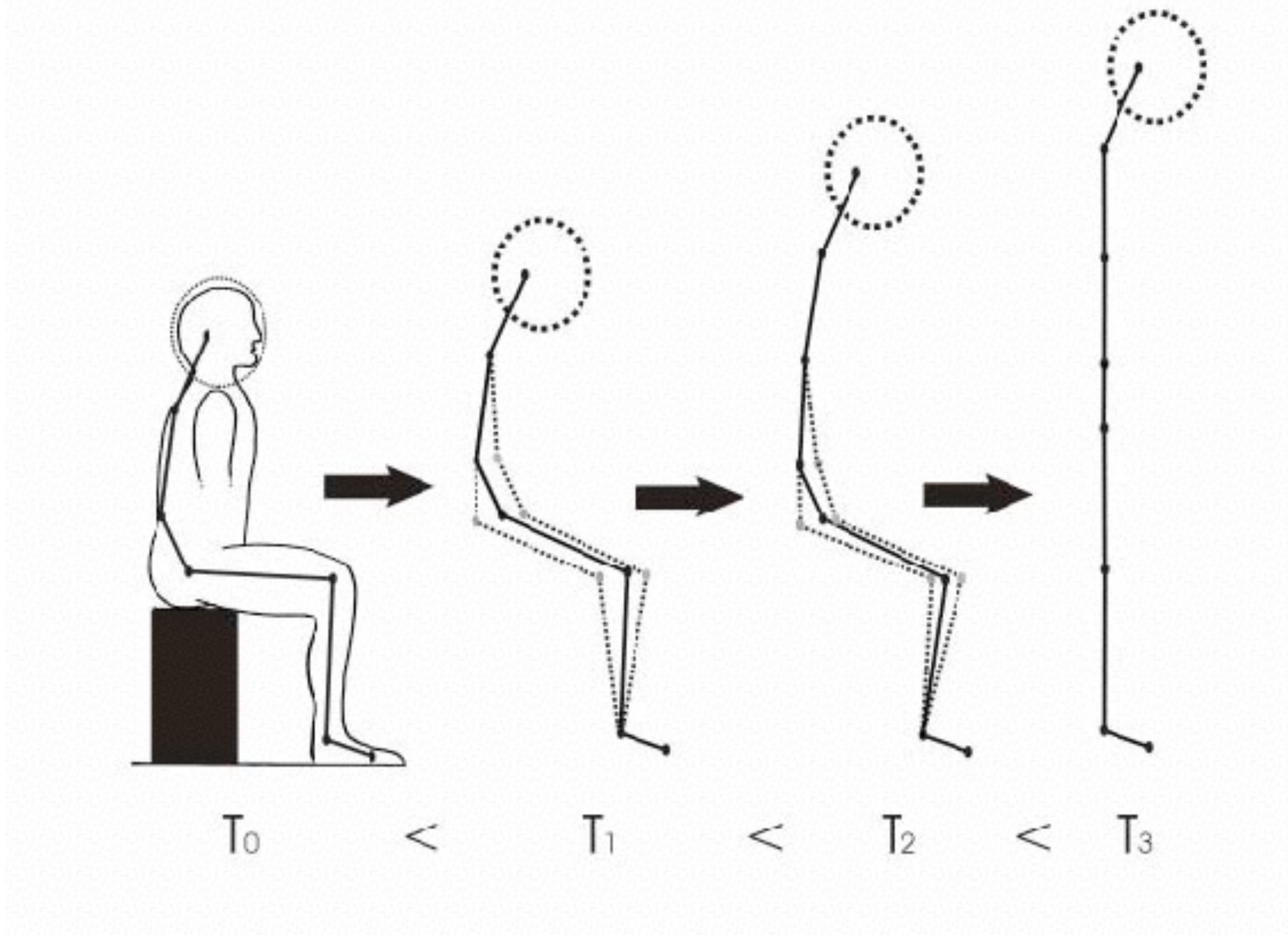


[from Scholz, Schöner, Latash: EBR 135:382 (2000)]

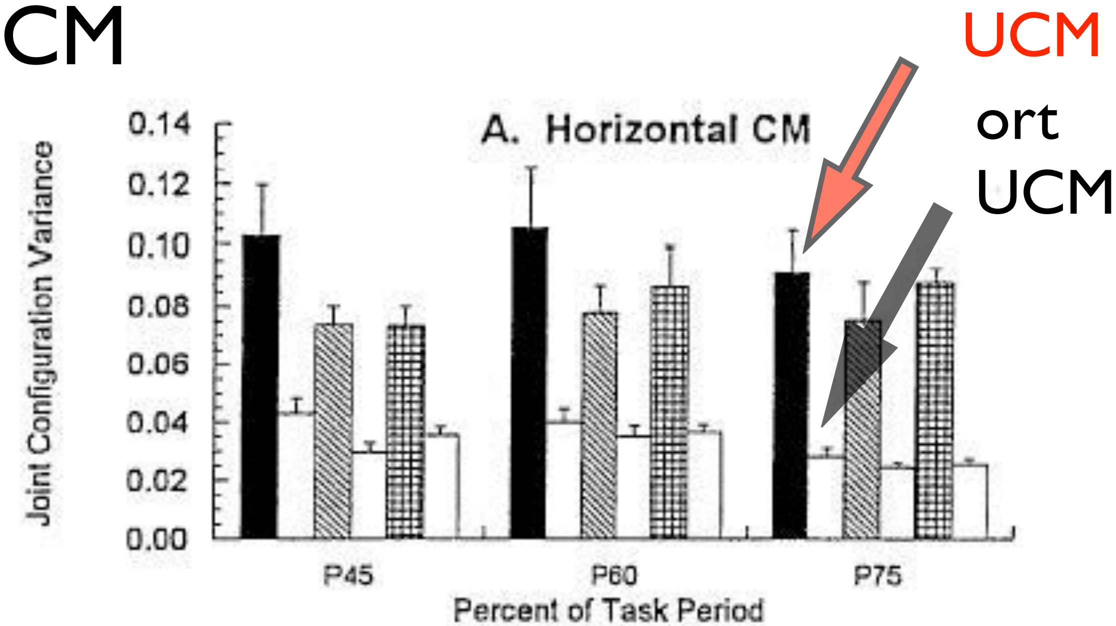
limits of redundancy

Example 3:

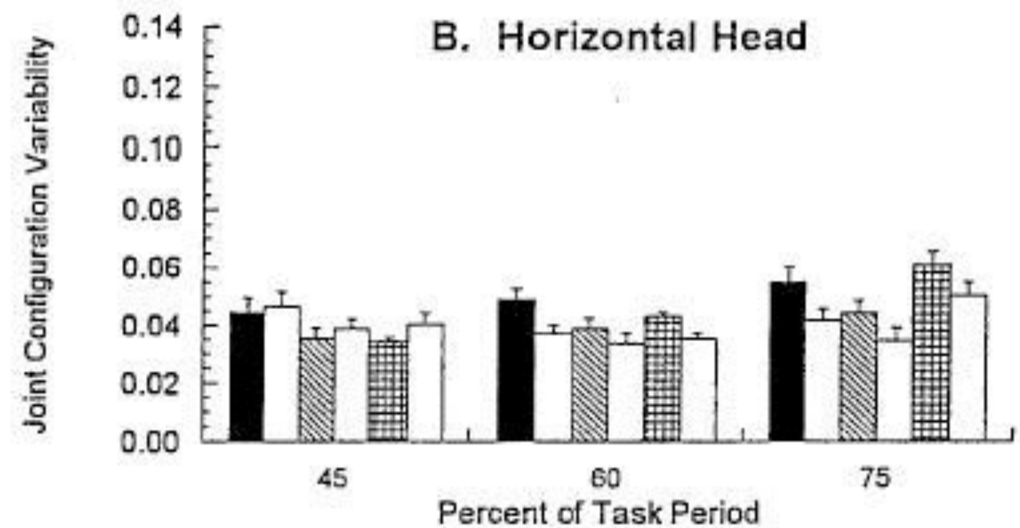
sit to stand transition as a whole body movement



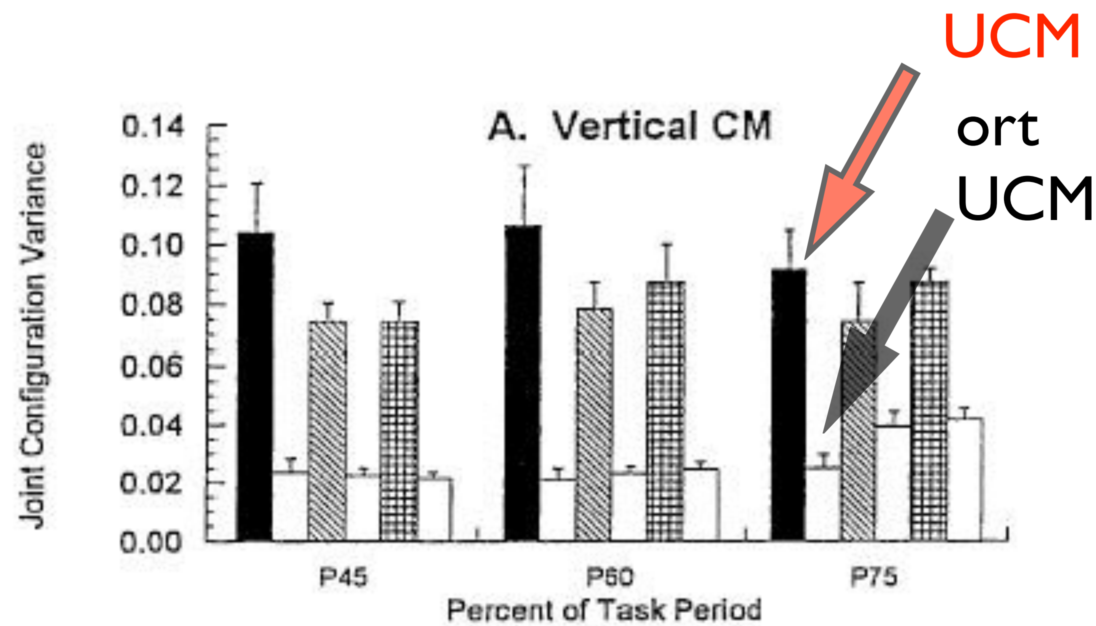
hypothesis: horizontal CM



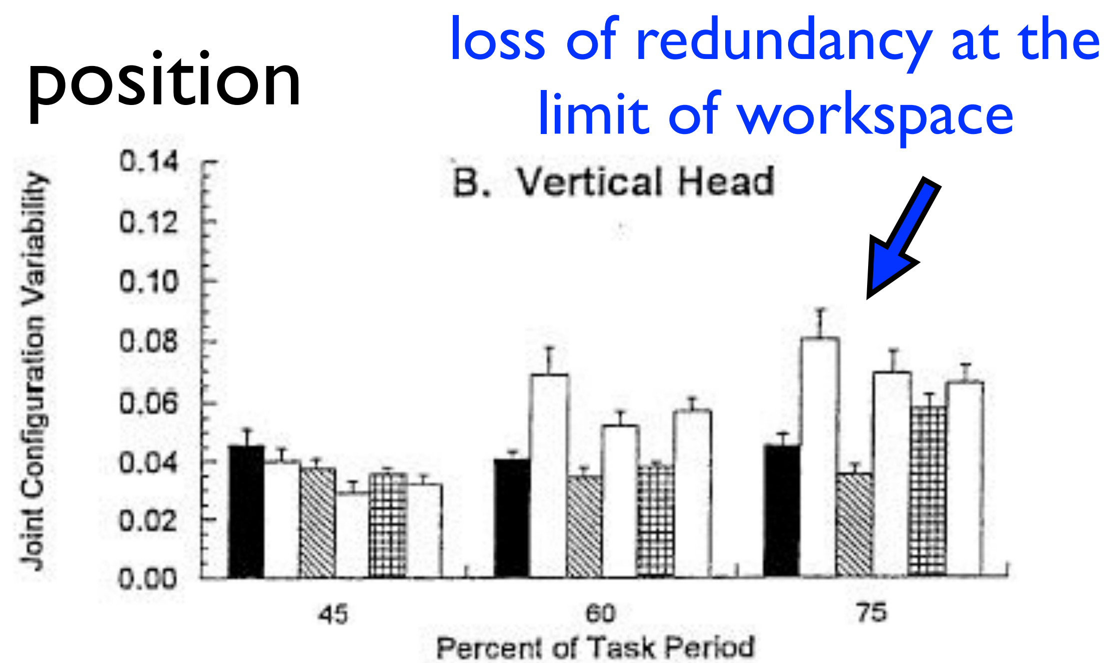
hypothesis: horizontal head position



[from: Scholz, Schöner EBR 126:289 (1999)] hypothesis: vertical CM



hypothesis: vertical head position



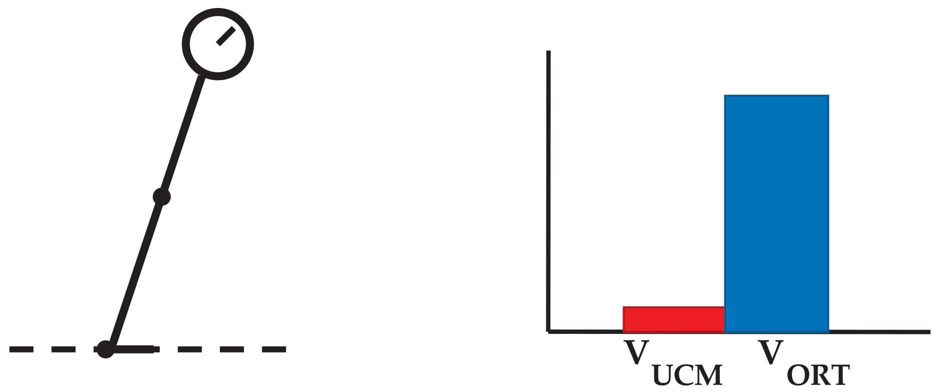
[from: Scholz, Schöner, EBR 126:289 (1999)]

UCM synergy: account

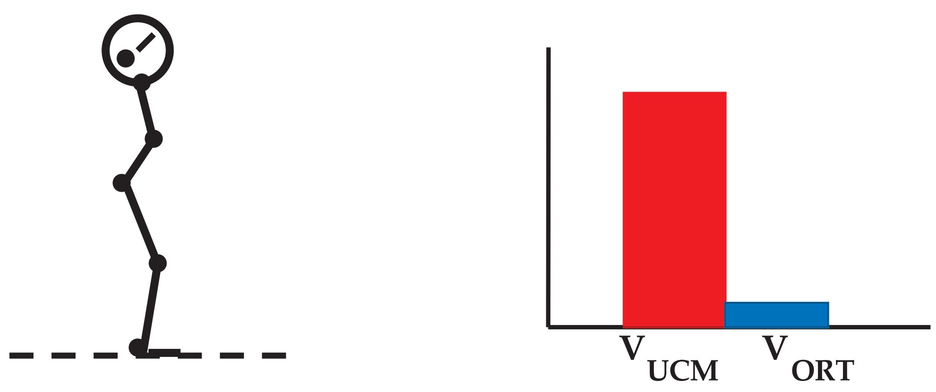
- more complex than for classical synergy... let’s go through case studies first

UCM synergy: accounts. Case study posture

- UCM non-trivial in posture because the classical inverted pendulum hypothesis predicts the opposite:

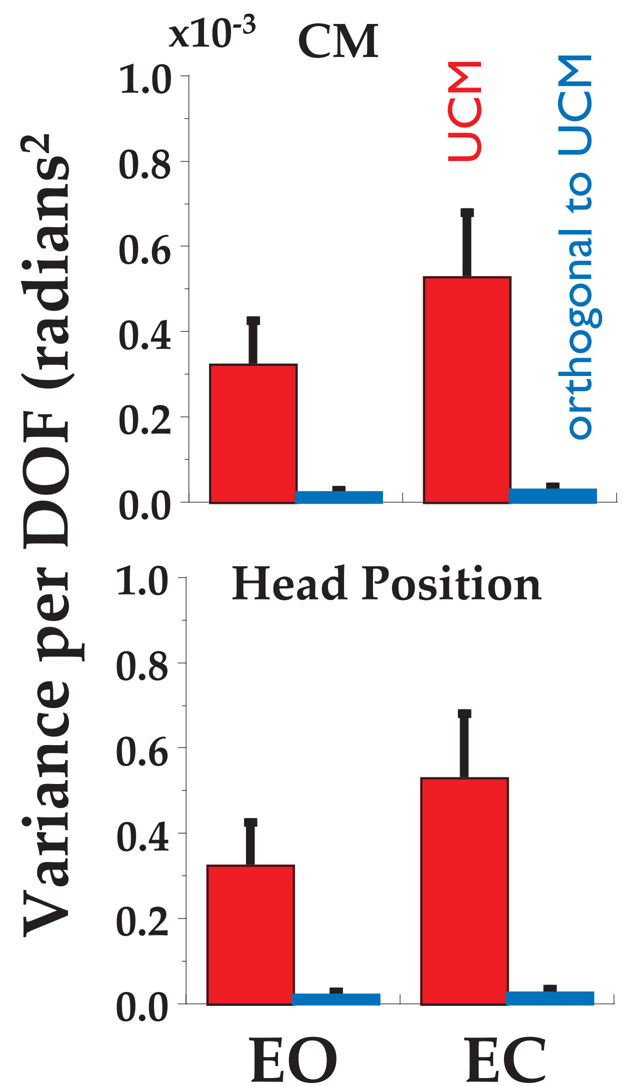


  - because the ankle moves the body in space, it lies orthogonal to the UCM predicting more variance in ORT than in UCM



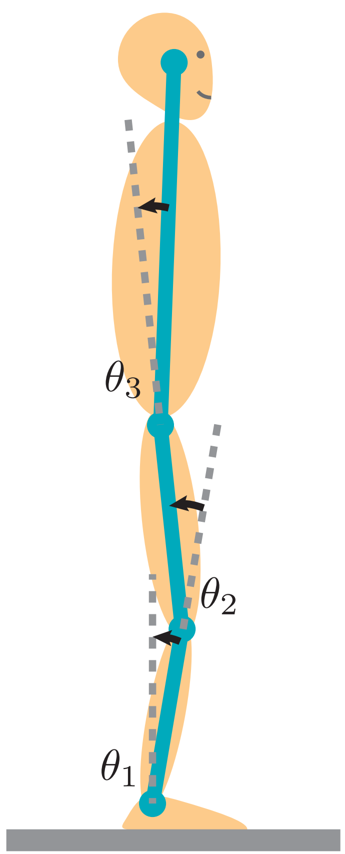
UCM synergy: accounts. Case study posture

