Dynamical systems tutorial: part 2

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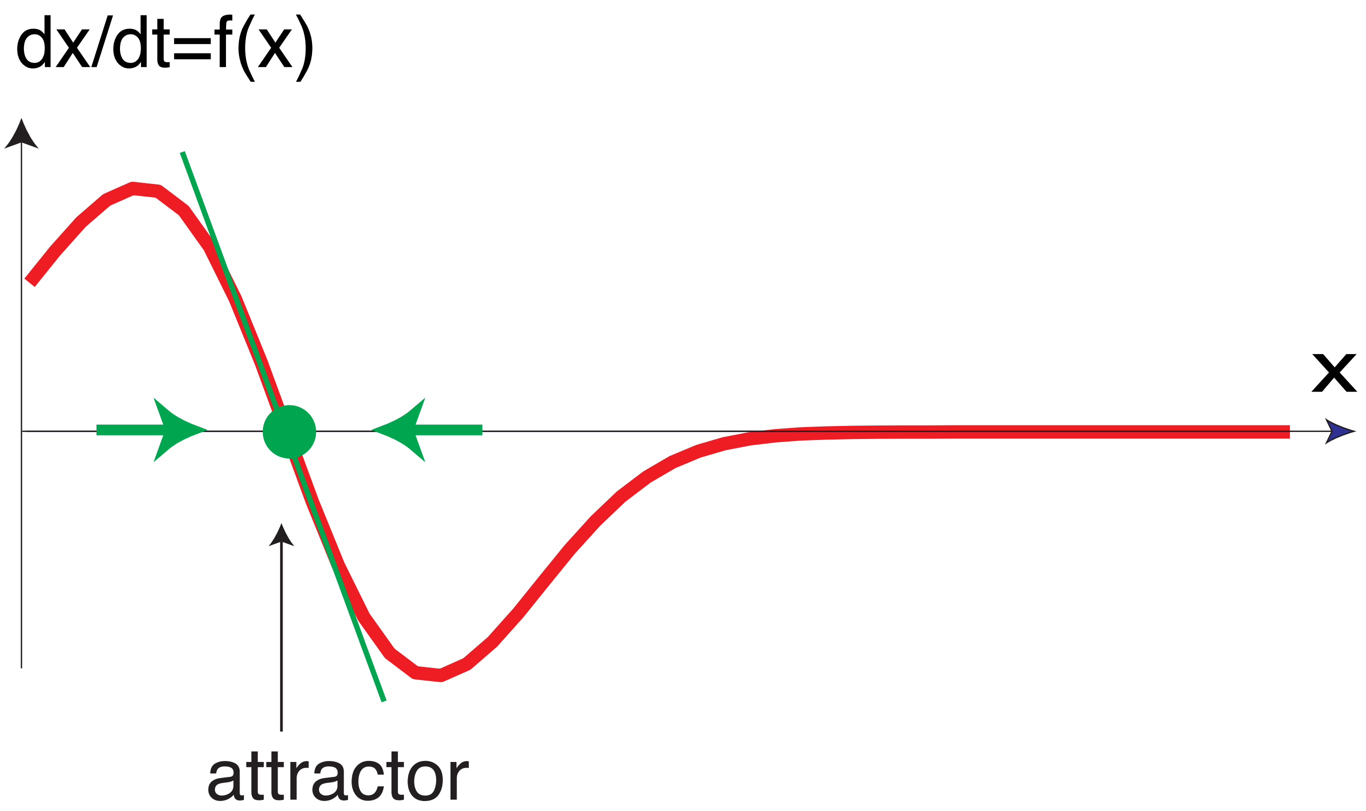
attractor

