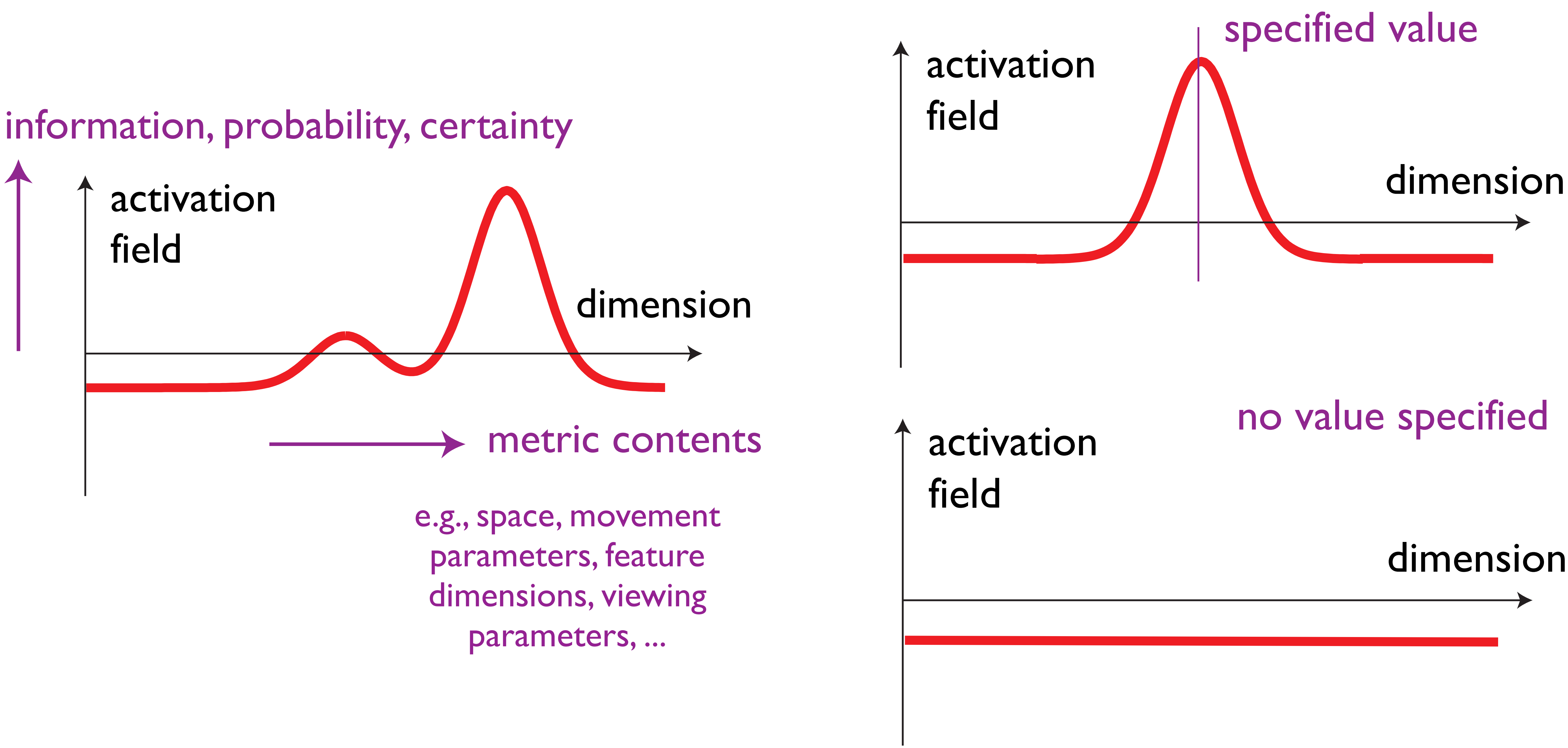
# Dynamic Field Theory

Gregor Schöner

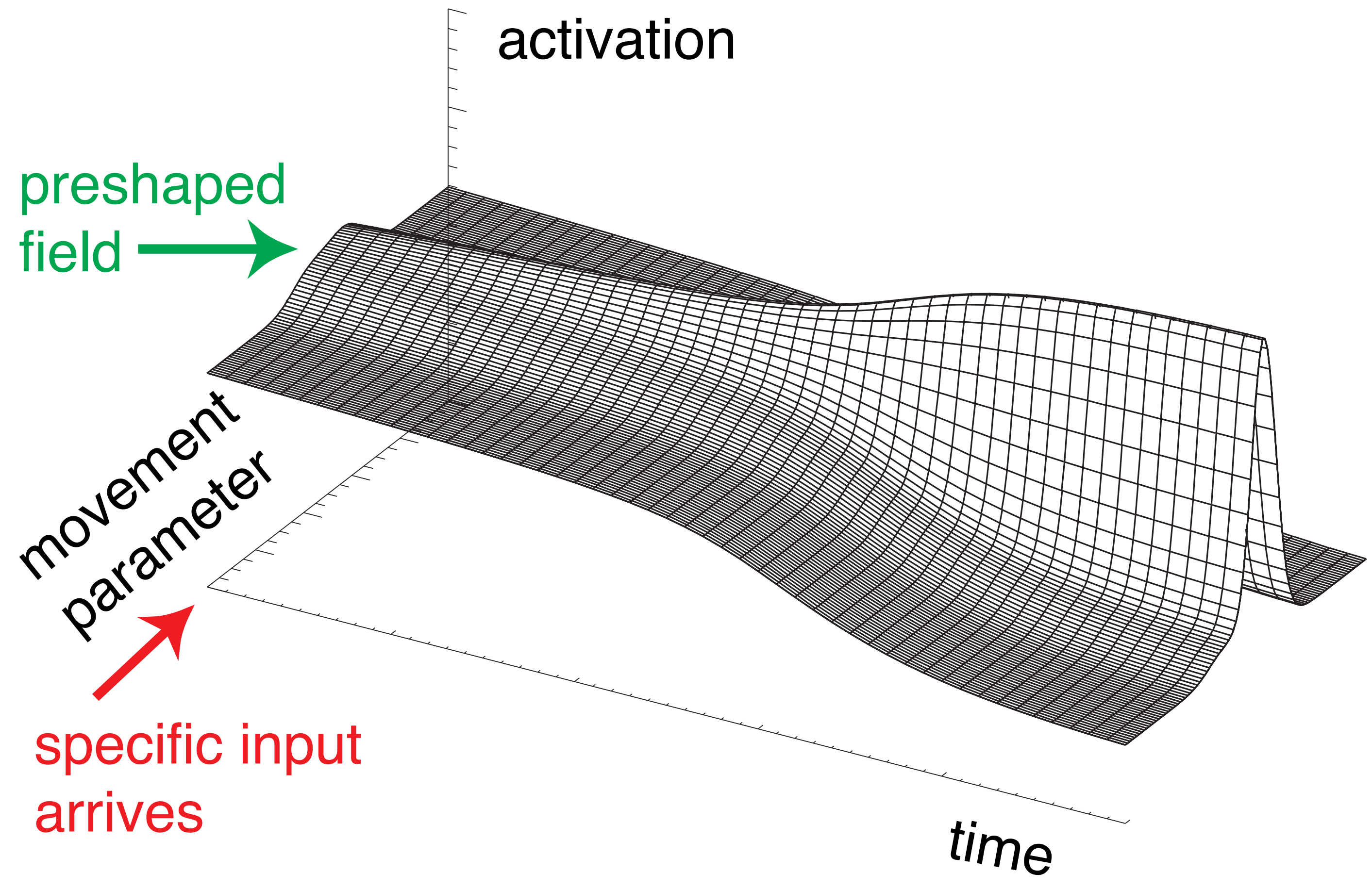
# Dynamic Field Theory

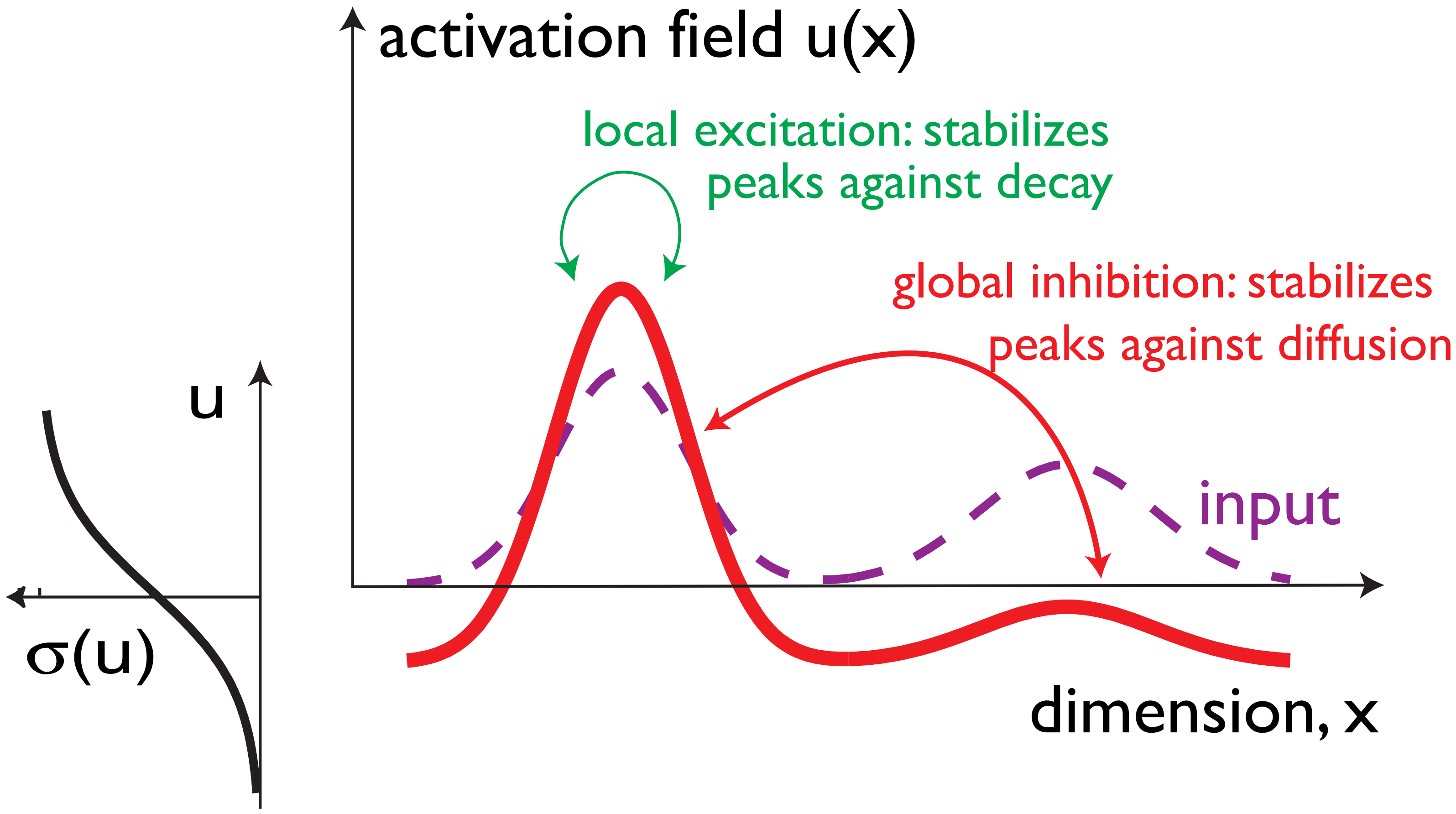
- dimensions
- activation fields
- field dynamics: peaks, instabilities

# activation fields



the dynamics such activation fields is structured so that localized peaks emerges as attractor solutions



Amari equation

$$\tau\dot{u}(x,t) = -u(x,t) + h + S(x,t) + \int w(x-x')\sigma(u(x',t))\;dx'$$

where

- time scale is $\tau$
- resting level is $h < 0$
- input is $S(x,t)$
- interaction kernel is

$$w(x-x') = w_i + w_e \exp\left[-\frac{(x-x')^2}{2\sigma_i^2}\right]$$

- sigmoidal nonlinearity is

$$\sigma(u) = \frac{1}{1+\exp[-\beta(u-u_0)]}$$

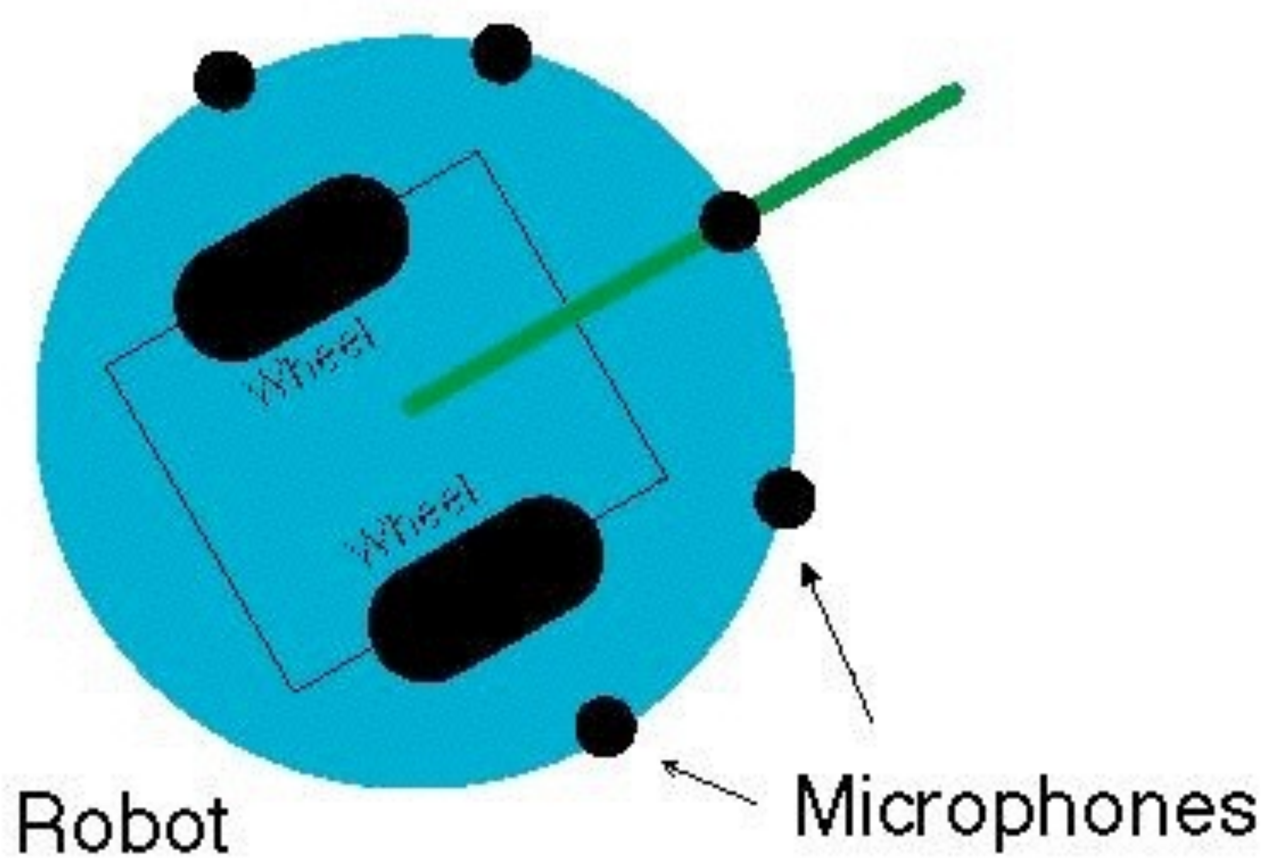
=> simulations

# instabilities

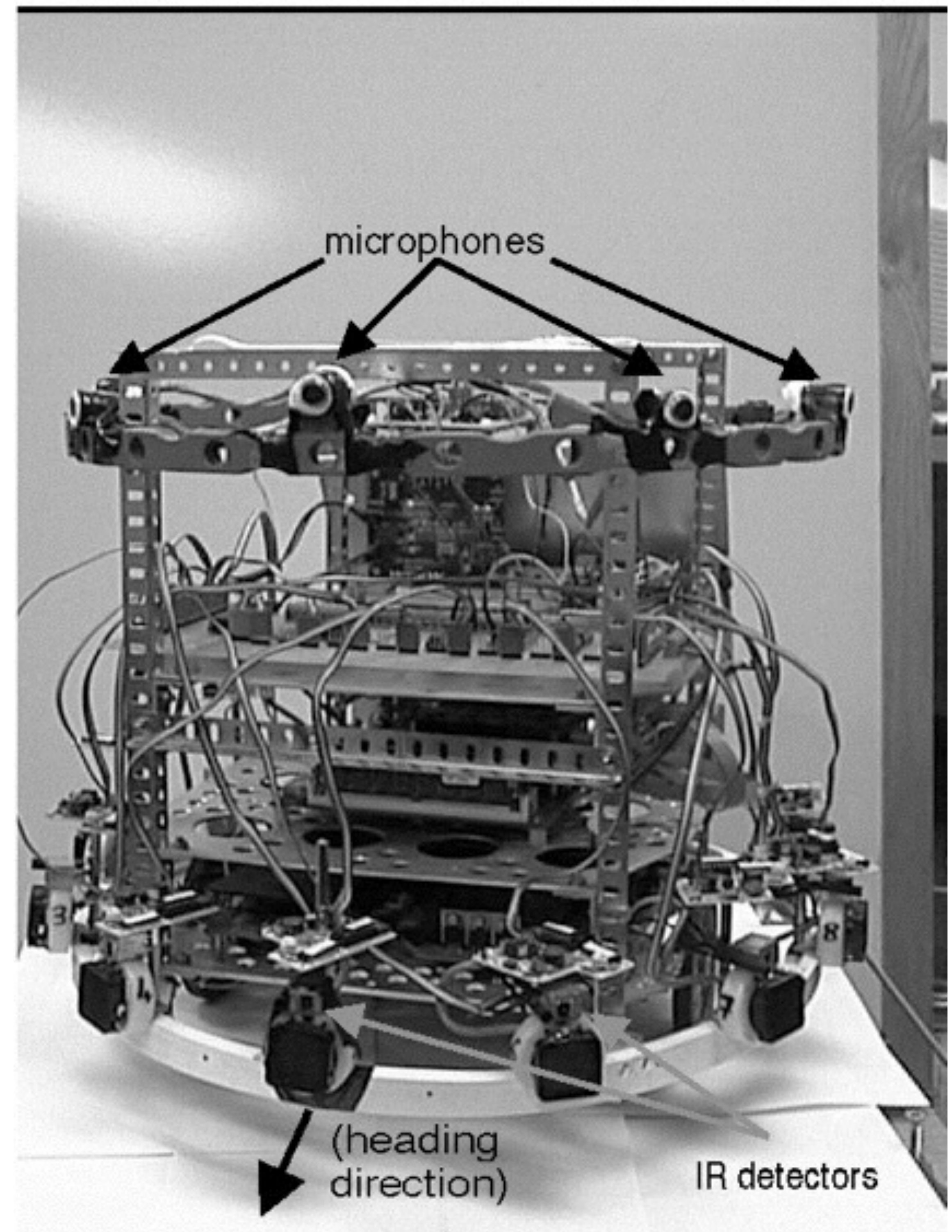
- self-stabilized or sustained peaks of activation vs. sub-threshold hills of activation
- detection instability, driven by localized input or boost
- selection instability
- memory instability

# illustration of the instabilities

# illustration of the instabilities

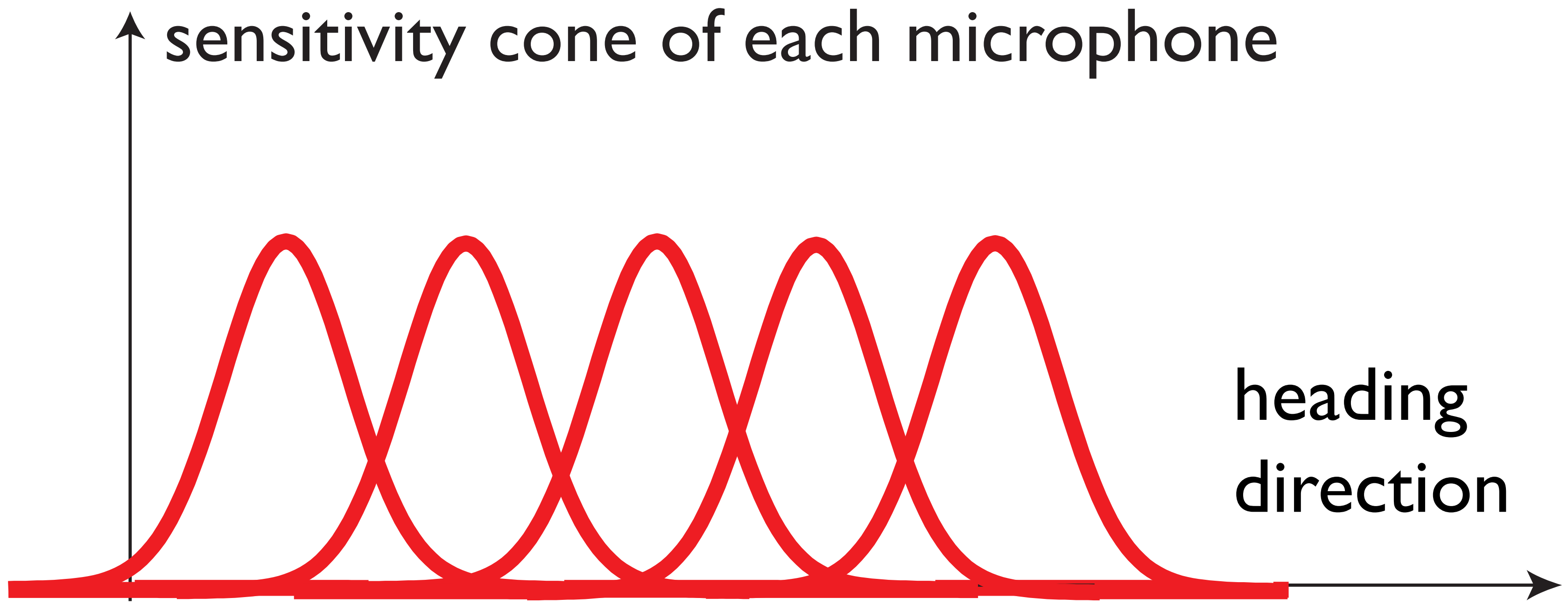



[from Bicho, Mallet, Schöner, Int J Rob Res,2000]