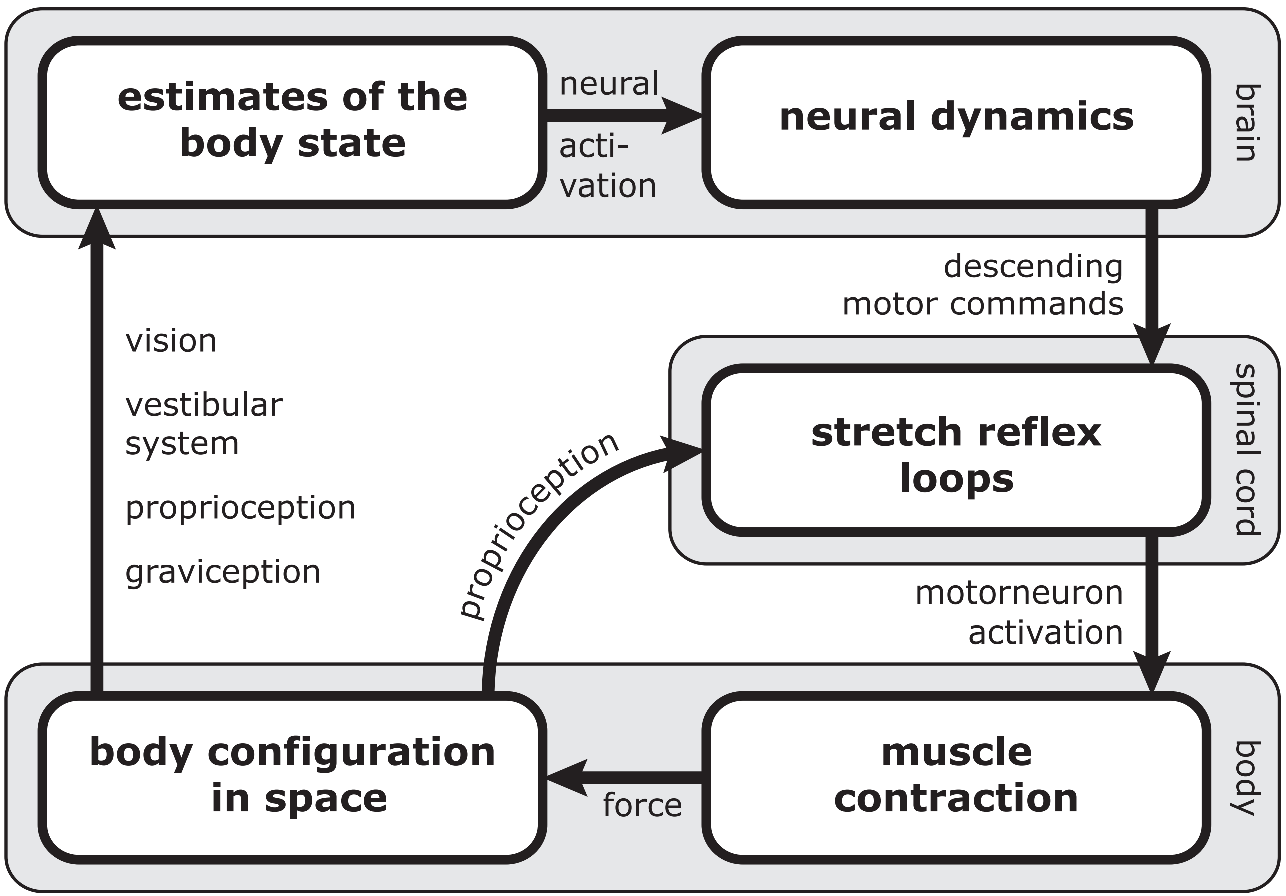
- but: find signature of UCM synergy



Hsu, Scholz, Schöner, Jeka, Kiemel, 2007

Multi-segment postural control model



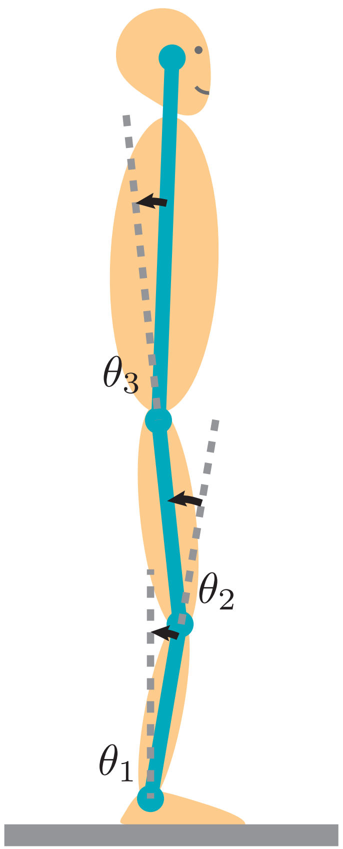


PhD thesis Hendrik Reimann

Reiman, Scholz, Schöner (in preparation)

Multi-segment postural control model

- bio-mechanical dynamics



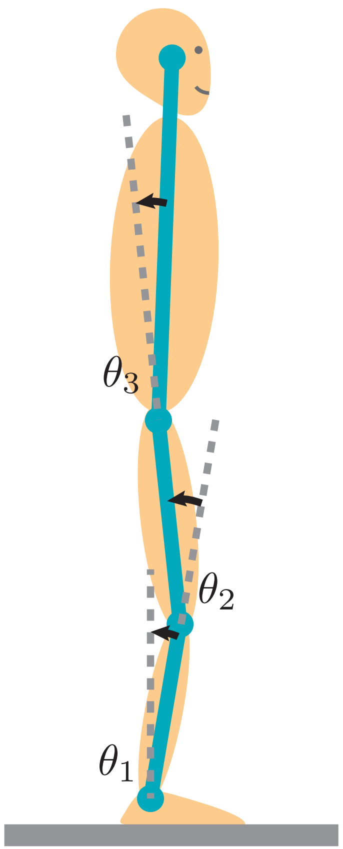
$$M(\theta)\ddot{\theta} + C(\theta, \dot{\theta})\dot{\theta} + N(\theta) = T$$

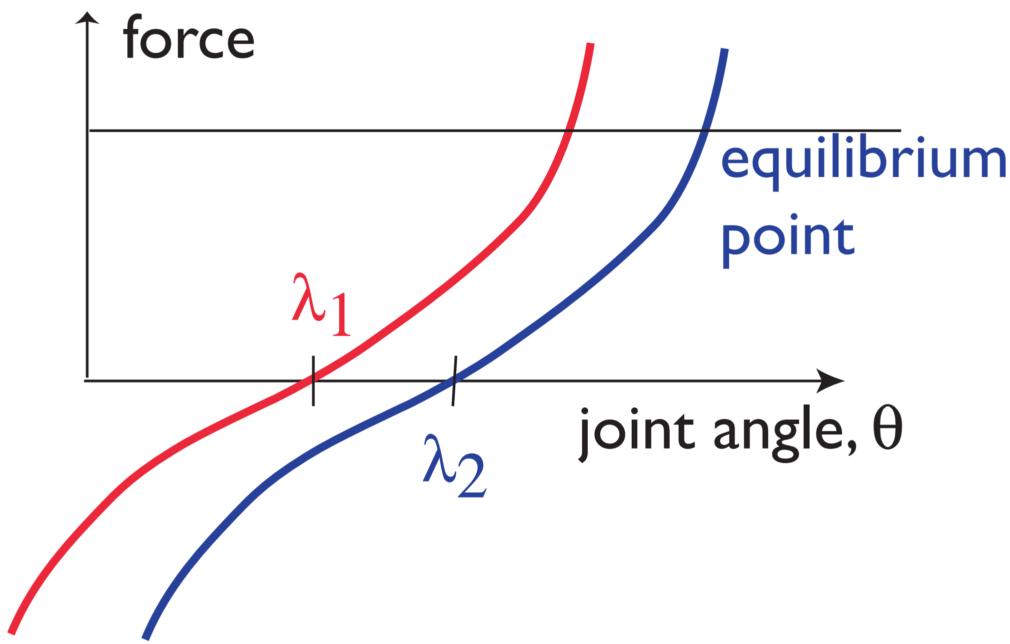
PhD thesis Hendrik Reimann

Reiman, Scholz, Schöner (in preparation)

Multi-segment postural control model

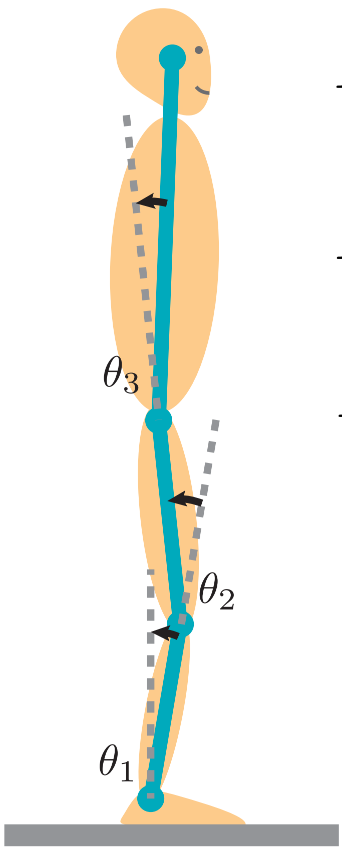
- muscle model





Multi-segment postural control model

- muscle model



$$E_{\mathrm{AG}} = e^{\left[\alpha_E\left(\widehat{\theta}-\lambda+\rho+\mu(\widehat{\dot{\theta}}-\dot{\lambda})\right)\right]^+} - 1,$$

$$E_{\mathrm{AN}} = e^{\left[-\alpha_E\left(\widehat{\theta}-\lambda-\rho+\mu(\widehat{\dot{\theta}}-\dot{\lambda})\right)\right]^+} - 1.$$

$$E = \left(-E_{\mathrm{AG}} + E_{\mathrm{AN}}\right)\eta_m$$

muscle activation

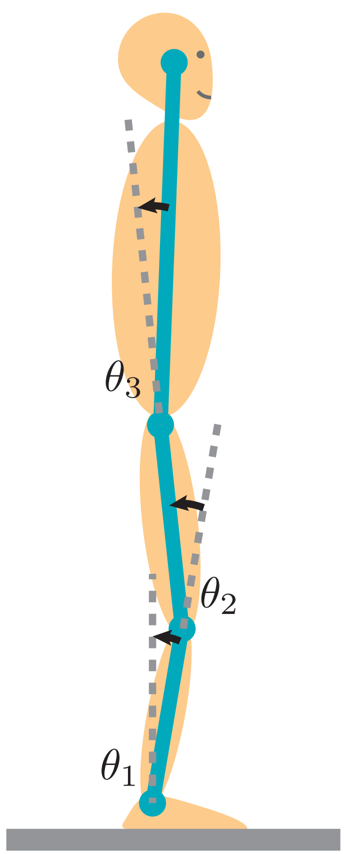
$$\widetilde{T}_{\mathrm{act}} = AE$$

$$\tau_m^2 \ddot{T}_{\mathrm{act}} + 2\tau_m \dot{T}_{\mathrm{act}} + T_{\mathrm{act}} = \widetilde{T}_{\mathrm{act}}$$

active muscle torque

Multi-segment postural control model

- muscle model



$$T_{\text{ela},j} = \exp(a_{j0} + \sum_{i=1}^{3} a_{ji}\theta_i) - \exp(b_{j0} + \sum_{i=1}^{3} b_{ji}\theta_i) + c_{ji}$$

$$T_{\text{vis}} = -B\dot{\theta}$$

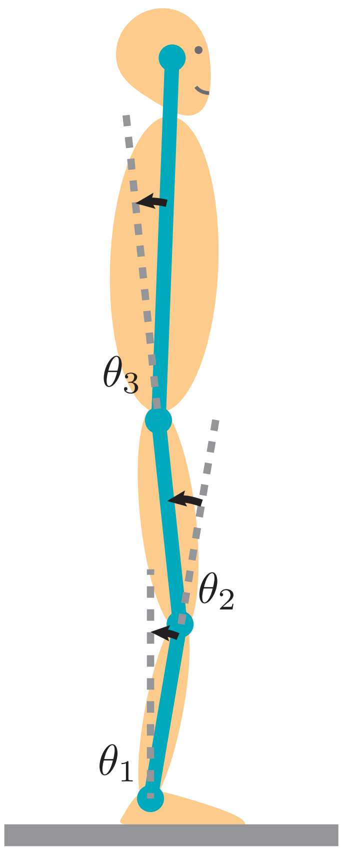
← passive torques

$$T = T_{\text{act}} + T_{\text{ela}} + T_{\text{vis}}$$

← total muscle torque

Multi-segment postural control model

- sensor model



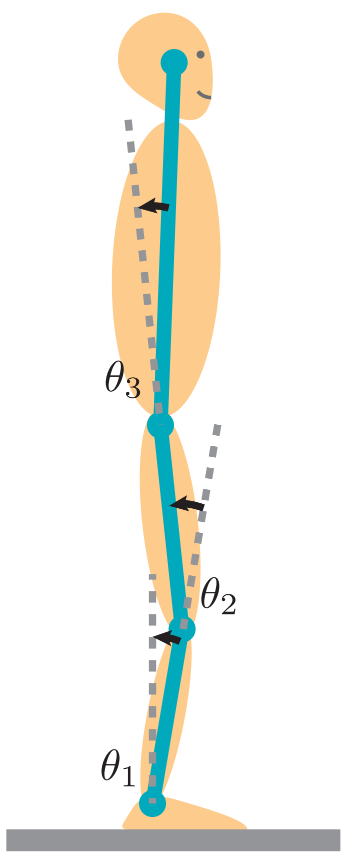
$$\widehat{\dot{c}}(t) = \dot{c}(t - d_c) + \eta_{\dot{c}},$$

$$\widehat{\ddot{c}}(t) = \ddot{c}(t - d_c) + \eta_{\ddot{c}},$$

body in space

Multi-segment postural control model

- control model



$$\dot{\lambda} = F_c = R^{-1} A^{-1} M J_c^{+} \left(-\alpha_{\dot{c}} \widehat{\dot{c}} - \alpha_{\ddot{c}} \widehat{\ddot{c}} \right)$$

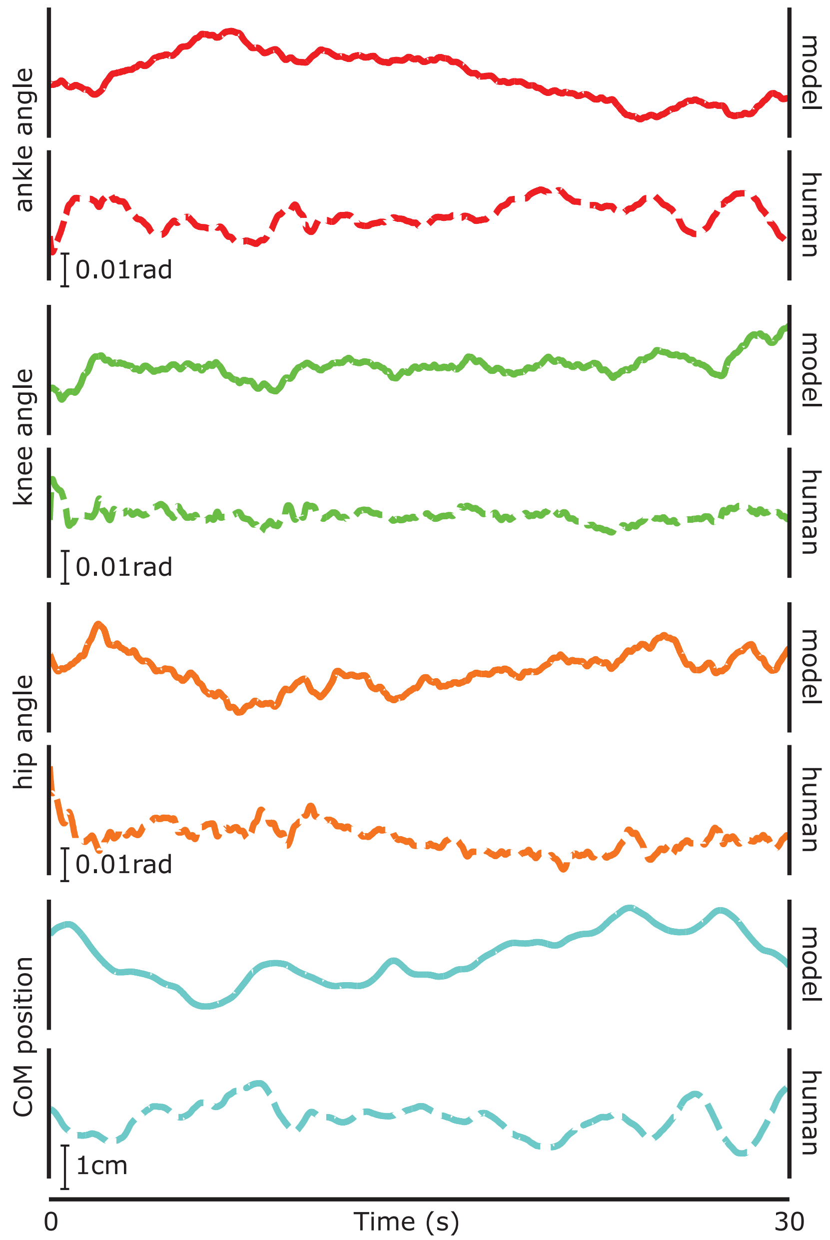
active stiffness (R^{-1})

inertial tensor (M)

kinematic pseudo-inverse (J_c^{+})

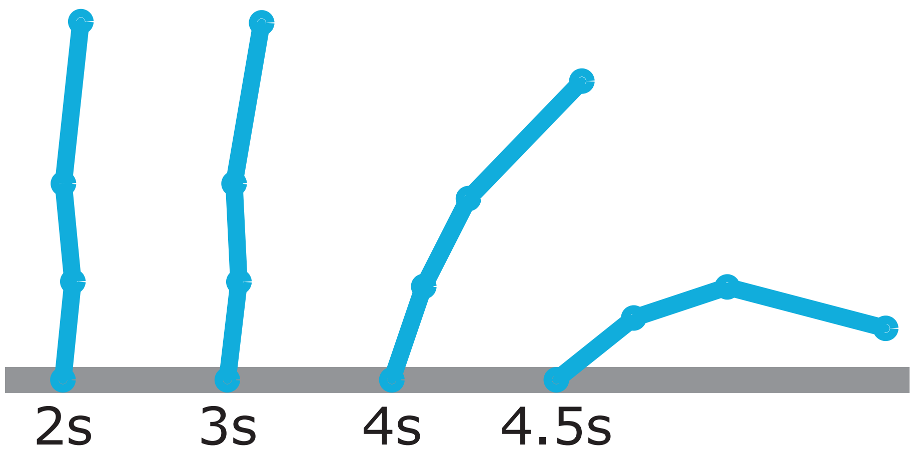
sensory estimate body in space ($\widehat{\ddot{c}}$)

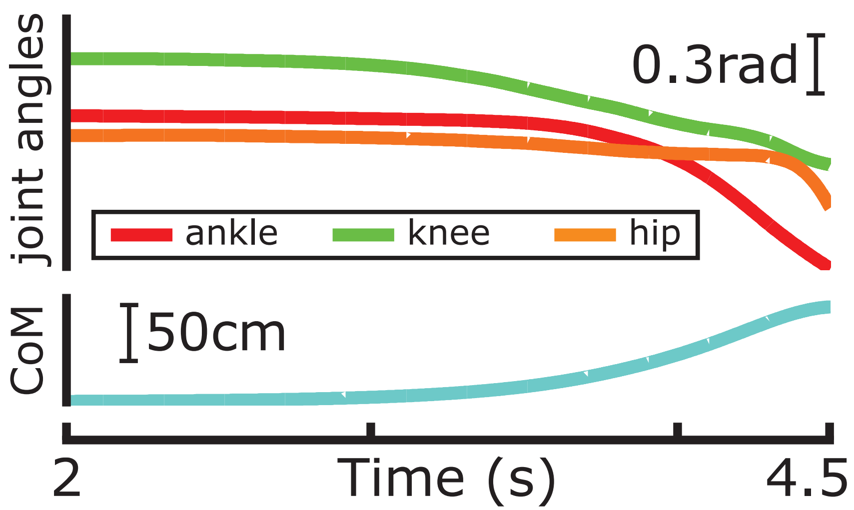
Results: model stands



Results: model falls

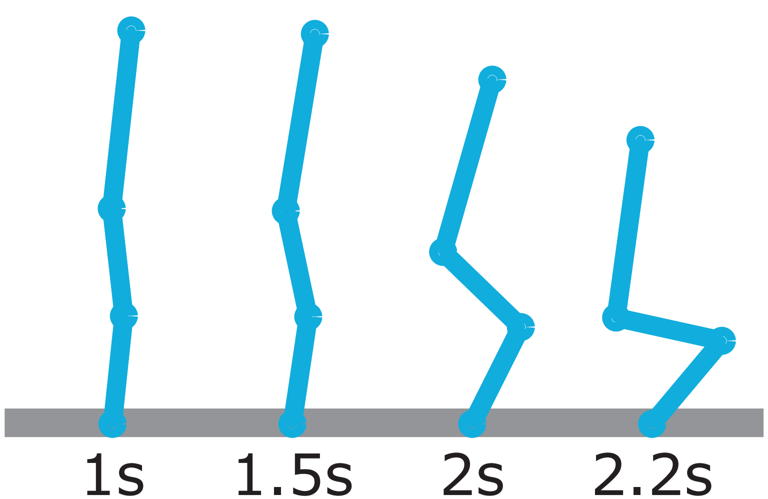
- when the sensory feedback loop about the body in space is removed