- fixed point, to which neighboring initial conditions converge = attractor



fixed point

- is a constant solution of the dynamical system

$$\dot{x} = f(x)$$

$$\dot{x} = 0 \Rightarrow f(x_0) = 0$$

stability

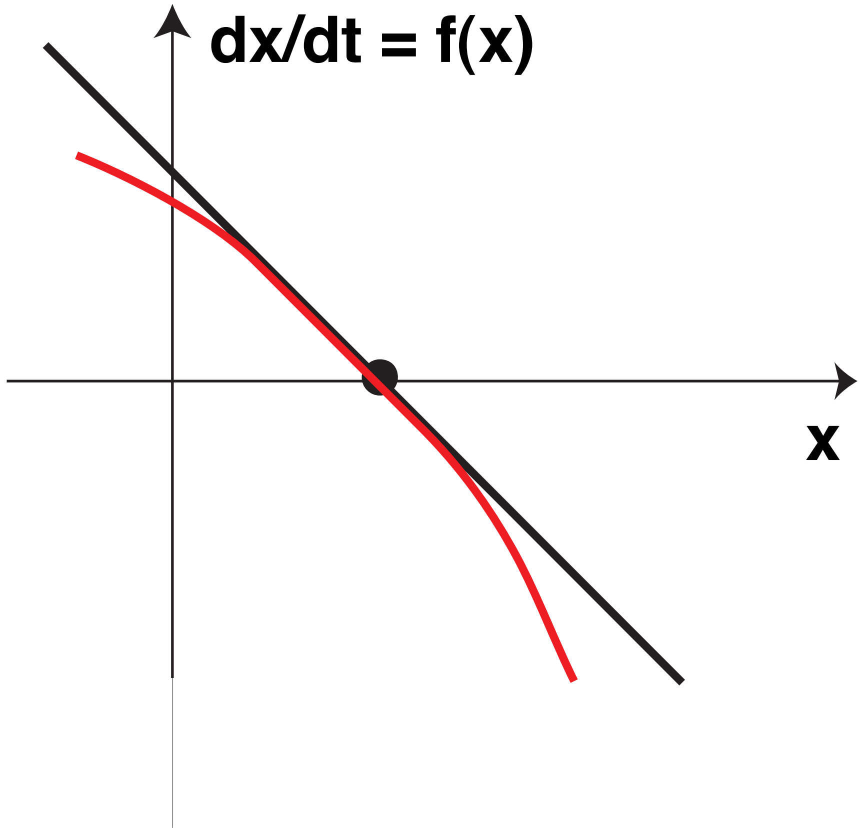
- mathematically really: asymptotic stability
- defined: a fixed point is asymptotically stable, when solutions of the dynamical system that start nearby converge in time to the fixed point

stability

- the mathematical concept of stability (which we do not use) requires only that nearby solutions stay nearby
- Definition: a fixed point is unstable if it is not stable in that more general sense,
  - that is: if nearby solutions do not necessarily stay nearby (may diverge)

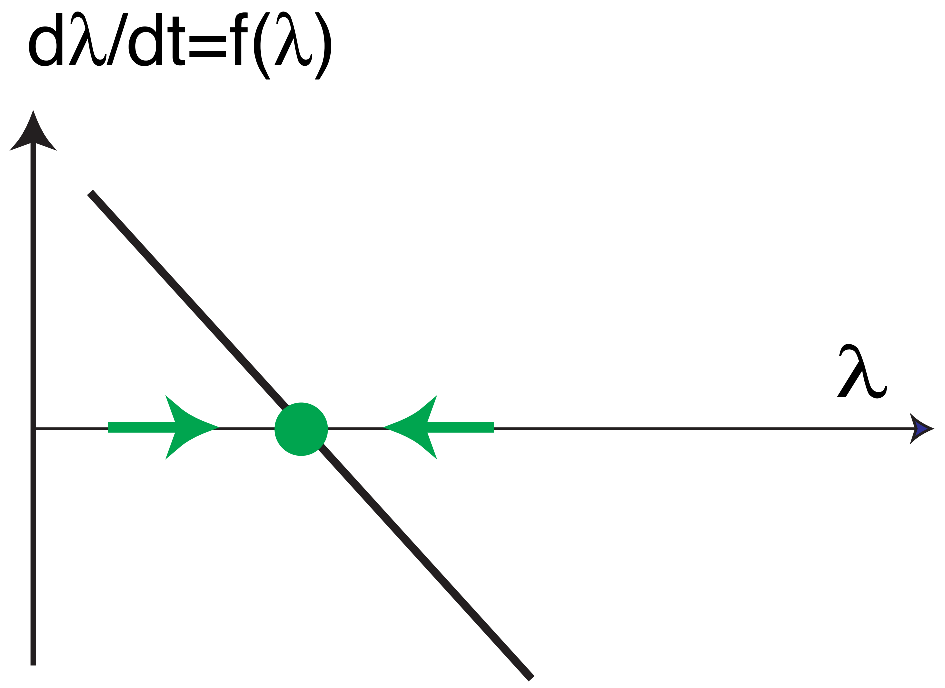
linear approximation near attractor

- non-linearity as a small perturbation/ deformation of linear system
- => non-essential non-linearity