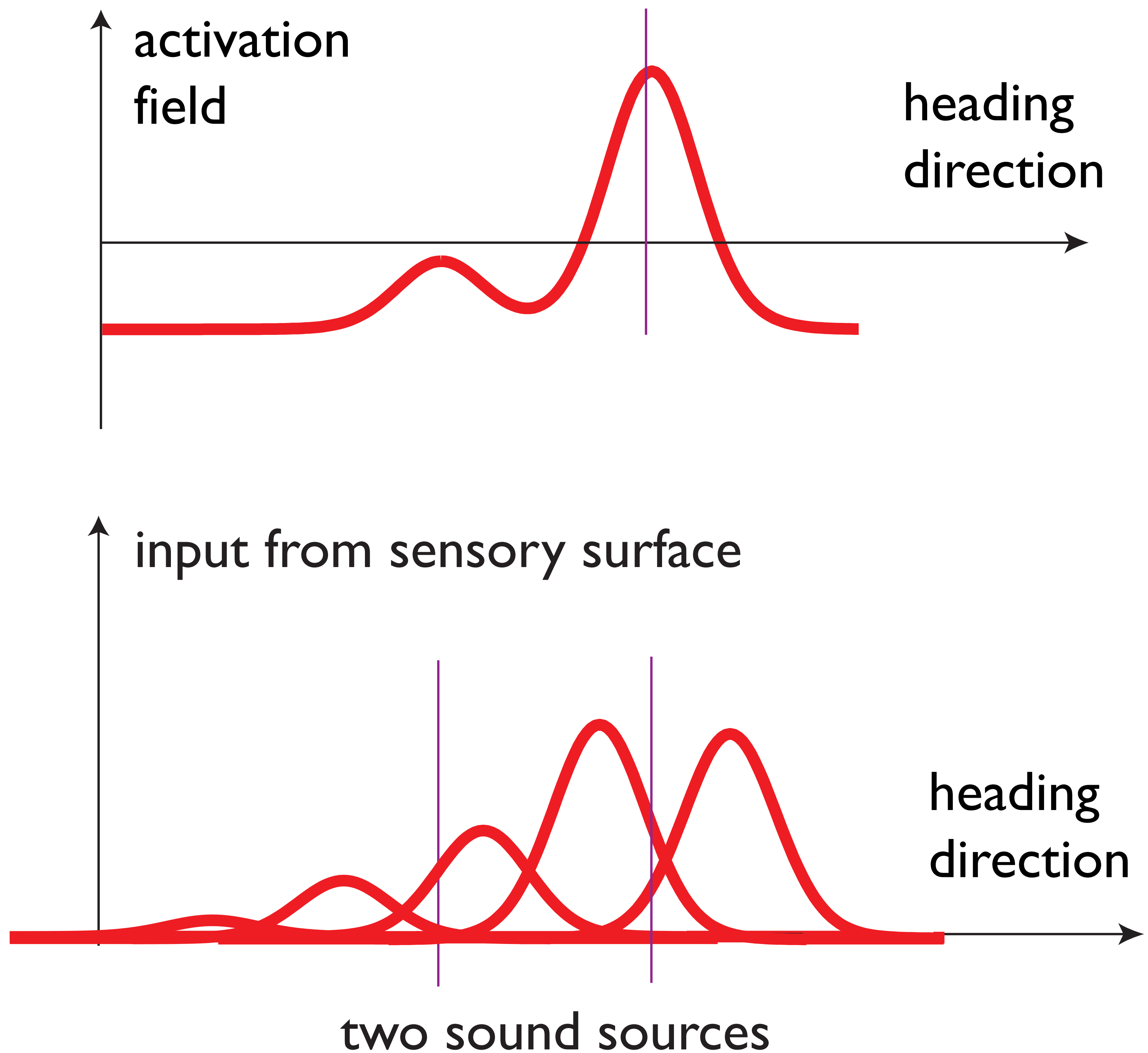


# sensory surface

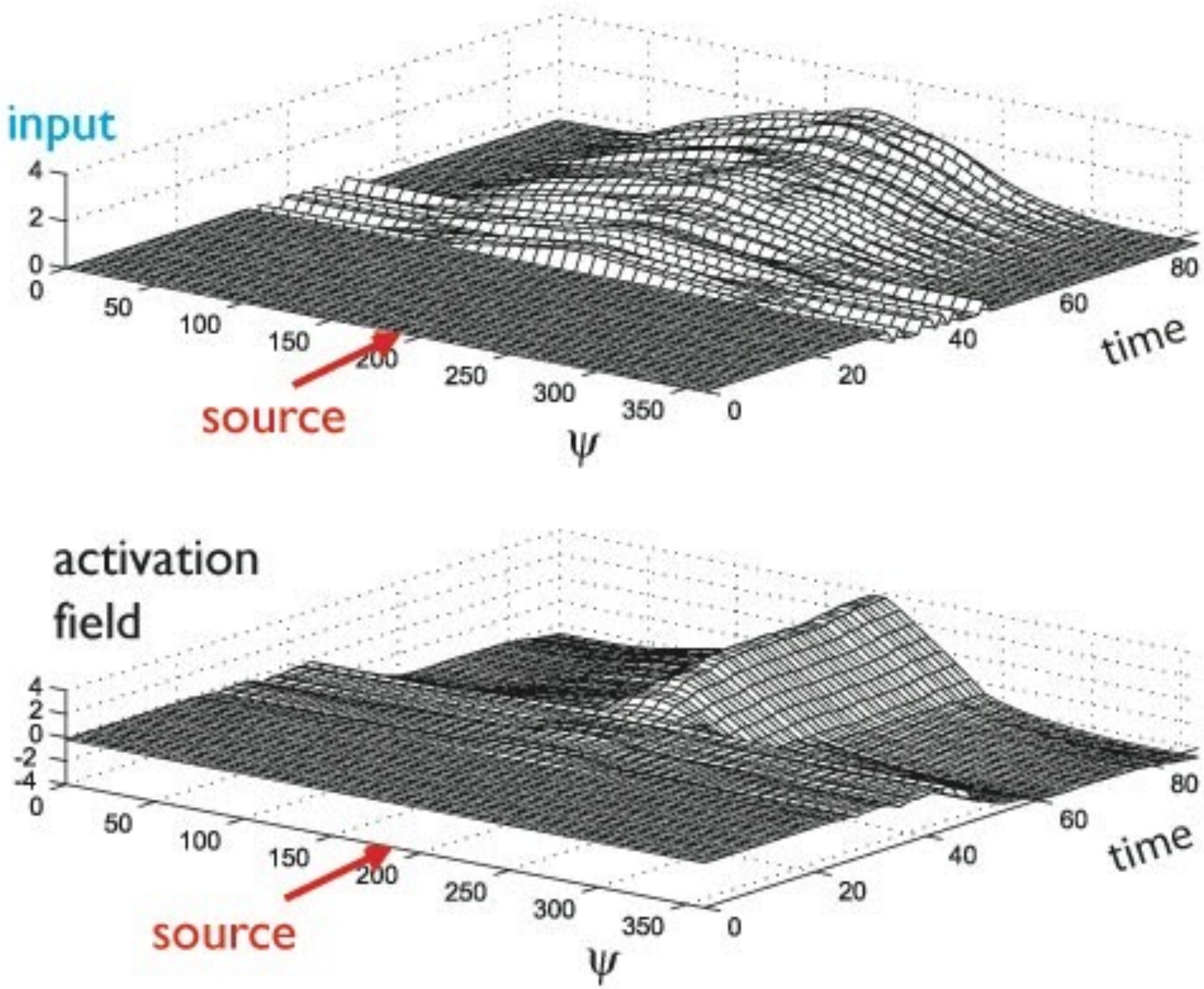
- each microphone samples heading direction

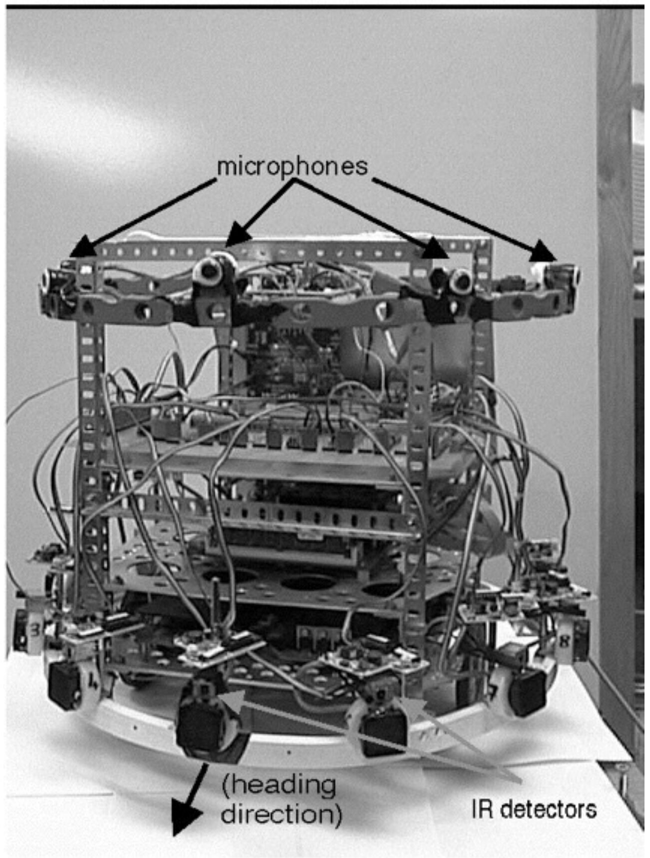


# and provides input to the field



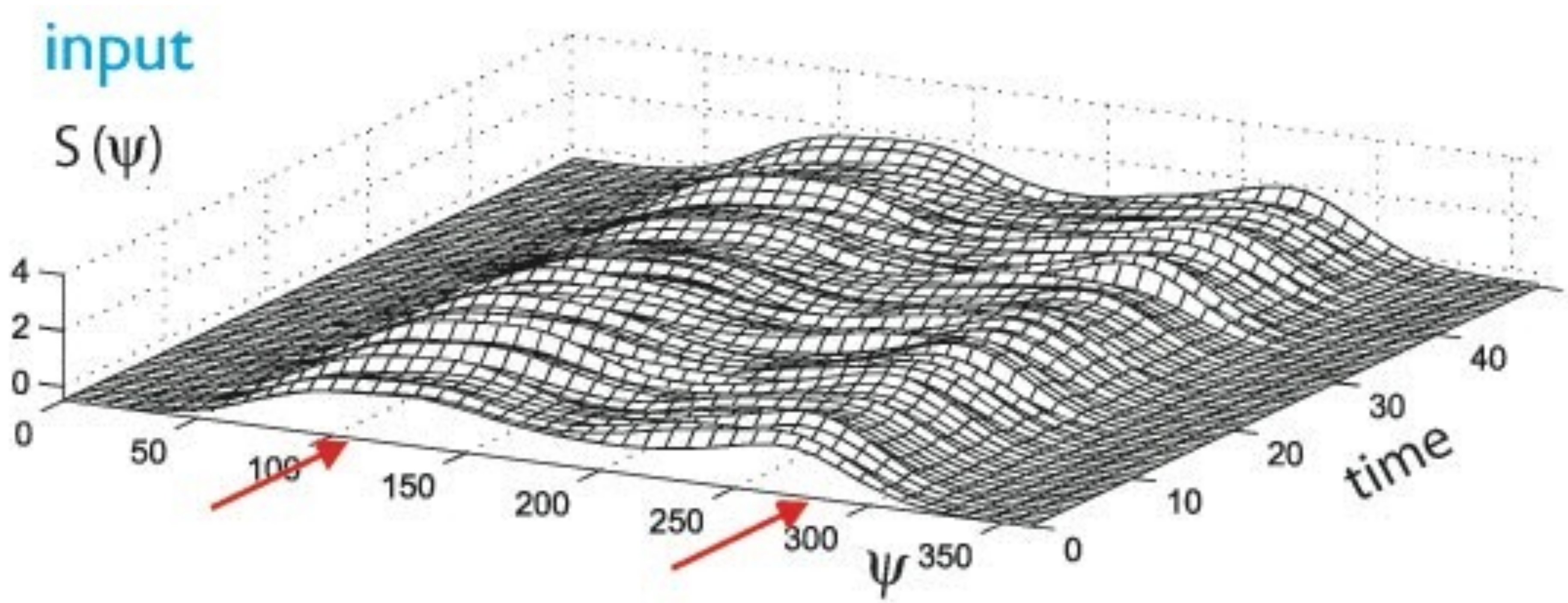
# detection instability on a phonotaxis robot

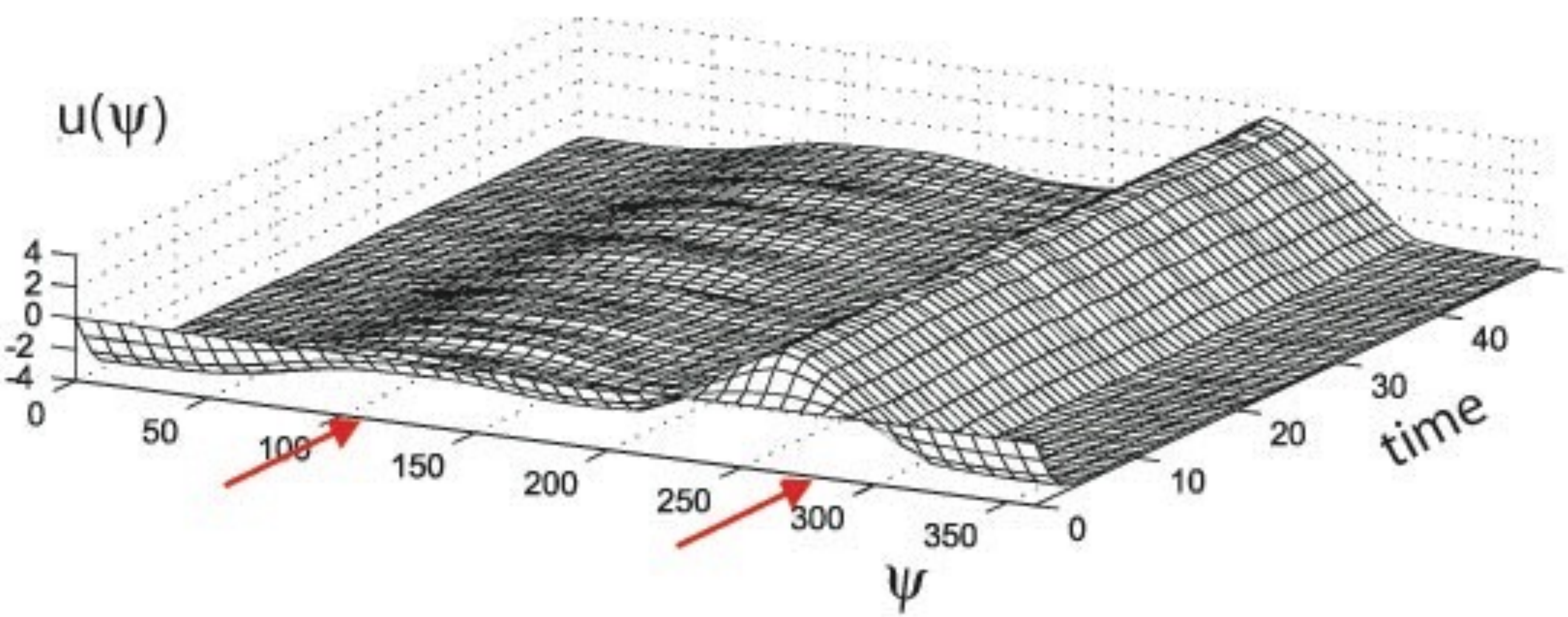


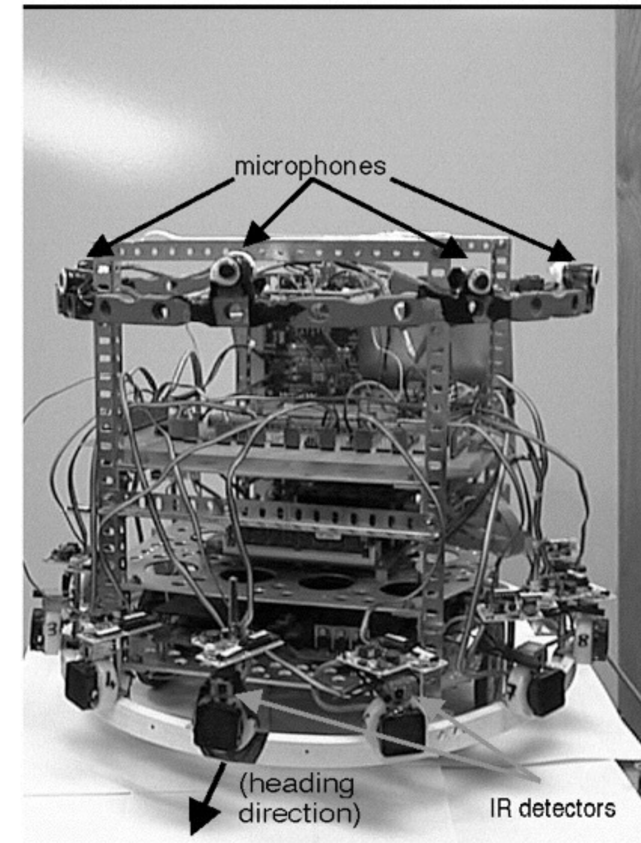




[from Bicho, Mallet, Schöner: Int. J. Rob. Res., 2000]

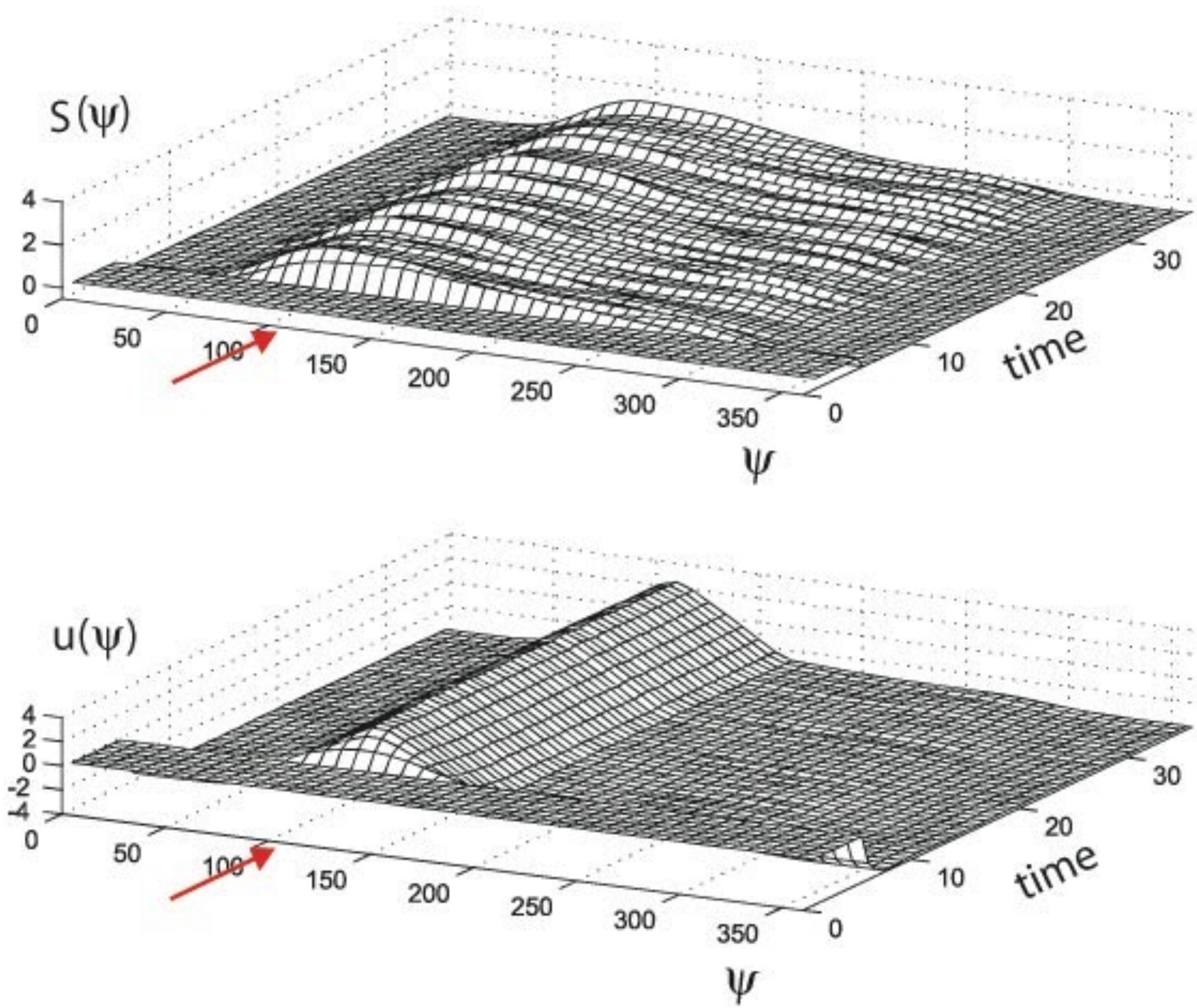
# target selection on phonotaxis vehicle



