

**Exercise 2, November 5, 2015, to be handed in November 12!**

1. Plot the function  $x(t) = x_0 \exp(-t/\tau)$  for one positive and for one negative value of  $x_0$ . You can use specific values for both  $\tau$  and  $x_0$  to compute a few points on the curve and then interpolate, or you can use your understanding of the curve to make a qualitative plot. In either case, mark the time axis with actual units (e.g., seconds) and choose a value for  $\tau$  in those units. Mark the levels of  $x_0$ .
2. Compute the values of that function  $x(t)$  for  $t = 0, t = \tau, t = 2\tau, t = 3\tau$ . You can do this for the abstract parameter,  $x_0$ , or by inserting an actual numerical value for  $x_0$ , whichever you find more intuitive. Now write down in a table the ratios:  $x(\tau)/x(0); x(2\tau)/x(\tau); x(3\tau)/x(2\tau)$ . State what you notice about those ratios. Interpret that observation by a kind of "theorem" that you can state about the "exponential decay" function  $\exp(-t/\tau)$
3. Do another plot of the function  $x(t) = x_0 \exp(-t/\tau)$ , but now for two values of  $\tau$ . State what the significance of the parameter  $\tau$  may be given how the functions vary with  $\tau$ .
4. Compute the derivative  $\dot{x}(t)$  of this same function,  $x(t) = x_0 \exp(-t/\tau)$ . (If you don't know how to do that, look it up in a book or on the internet). Plot that derivative in the same way for just one value of  $x_0$ . Make sure the time axis has the same units as in the plots above, so you can compare across the plots of  $x$  and  $\dot{x}$ . Form pairs of values for  $x(t_i)$  and  $\dot{x}(t_i)$  for a few moments in time,  $t_i$ , (where  $i$  is 1, 2, 3, etc.) and plot these values against each other:  $\dot{x}$  against  $x$ . State what functional relationship you are guessing. Check that by computing analytically the ratio  $\dot{x}(t)/x(t)$  and write down the differential equation that now follows.