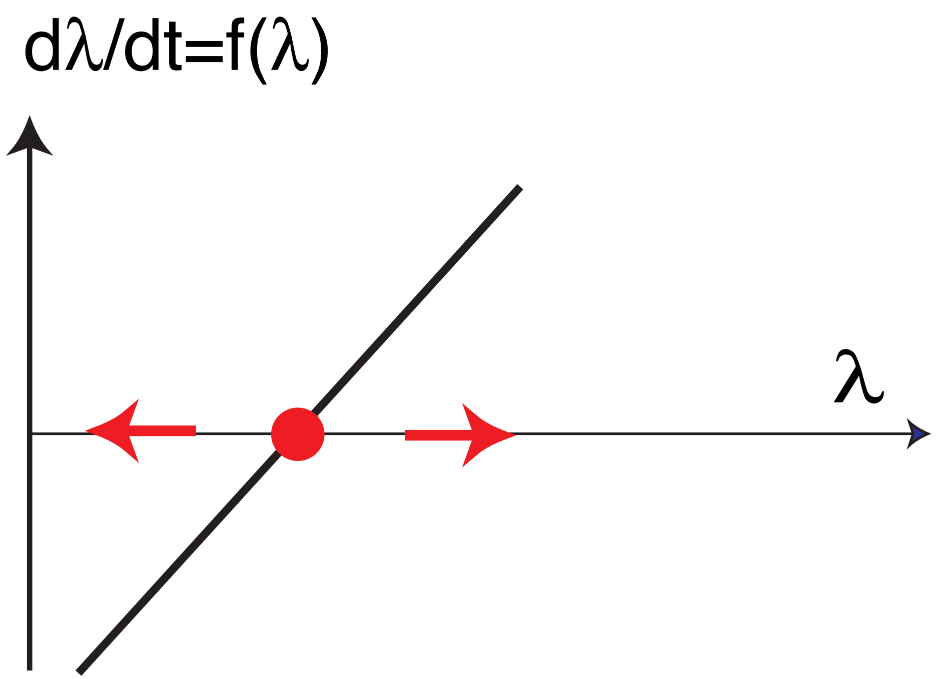
stability in a linear system

- if the slope of the linear system is negative, the fixed point is (asymptotically stable)



