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June 24, 2016

Exercise 6 – Differential kinematics

In the lecture we saw how differential inverse kinematics can be used to generate movements by transforming Cartesian velocities $\dot{\vec{x}}$ into joint velocities $\dot{\vec{\theta}}$ and then applying these. This is useful because oftentimes tasks are defined in the Cartesian space but robotic arms are operated in joint space. The mapping between these two spaces is given by the kinematics of the robot.

1. Make a sketch of the planar three revolute joint arm, with the forward kinematics:

$$\vec{x}(\vec{\theta}) = \begin{pmatrix} l_1 cos(\theta_1) + l_2 cos(\theta_1 + \theta_2) + l_3 cos(\theta_1 + \theta_2 + \theta_3) \\ l_1 sin(\theta_1) + l_2 sin(\theta_1 + \theta_2) + l_3 sin(\theta_1 + \theta_2 + \theta_3) \end{pmatrix}$$

   where x is the position of the end-effector, l_i are the lengths of the links and θ_i the joint angles are defined such that all links are colinear if $\vec{\theta} = 0$.

2. Make a sketch of the configuration at $\vec{\theta}_a = \begin{pmatrix} 0 \\ \pi/2 \\ \pi \end{pmatrix}$ for links with link lengths $l_i = 1 \; \forall i$.

3. Calculate the corresponding Jacobian matrix J in the configuration θ_a and for the same unit link lengths.

4. Thinking of the differential kinematic relation $\dot{\vec{x}} = J\dot{\vec{\theta}}$, show formally how the joint velocities $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ affect the Cartesian velocities of the end-effector at θ_a. Explain your results in full sentences.