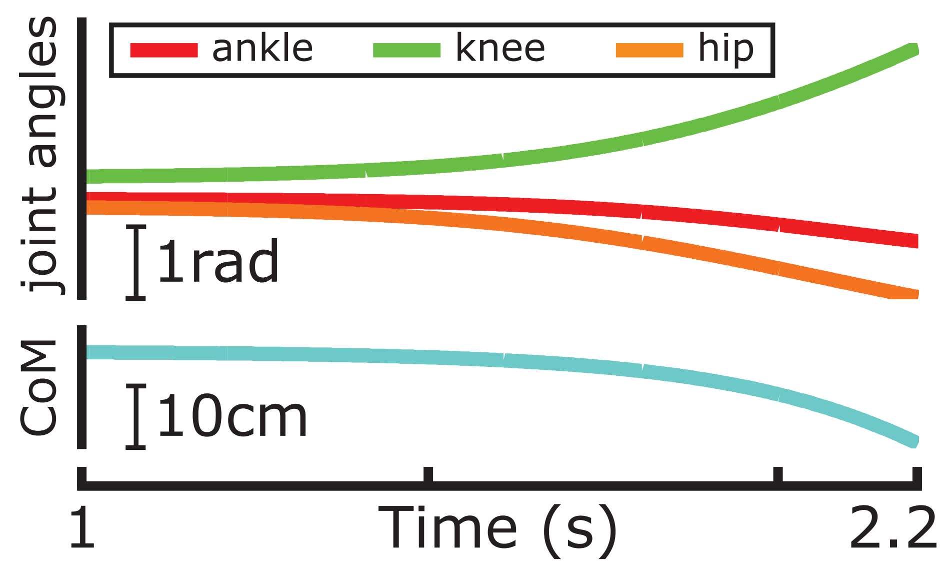




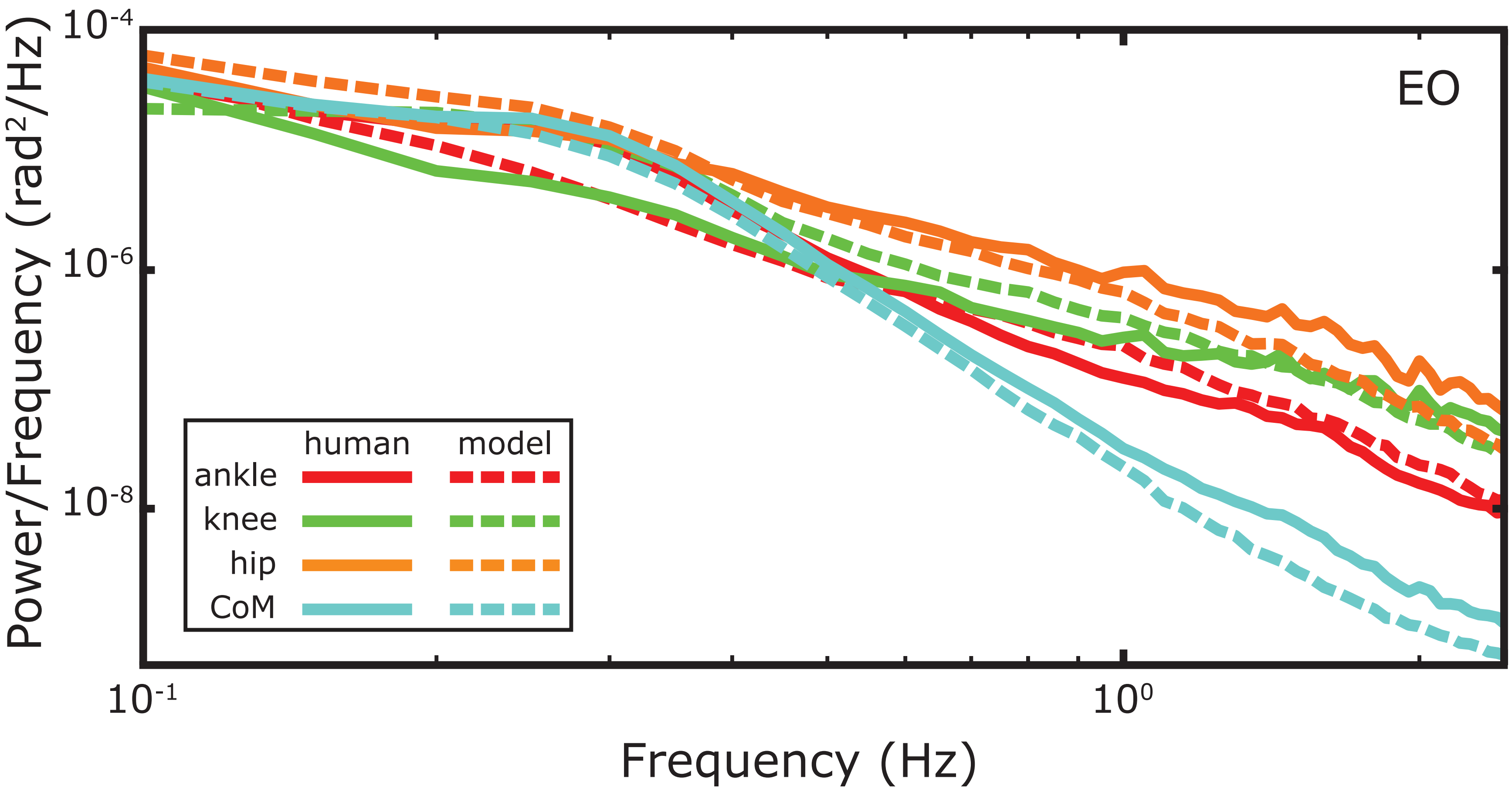
Results: model falls

- when the spinal reflex loop within muscle model is removed (constant activation level of motor neurons)

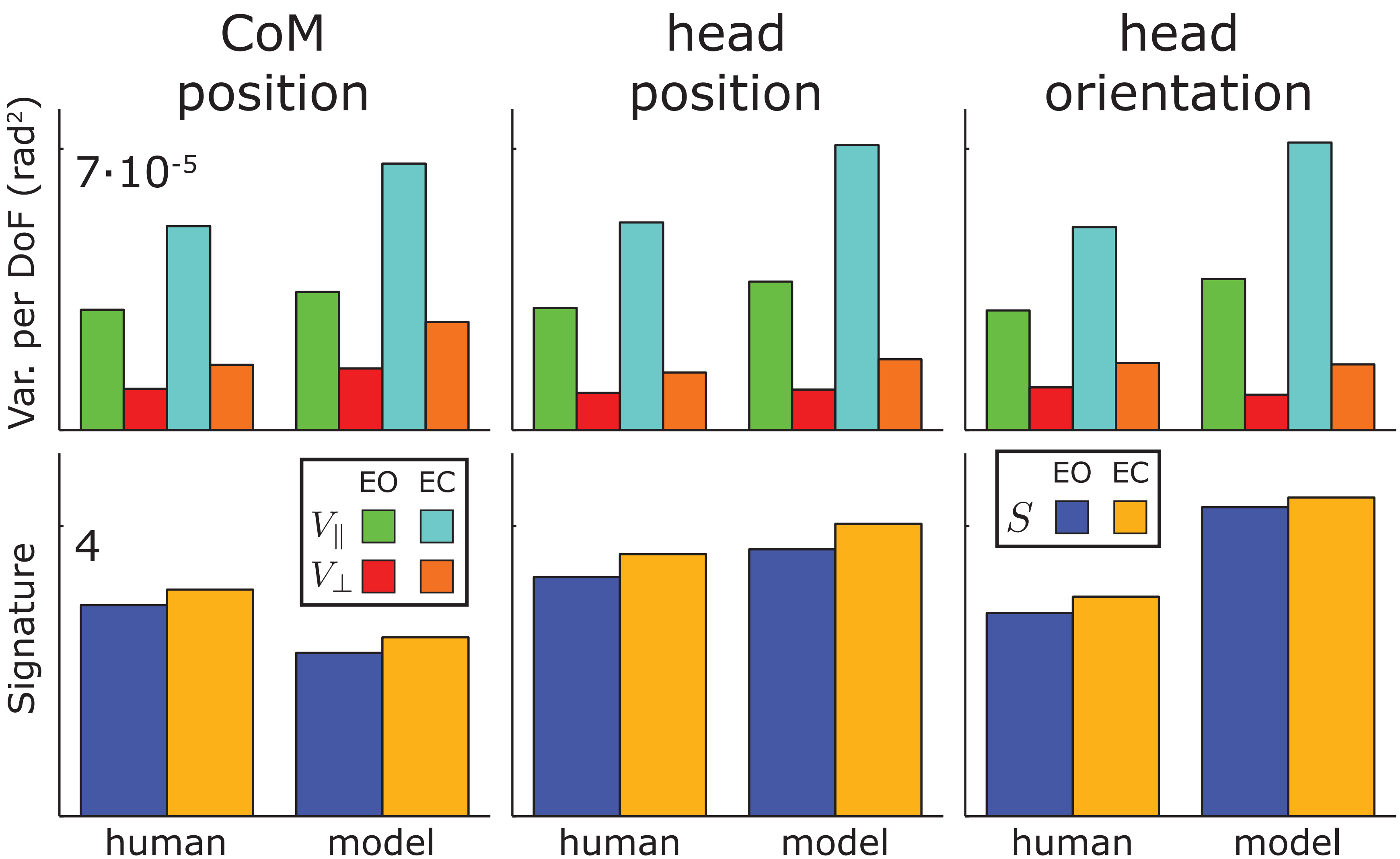




Results: model predicts joint spectra



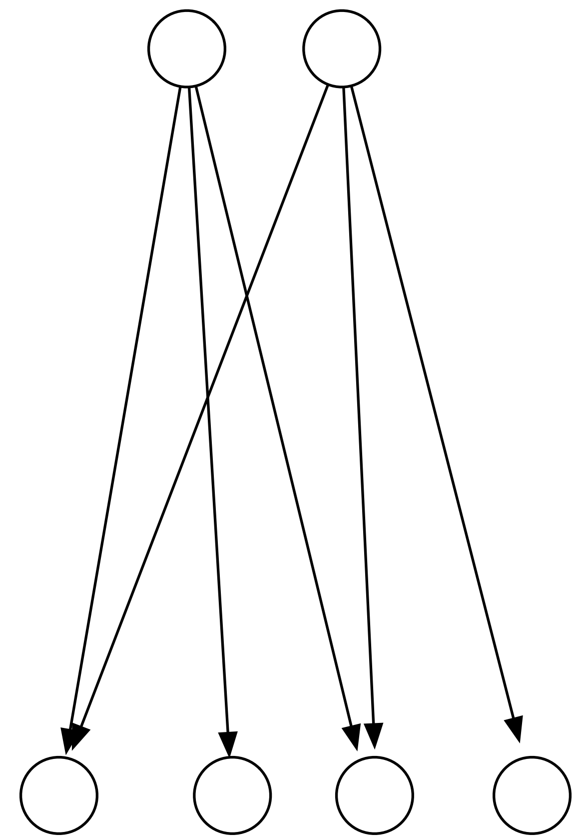
Results: model predicts UCM signature



Why does this work?

$$\dot{\lambda} = F_c = R^{-1}A^{-1}MJ_c^{+}\left(-\alpha_{\dot{c}}\widehat{\dot{c}} - \alpha_{\ddot{c}}\widehat{\ddot{c}}\right)$$

$\widehat{\dot{c}}(t)$ $\widehat{\ddot{c}}(t)$ → motor commands



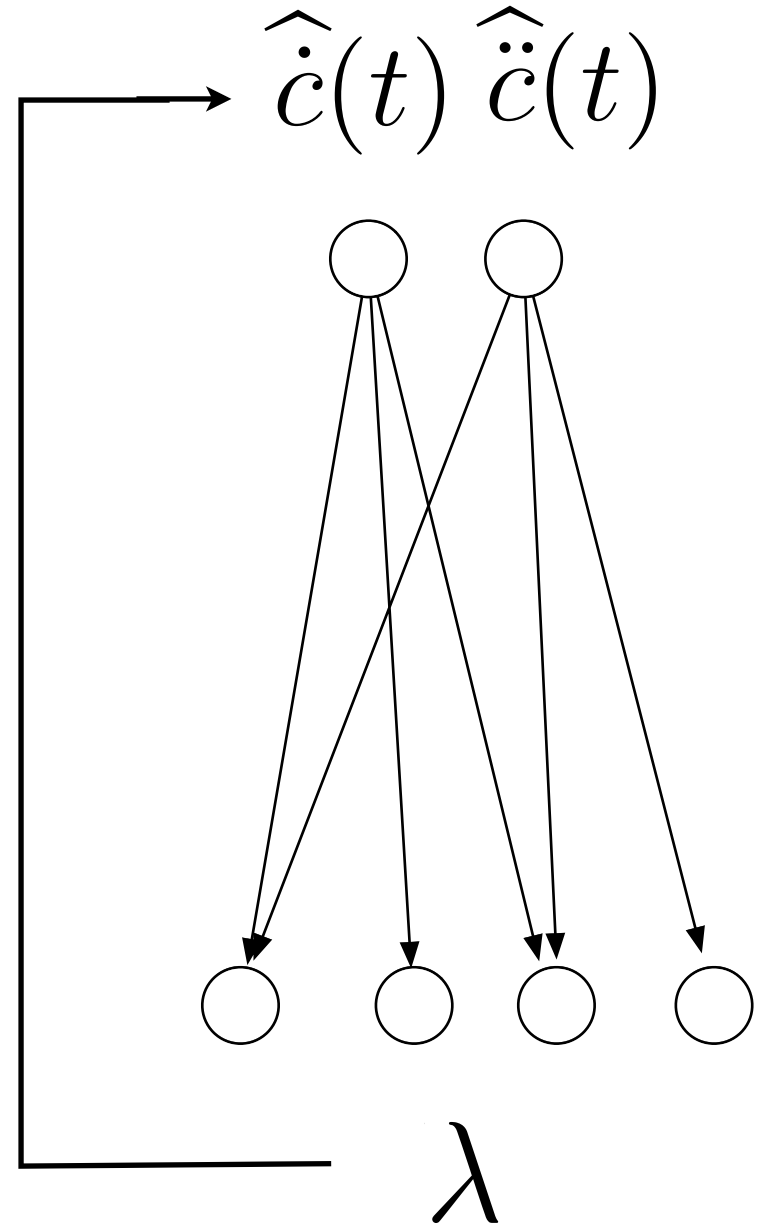
- model looks like a feed-forward neural network
- => should not have a UCM signature: classical synergy?

λ →

DoF/muscles

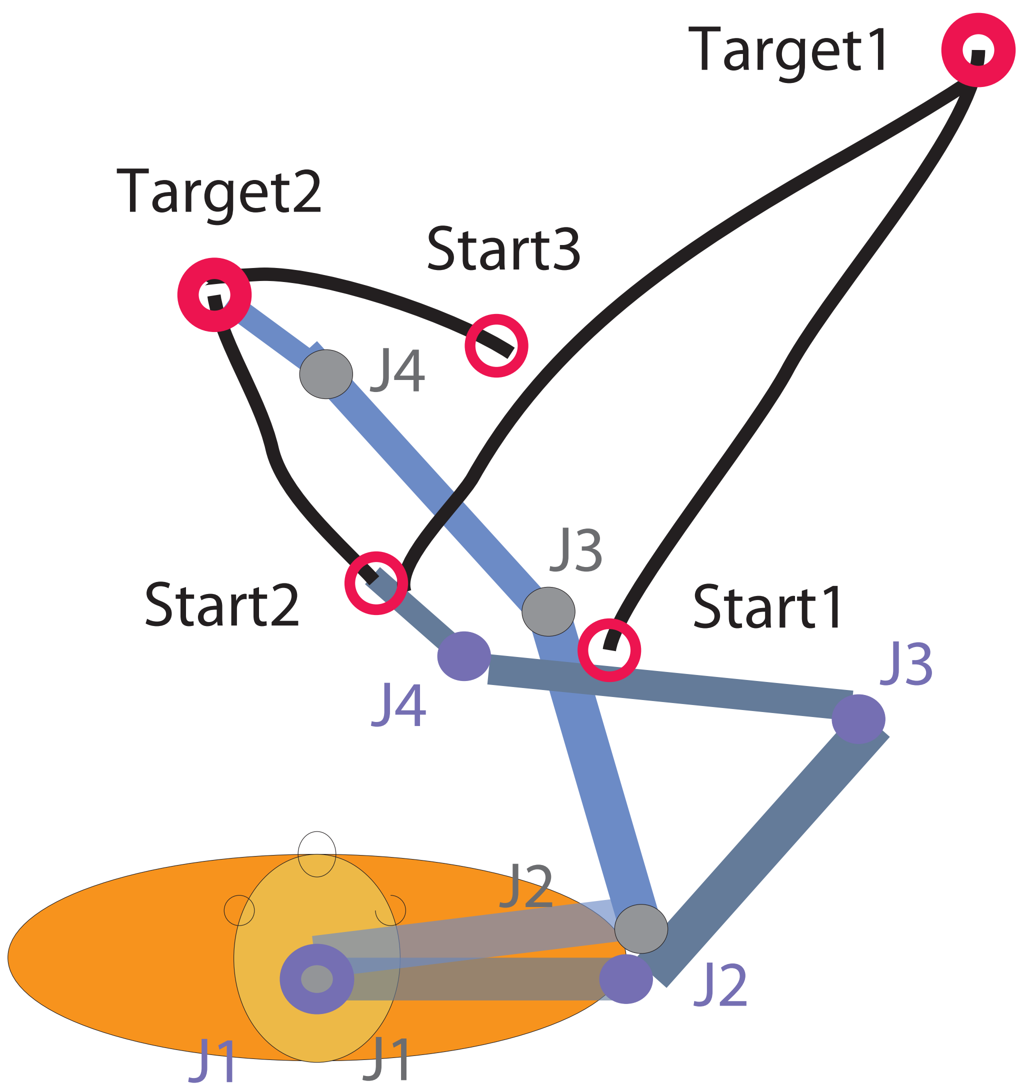
Why does this work?