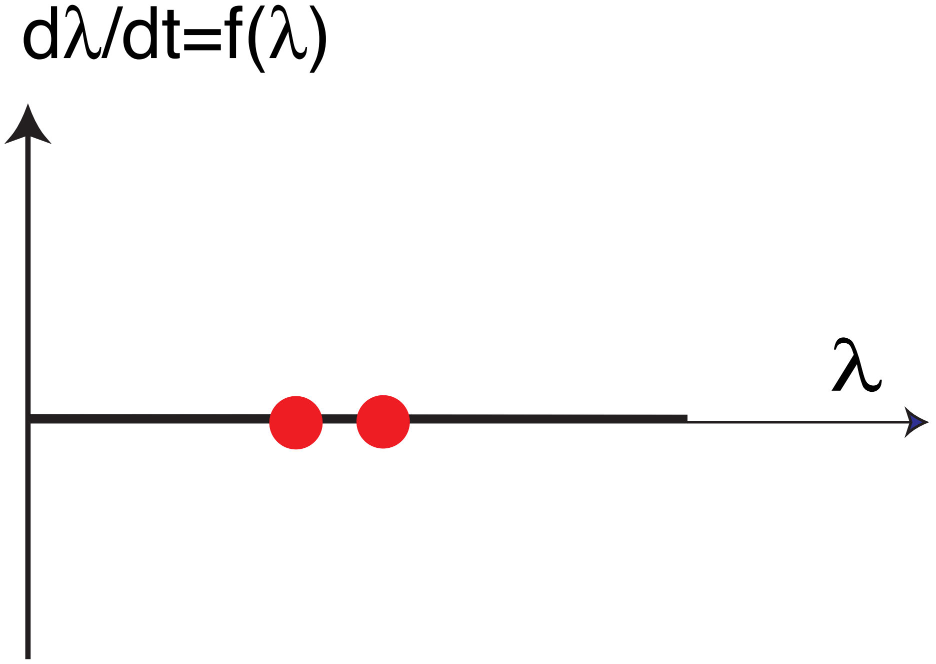
stability in a linear system

- if the slope of the linear system is positive, then the fixed point is unstable



stability in a linear system

- if the slope of the linear system is zero, then the system is indifferent (marginally stable: stable but not asymptotically stable)



stability in linear systems

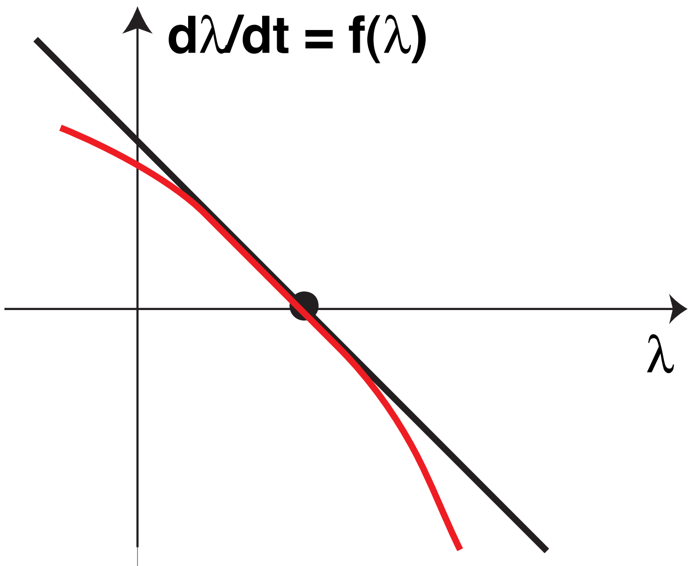
- generalization to multiple dimensions
  - if the real-parts of all Eigenvalues are negative: stable
  - if the real-part of any Eigenvalue is positive: unstable
  - if the real-part of any Eigenvalue is zero: marginally stable in that direction (stability depends on other eigenvalues)

stability in nonlinear systems

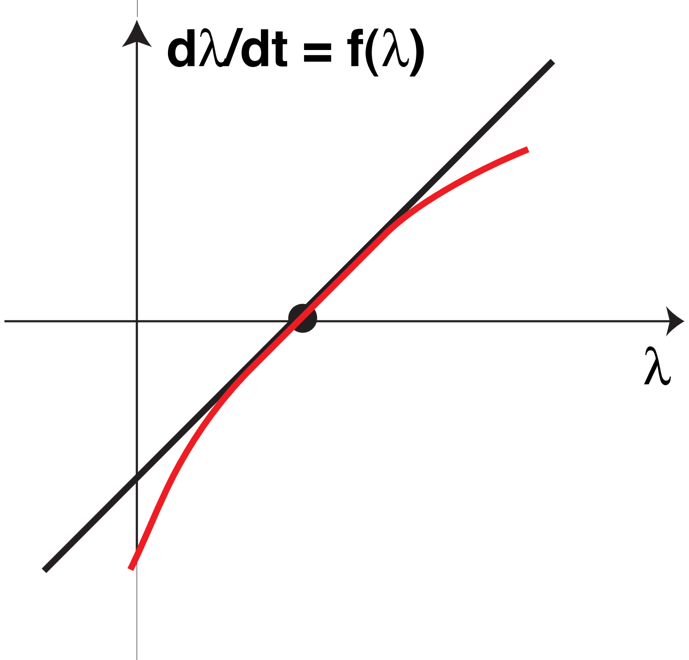
- stability is a local property of the fixed point
- => linear stability theory
  - the eigenvalues of the linearization around the fixed point determine stability
  - all real-parts negative: stable
  - any real-part positive: unstable
  - any real-part zero: undecided: now nonlinearity decides (non-hyberpolic fixed point)