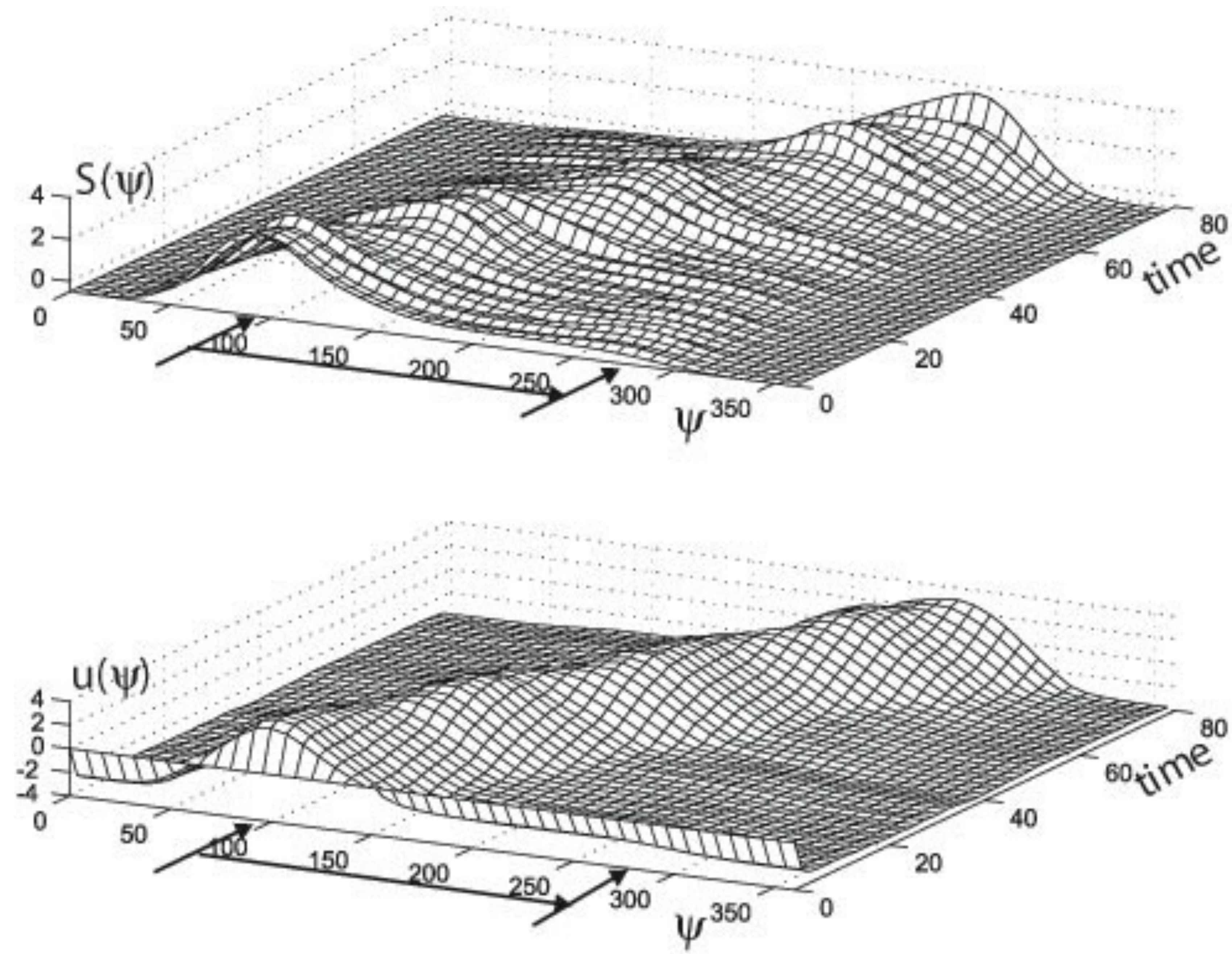



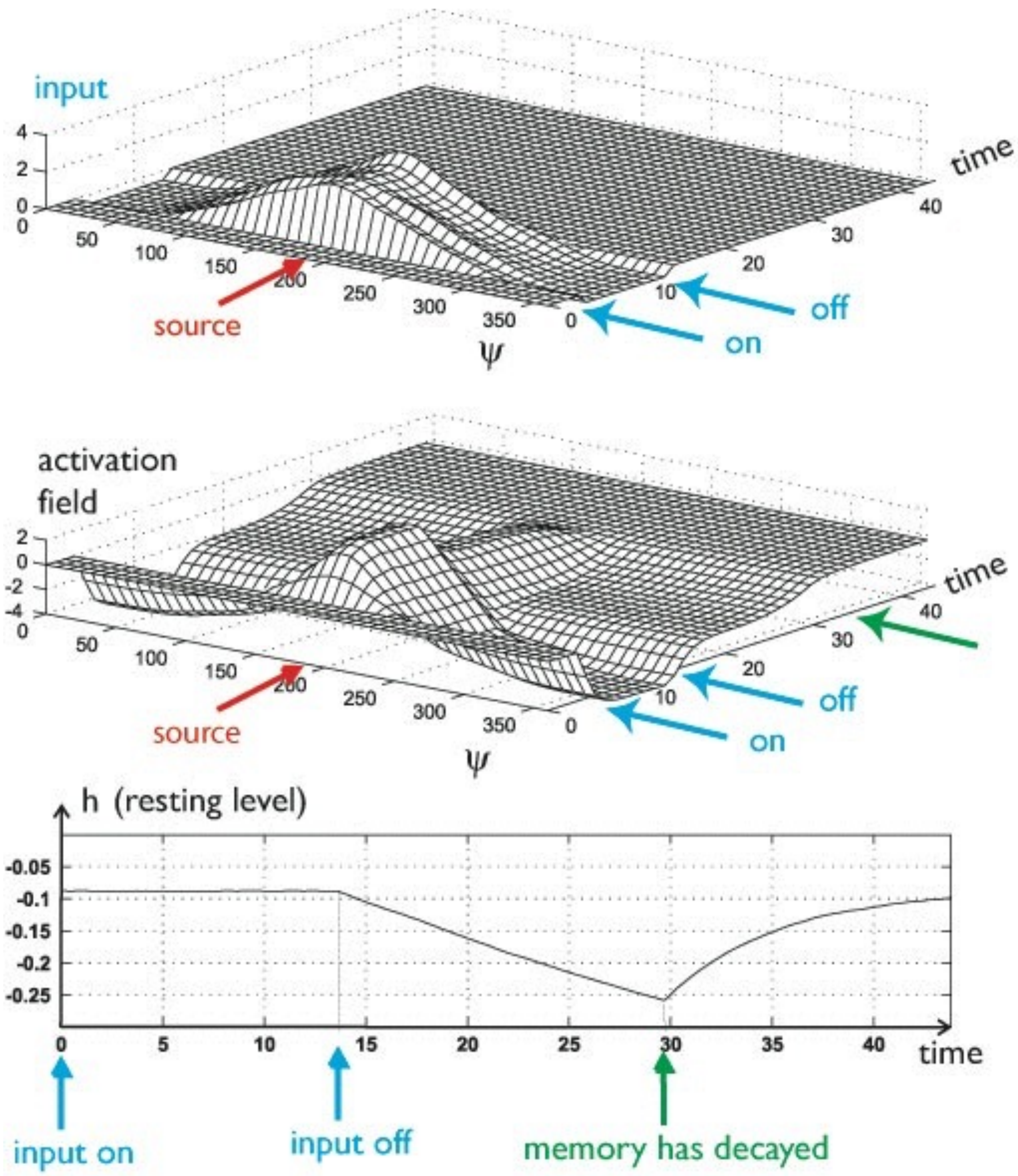
# robust estimation

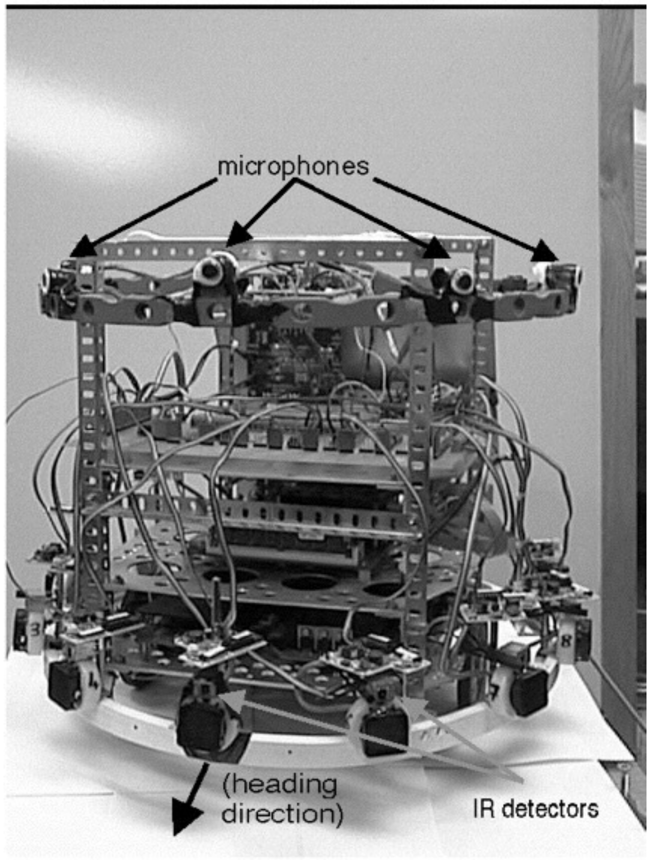


# tracking


S(ψ)
4
2
0
0
50
100
150
200
250
300
350
ψ
0
20
40
60
80
time
u(ψ)
4
2
0
-2
-4
0
50
100
150
200
250
300
350
ψ
0
20
40
60
80
time

# memory & forgetting on phonotaxis vehicle



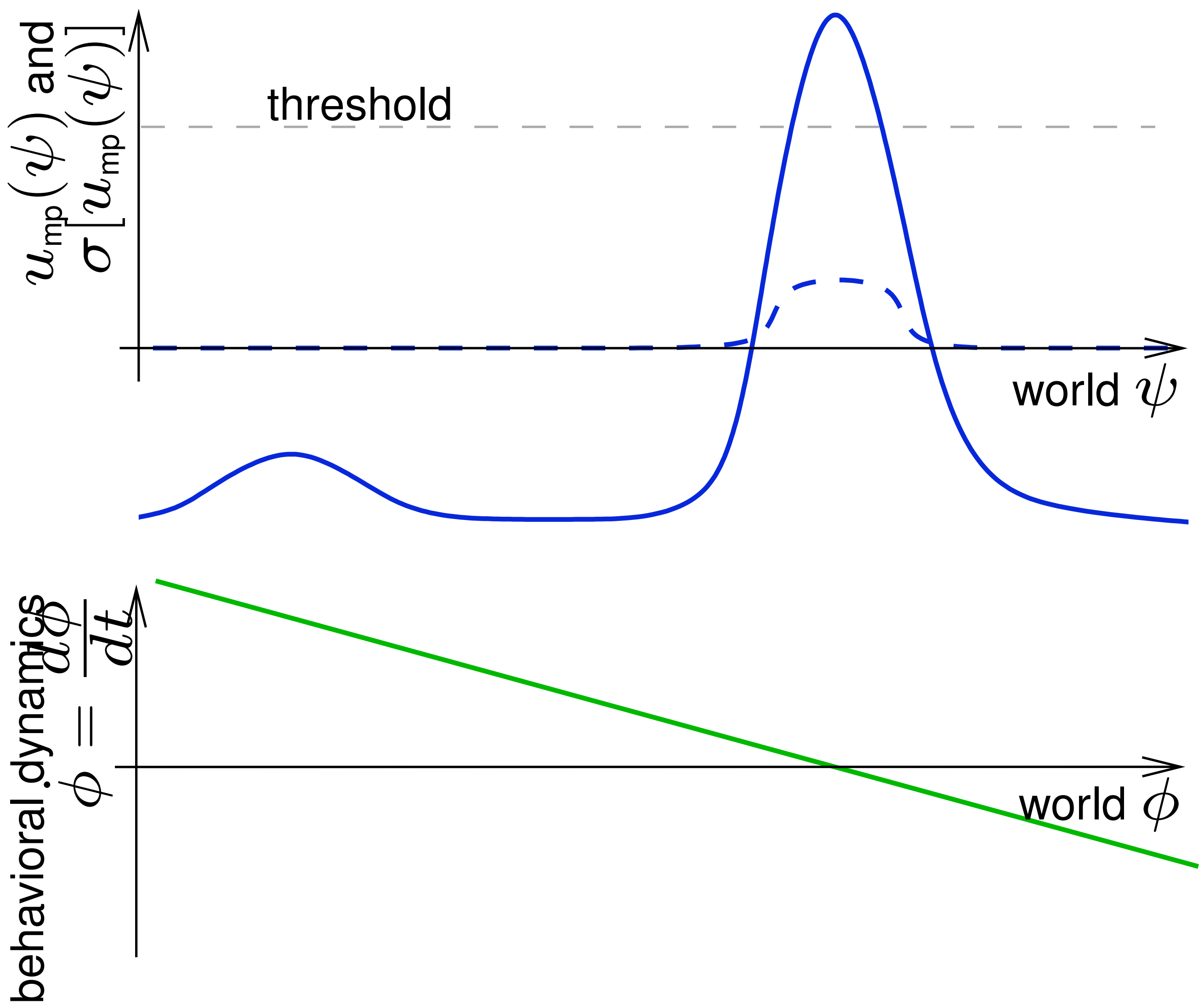




[from Bicho, Mallet, Schöner: Int J Rob Res 19:424(2000)]

# a robotic demo of all of instabilities

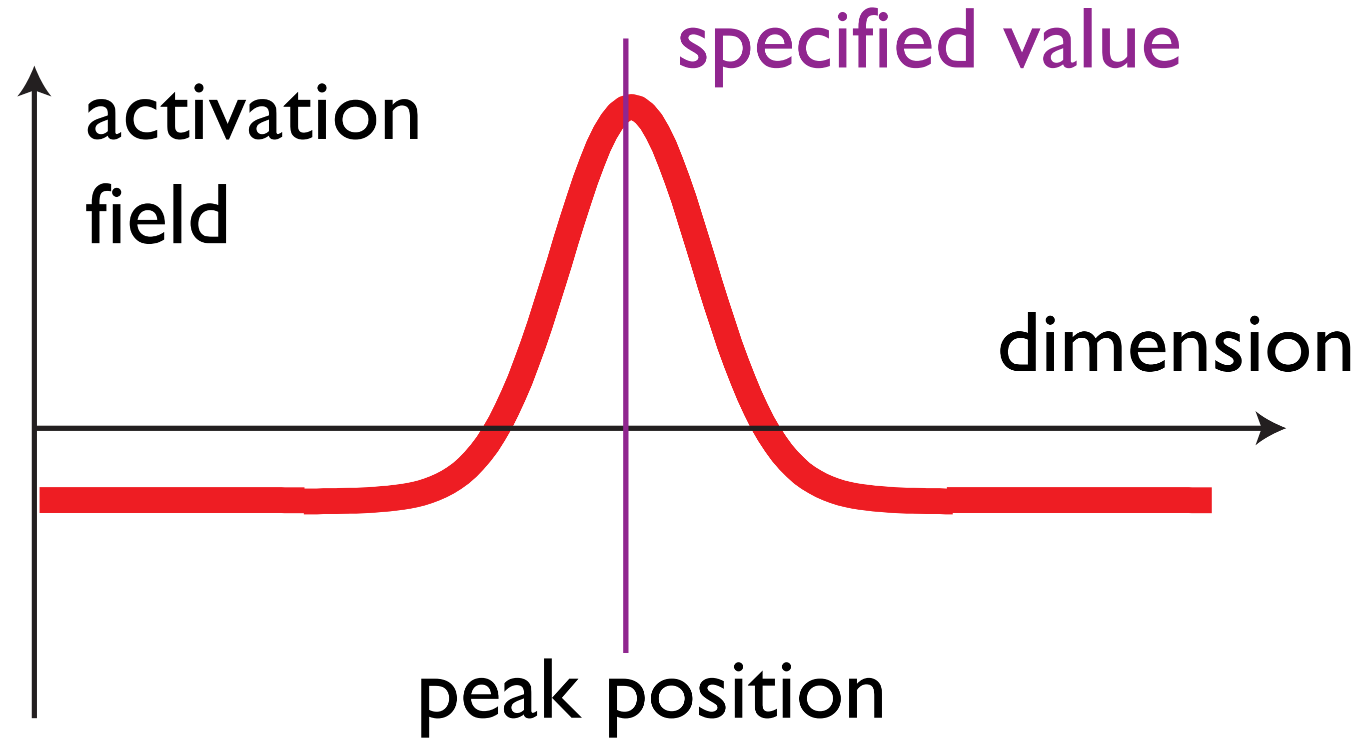
# motor dynamics

- couple peak in direction field into dynamics of heading direction as an attractor



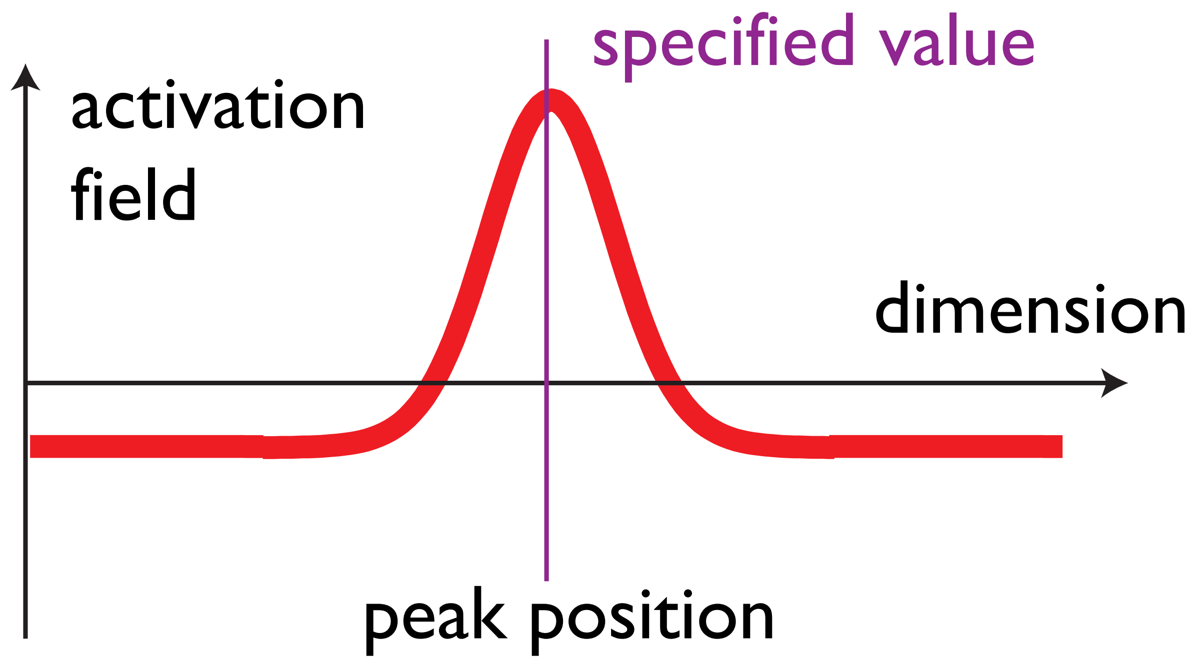
# => transition from DFT to DST

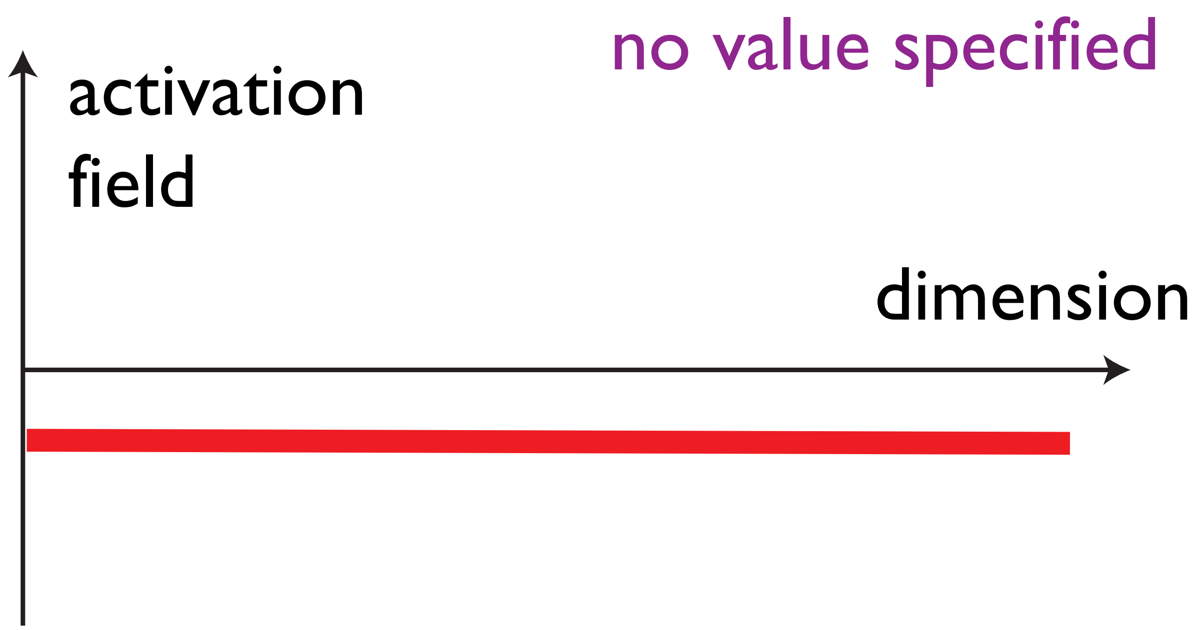
- peak specifies value for a dynamical variable that is congruent to the field dimension



# from DFT to DST

- treating sigmoided field as probability: need to normalize
  - => problem when there is no peak: devide by zero!



