Dynamical systems tutorial

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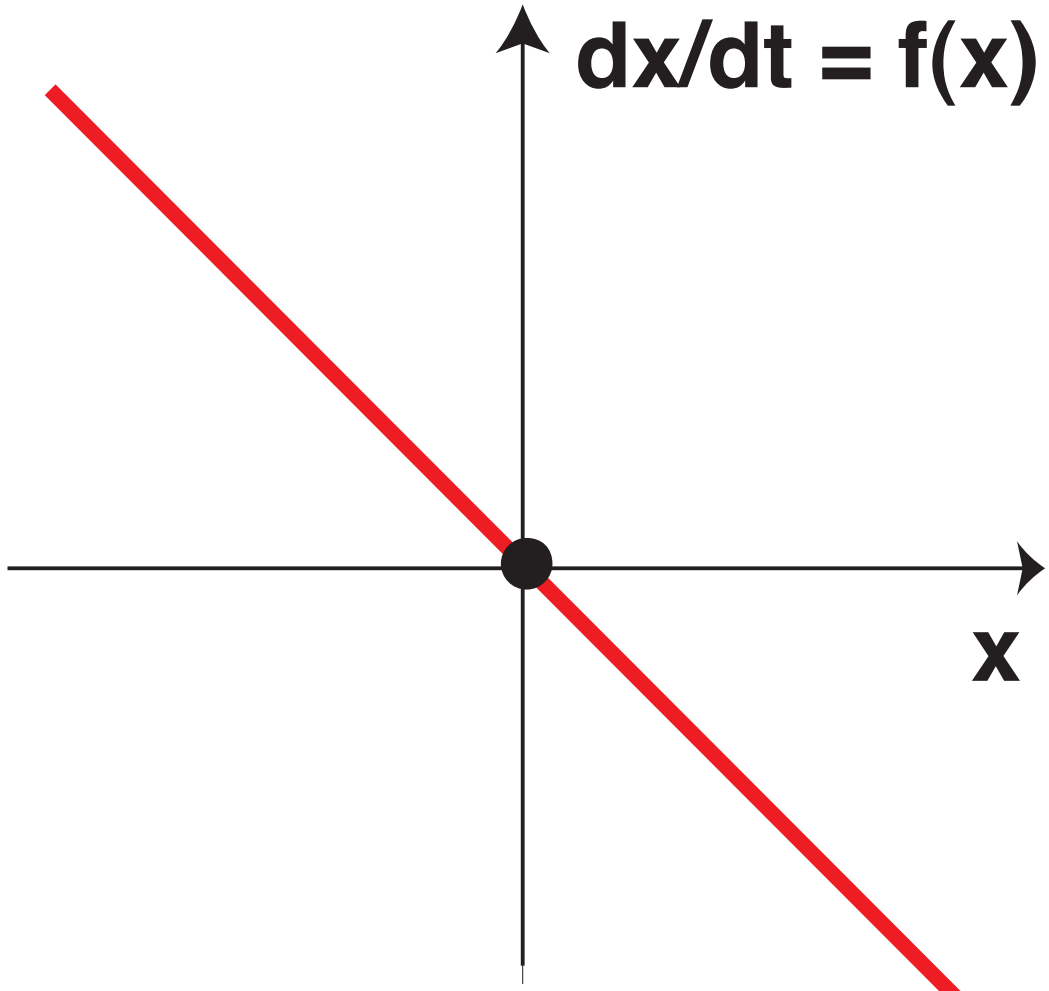
Dynamical systems: Tutorial

- the word "dynamics"
  - time-varying measures
  - range of a quantity
  - forces causing/accounting for movement => dynamical systems
- dynamical systems are the universal language of science
  - physics, engineering, chemistry, theoretical biology, economics, quantitative sociology, ...