stability in nonlinear systems

- all real-parts negative: stable

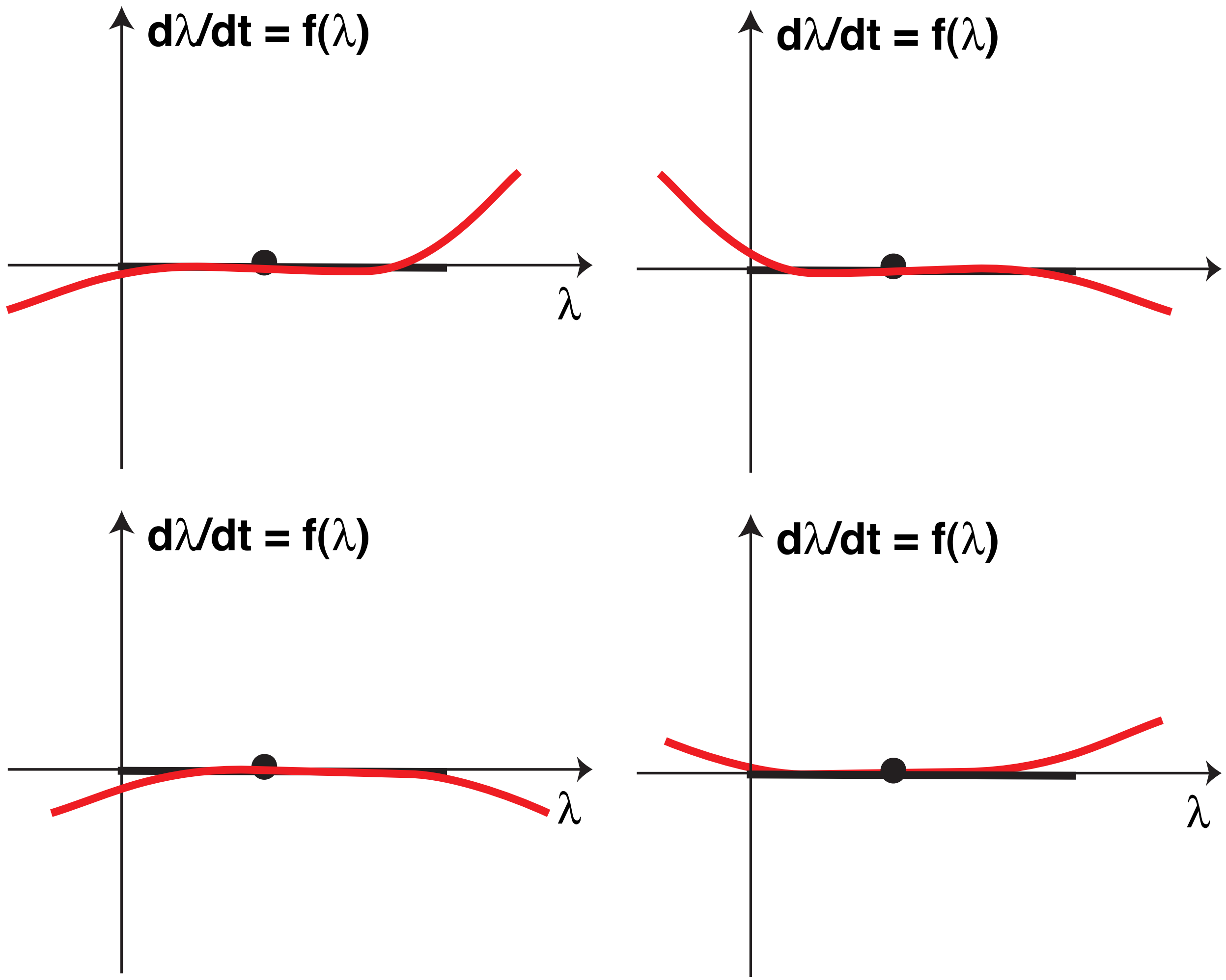


- any real-part positive: unstable



