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Neural Dynamics, Exercise 3, October. 30, 2014

Consider this simple linear dynamical system:

$$\dot{u} = -\alpha u + h + S(t)$$

where u is the dynamical variable and α and $h < 0$ are two parameters and $S(t)$ is a potentially time-varying input function.

1. Make a plot of the dynamics.
2. What is the fixed point of this dynamics (assume $S = constant$) ?
3. Discuss whether the fixed point is an attractor or a repellor based on the sign of α.
4. Write down the general solution of this equation for $S(t) = constant = S_0$ and discuss its long term behavior. If you don't know, look this up in a math textbook for differential equations, it is in every book you'll find.
5. Plot the time courses of the solution for $\alpha > 0$ and $S = constant$ from a few different initial conditions.
6. Illustrate the time scale of that solution ($\alpha > 0$) by comparing the times it takes to decrease the initial distance from the stable fixed point by a factor of $e = 2,73...$ and by a factor $e^2 = 7,37....$ How does that time scale vary with α?
7. Consider input functions that change stepwise from 0 to a positive value and back. Plot the solution.
8. Bonus question: If you have Matlab, use the simulator available at
   `http://www.robotics-school.org`
   under: exercises: neural dynamics simulations
   to establish all facts you have addressed here so far.
9. Bonus question: Can you write down the general solution of the equation when input is a time-function, $S(t)$?