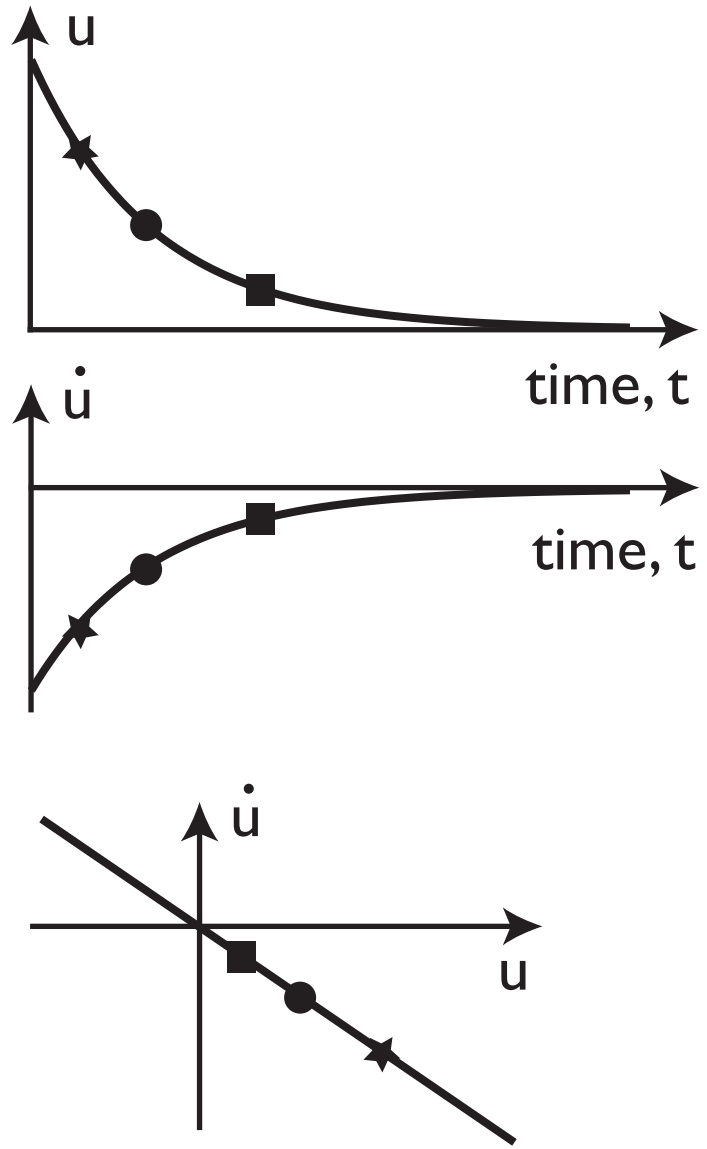
time-variation and rate of change

- variable x(t);
- variable as function of time x(t)
- rate of change dx/dt

dynamical system

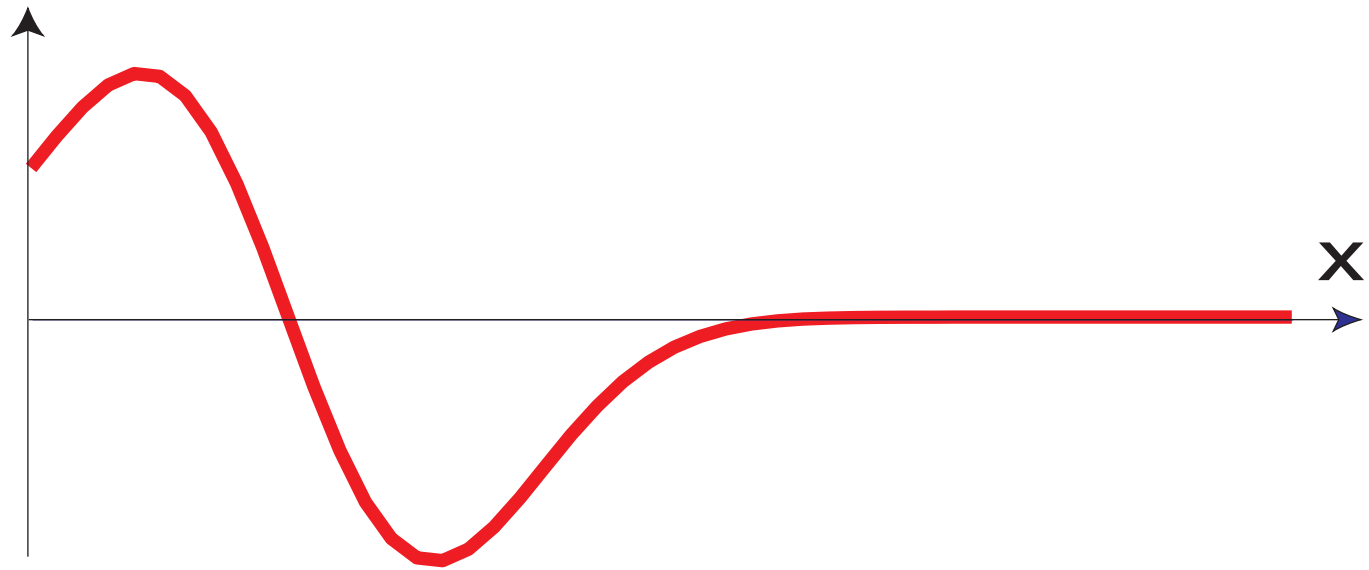


dynamical system: relationship between a variable and its rate of change



dynamical system: nonlinear

dx/dt=f(x)