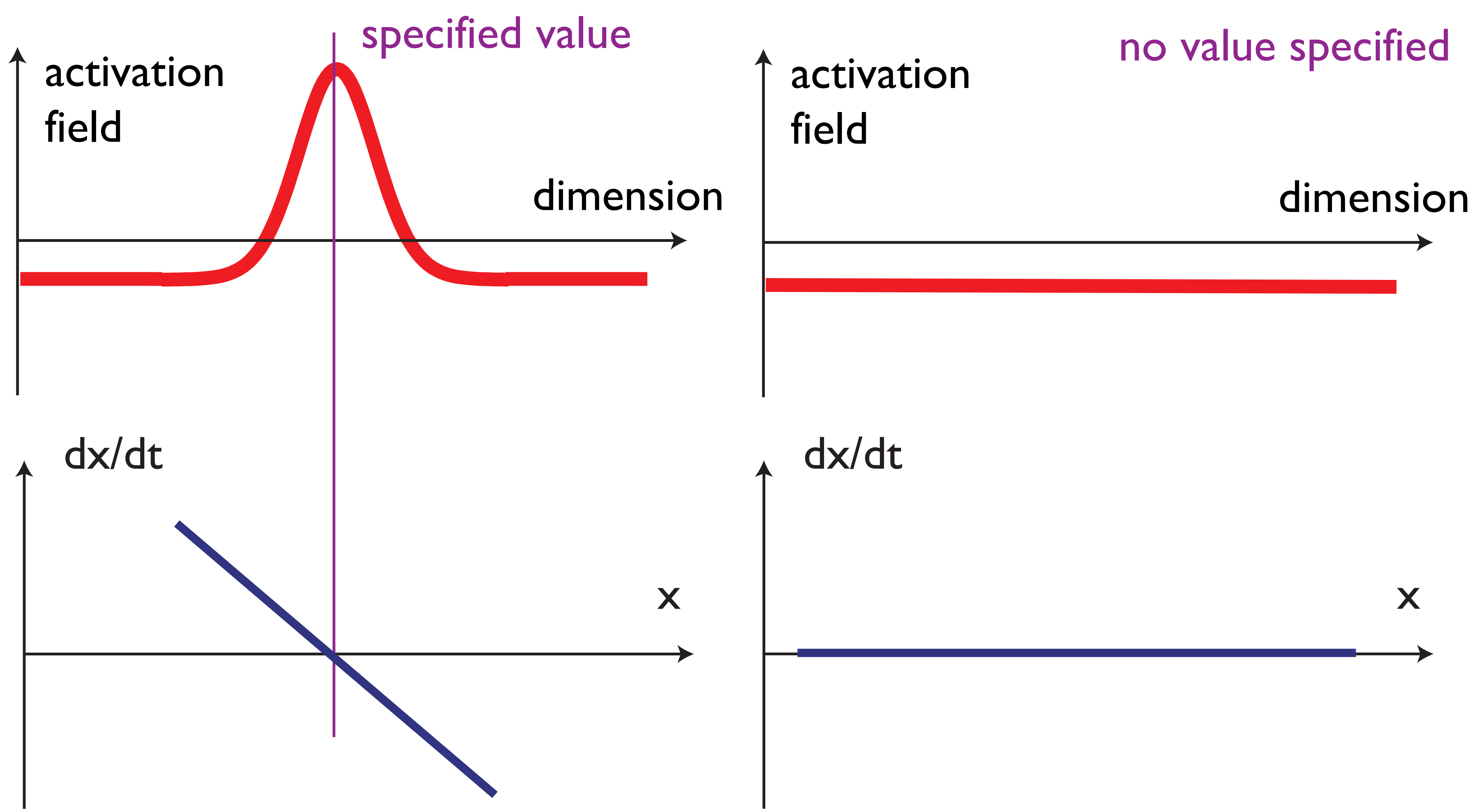



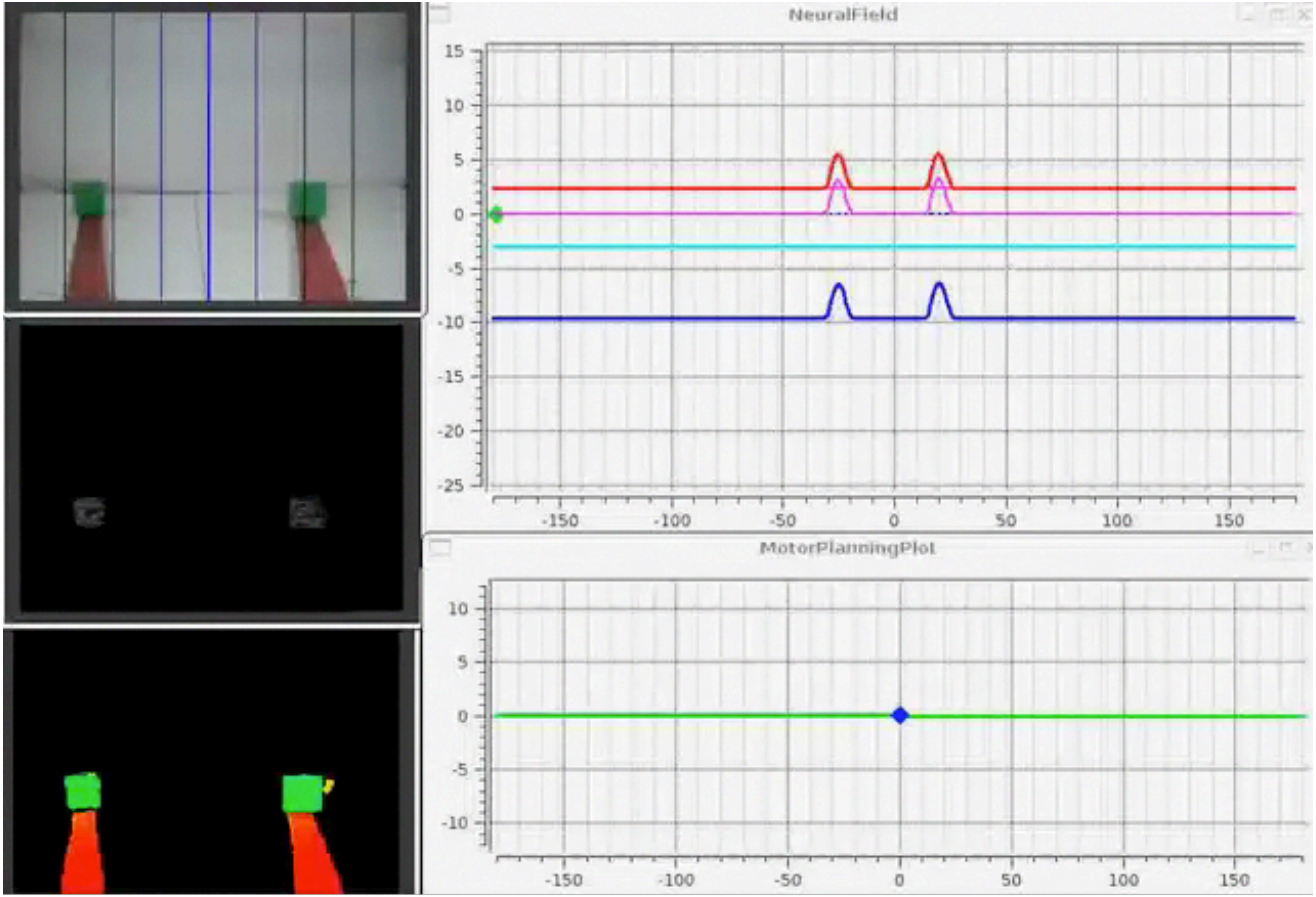
# from DFT to DST

- solution: peak sets attractor
  - location of attractor: peak location
  - strength of attractor: summed supra-threshold activation

$$
\begin{aligned}
x_{\text{peak}} &= \frac{\int dx\ x\ \sigma(u(x,t))}{\int dx\ \sigma(u(x,t))} \\
\dot{x} &= -\left[\int dx\ \sigma(u(x,t))\right](x - x_{\text{peak}}) \\
\Rightarrow \dot{x} &= -\left[\int dx\ \sigma(u(x,t))\right]\ x + \left[\int dx\ x\ \sigma(u(x,t))\right]
\end{aligned}
$$

# from DFT to DST



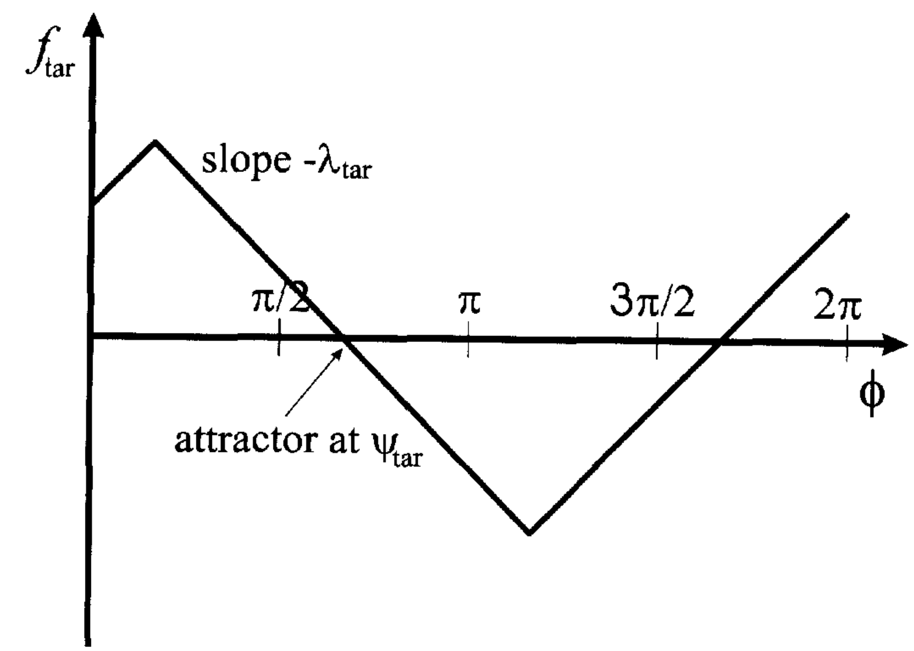

NeuralField
15
10
5
0
-5
-10
-15
-20
-25
-150
-100
-50
0
50
100
150
MotorPlanningPlot
10
5
0
-5
-10
-150
-100
-50
0
50
100
150

# => Bicho, Mallet, Schöner (2000)

- this is how target acquisition is integrated into obstacle avoidance on the robot

$$\frac{d\phi}{dt} = \sum_{i=1}^{7} f_{\text{obs},i} + f_{\text{tar}}.$$

$$\psi_{\text{tar}} = \int_0^{2\pi} \psi H(u(\psi)) d\psi / N_u$$



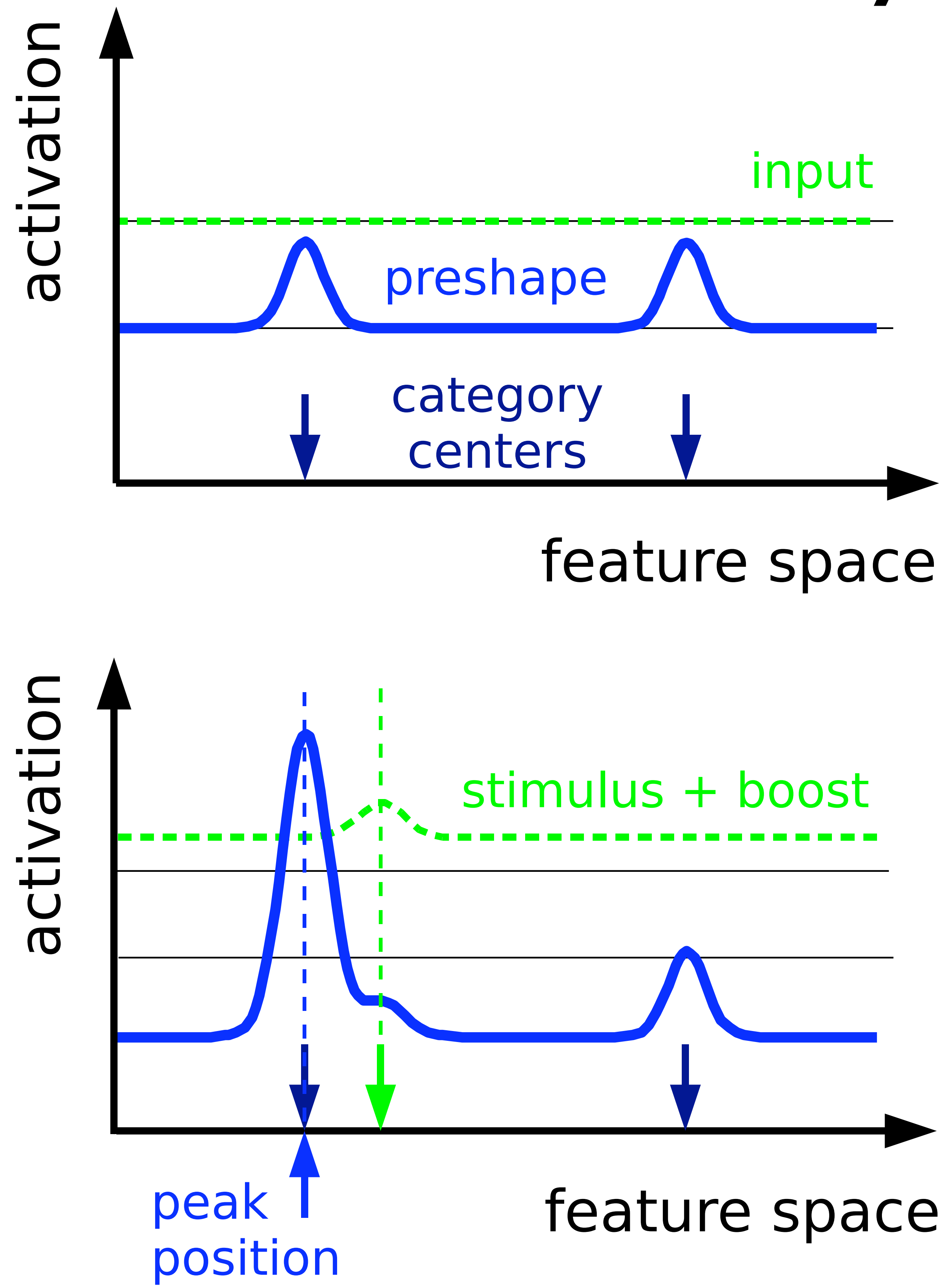

$$f_{\text{tar}} = \begin{cases} -\lambda'_{\text{tar}}(N_u\phi - \int_0^{2\pi}(H(u(\psi))\psi)d\psi) \\ \text{for } \psi_{\text{tar}} - \pi/2 < \phi \le \psi_{\text{tar}} + \pi/2 \\ \lambda'_{\text{tar}}(N_u(\phi - \pi) - \int_0^{2\pi}(H(u(\psi))\psi)d\psi) \\ \text{for } \psi_{\text{tar}} + \pi/2 < \phi \le \psi_{\text{tar}} + 3\pi/2 \end{cases}$$

# boost-driven detection instability

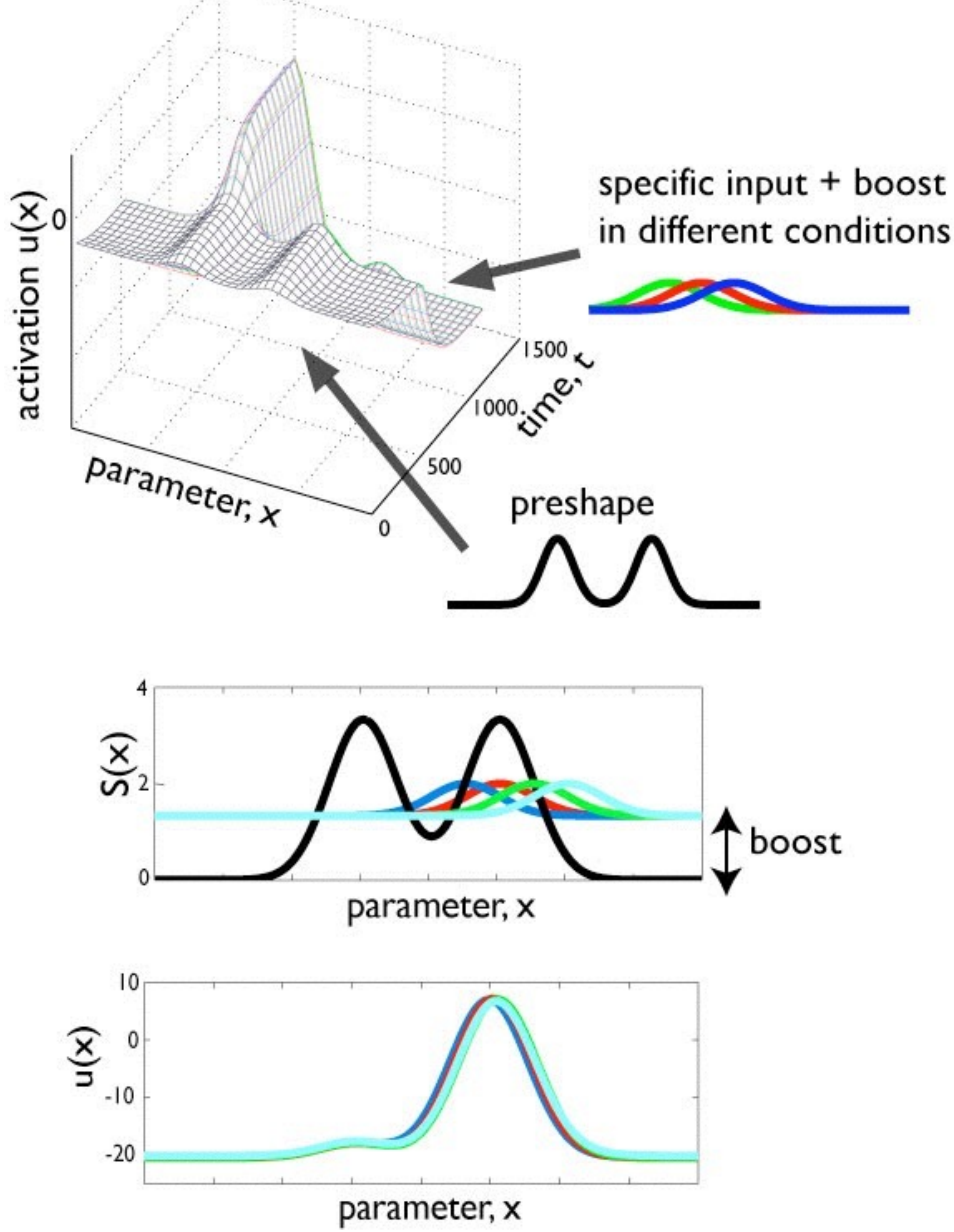
- inhomogeneities in the field existing prior to a signal/stimulus that leads to a macroscopic response="preshape"
- the boost-driven detection instability amplifies preshape into macroscopic selection decisions

# boost-induced detection instability

- transforms graded patterns, learned inhomogeneities into macroscopic decisions: categorical states!