- feedback loop through the world stabilizes configuration in ORT space
- DoF are effectively coupled through that loop to generate the compensatory signature

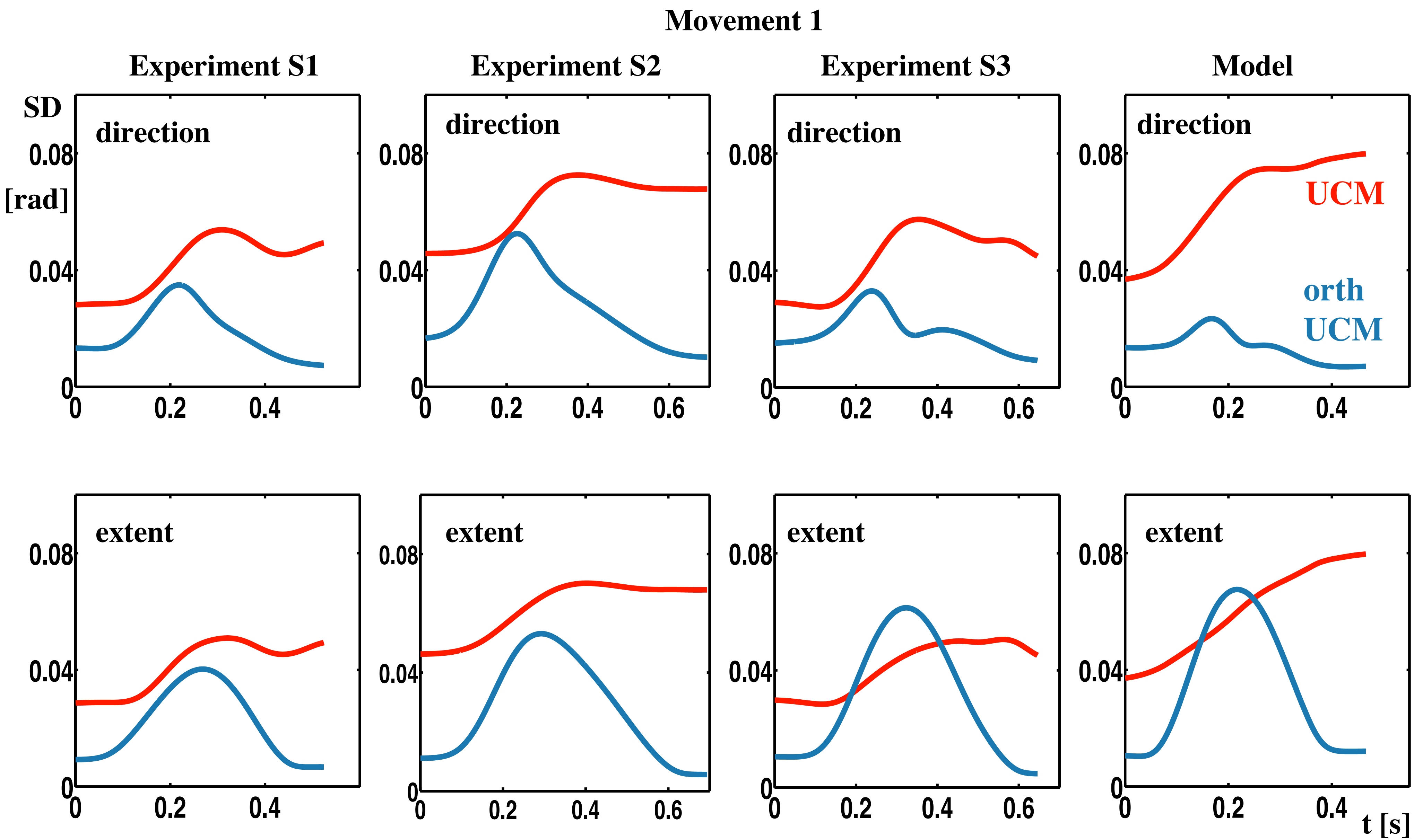


UCM synergy accounts: Case study: Reaching movements

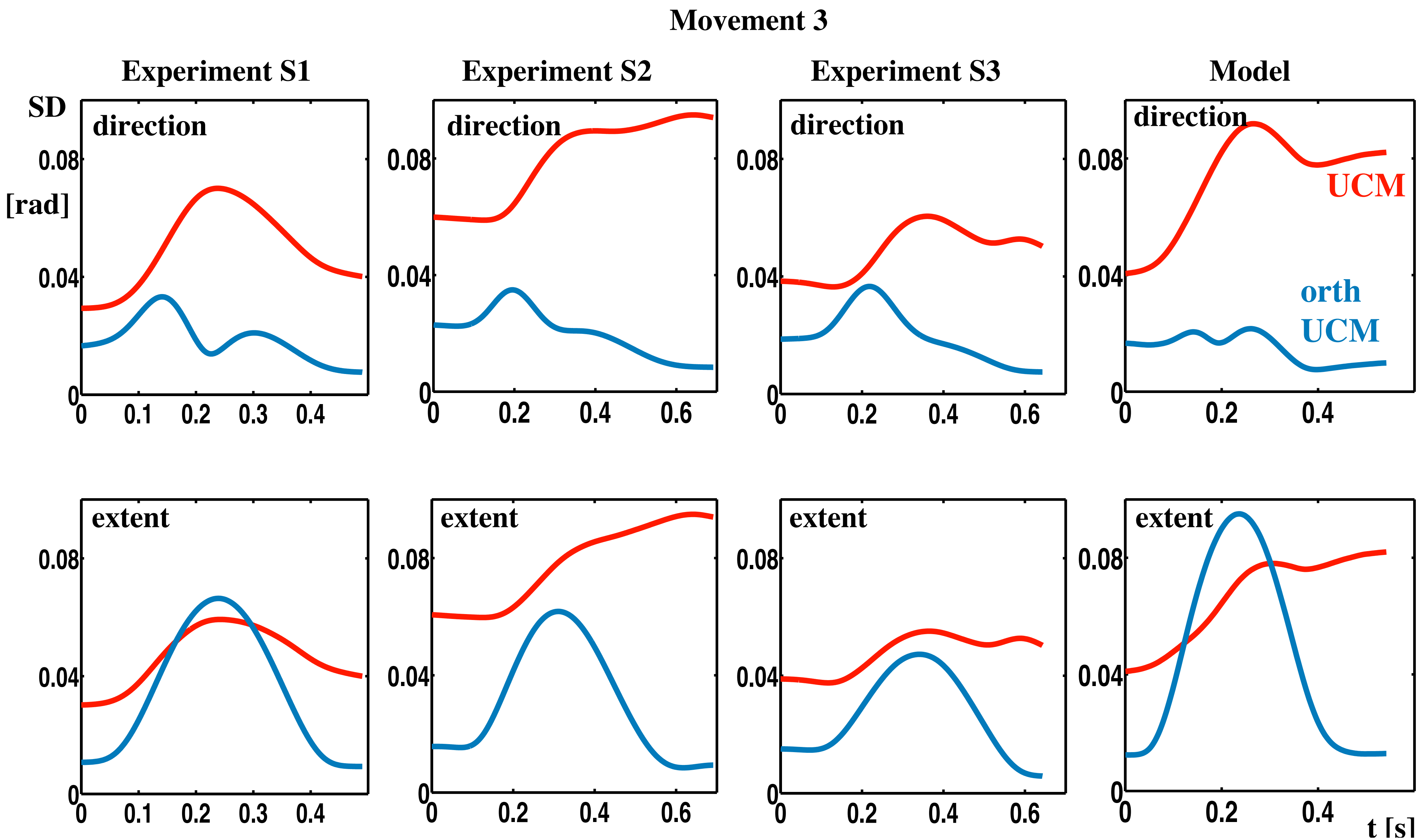
- Experiment from John Scholz's lab:
  - reaching with 4DoF in 2D



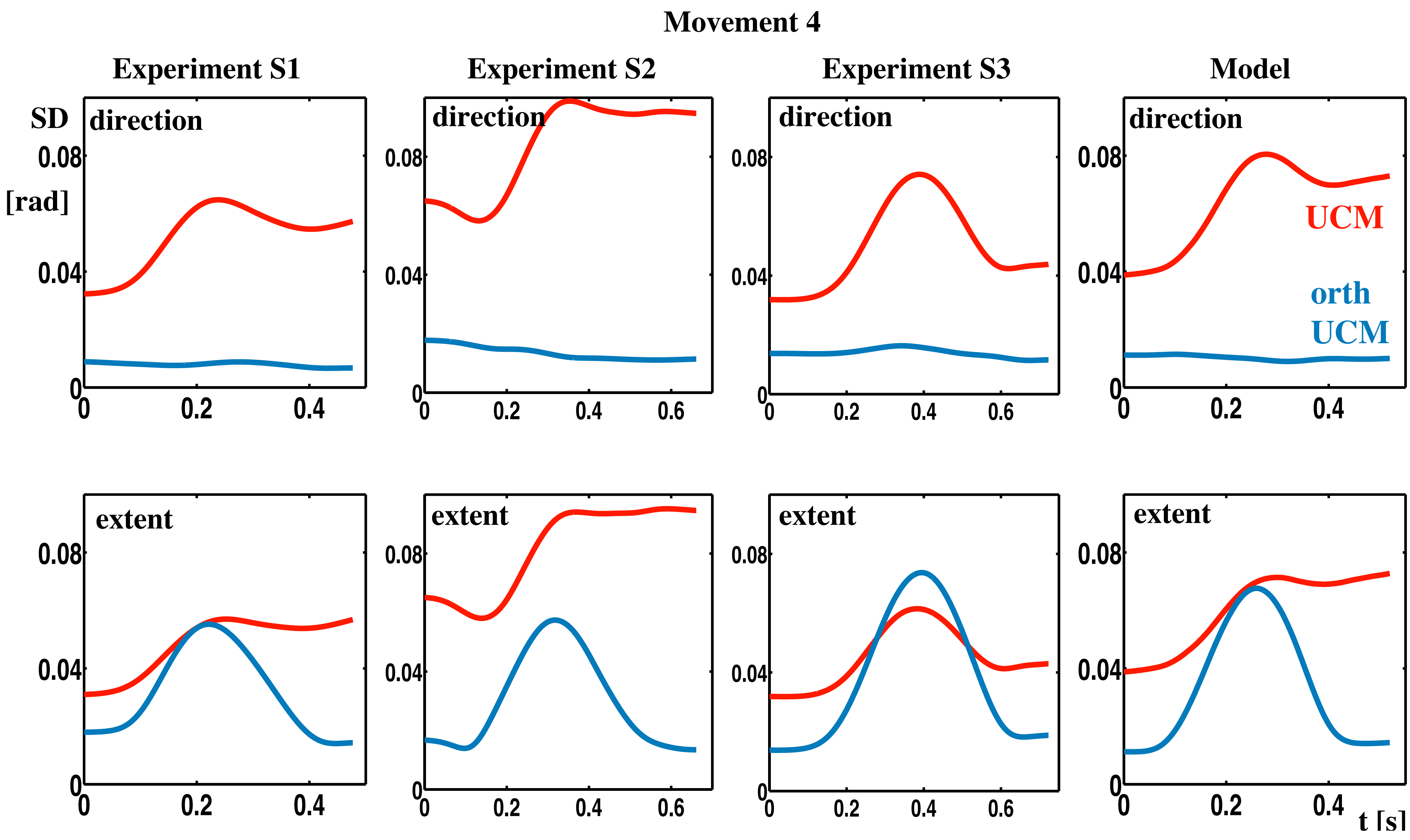
UCM structure of variance



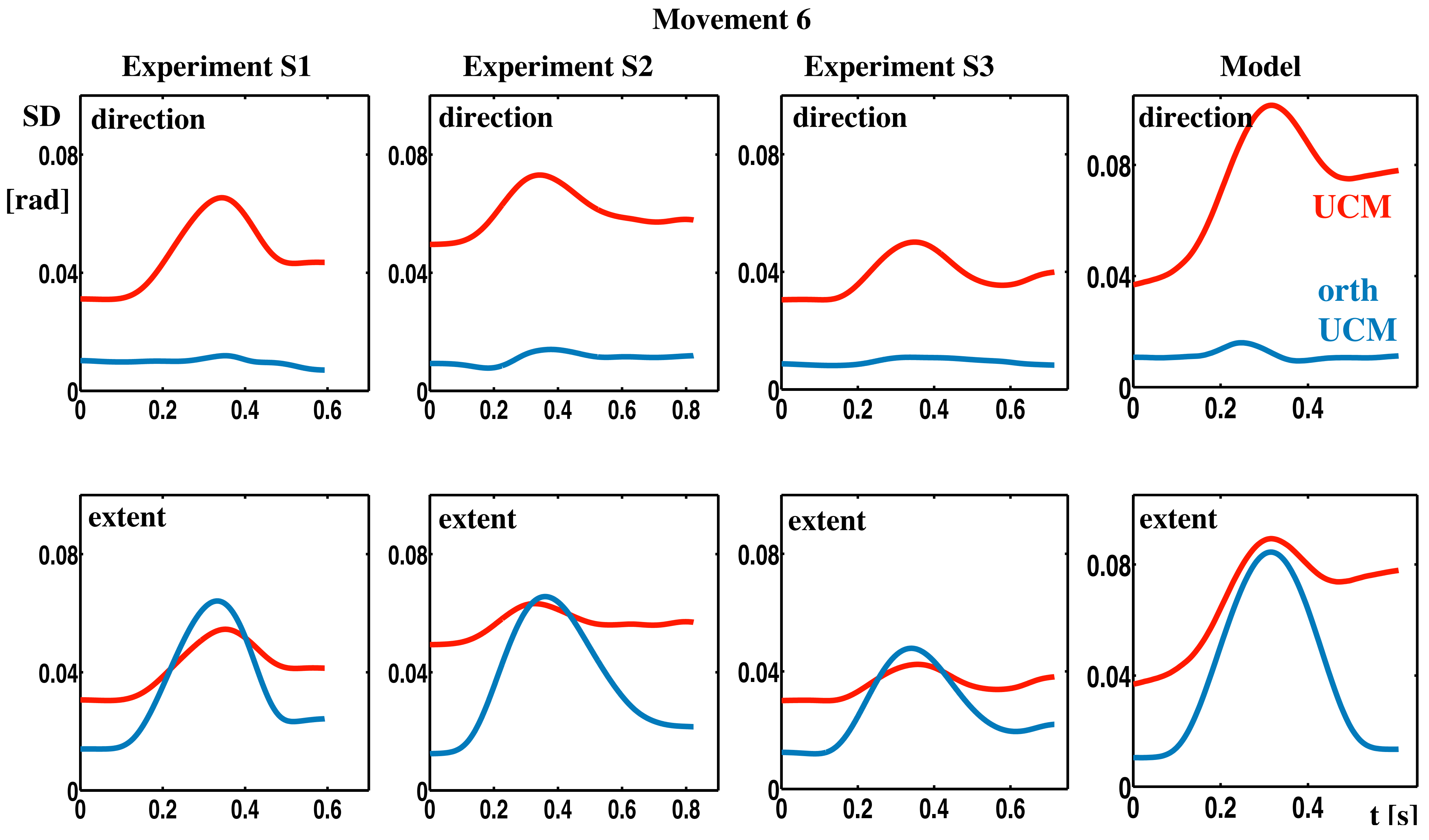
UCM structure of variance



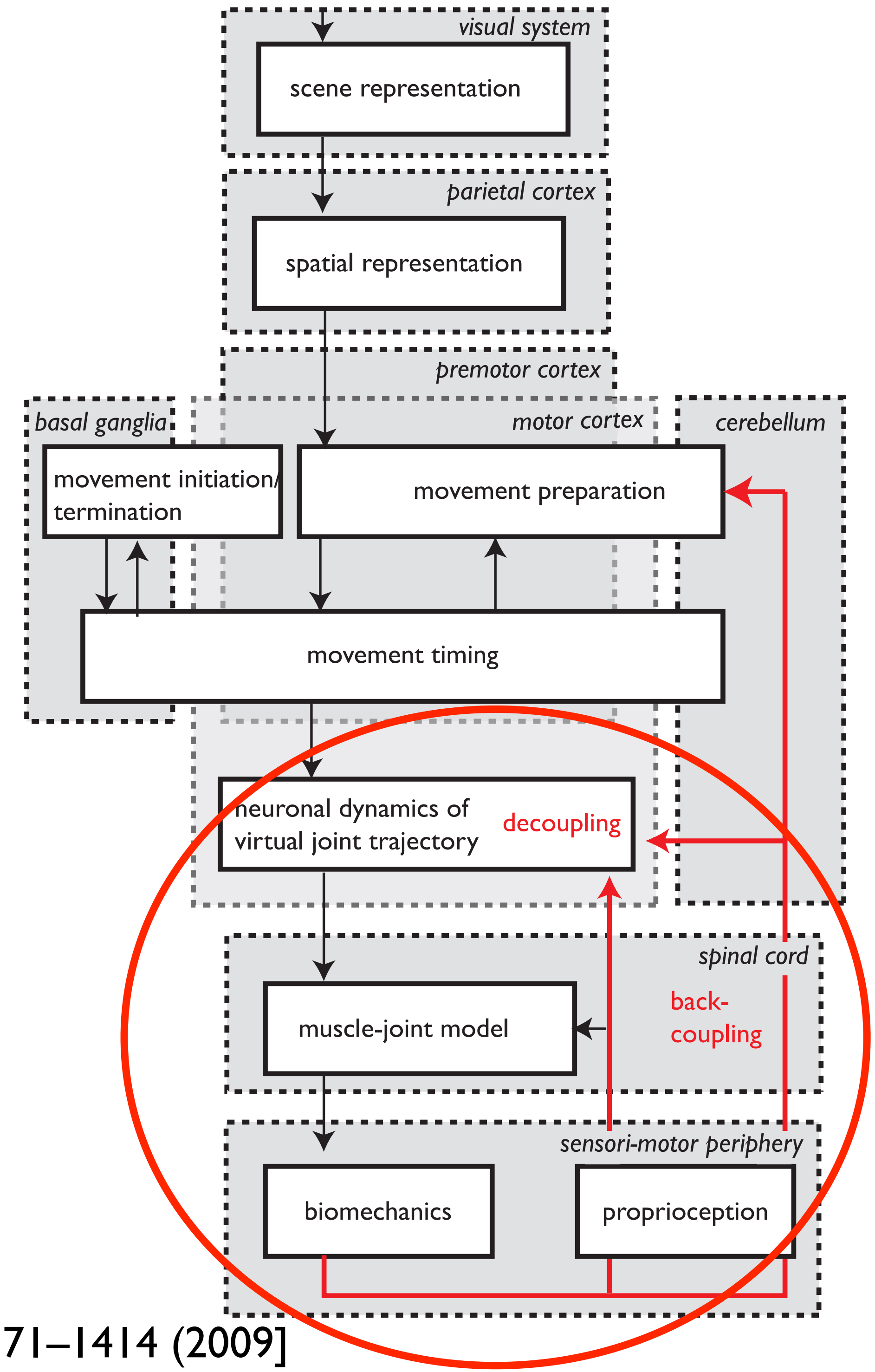
UCM structure of variance



UCM structure of variance



Neural process model of 4DoF reaching



[Martin, Scholz, Schöner. *Neural Computation* 21, 1371–1414 (2009)]

model

biomechanical dynamics

$$M(\boldsymbol{\theta}) \cdot \ddot{\boldsymbol{\theta}} + H(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) = \mathbf{T}_{\mathrm{m}}$$

muscle models

$$\begin{aligned} T_i &= K_l \cdot \Big((e^{[K_{nl} \cdot (\theta_i - \lambda_i^p)]^+} - 1) - (e^{-[K_{nl} \cdot (\theta_i - \lambda_i^m)]^-} - 1) \Big) \\ &+ \mu_{bl} \cdot \mathrm{asinh}(\dot{\theta}_i - \dot{\lambda}_i) + \mu_{rl} \cdot \dot{\theta}_i . \end{aligned}$$

neural dynamics of lambda

$$\dot{\mathbf{v}} = -\beta_v(\mathbf{v} - \mathbf{u}(t)),$$

timing signal

$$\mathbf{v}(t) = \mathbf{J}[\boldsymbol{\lambda}(t)] \cdot \dot{\boldsymbol{\lambda}}(t),$$

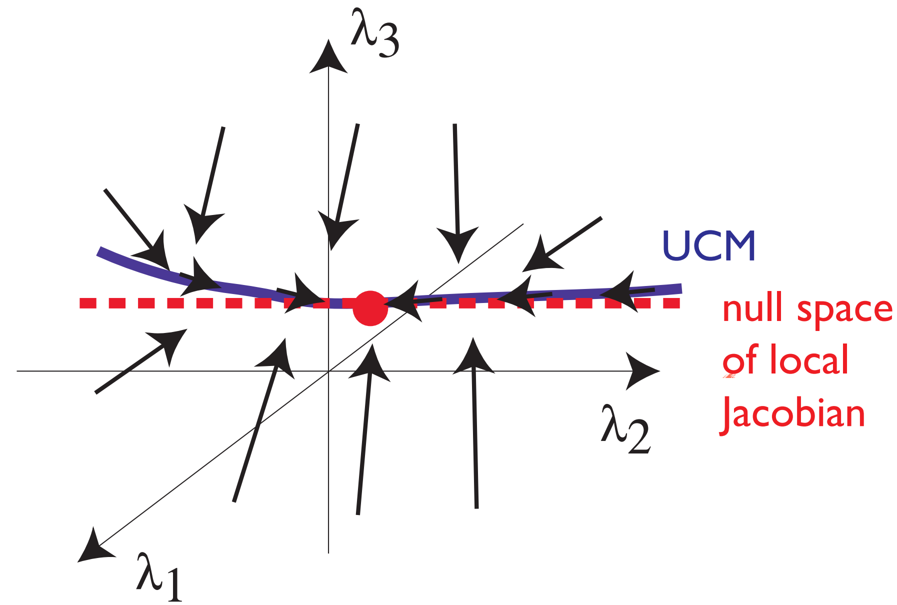
$$\ddot{\boldsymbol{\lambda}} = (\mathbf{J}^+ \ \mathbf{E}) \cdot \begin{pmatrix} -\beta_v \mathbf{J} \cdot \dot{\boldsymbol{\lambda}} + \beta_v \mathbf{u} - \cancel{\dot{\mathbf{J}} \cdot} \dot{\boldsymbol{\lambda}} \\ -\beta_{s1}\mathbf{E}^T \cdot (\boldsymbol{\lambda} - \boldsymbol{\theta}_d) - \beta_{s2}\mathbf{E}^T \cdot (\dot{\boldsymbol{\lambda}} - \dot{\boldsymbol{\theta}}_d). - \cancel{\dot{\mathbf{E}}^T \cdot} \dot{\boldsymbol{\lambda}} \end{pmatrix}$$