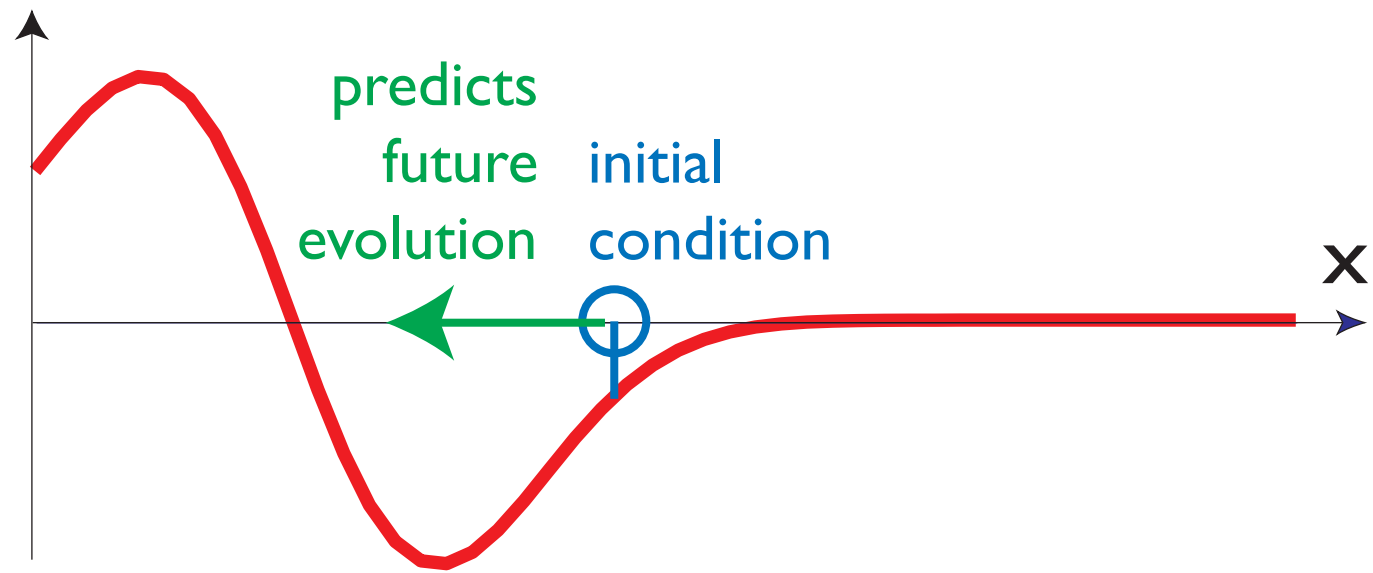
notions

- variable, equation, solution
- function, functional equation, solution

dynamical system

- present determines the future
  - given initial condition
  - predict evolution (or predict the past)

$$dx/dt=f(x)$$



dynamical systems

- x: spans the state space (or phase space)
- f(x): is the "dynamics" of x (or vector-field)
- x(t) is a solution of the dynamical systems to the initial condition x_0
  - if its rate of change = f(x)
  - and x(0)=x_0

notions

- simple examples of differential equations
- and their solutions

other functional equations

- delayed (functional) differential equations
- partial differential equations
- integro-differential equations

numerics

- sample time discretely
- compute solution by iterating through time

$$\dot{x} = f(x)$$

$$t_i = i * \Delta t; \qquad x_i = x(t_i)$$

$$\dot{x} = \frac{dx}{dt} \approx \frac{\Delta x}{\Delta t} = \frac{x_{i+1} - x_i}{\Delta t}$$

$$x_{i+1} = x_i + \Delta t * f(x_i)$$

[forward Euler]

linear dynamics

- => simulation

outlook

- fixed points, stability, attractors
- instabilities
- inverse dynamics