Exercise 4

1. Consider this simplified dynamical node without self-excitation (a linear dynamical system):

\[ \tau \dot{u} = -u + h + S(t), \]

where \( u \) is the dynamical variable, \( h < 0 \) is a parameter for the resting level, \( \tau \) is a time constant, and \( S(t) \) is a potentially time-varying input function.

(a) Make a plot of the dynamics and mark its fixed points, assuming an input \( S \) that is constant over time. (Hint: It is not necessary to determine the fixed points analytically.)

(b) Consider an input function that changes stepwise from 0 to a positive value, remains at that value for a while and then changes back to 0. Plot the input function \( S(t) \) as well as the activation \( u(t) \) of the node over time.

2. Now consider a dynamical node with self-excitation:

\[ \tau \dot{u} = -u + h + S(t) + c\sigma(u), \]

where \( \sigma(u) \) is the sigmoided output of the node and the constant \( c \geq 0 \) determines the strength of the self-excitation.

(a) Plot a generic sigmoid function, \( \sigma(u) \).

(b) Make three plots of the dynamics for different (constant) values of the input \( S \) and mark the fixed points in each plot. For each case, choose a current value of \( u \), mark it in the plot, and mark whether the node is currently active or inactive. (Hint: The three plots should show different numbers and positions of fixed points.)

(c) Now consider the time-varying, step-wise input function from Task 1b. Assume a large value of \( c \geq 0 \). Make a plot of the input function \( S(t) \) as well as the activation \( u(t) \) and the output \( \sigma(u) \) of the node over time. Describe what this node represents.

3. Verbally describe the concepts of intention and condition of satisfaction for the case of a movement behavior, for instance moving your hand to an object.

Another hint: I like clear plots whose axes are all marked. Use different colors where it helps. If you assume specific values for any variables, write them down next to the plot.