back-coupling

approximation

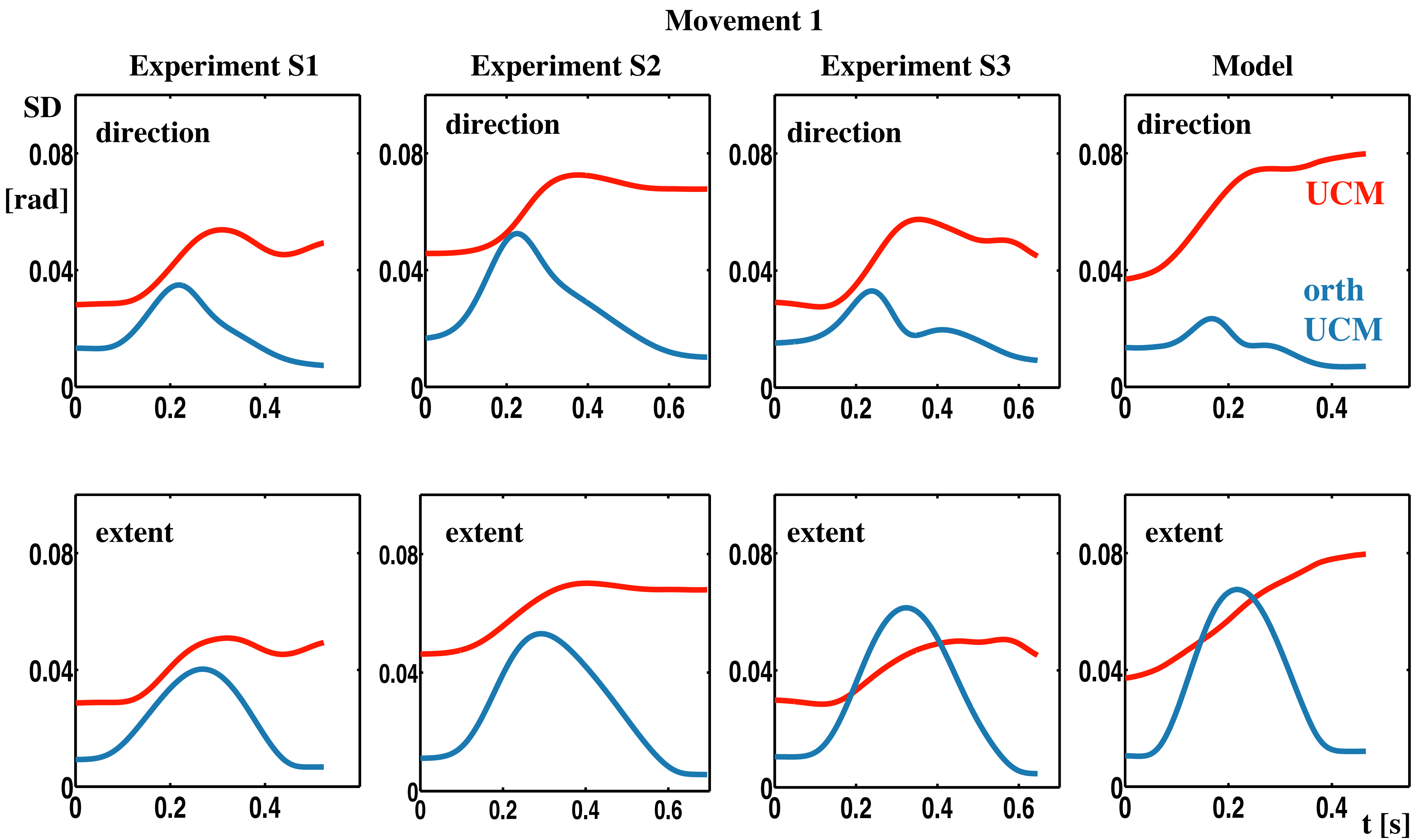
timing signal

$$\ddot{\boldsymbol{\lambda}} = (\mathbf{J}^{+} \ \mathbf{E}) \cdot \begin{pmatrix} -\beta_v \mathbf{J} \cdot \dot{\boldsymbol{\lambda}} + \beta_v \mathbf{u} \\ 0 \end{pmatrix}$$

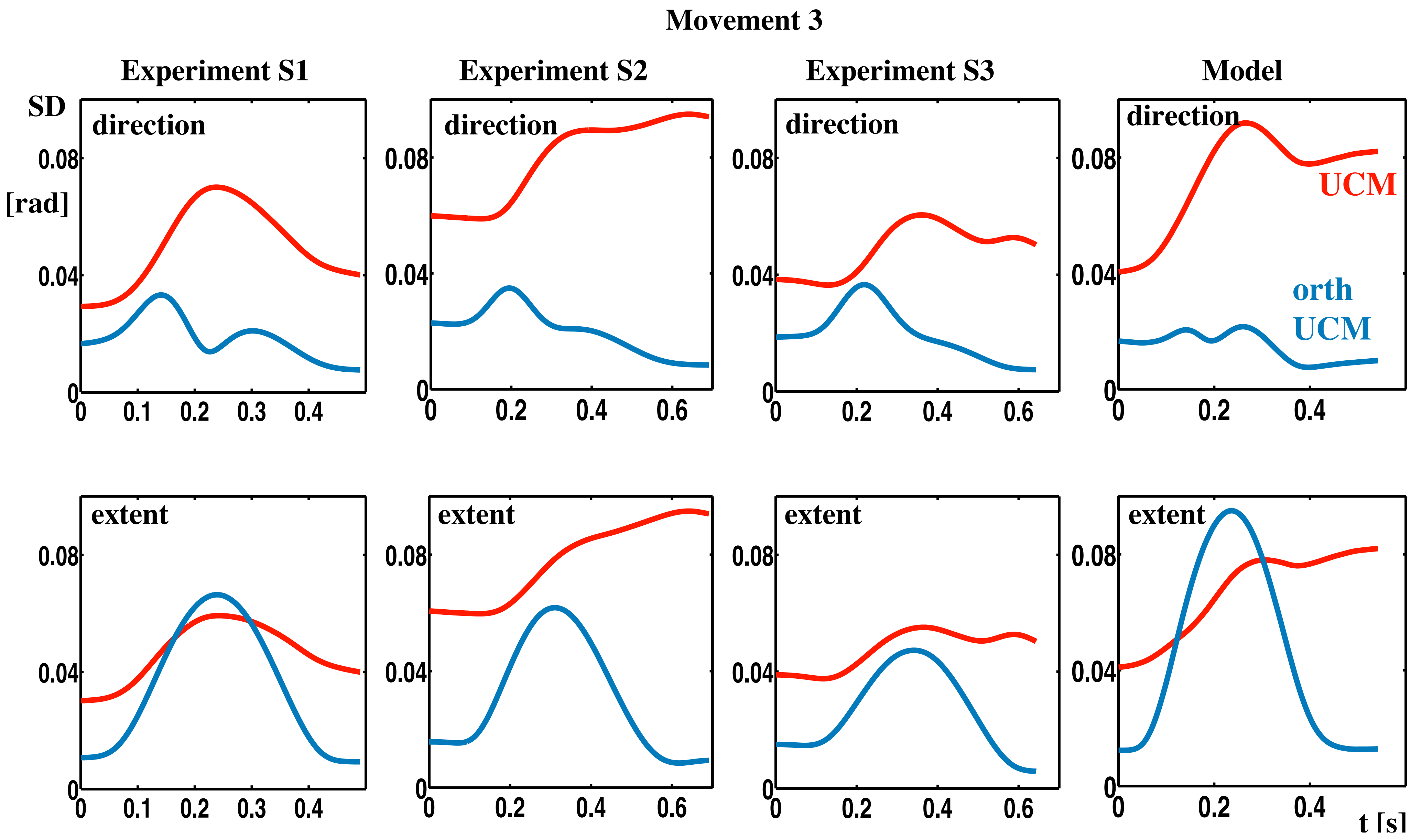


- => control is stable in range space
- => marginally stable in UCM/null space

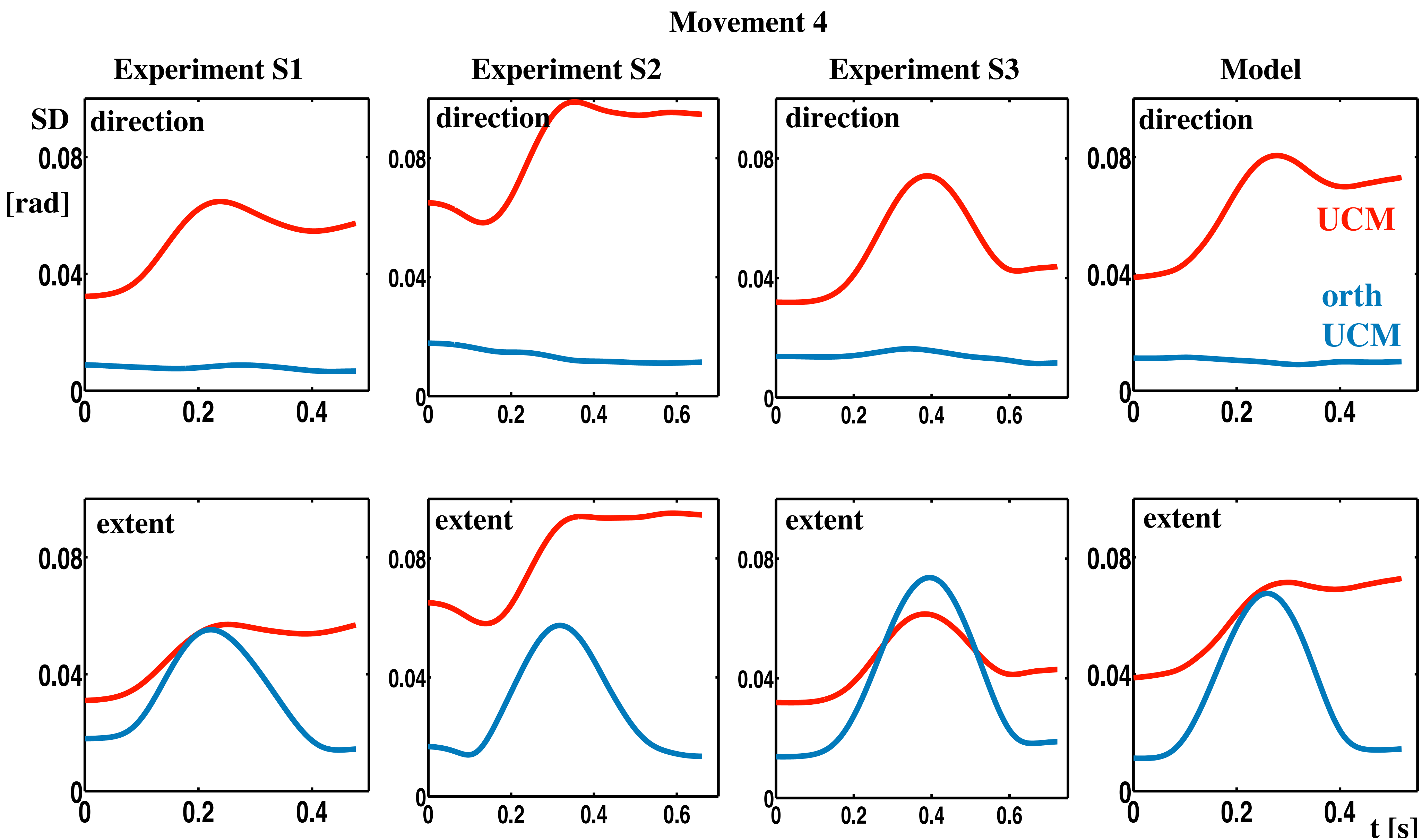
UCM structure of variance



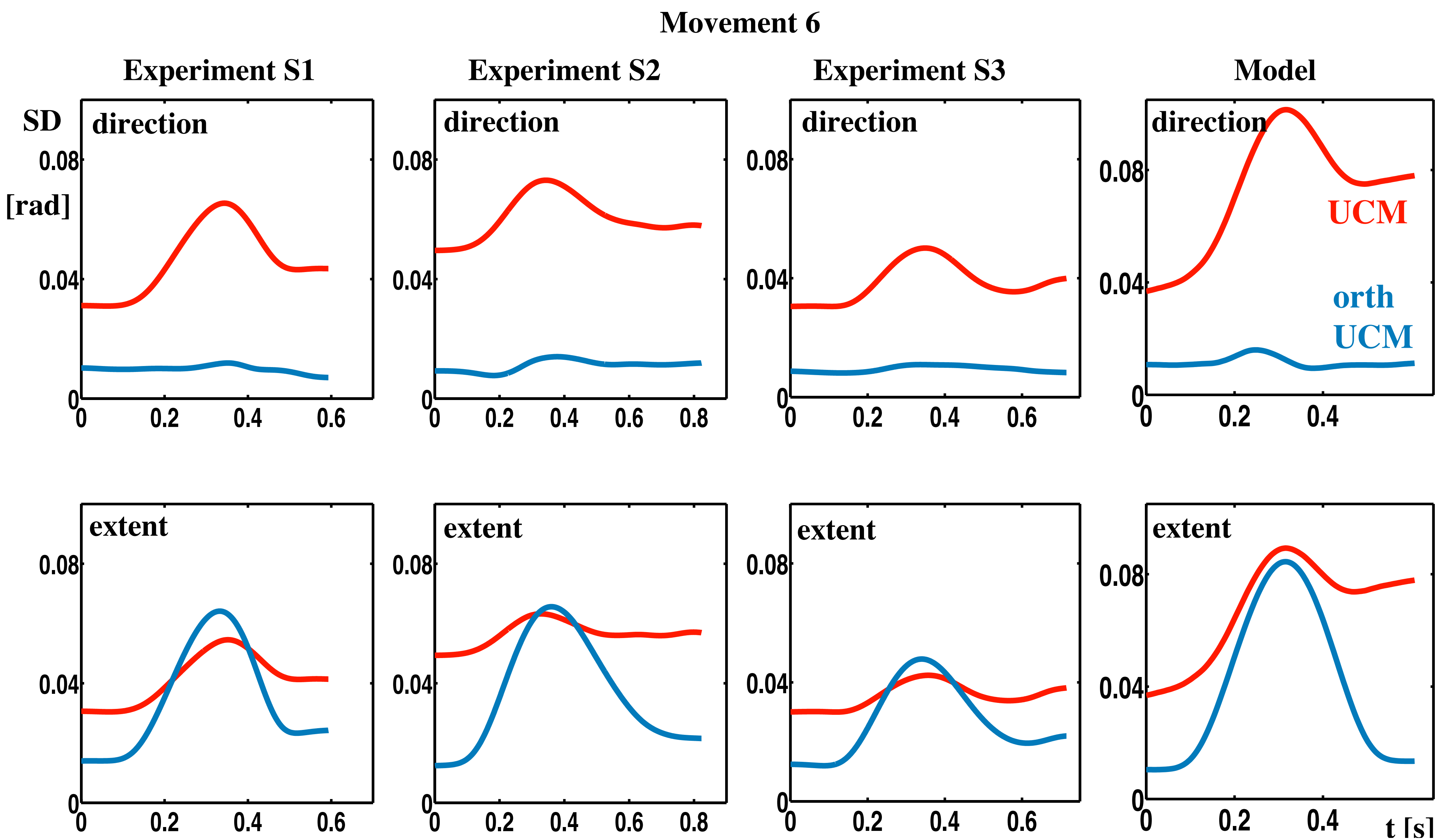
UCM structure of variance



UCM structure of variance



UCM structure of variance



where does this come from?

start with pseudo-inverse of: $v = J\dot{\lambda}$

$$\dot{\lambda} = J^{+}v$$

$$\ddot{\lambda} = J^{+}\dot{v} \quad [+\dot{J}^{+}v \approx 0]$$

a neuron, n, encoding rate of change of λ : $n = \dot{\lambda}$

$\dot{n} = J^{+}\dot{v}$ <= insert timing signal $\dot{v} = -v + u$

$\dot{n} = J^{+}(-v + u)$ <= insert $v = J\dot{\lambda}$

$\dot{n} = J^{+}(-J\dot{\lambda} + u)$ <= replace $n = \dot{\lambda}$

$$\dot{n} = J^{+}(-Jn + u)$$

$$\dot{n} = -J^{+}Jn + J^{+}u$$

where does this come from?

$$\dot{n} = -J^+ J n + J^+ u$$

$$\dot{n} = -n + n - J^+ J n + J^+ u$$

$$\dot{n} = -n + (1 - J^+ J) n + J^+ u$$

projection onto null-space

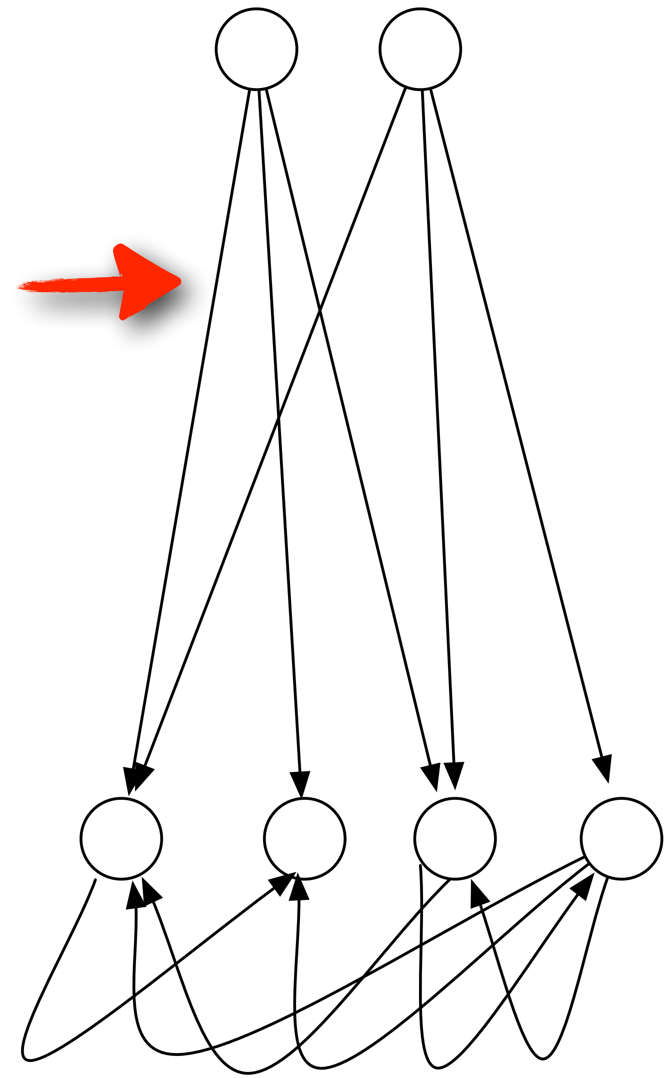
feed-forward from timing command

where does this come from?

feed-forward from timing command

$$\dot{n} = -n + (1 - J^{+}J)n + J^{+}u$$

projection onto null-space



how does this do the UCM effect?

projection onto null-space

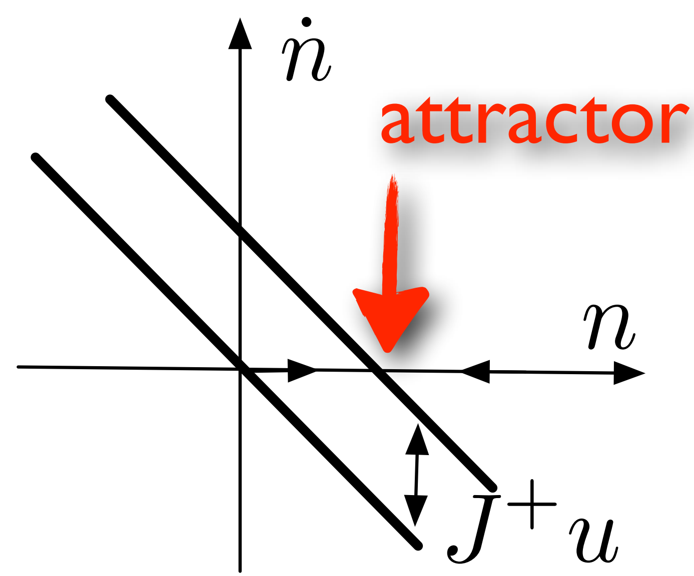
feed-forward from timing command

$$\dot{n} = -n + (1 - J^+ J)n + J^+ u$$

within the range-space

$$\dot{n} = -n + J^+ u$$

=> stability within the range-space



how does this do the UCM effect?

