Dynamical systems tutorial

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Dynamical systems: Tutorial

- the word “dynamics”
  - time-varying measures
  - range of a quantity
  - forces causing/accounting for movement => dynamical systems

- dynamical systems are the universal language of science
  - physics, engineering, chemistry, theoretical biology, economics, quantitative sociology, ...
time-variation and rate of change

- variable $x(t)$;
- variable as function of time $x(t)$
- rate of change $\frac{dx}{dt}$
dynamical system

\[
dx/dt = f(x) \]
dynamical system: relationship between a variable and its rate of change
dynamical system: nonlinear

\[ \frac{dx}{dt} = f(x) \]
notions

- variable, equation, solution
- function, functional equation, solution
dynamical system

- present determines the future
  - given initial condition
  - predict evolution (or predict the past)

\[
dx/dt = f(x)\]
dynamical systems

- $x$: spans the state space (or phase space)
- $f(x)$: is the “dynamics” of $x$ (or vector-field)
- $x(t)$ is a solution of the dynamical systems to the initial condition $x_0$
  - if its rate of change = $f(x)$
  - and $x(0) = x_0$
notions

- simple examples of differential equations
- and their solutions
other functional equations

- delayed (functional) differential equations
- partial differential equations
- integro-differential equations
Sample time discretely

Compute solution by iterating through time

\[ \dot{x} = f(x) \]

\[ t_i = i \cdot \Delta t; \quad x_i = x(t_i) \]

\[ \dot{x} = \frac{dx}{dt} \approx \frac{\Delta x}{\Delta t} = \frac{x_{i+1} - x_i}{\Delta t} \]

\[ x_{i+1} = x_i + \Delta t \cdot f(x_i) \]

[forward Euler]
linear dynamics

=> simulation
outlook

- fixed points, stability, attractors
- instabilities
- inverse dynamics