stability in nonlinear systems

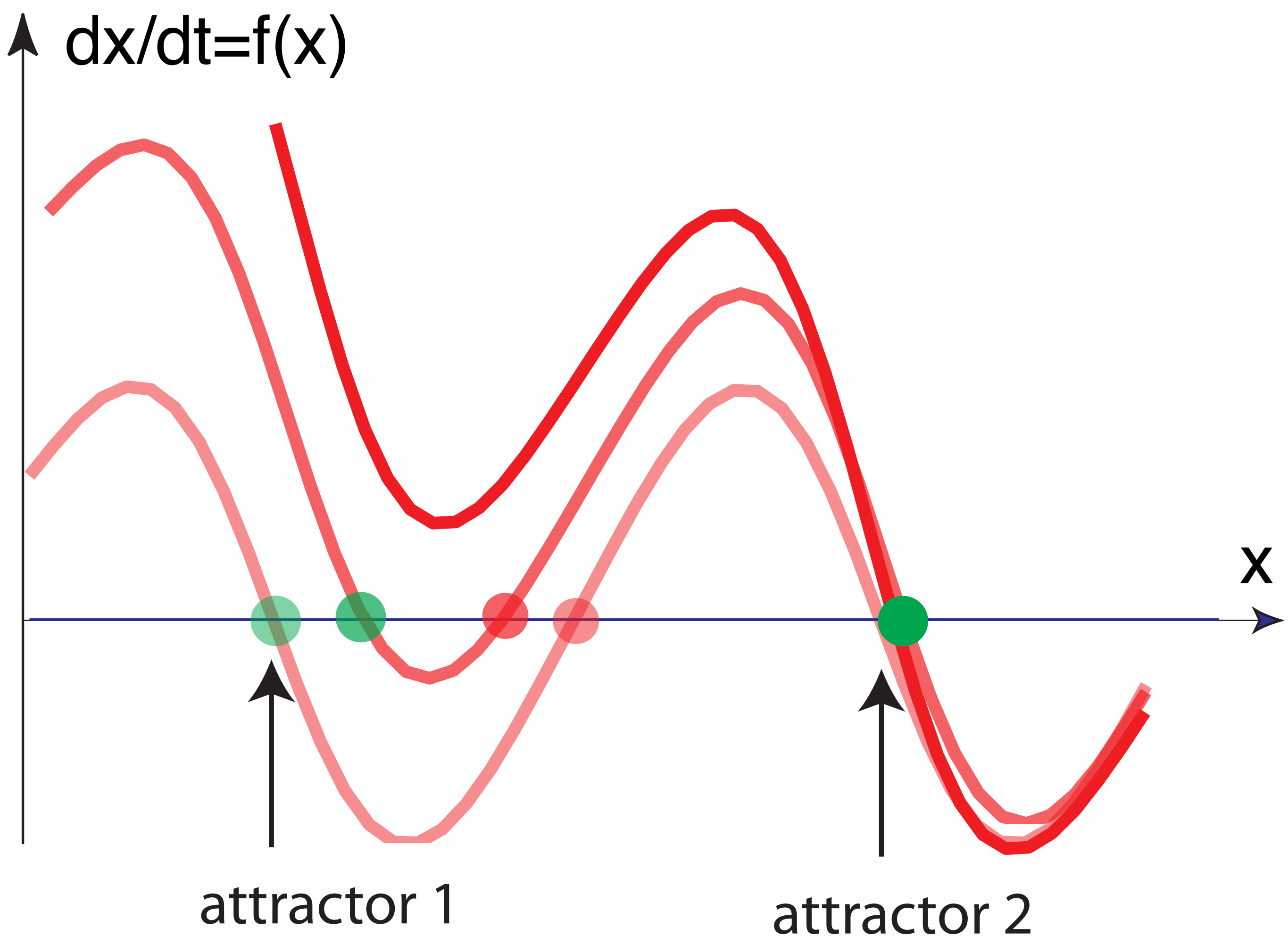
- any real-part zero: undecided: now nonlinearity decides (non-hyberpolic fixed point)