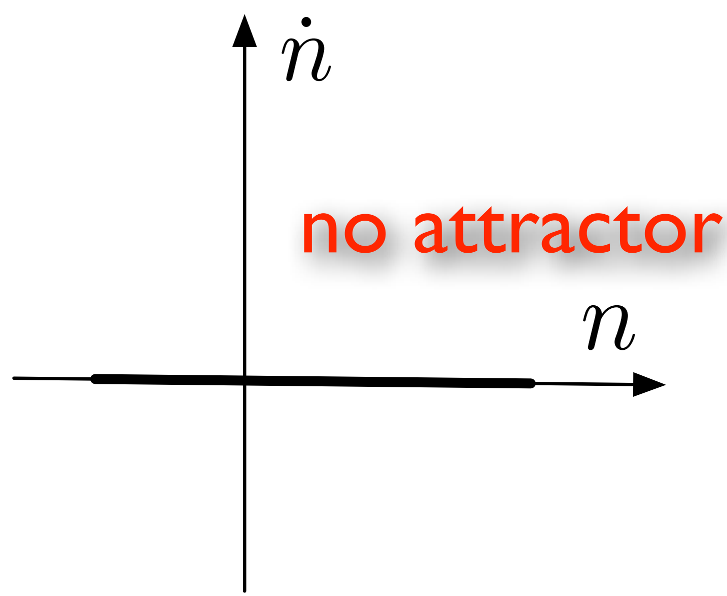
projection onto null-space → feed-forward from timing command

$$\dot{n} = -n + (1 - J^{+}J)n + J^{+}u$$

within the null-space

$$\dot{n} = -n + n + 0$$

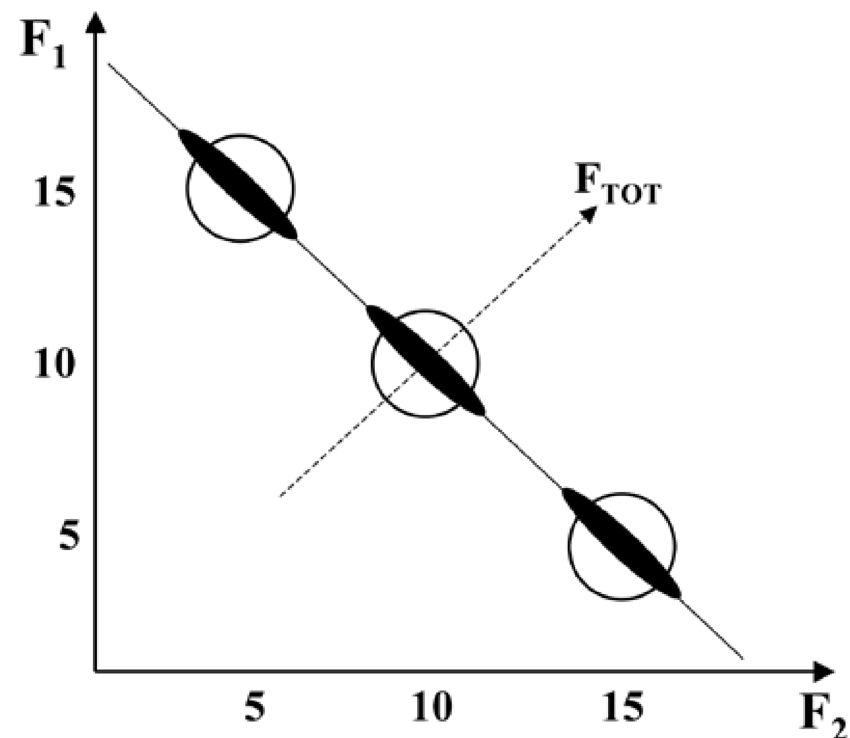
$$\dot{n} = 0$$



=> no stability within the null-space

UCM synergy accounts: case study finger movements

- Mark Latash et al: press with two fingers to produce fixed total force



model

task variable F

$$F = F_1 + F_2$$

Jacobian

$$J = (1\ 1)$$

Pseudo-inverse

$$J^+ = \begin{pmatrix} 1/2 \\ 1/2 \end{pmatrix}$$

projection onto null space

$$1 - J^+ J = \begin{pmatrix} 1/2 & -1/2 \\ -1/2 & 1/2 \end{pmatrix}$$

model

two neurons to represent forces

$$n = \begin{pmatrix} F1 \\ F2 \end{pmatrix} \qquad J^+ = \begin{pmatrix} 1/2 \\ 1/2 \end{pmatrix}$$

dynamics

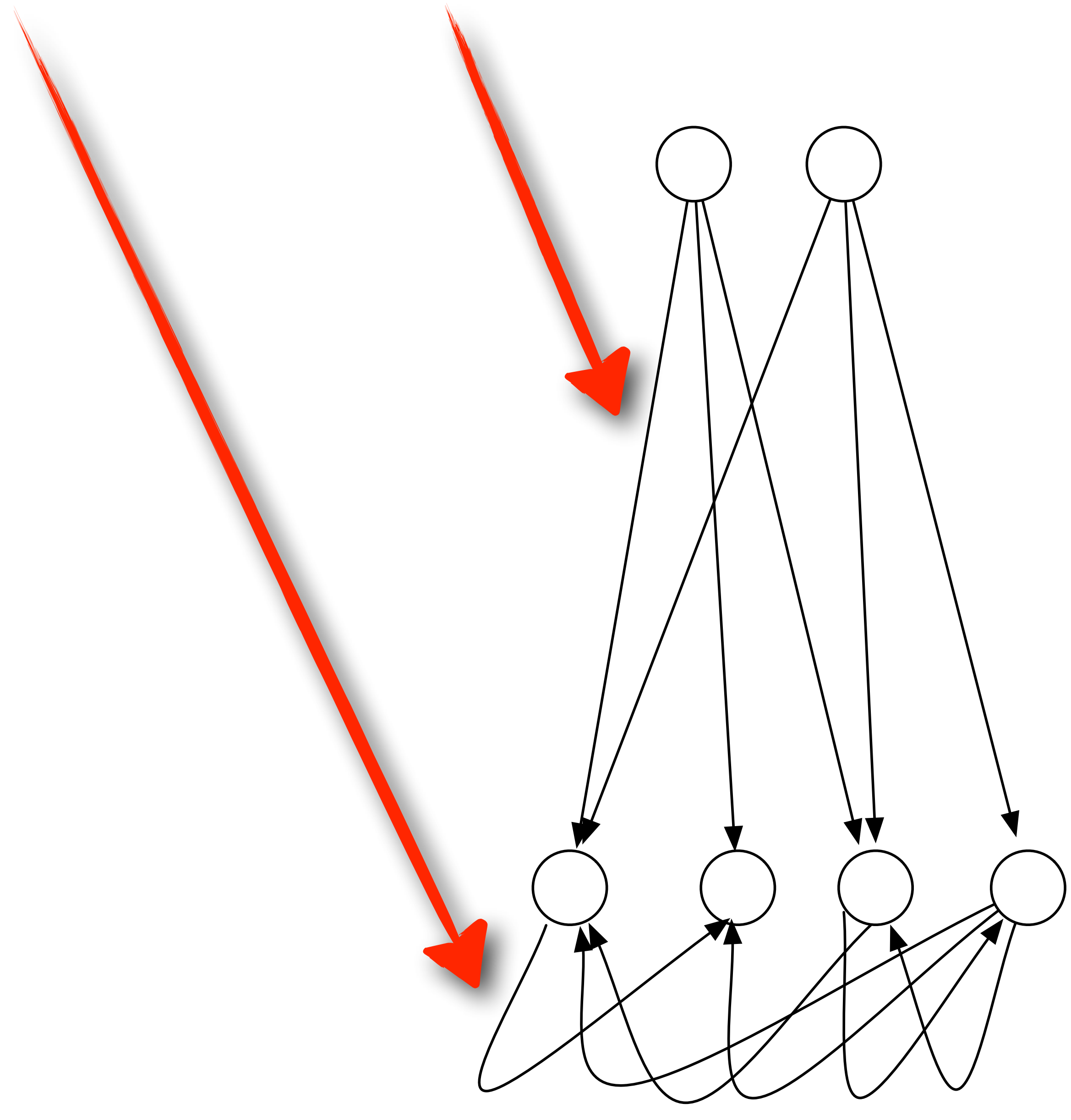
$$\dot{n} = -n + (1 - J^+ J)n + J^+ u$$

$$1 - J^+ J = \begin{pmatrix} 1/2 & -1/2 \\ -1/2 & 1/2 \end{pmatrix}$$

leads to

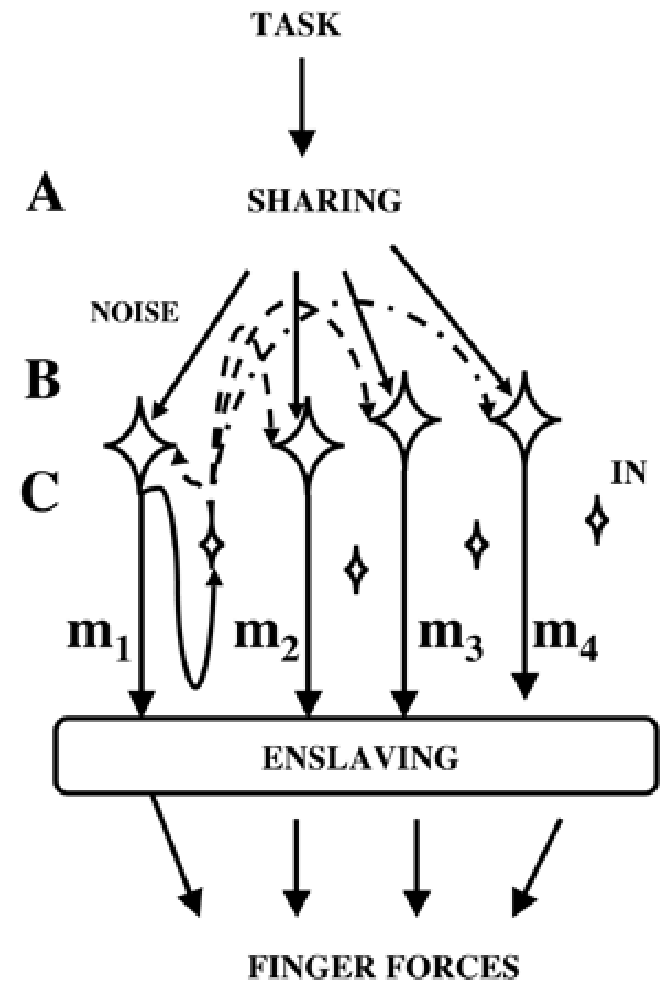
$$\dot{n} = -\begin{pmatrix} n_1 \\ n_2 \end{pmatrix} + \begin{pmatrix} n_1/2 - n_2/2 \\ n_2/2 - n_1/2 \end{pmatrix} + \begin{pmatrix} u/2 \\ u/2 \end{pmatrix}$$

$$\dot{n} = -\begin{pmatrix} n_1 \\ n_2 \end{pmatrix} + \begin{pmatrix} n_1/2 - n_2/2 \\ n_2/2 - n_1/2 \end{pmatrix} + \begin{pmatrix} u/2 \\ u/2 \end{pmatrix}$$



compare to Latash et al 2005

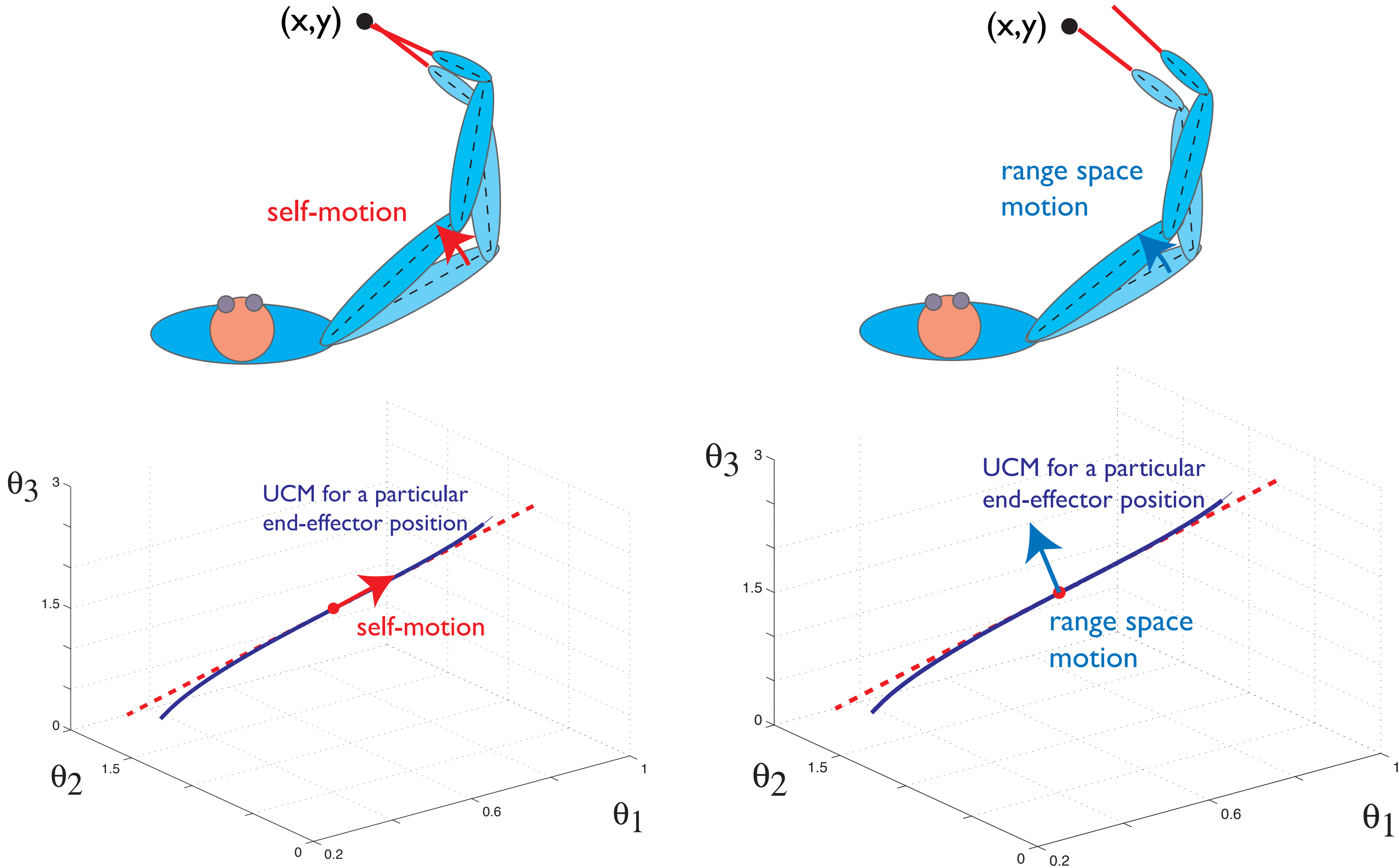
- candidate for recurrent inhibitory interaction: Renshaw system



Self-motion

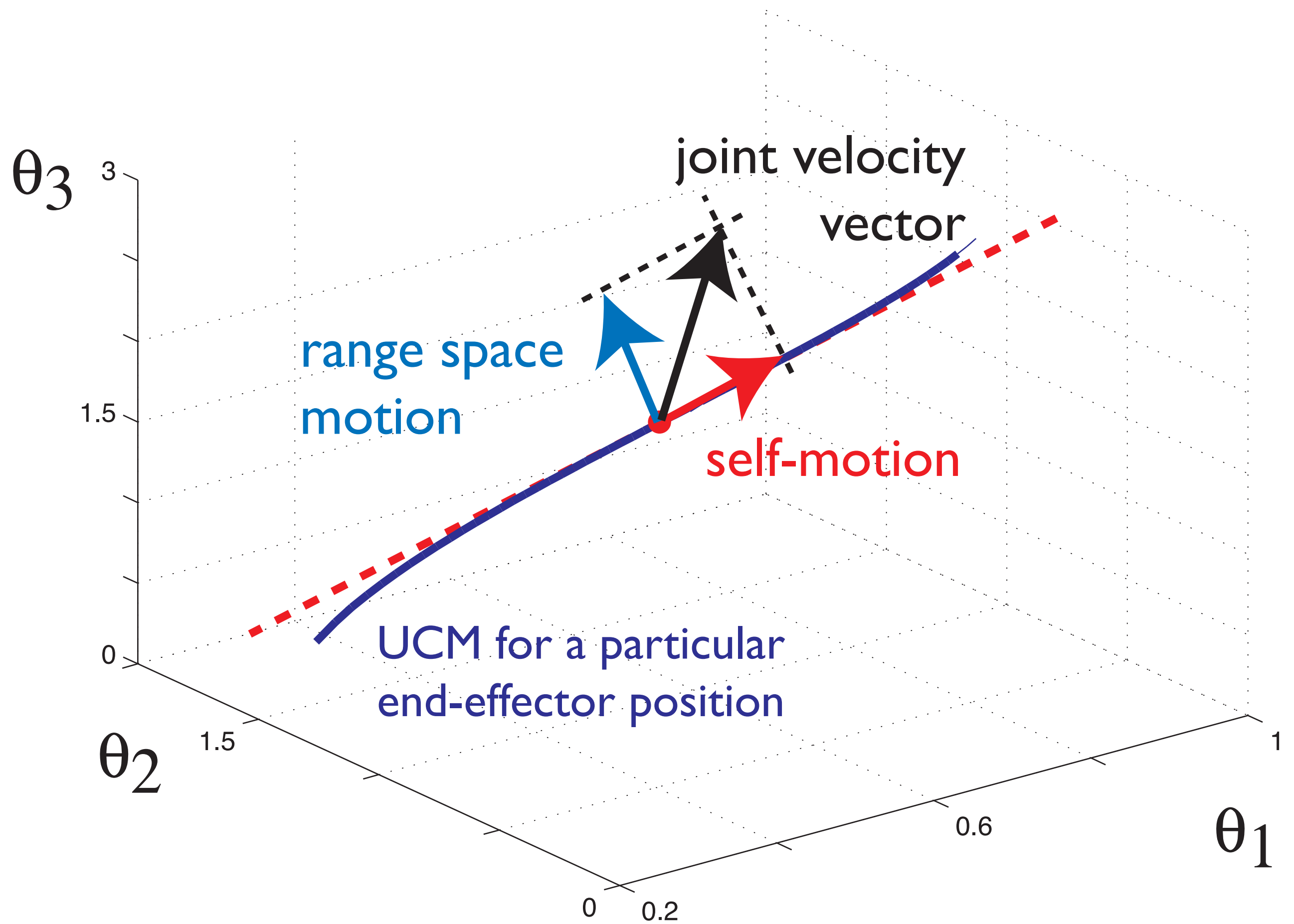
- all this was about variation/variance…
- how about the motion itself, the mean motion… does that reveal the DoF problem and its solution?
- => self-motion

Self motion



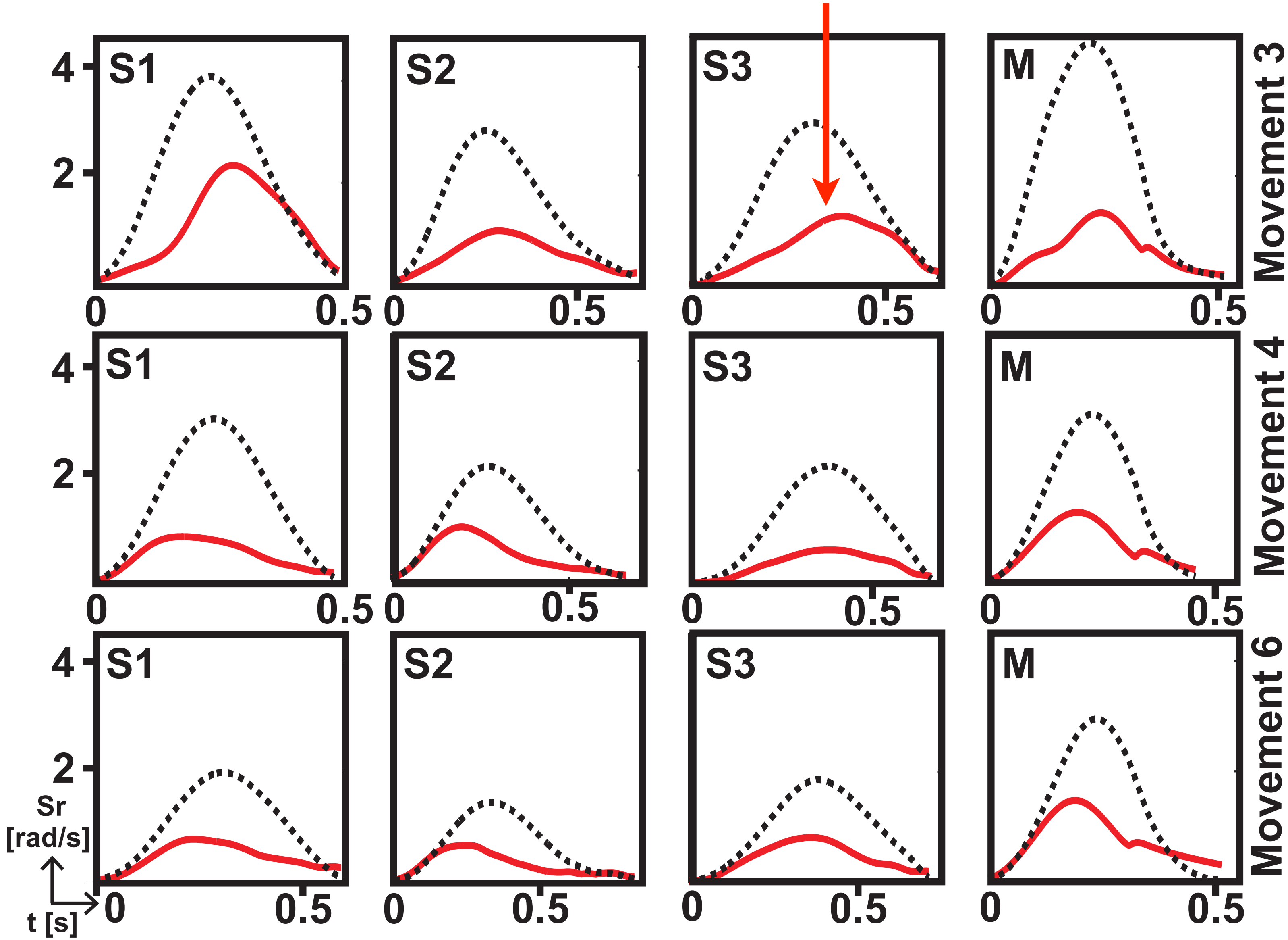
(x,y)
self-motion
(x,y)
range space
motion
θ3
3
1.5
0
UCM for a particular
end-effector position
self-motion
θ2
1.5
0
0.2
0.6
1
θ1
θ3
3
1.5
0
UCM for a particular
end-effector position
range space
motion
θ2
1.5
0
0.2
0.6
1
θ1