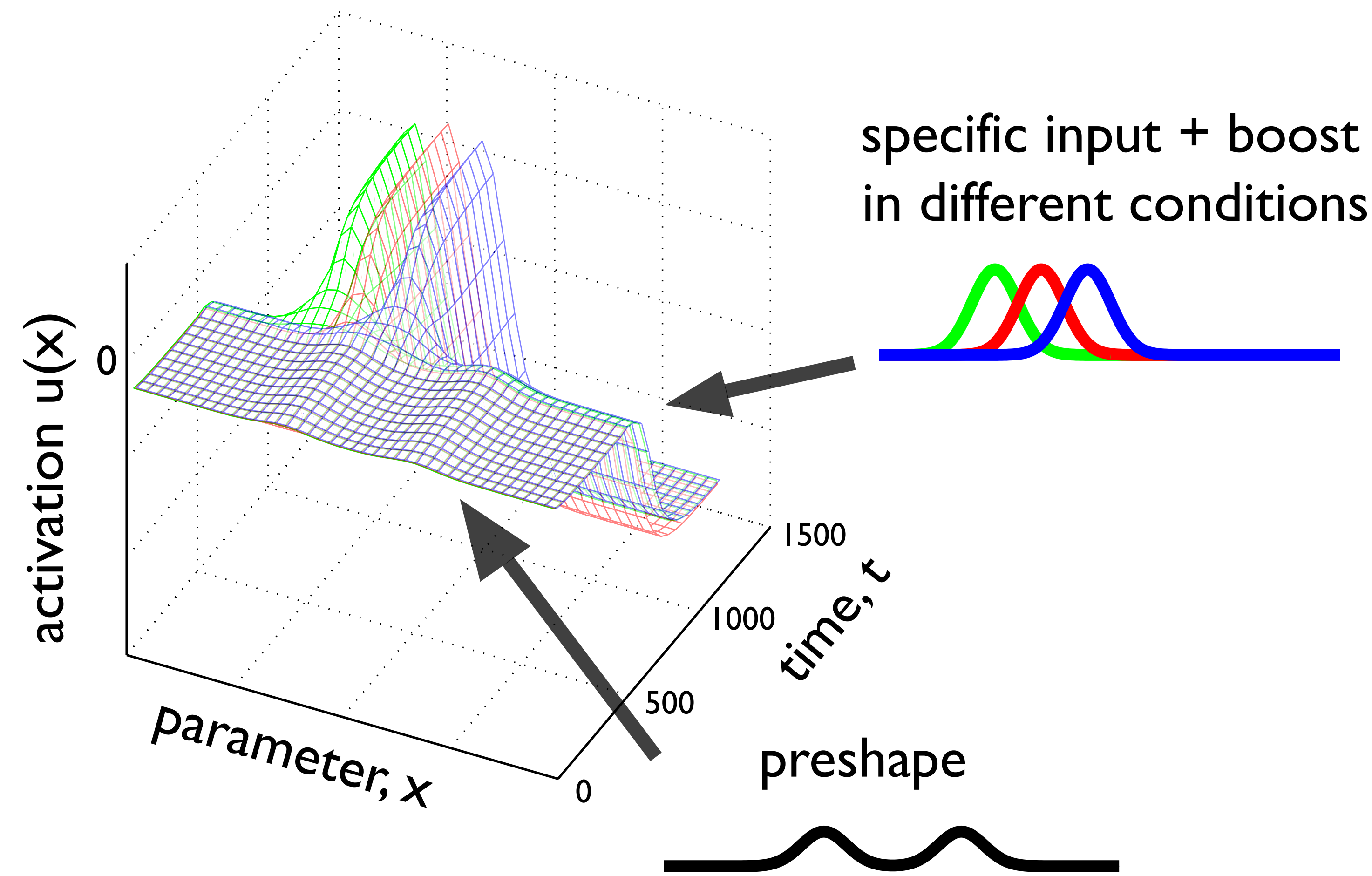
# this supports categorical behavior



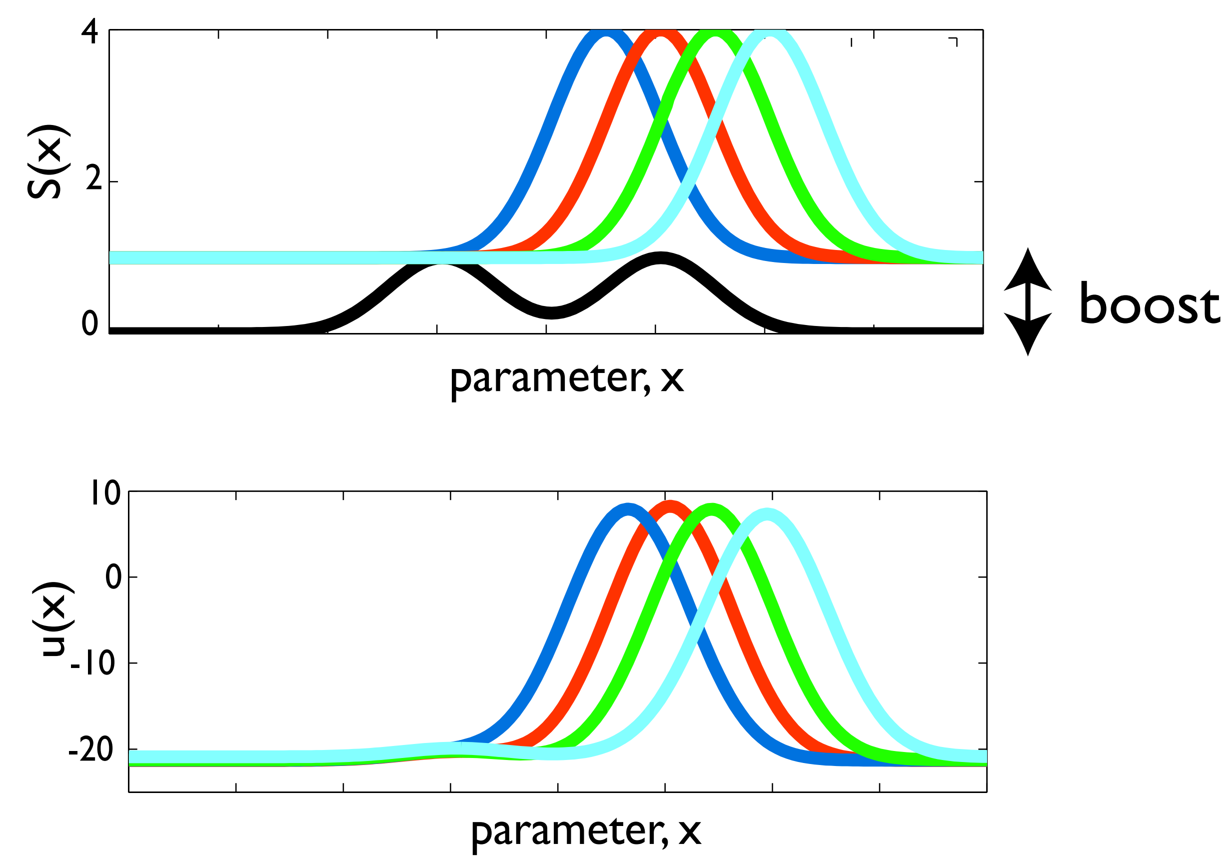

- when preshape dominates

[Wilimzig, Schöner, 2006]

# continuous responding for weak preshape

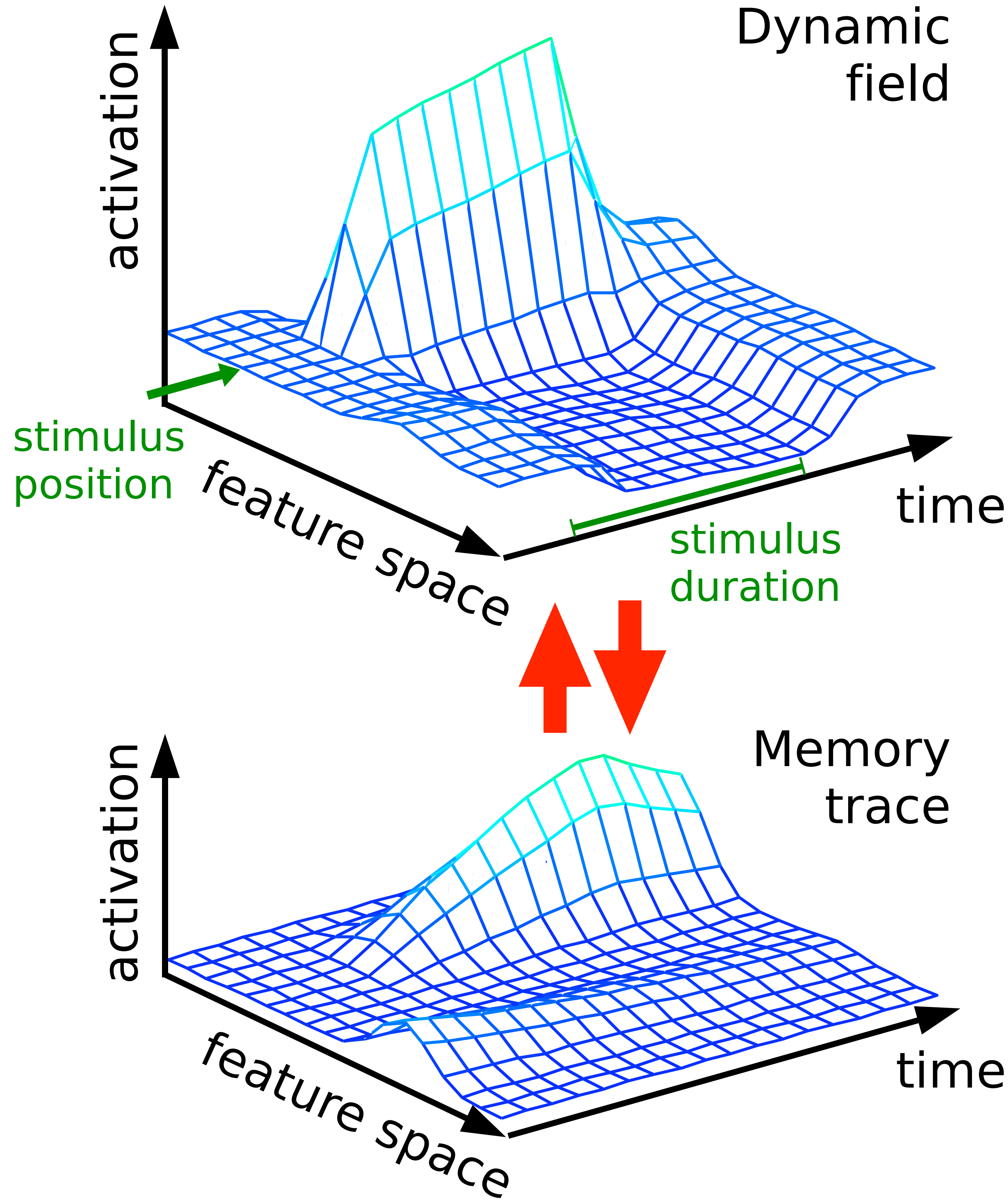



- specific (imperative) input dominates and drives detection instability




[Wilimzig, Schöner, 2006]

# How does a preshape/memory trace arise?



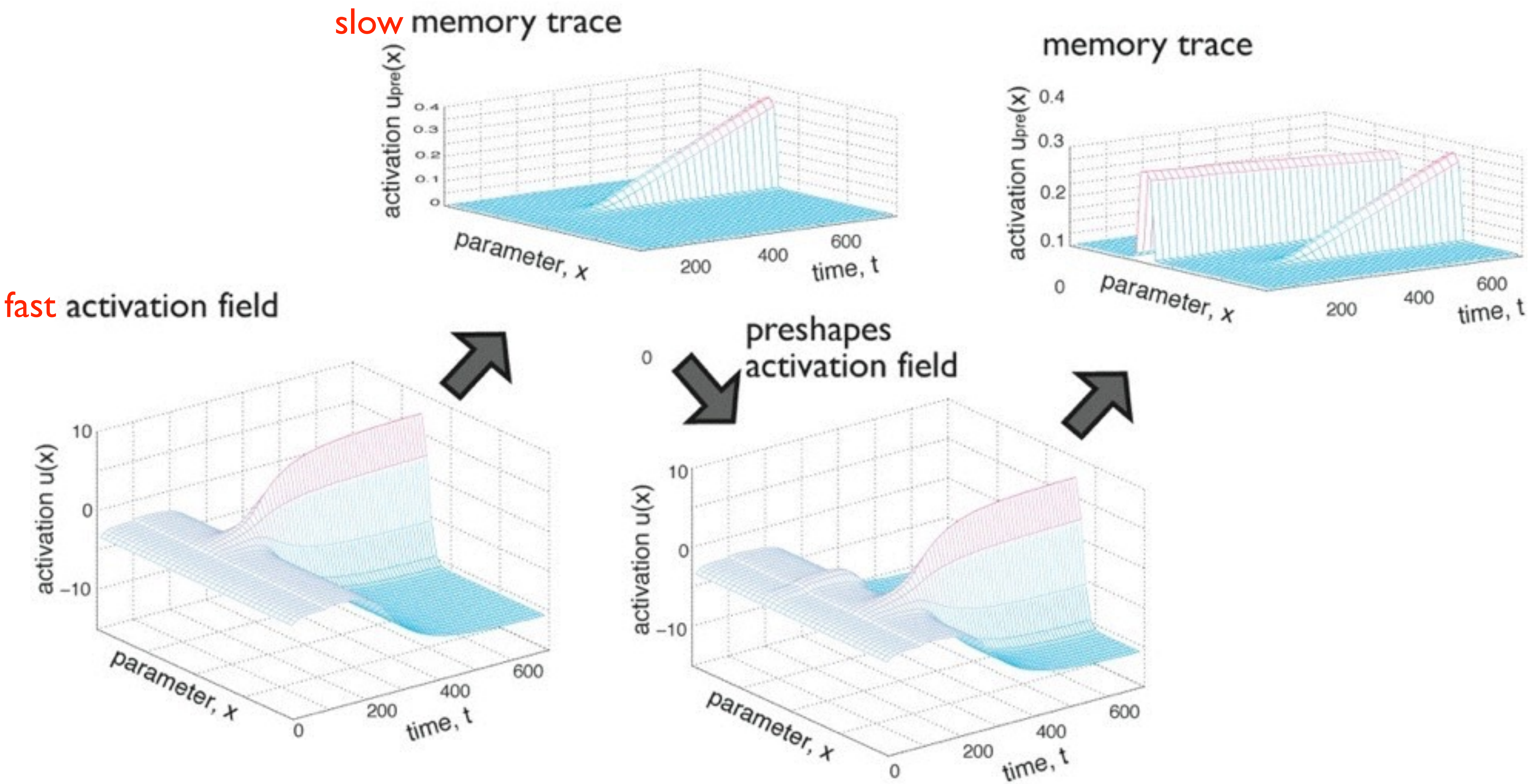

- memory trace dynamics
- a form of learning that captures habit formation by stabilizing activation patterns

# mathematics of the memory trace

$$\begin{aligned}\tau \dot{u}(x,t) &= -u(x,t) + h + S(x,t) + u_{\text{mem}}(x,t) \\ &+ \int dx' \; w(x-x') \; \sigma(u(x'))\end{aligned}$$

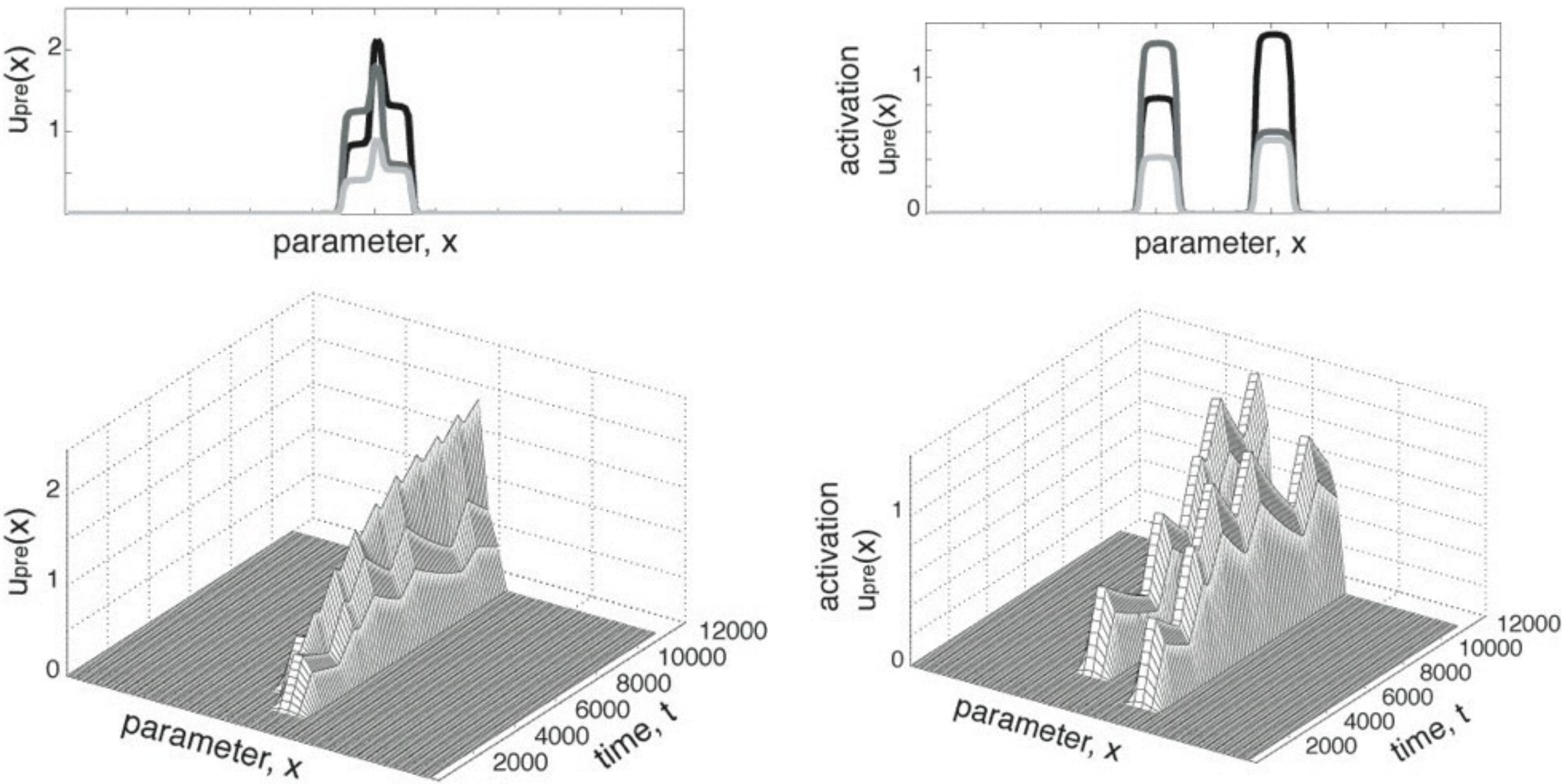
$$\begin{aligned}\tau_{\text{mem}} \; \dot{u}_{\text{mem}}(x,t) &= -u_{\text{mem}}(x,t) \\ &+ \int dx' \; w_{\text{mem}}(x-x')\sigma(u(x',t))\end{aligned}$$

- memory trace only evolves while activation is excited
- potentially different growth and decay rates




Wilimzig, Schöner 2006

# categories may emerge ...



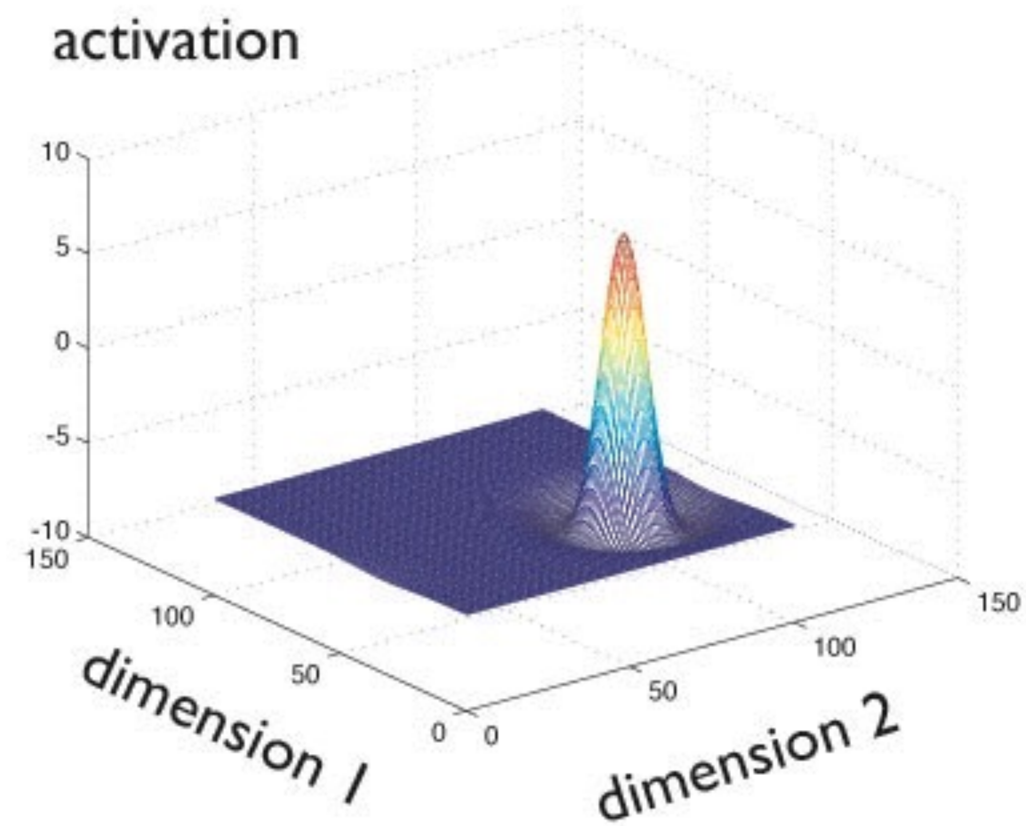

Wilimzig, Schöner 2006

# Higher-dimension dynamic fields

- provide new functions

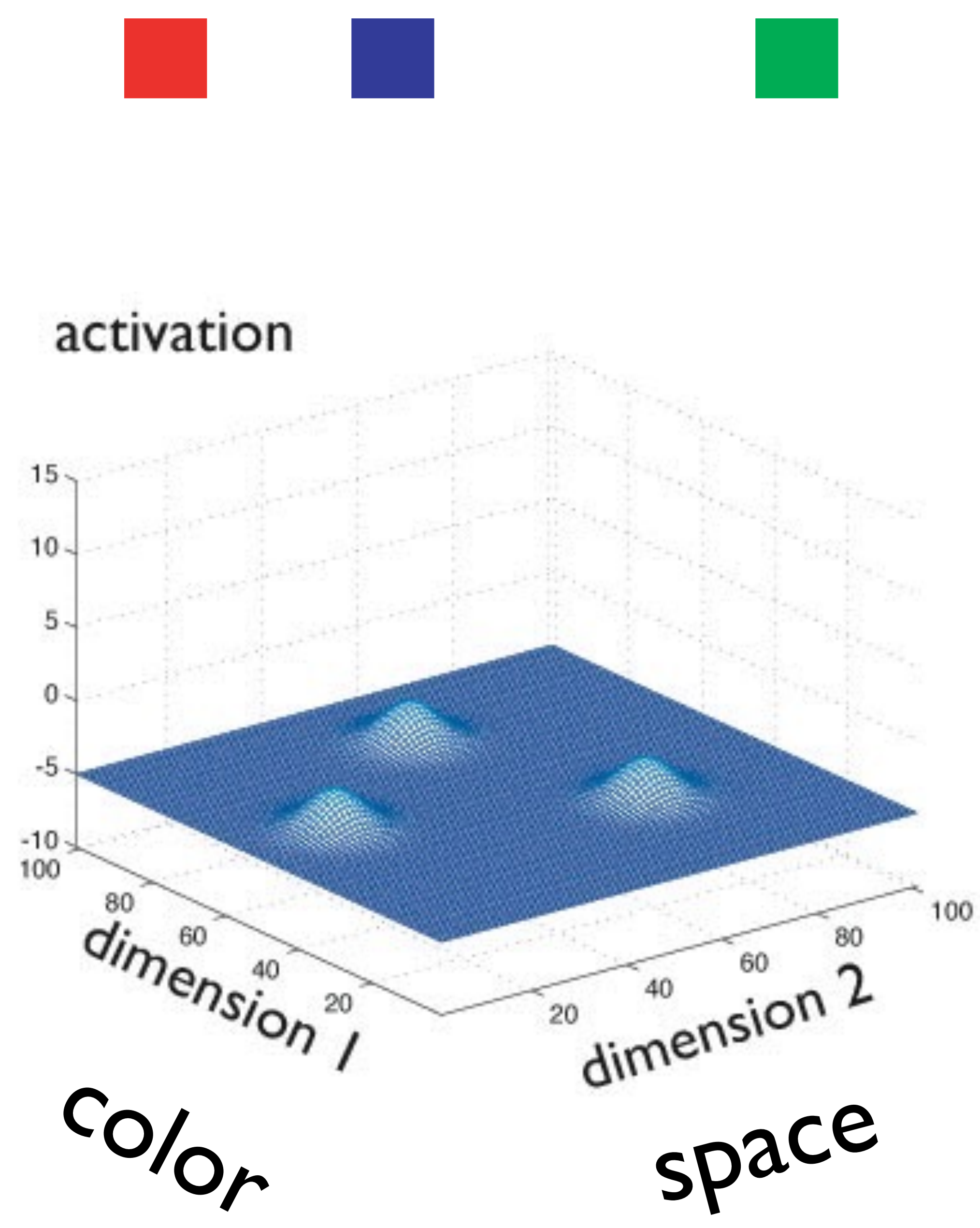
# mathematics of 2D fields

- => simulation
- no problem ... self-stabilized peaks work just fine...



