Exercise 3, November 12, 2015, to be handed in November 19.

Consider this dynamical system:

$$\dot{x} = f(x) = \alpha - x^2$$

where x is the dynamical variable and α is a parameter.

1. Compute the fixed points, x_i ($i =$ natural number) of this dynamics and determine for which values of α they exist.

2. Make a plot of the dynamics ($\dot{x}$ against x) for $\alpha < 0$ and $\alpha > 0$. To do this, do three things:

   (a) Compute the maximum of this function by solving $df/dx = 0$ and then computing the value of f at that location.

   (b) Determine the limit when $x \to \infty$

   (c) Determine the limit when $x \to -\infty$

3. Determine the stability of the fixed points by computing $df/dx(x = x_i)$ for each fixed point, x_i. Examine the sign of that slope and state which stability it implies for which fixed point and which value of α.

4. Make a bifurcation diagram in which you plot the fixed point against the bifurcation parameter, α, and mark the fixed point as stable or unstable.

5. Bonus: What if you added a linear function to this equation

$$\dot{x} = f(x) = \alpha + \beta x - x^2$$

   Make a plot. Would this be qualitatively different? Make the analogous computations as far as you can.

You can make use a textbook of dynamical systems for help with this exercise. This is the normal form of the tangent bifurcation... search for that term.