Self-motion



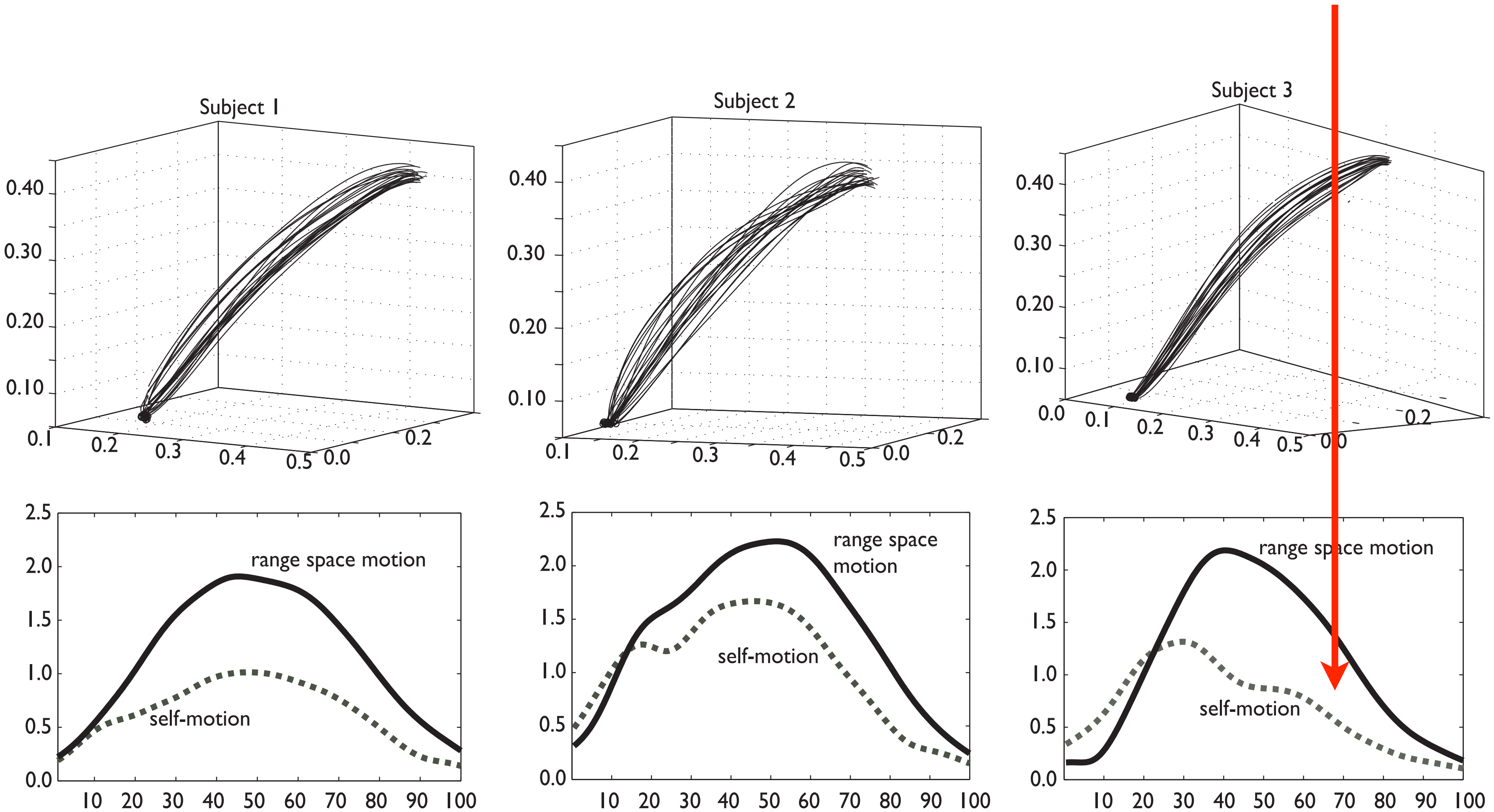
joint velocity
vector
range space
motion
self-motion
UCM for a particular
end-effector position
θ3
3
1.5
0
θ2
1.5
0
0.2
0.6
1
θ1

reaching in 2D, 4DoF: considerable amount of self-motion!



[Martin, Scholz, Schöner. *Neural Computation* 21, 1371–1414 (2009)]

reaching movement in 3D, 10 DoF also shows considerable amount of self-motion



[Martin, Scholz, Schöner. *Neural Computation* 21, 1371–1414 (2009)]

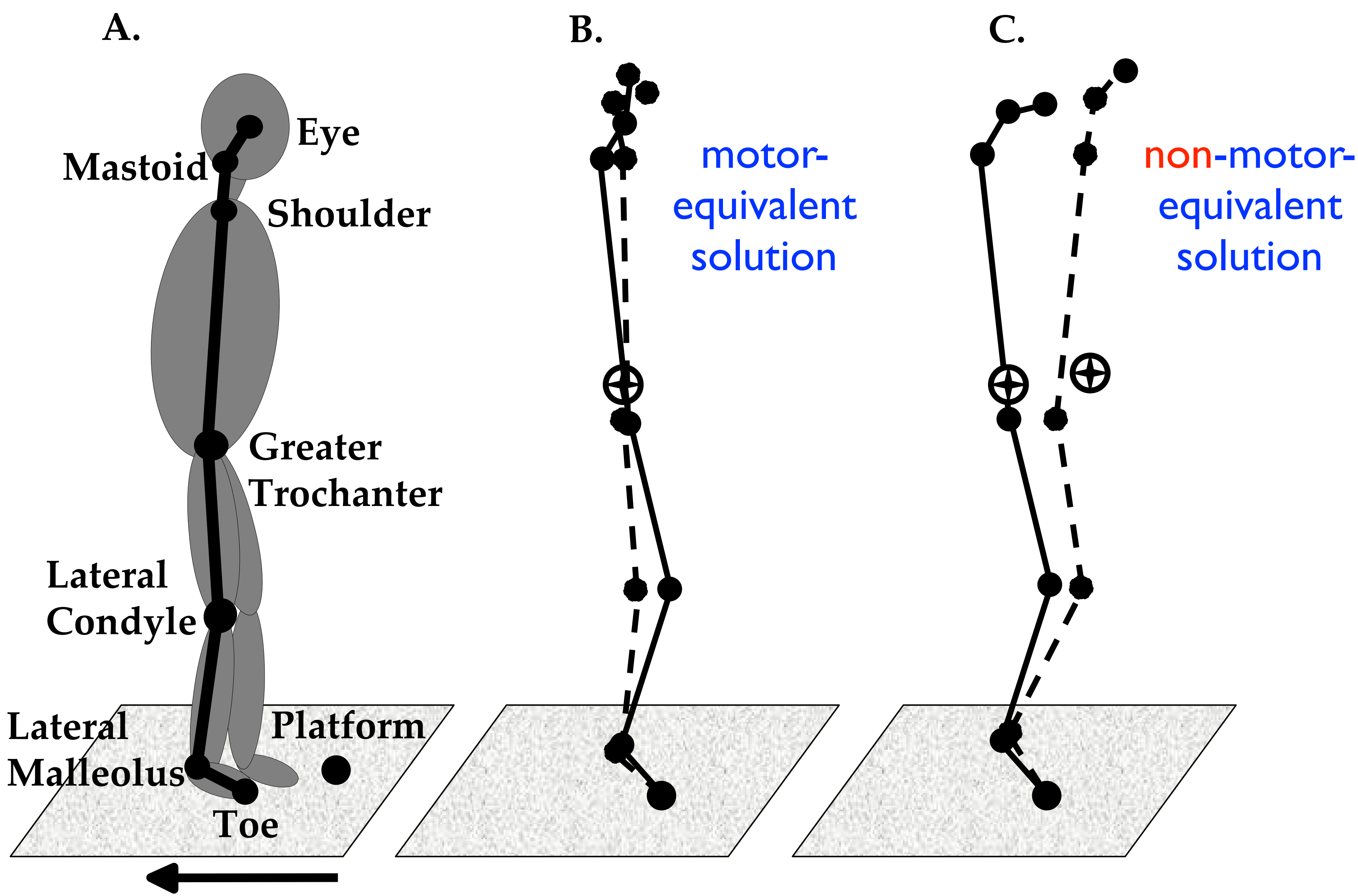
Motor equivalence

- can we see directly the use of the redundant/abundant DoF to solve some problem?
- motor equivalence: "task achieved with a new joint configuration following perturbation, different initial condition, or changed conditions"

Motor equivalence

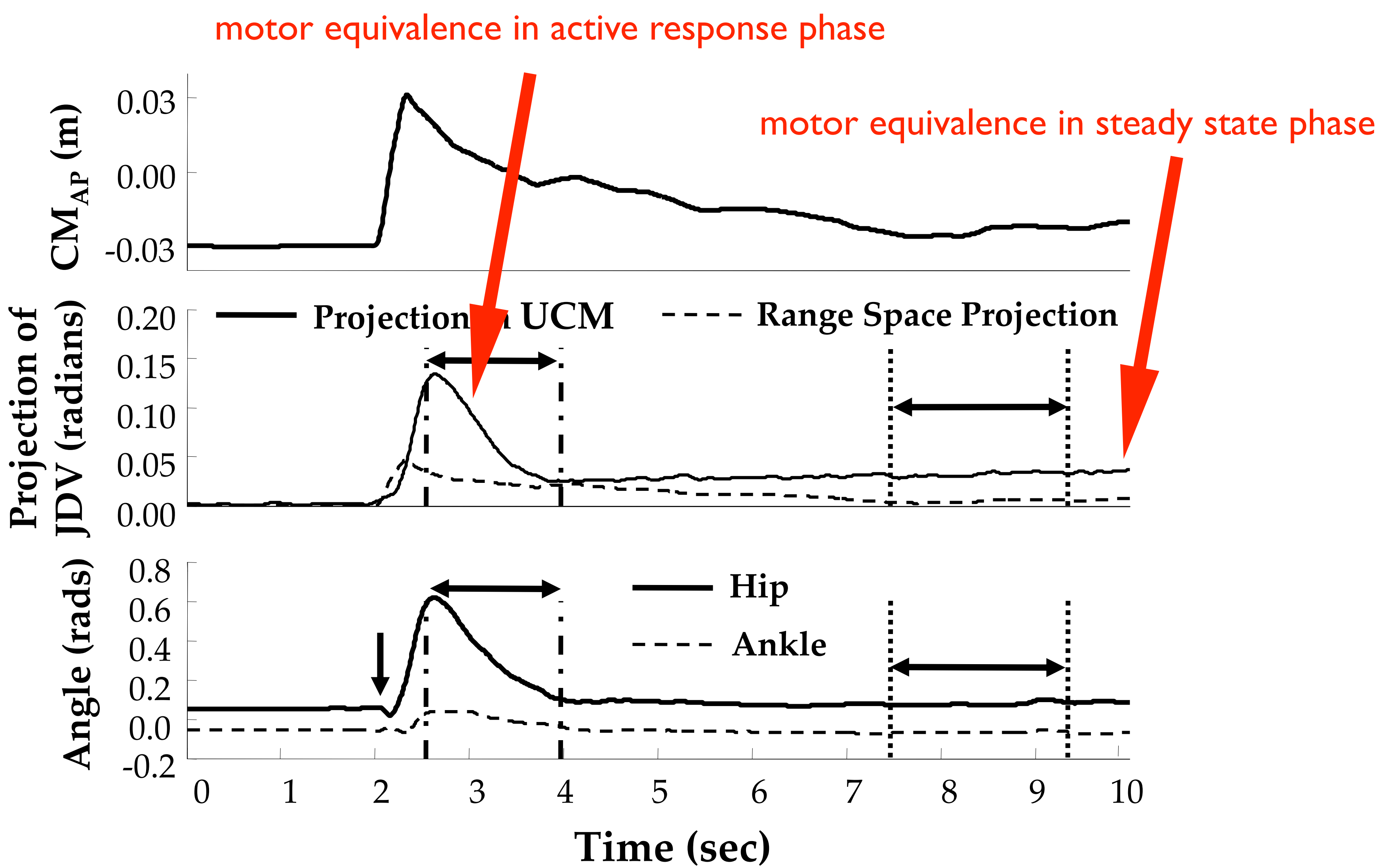
- “task achieved with other than standard joint configuration following perturbation or other change”
- but: task never achieved 100 percent
- how much error on task level compared to how much error at joint level? how do you compare?
- answer: error lies more within UCM than perpendicular!

Motor equivalence in quiet stance



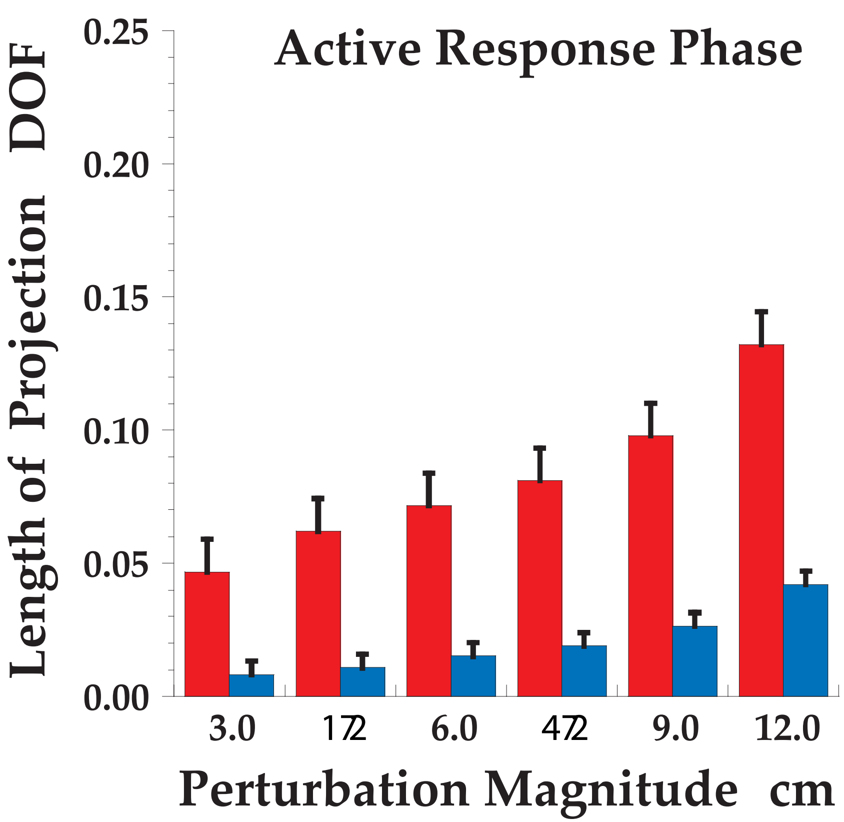
[Scholz, Schöner, Hsu, Jeka, Horak, Martin. *Exp Brain Res* (2007)]

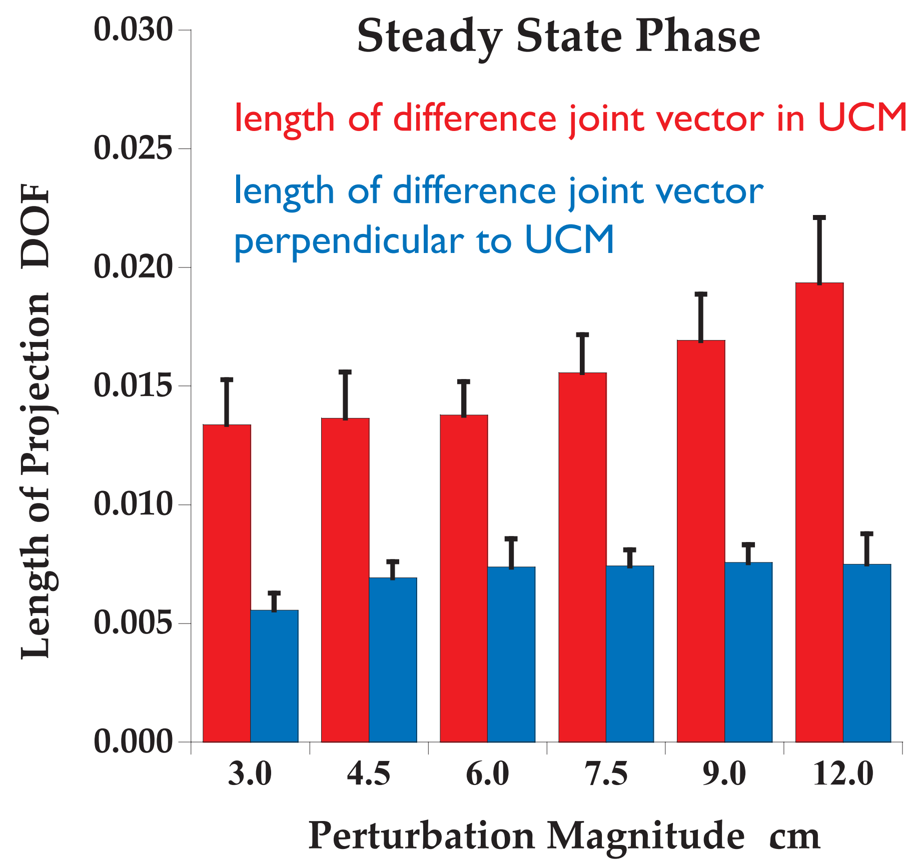
Motor equivalence in quiet stance



[Scholz, Schöner, Hsu, Jeka, Horak, Martin. *Exp Brain Res* (2007)]