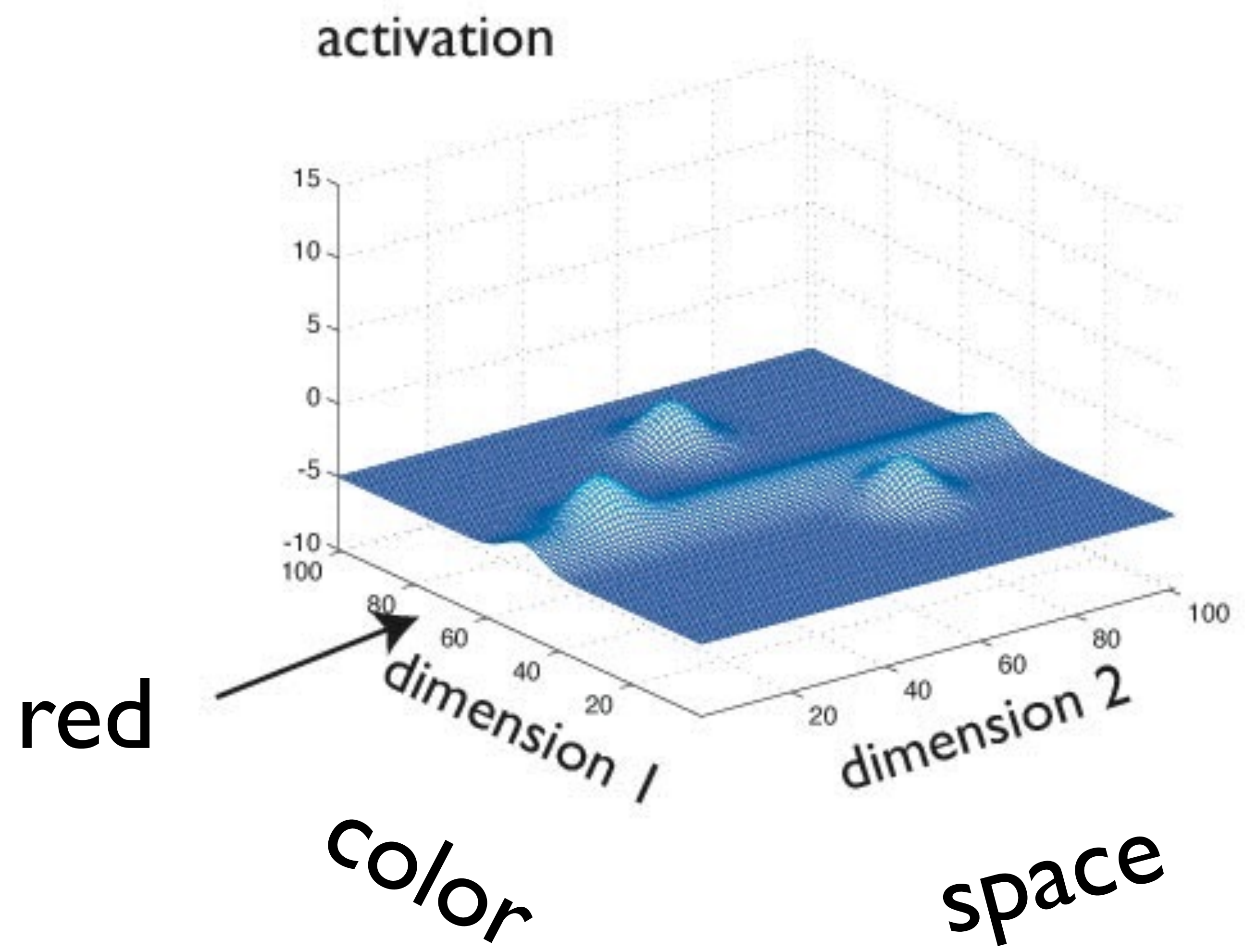
# dimensional cuing

- e.g., three inputs at three location with three different colors
- answer: "where is the red square"



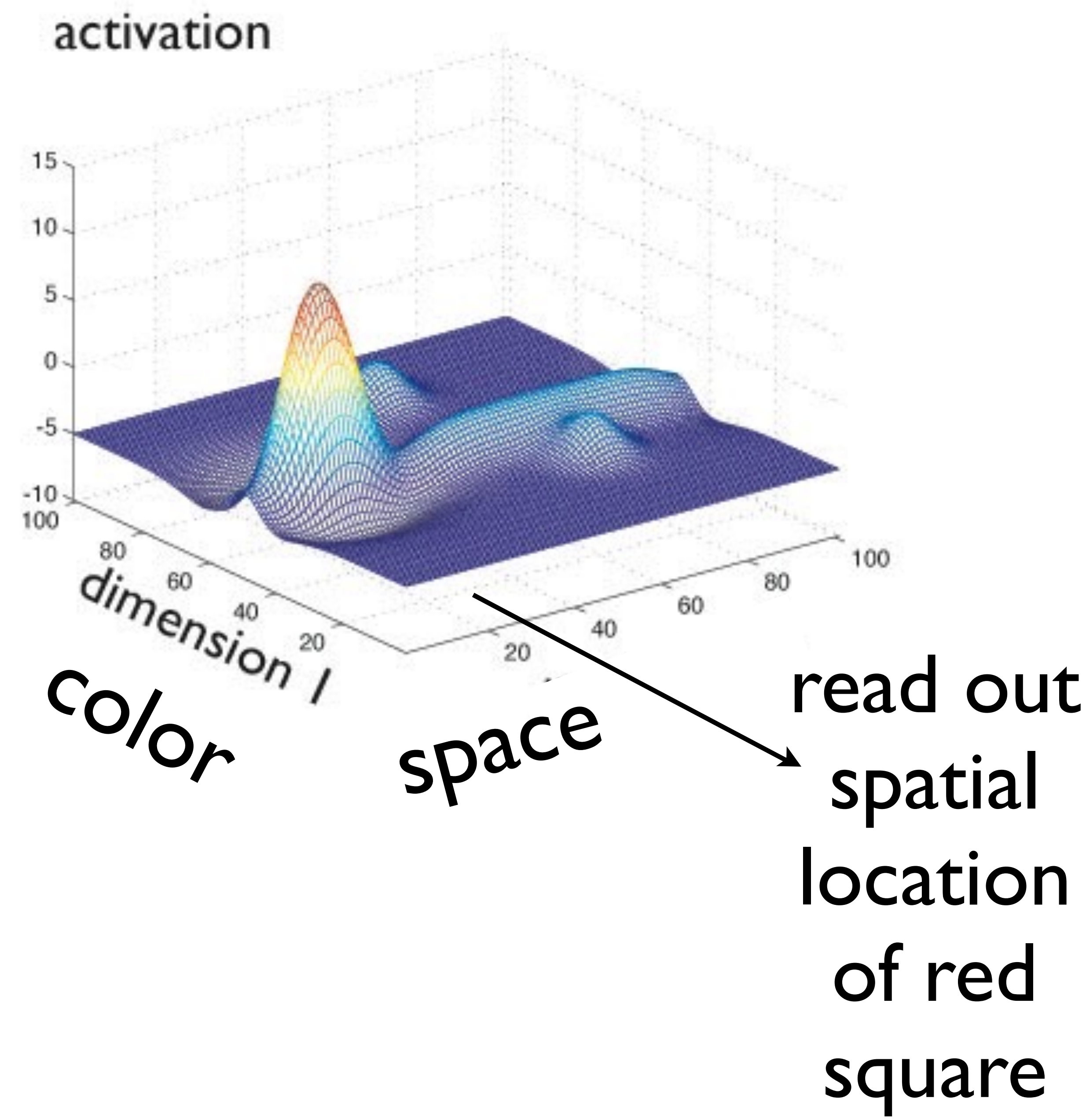
# dimensional cuing

- supply ridge input along the cued color dimension



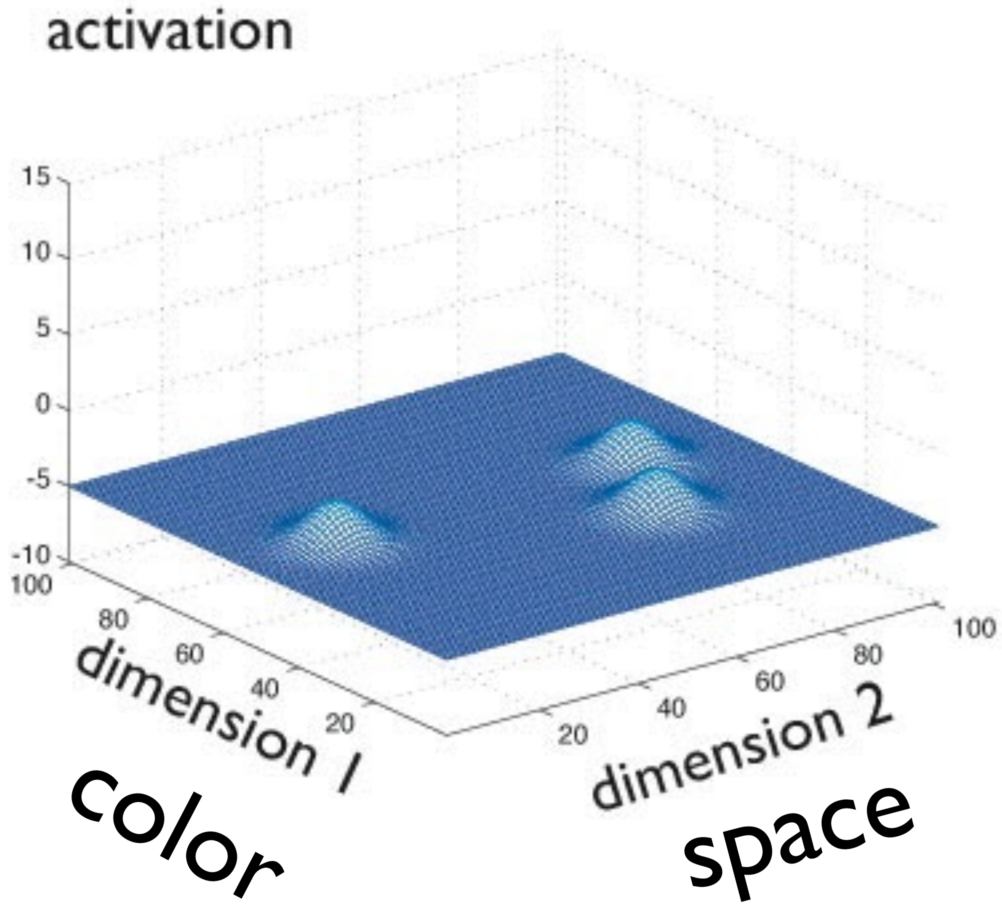
# dimensional cuing

- peak comes up where stimulus input and cue overlap
- read out spatial location at which peak is located



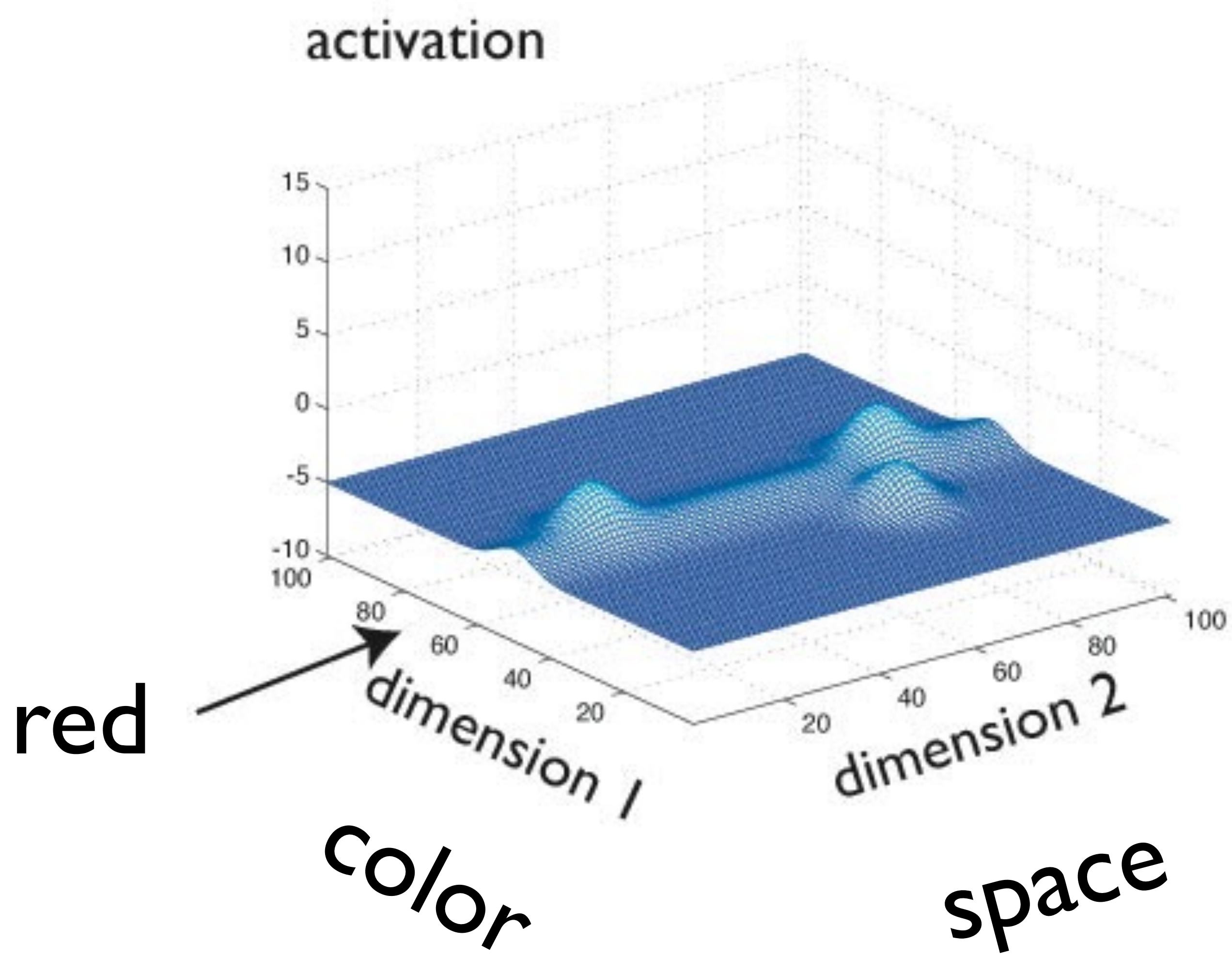
# dimensional cuing

- three colored objects including two red ones
- answer: "where are the red ones"?



