Exercise 4

1. Consider this simplified dynamical node without self-excitation (a linear dynamical system):

$$\tau \dot{u} = -u + h + S(t),$$

where u is the dynamical variable, h < 0 is a parameter for the resting level, τ is a time constant, and S(t) is a potentially time-varying input function.

- (a) Make a plot of the dynamics and mark its fixed points, assuming an input S that is constant over time. (*Hint:* It is not necessary to determine the fixed points analytically.)
- (b) Consider an input function that changes stepwise from 0 to a positive value, remains at that value for a while and then changes back to 0. Plot the input function S(t) as well as the activation u(t) of the node over time.
- 2. Now consider a dynamical node with self-excitation:

$$\tau \dot{u} = -u + h + S(t) + c\sigma(u),$$

where $\sigma(u)$ is the sigmoided output of the node and the constant $c \ge 0$ determines the strength of the self-excitation.

- (a) Plot a generic sigmoid function, $\sigma(u)$.
- (b) Make three plots of the dynamics for different (constant) values of the input S and mark the fixed points in each plot. For each case, choose a current value of u, mark it in the plot, and mark whether the node is currently active or inactive. (*Hint:* The three plots should show different numbers and positions of fixed points.)
- (c) Now consider the time-varying, step-wise input function from Task 1b. Assume a large value of $c \ge 0$. Make a plot of the input function S(t) as well as the activation u(t) and the output $\sigma(u)$ of the node over time. Describe what this node represents.
- 3. Verbally describe the concepts of *intention* and *condition of satisfaction* for the case of a movement behavior, for instance moving your hand to an object.

Another hint: I like clear plots whose axes are all marked. Use different colors where it helps. If you assume specific values for any variables, write them down next to the plot.