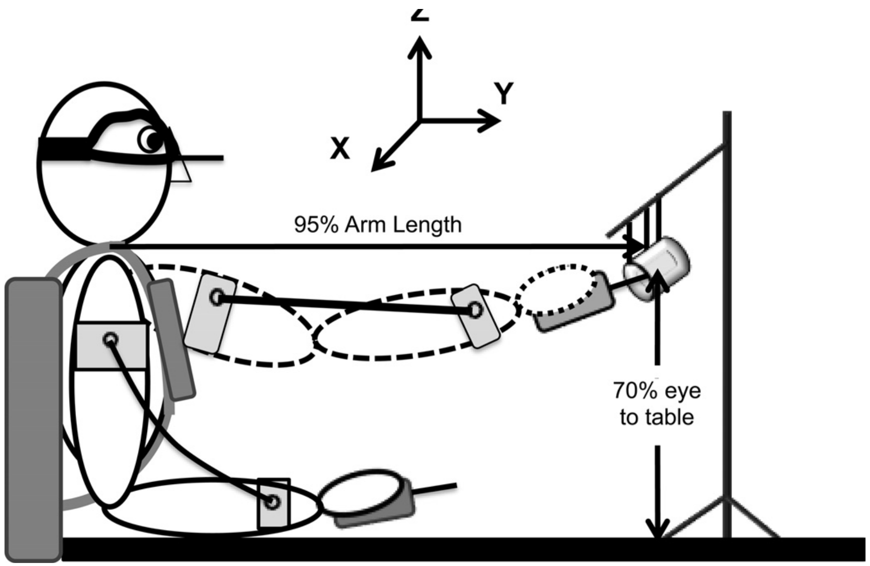
Motor equivalence in quiet stance

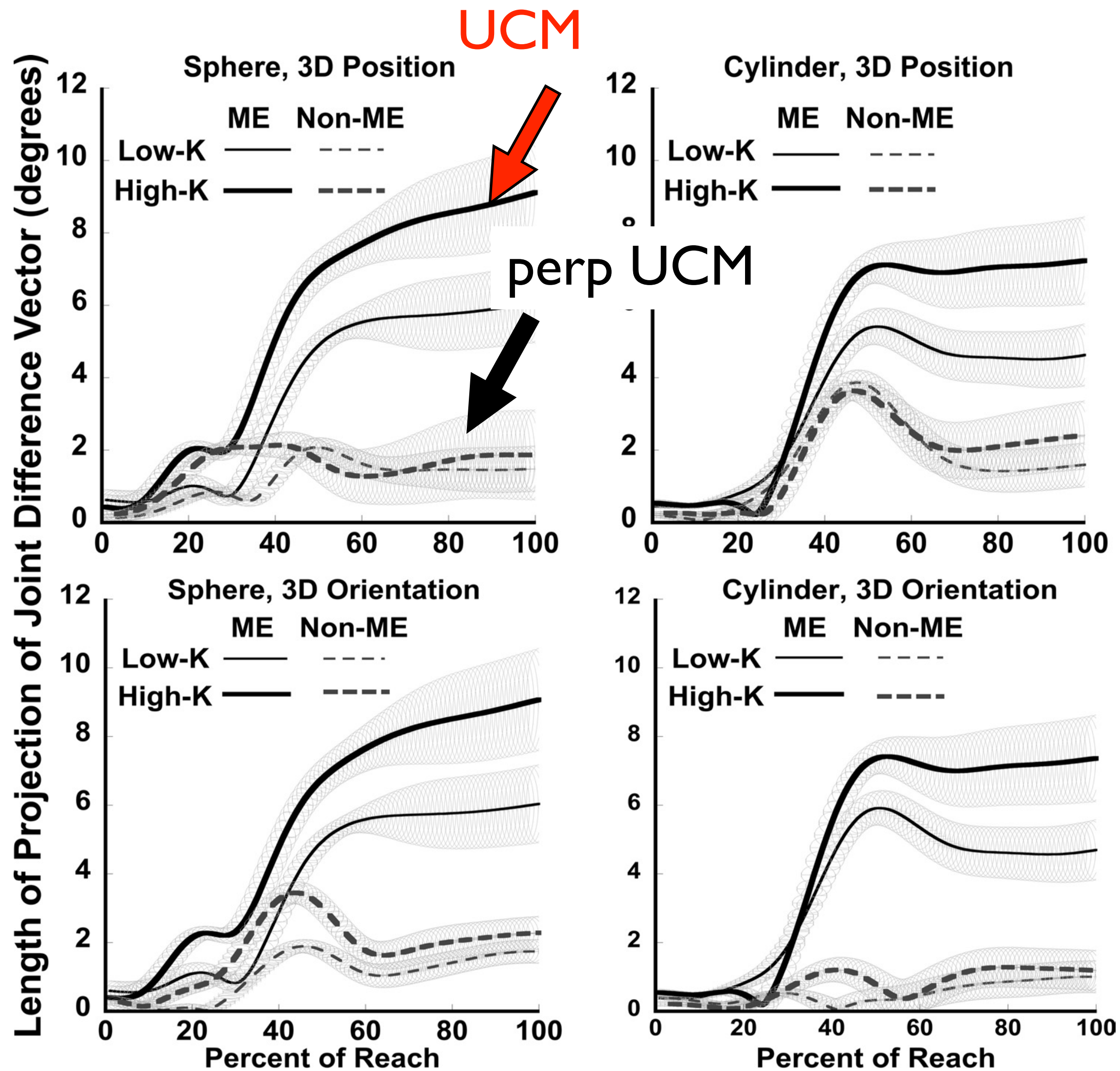




[Scholz, Schöner, Hsu, Jeka, Horak, Martin. *Exp Brain Res* (2007)]

Motor equivalence in reaching





[D. J. S. Mattos, M. L. Latash, E. Park, J. Kuhl, J. P. Scholz J Neurophysiol 106:1424 (2011)]

Model of UCM with back-coupling

decoupling

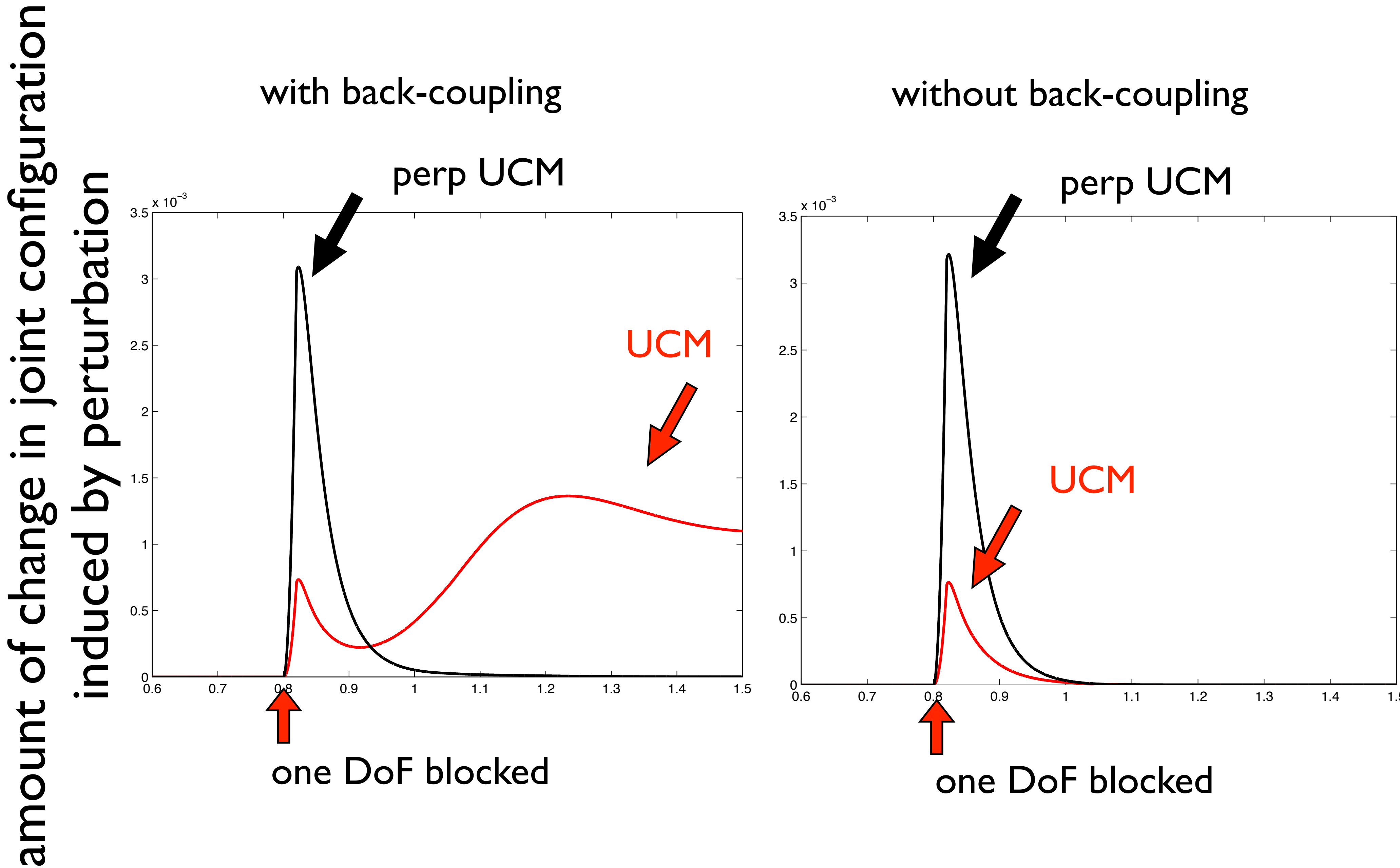
end-effector virtual velocity

$$\dot{\boldsymbol{\lambda}} = \begin{pmatrix} \mathbf{J} \\ \mathbf{E}^T \end{pmatrix}^{-1} \cdot \begin{pmatrix} \mathbf{v} \\ \mathbf{s} \end{pmatrix} = \begin{pmatrix} \mathbf{J}^+ & \mathbf{E} \end{pmatrix} \cdot \begin{pmatrix} \mathbf{v} \\ \mathbf{s} \end{pmatrix} = \mathbf{J}^+ \cdot \mathbf{v} + \mathbf{E} \cdot \mathbf{s}$$

back-coupling into null space

$$\dot{\mathrm{s}} = -\beta_{s1}\mathbf{E}^T \cdot (\boldsymbol{\lambda} - \boldsymbol{\theta_d}) - \beta_{s2}\mathbf{E}^T \cdot (\dot{\boldsymbol{\lambda}} - \dot{\boldsymbol{\theta}}_d).$$

Motor equivalence: model



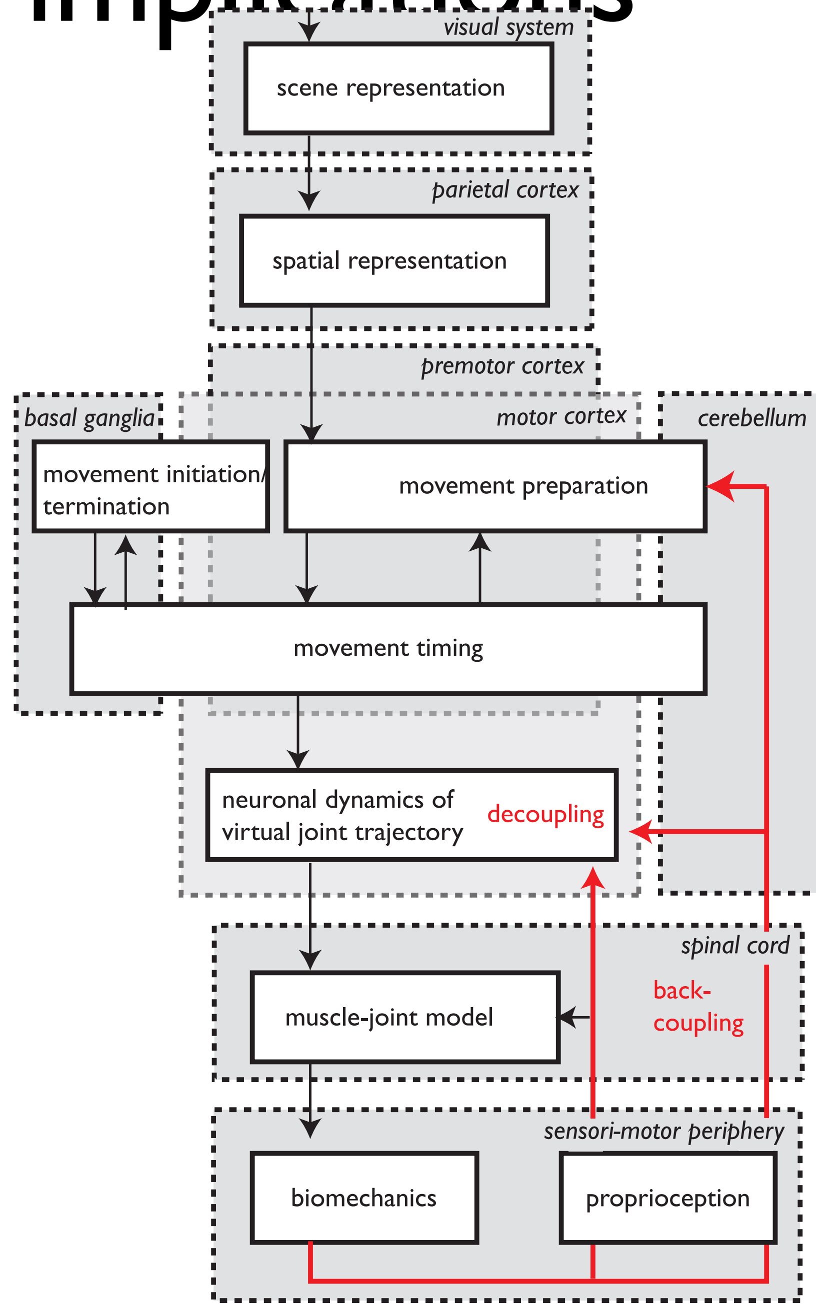
[Martin, Scholz, Schöner, unpublished]

Motor equivalence: implications

- UCM structure of variance does not necessarily predict Motor Equivalence: a model that accounts for UCM variance does not predict Motor Equivalence
- But the mechanism that is critical for ME, back-coupling, also contributes to UCM variance.

Motor equivalence: implications

- back-coupling reflects that movement plans are in a loop, in which they "yield" to sensory information about the periphery
- => we need a better understanding of back-coupling

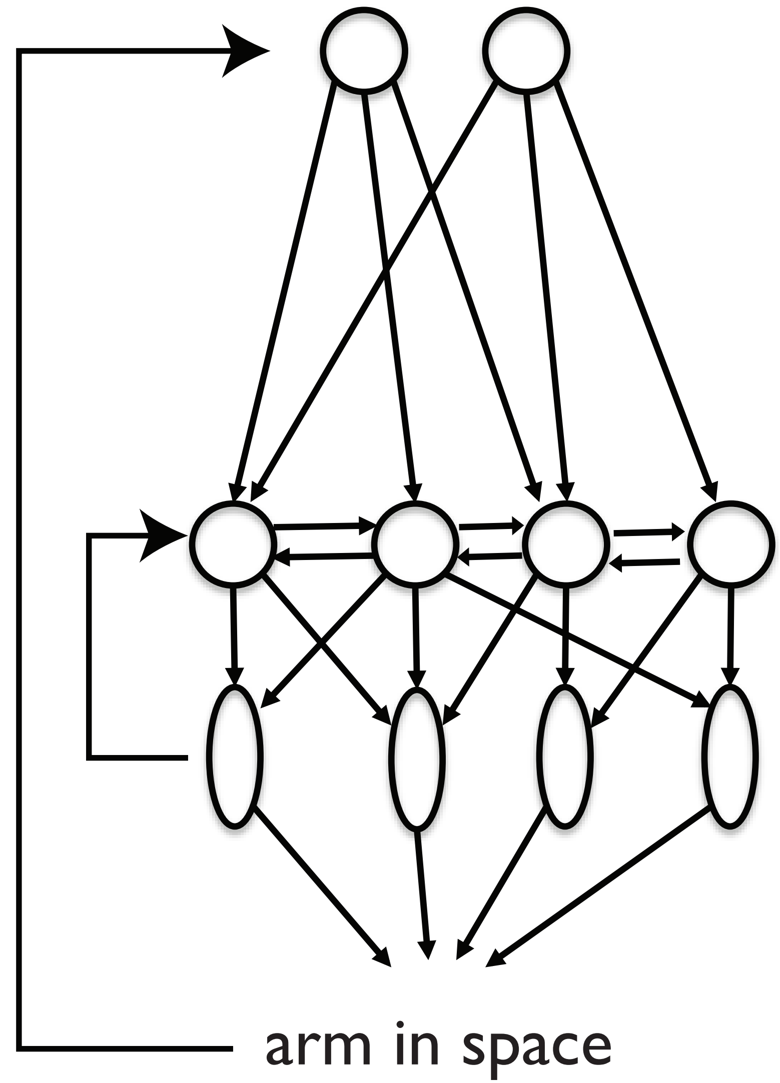


Conclusions

- Synergy has two aspects:
  - descending neural organization induces co-variation
  - recurrent coupling induces UCM structure
- these are caused by two different portions of a neural network
  - the feed-forward projection from motor command to DoF
  - and recurrent connections and/or feedback to the motor command level
- Back-coupling
  - a new hypothesis that goes beyond UCM and synergy

Account for both/all

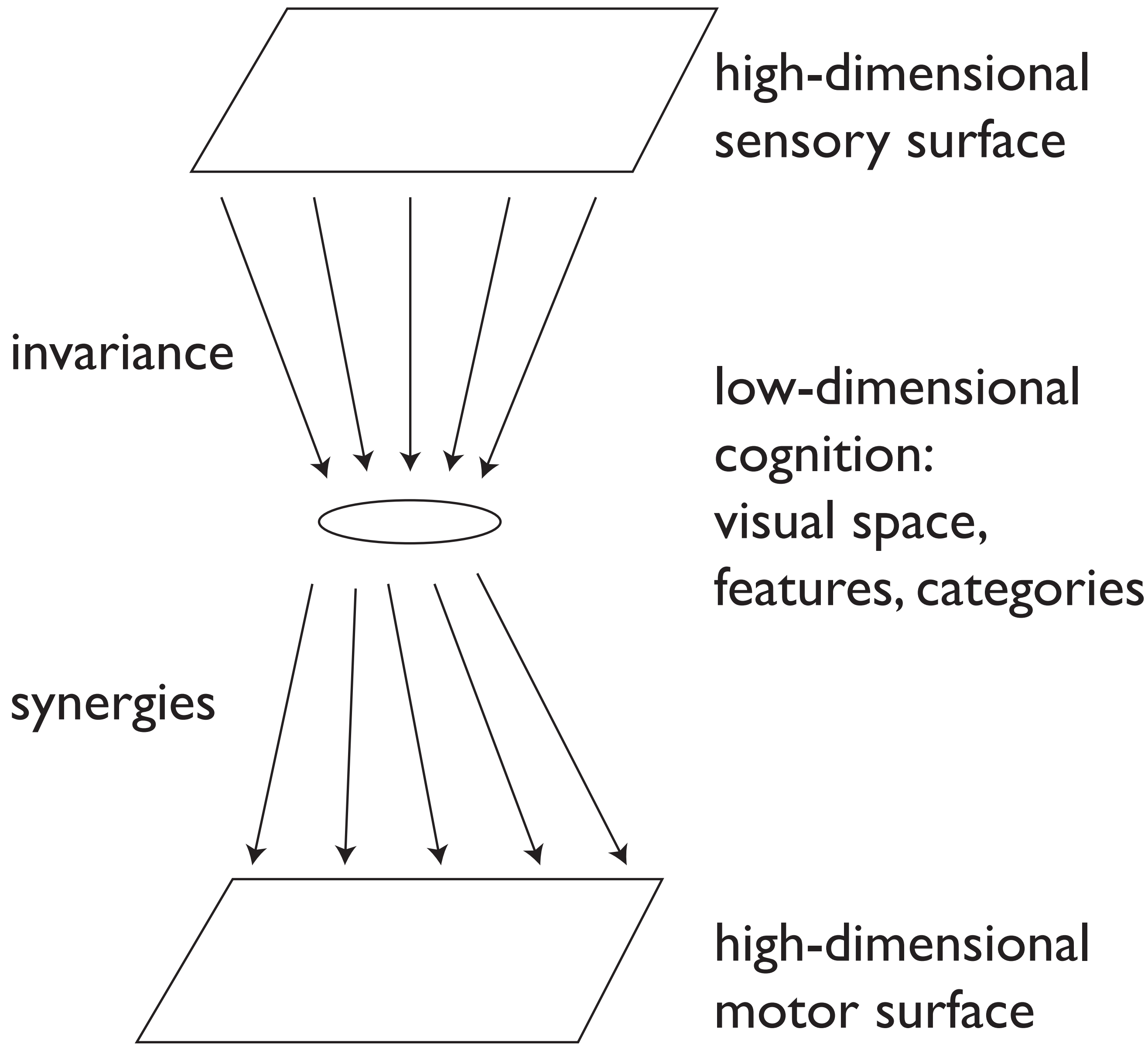
- forward projection plus
  - external or
  - internal feedback loop
  - back-coupling
- accounts for
  - structure of variance
  - self-motion
  - motor equivalence



[Reimann, Schöner, submitted]

Conclusion

- the "hour-glass" metaphor



Principles