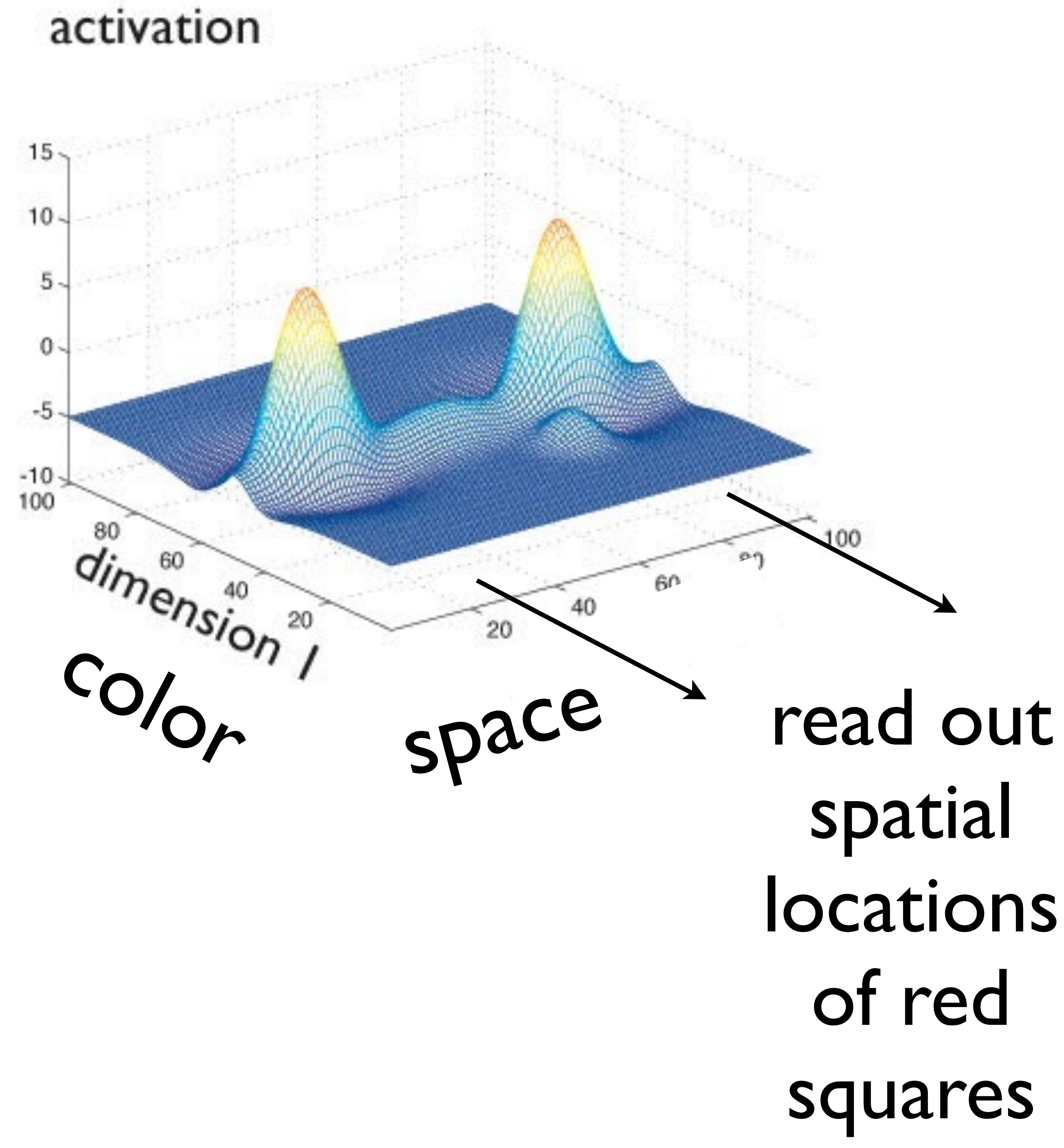
# dimensional cuing

- same idea: cue at read through ridge input



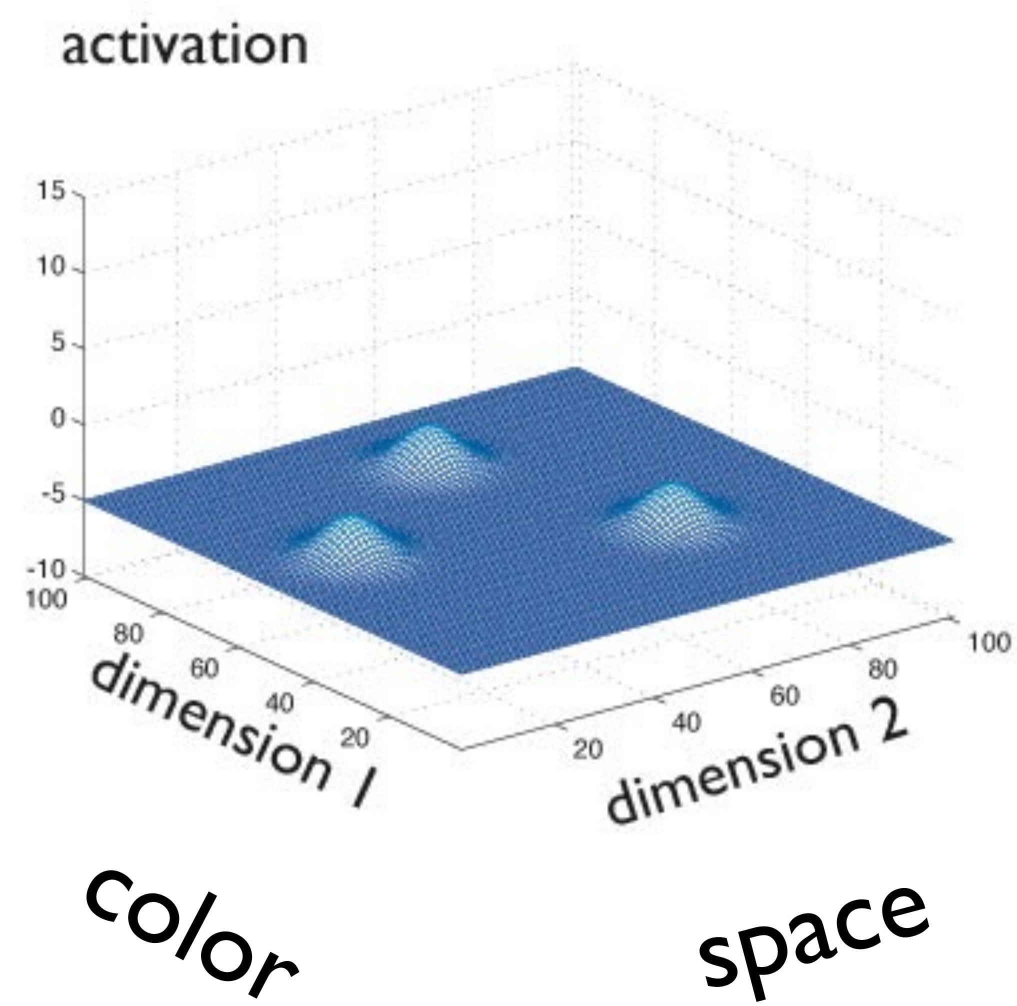
# dimensional cuing

- => both red squares generate peaks
- and their locations can be read out



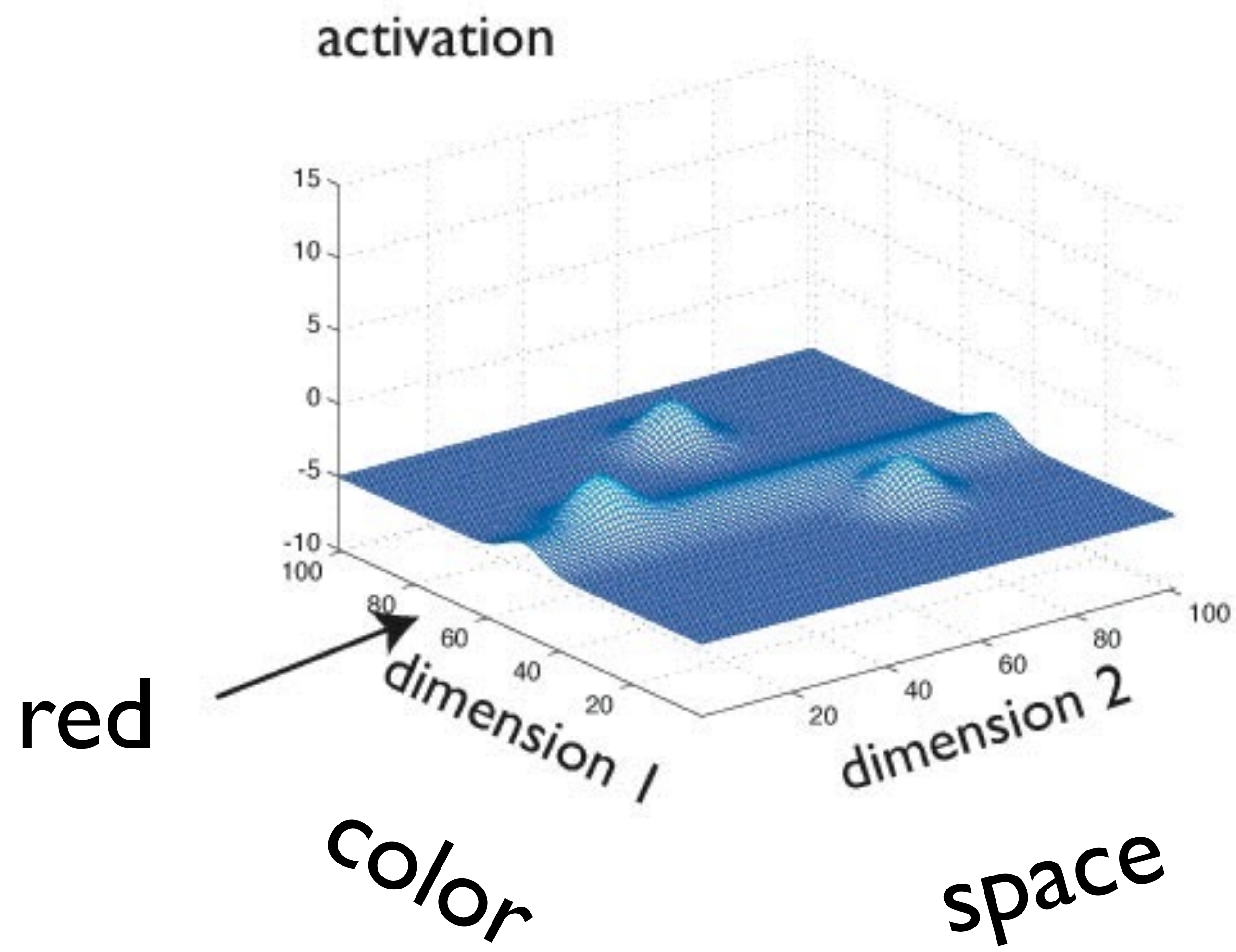
# dimensional cuing from long-term memory: cued recall

- not input, but a memory trace from previous exposures to colored squares at particular locations provides preshape
- "where was the red square"



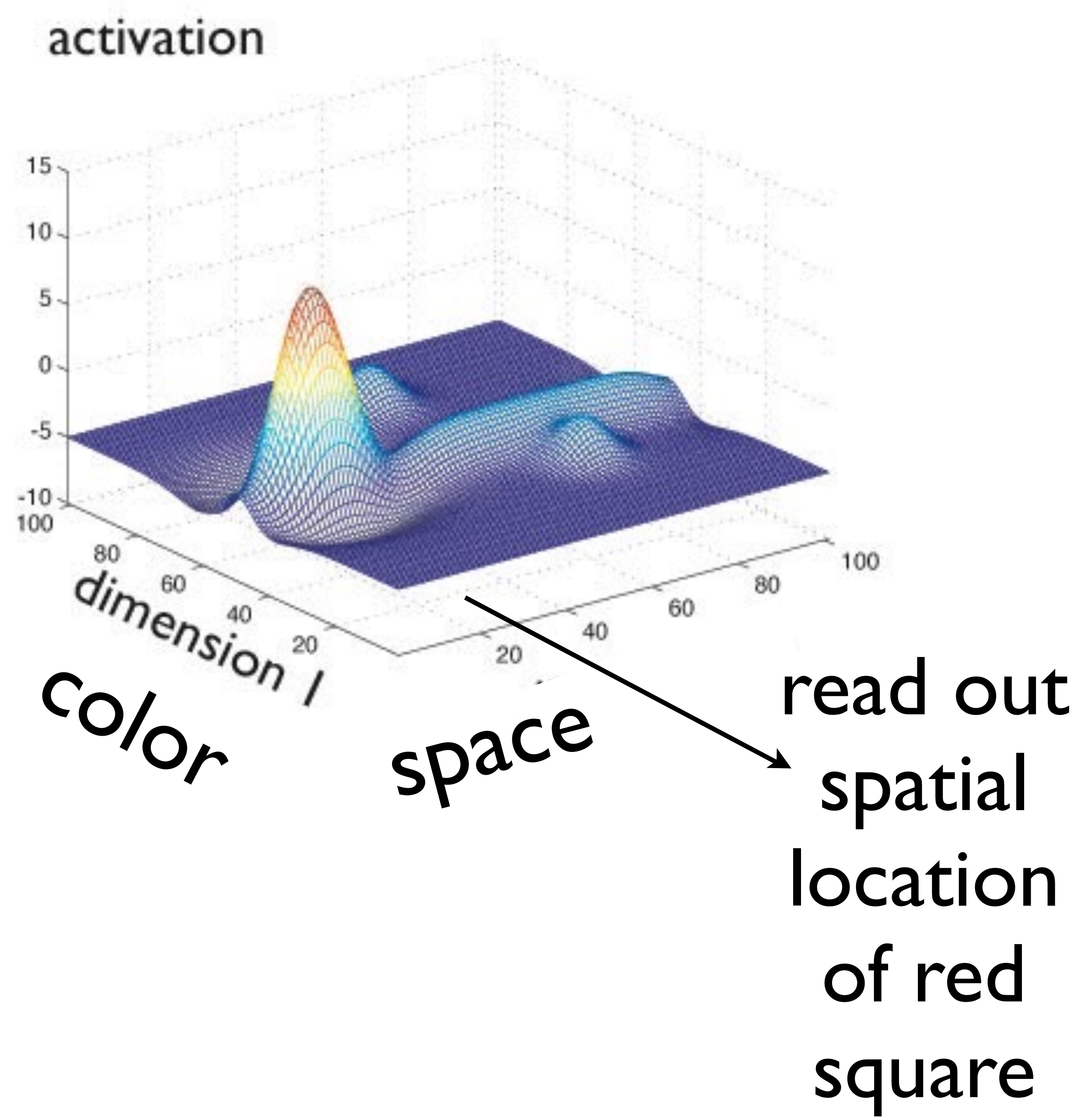
# dimensional cuing from long-term memory: cued recall

- => same mechanism applies



# dimensional cuing from long-term memory: cued recall

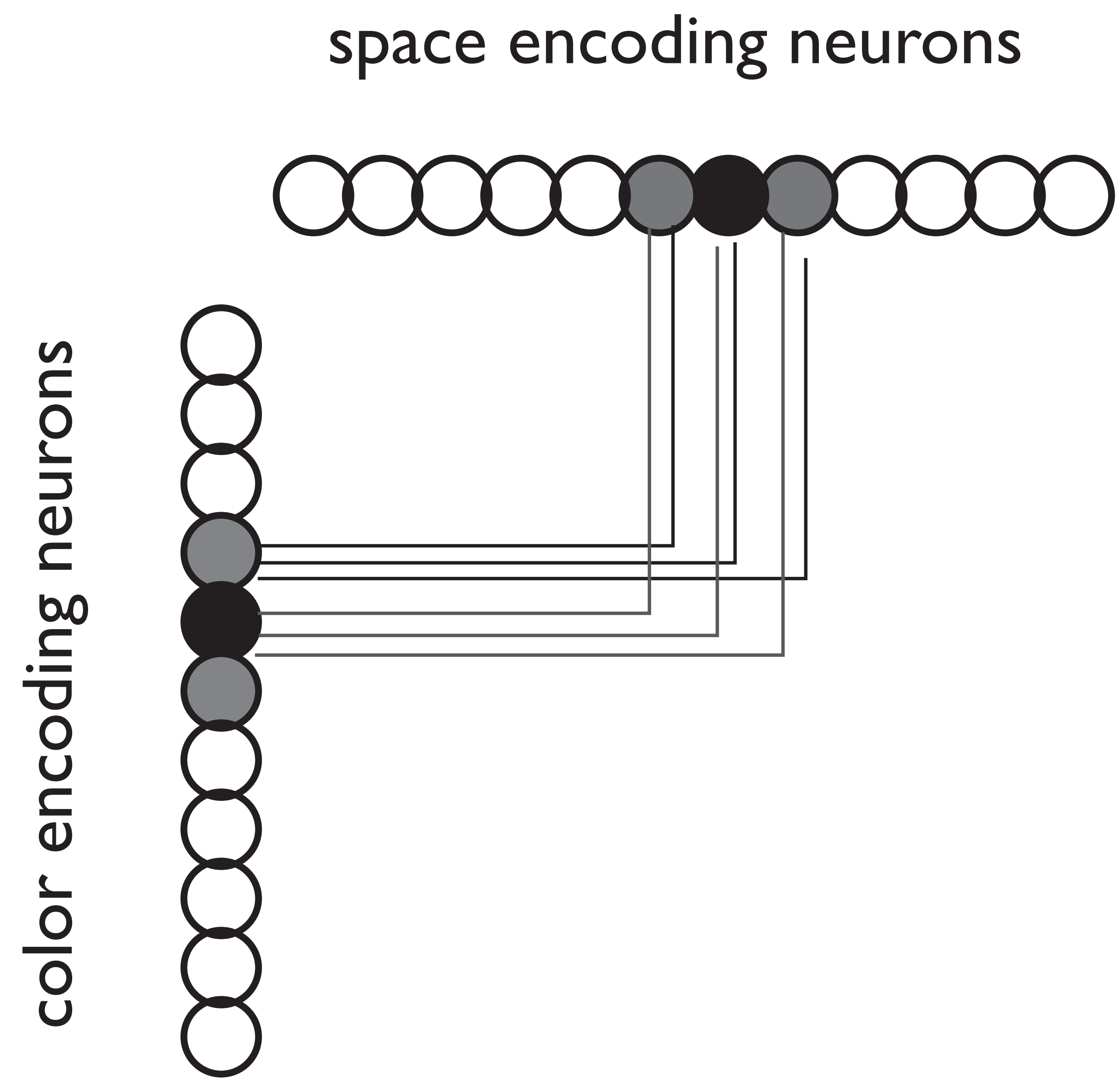
- peak comes up where preshape and cue overlap
- read out spatial location at which peak is located




\

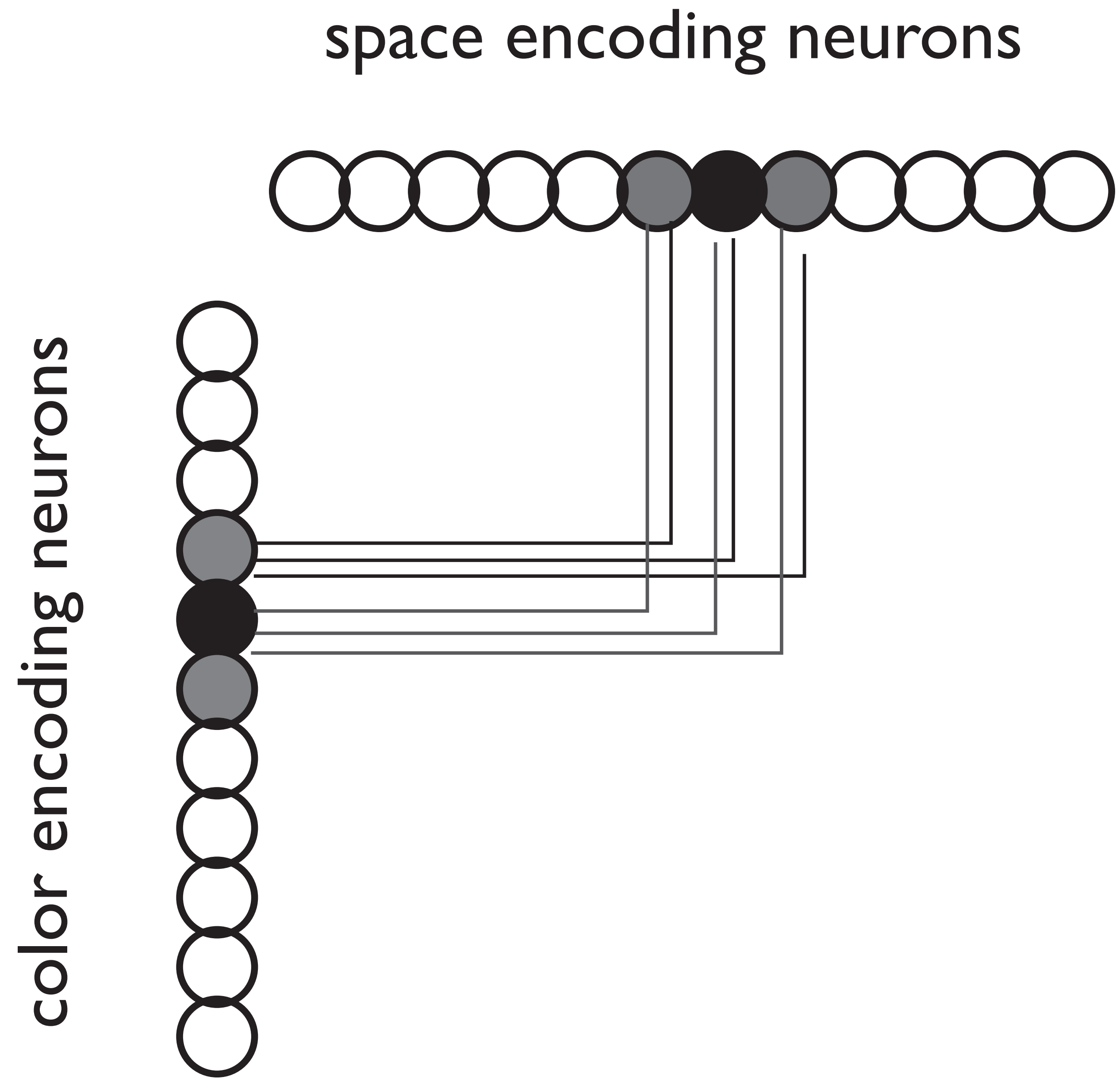
# synaptic association

- in conventional connectionist networks associative relationships are learned by adjusting synapses between those color and space neurons that have been co-activated



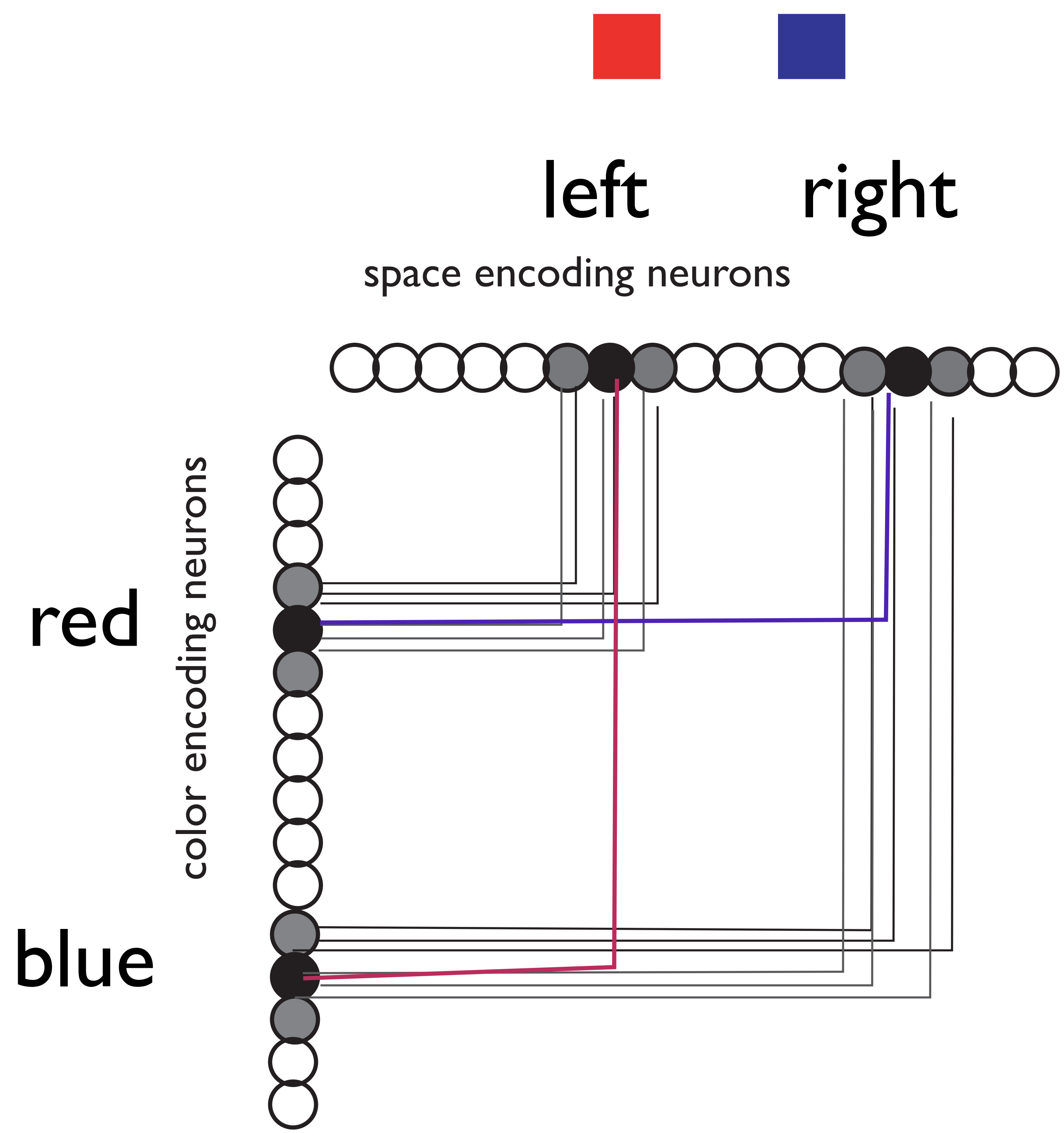
# limitations of synaptic association

- connections must be learned, so does not account for how "where is the red square" works from current stimulation (seen for the first time ever)



