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Exercise 6

The first assignment may look familiar to you; we go through this dynamics again as additional practice, and to link it to the dynamics of neural fields. Because of this, you can achieve up to 25% of the grade with the first assignment, and 75% (plus 25% bonus) for the second one.

1. A single activation variable with self-excitation is governed by the dynamic equation
$$\dot{u} = -u + h + s + g(u)$$
where $h = -1$ is the resting level, $s \geq 0$ is input and $g(\cdot)$ is a sigmoidal function that ranges from zero to one.
   (a) Assume a smooth sigmoid; make a plot of the dynamics ($\dot{u}$ vs. u) for $s = 2$, $s = 1$, and $s = 0$. [Hint: make a plot of the components, $-u + h + s$ and of the sigmoid, $g(u)$, then add the two.]
   (b) Mark the attractors of the dynamics and discuss the instabilities that occur in this dynamics as input, s, increases and decreases. (Discuss means: name them, describe which solutions of which stability fuse or split as which parameter is changed in which direction...). Plot the bifurcation diagram (plot of the fixed points against the bifucation parameter, marking the stability of each fixed point; the plot can be qualitative, the numbers above are given only to help you deal with the relative size of input and resting level).
   (c) Assume that the sigmoid function is a step function (0 for $u < 0$ and 1 for $u > 0$). Compute the fixed points of the equation. Verify the result by comparing to the figure of part (a).

2. Dynamic fields governed by
$$\dot{u}(x,t) = -u(x,t) + h + s(x,t) + \int dx' w(x - x') g(u(x'))$$
go through the detection instability, when input, $s(x,t)$ pushes activation to levels that reach above the threshold level of the sigmoidal nonlinearity. For the following, assume a field with strong global inhibition.
   (a) Plot the interaction kernel of the field as a function of the distance between field locations. Assume local excitation and global inhibition are present in the kernel.
   (b) Plot the sub-threshold activation pattern in the field when a weak, localized input, $s(x)$, is present ($s(x) + h < 0$ for all x). Mark the resting level and the maximal activation level $s(x) + h$.

(c) Plot the self-excited activation peak as an activation pattern of the field when the localized input is stronger ($s(x) + h > 0$). Include a line that indicates the resting level, $h < 0$, so that we can compare to that level. Also include a drawing of the input level itself ($s(x)$, you may also plot $s(x) + h$, which may help interpret the figure).

(d) Describe in words what the detection instability consists of (which solution becomes unstable when what parameter changes, what happens beyond the instability).

(e) Describe in the same style the reverse detection instability and explain the notion of a bistable regime (regime in what parameter space?).

bonus The dynamics of the neural field are similar to the one of the system in assignment 1. Discuss how the fixed points of the first system are related to the stable states of the neural field.