bifurcations

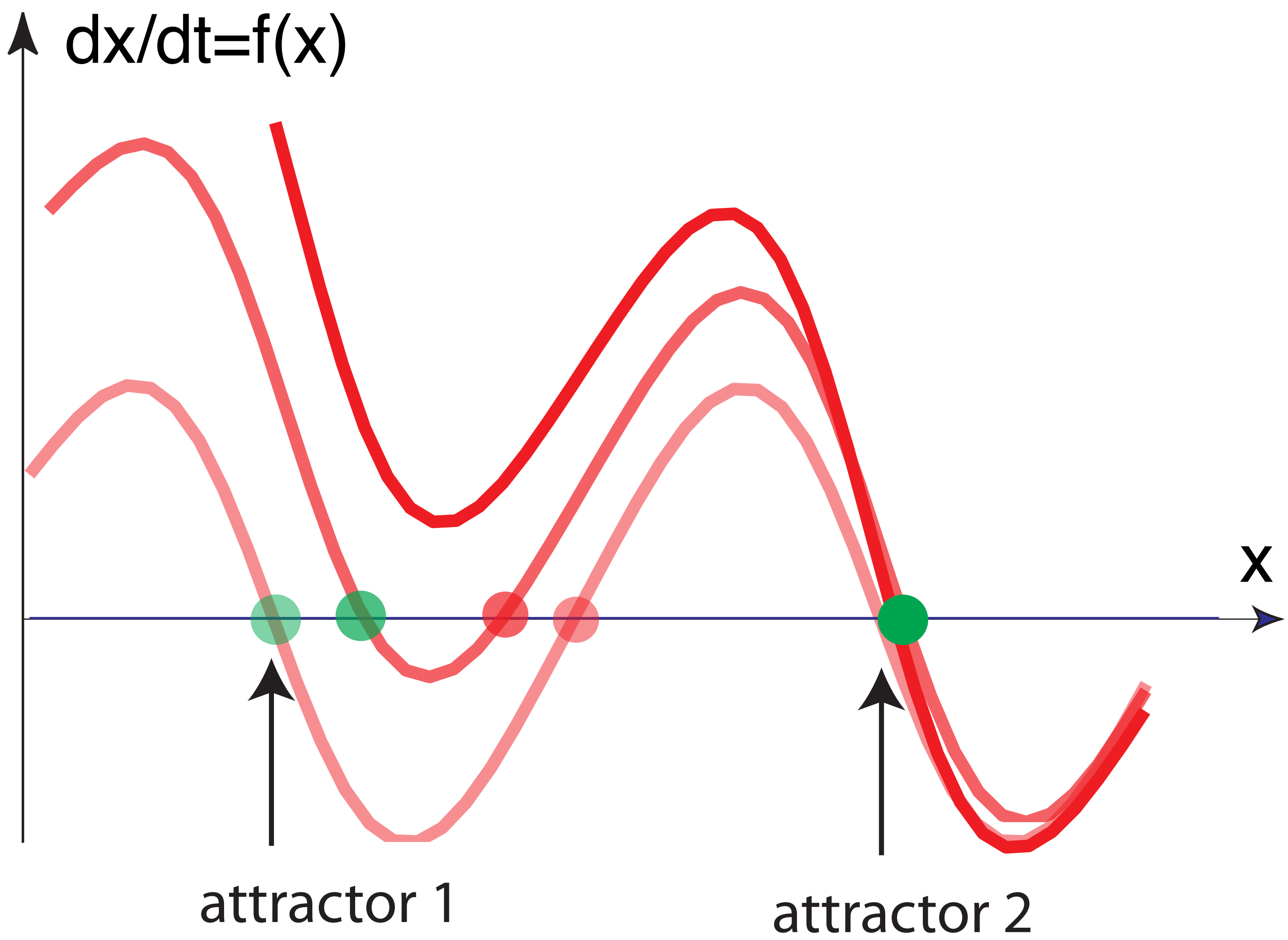
- look now at families of dynamical systems, which depend (smoothly) on parameters
- ask: as the parameters change (smoothly), how do the solutions change (smoothly?)
  - smoothly: topological equivalence of the dynamical systems at neighboring parameter values
  - bifurcation: dynamical systems NOT topological equivalent as parameter changes infinitesimally

bifurcation



bifurcation

- bifurcation=qualitative change of dynamics (change in number, nature, or stability of fixed points) as the dynamics changes smoothly



tangent bifurcation

- the simplest bifurcation (co-dimension 0): an attractor collides with a repellor and the two annihilate