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Exercise 1

1. In the lecture we saw how to generate movement by generating a time course of vehicle heading, $\phi(t)$ from a dynamical system defined over ϕ. The contribution of a single obstacle to this dynamics is given by

$$\dot{\phi} = \alpha(\phi - \psi) \exp\left[-\frac{(\phi-\psi)^2}{2\sigma^2}\right]$$

where ψ is the direction in which an obstacle lies.

   (a) Plot the first factor and describe the geometrical meaning of the two parameters, ψ and α.

   (b) Plot the second factor and describe the geometrical meaning of the two parameters, ψ and σ.

   (c) Plot the product. Is the slope of the dynamics at $\phi = \psi$ affected by the second factor? Why or why not?

   (d) Plot the time course of heading direction that results from this dynamics when the initial heading direction, $\phi(0)$ is (a) $< \psi$, (b) $> \psi$, (c) $= \psi$. These plots are qualitative based on your mental "simulation" of the dynamics.

   (e) Plot the same time courses when α is larger.

   (f) State what happens when the initial heading, $\phi(0)$ is far from ψ: $|\phi(0) - \psi| >> \sigma$

2. The low-level variant of obstacle avoidance uses force-lets like the one above for every sensor. A vehicle with five distance sensors has five contribution ($i = 1, ..., 5$) of this kind:

$$f_i(\phi) = \lambda(d_i)(\phi - \psi_i) \exp\left[-\frac{(\phi-\psi_i)^2}{2\sigma(d_i)}\right].$$

These repel from the heading directions, $\psi_i = \phi + \theta_i$, where θ_i is the angle at which the sensor is mounted on the vehicle (counted from vehicle's "nose" from which heading direction, ϕ, is also counted). The strength of repulsion is a decreasing function of the sensed distance, d_i, at each sensor:

$$\lambda(d_i) = \beta_1 \exp\left[-\frac{d_i}{\beta_2}\right]$$

where β_1 and β_2 are parameters. The range of repulsion corrects the sensor sensitivity cone, $\Delta\theta_i$, for the angle subtended by the vehicle of radius, R_{robot}, at the sensed distance:

$$\sigma(d_i) = \arctan\left[\tan\left(\frac{\Delta\theta}{2}\right) + \frac{R_{\text{robot}}}{R_{\text{robot}} + d_i}\right].$$

(a) Make a drawing of the robot and mark angles ϕ, θ_i, and ψ_i (for one sensor, i, only).

(b) Make a qualitative drawing of the force-let at two difference distances from an obstacle.

(c) Draw the superposition of two such force-lets when they are metrically close (overlap) and when they are metrically far (overlap little). Discuss the difference.

(d) Convince yourself that the contributions of two such obstacles sensors can go through a bifurcation from having an attractor between the two sensor directions, ψ_i and ψ_{i+1} to having a repellor there as the distance between the vehicle and a wall decreases. Use drawings and discussion to make the point.