# limitations of synaptic association

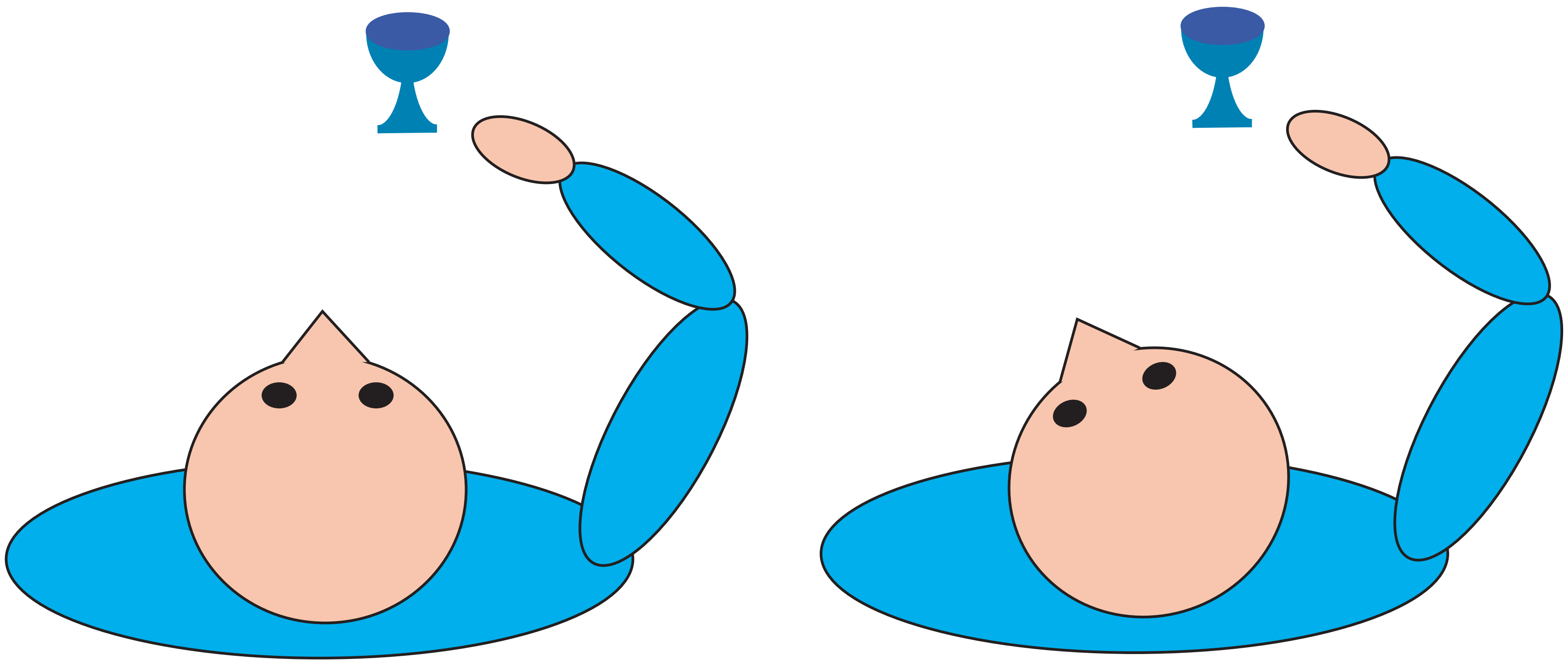
- learning multiple associations poses a binding problem:
- connectionist associators learn one item at a time and need separate presentation of individual items!




the network may associate blue with left and read with right

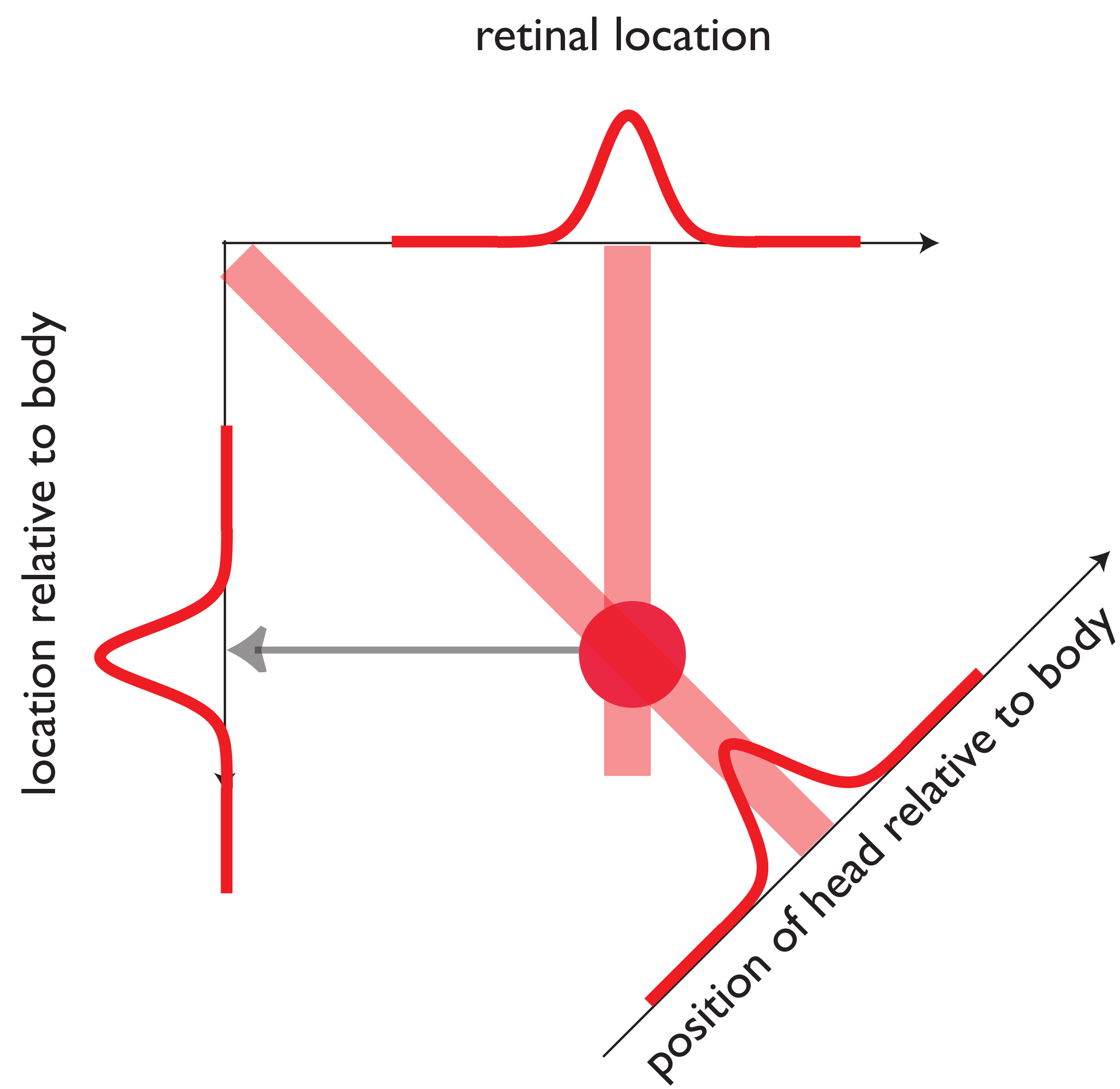
# Coordinate transformations

- Example: transform visual target from retinal representation to body-centered representation for reaching



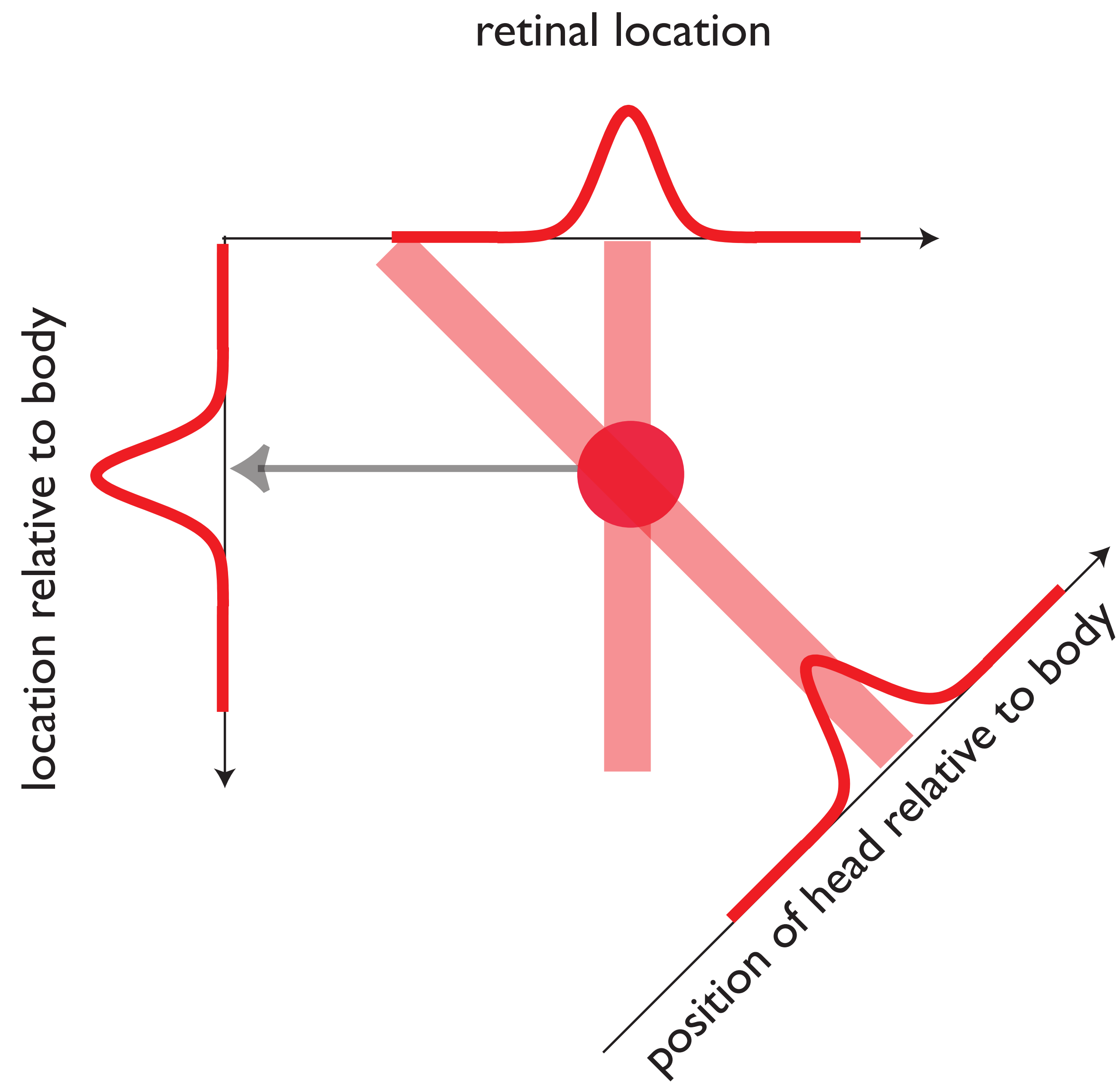
# coordinate transformations

- 2D field enables representation of associated retinal location and head position
- => project to extract body related location



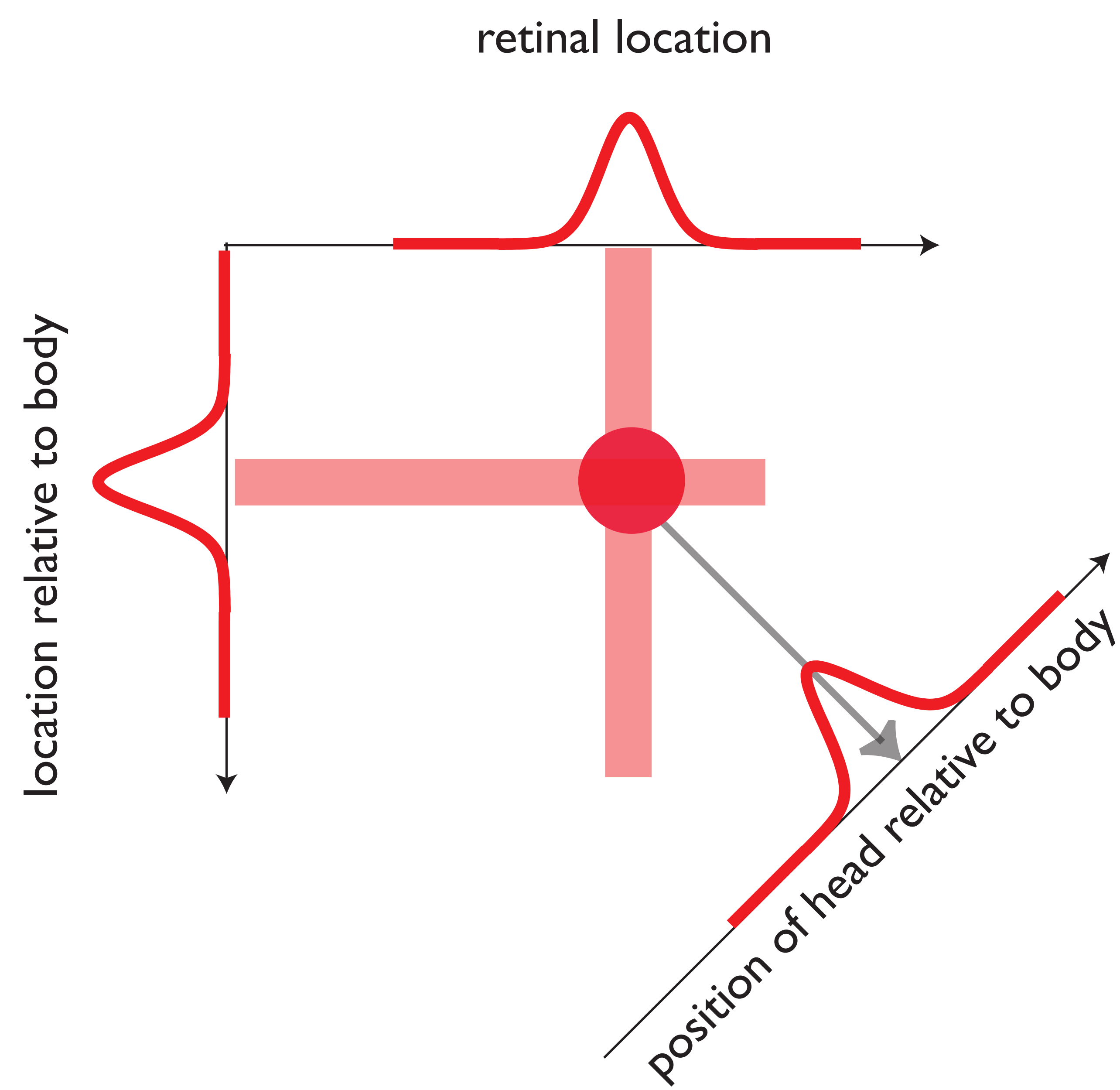
# coordinate transformations

- peak in body relative coordinates tracks changes of head position



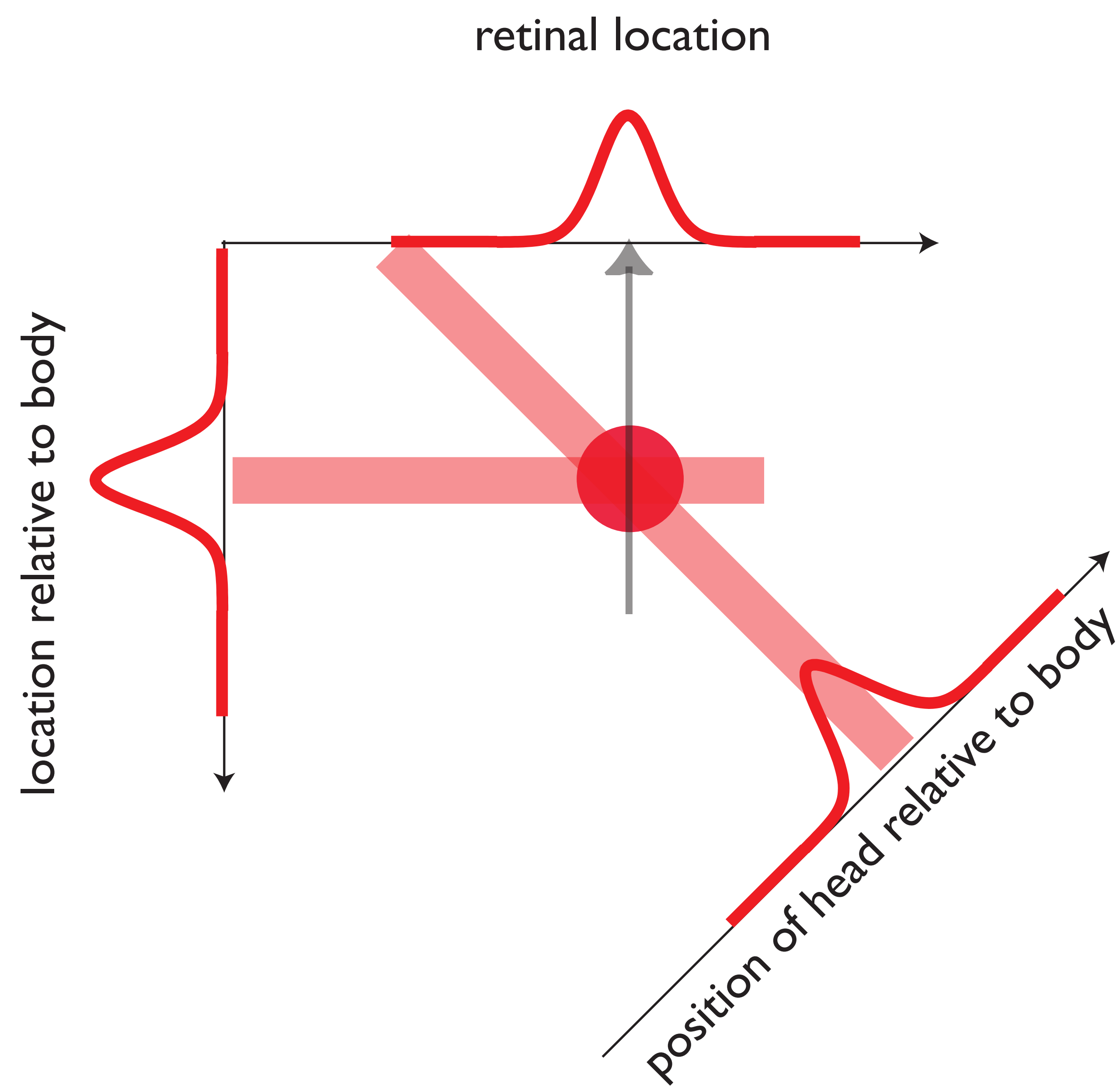
# coordinate transformations

- use same 2D field to reciprocally estimate head position from retinal position and position relative to body (e.g., while holding object in hand)



# coordinate transformations

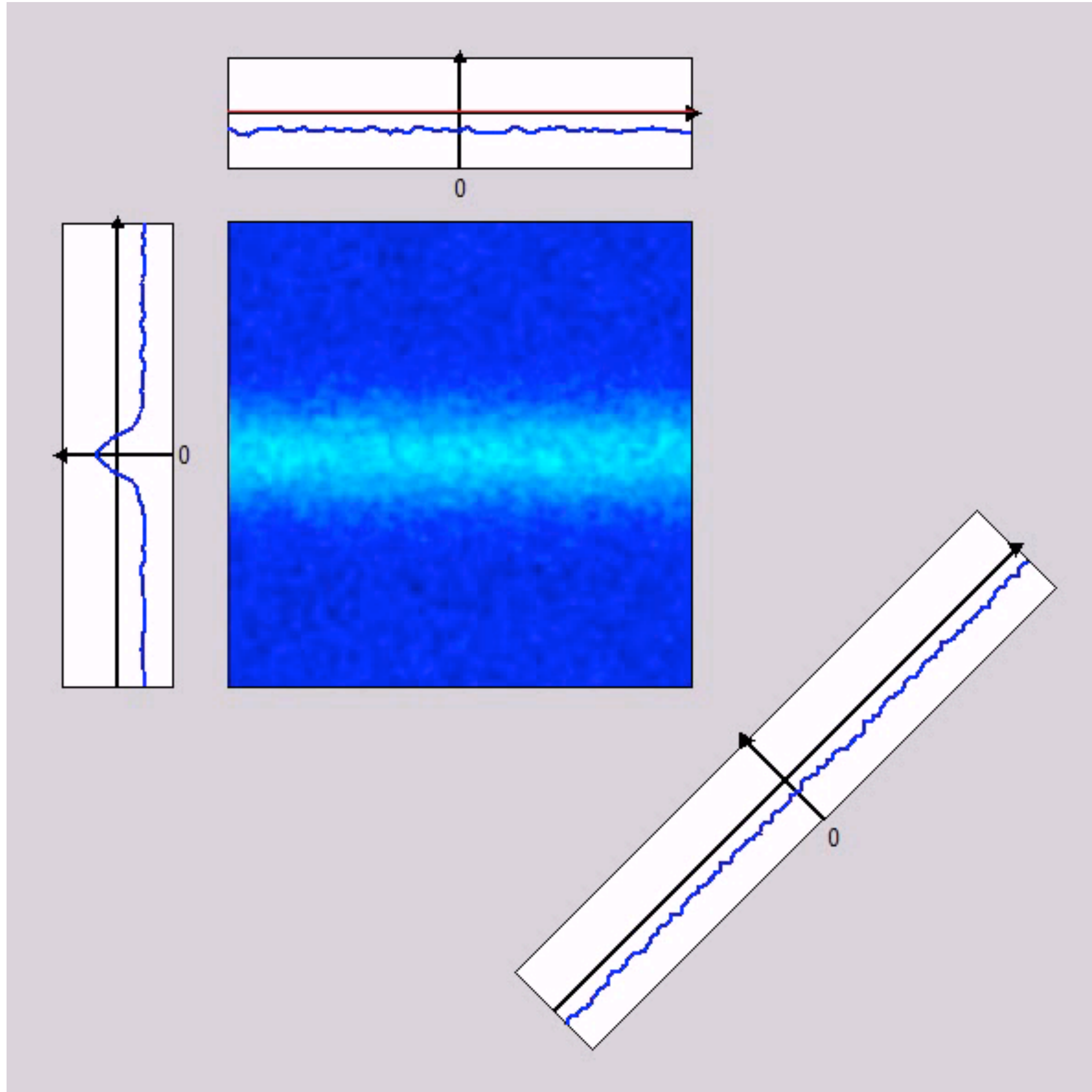
- or predict retinal position from location of object relative to body and head position




=> project Sebastian Schneegans

# Coordinate transformations

- predict retinal location following gaze shift




[Schneegans, Schöner, BC in press]

# … in next lecture

- you will see how higher dimensional fields make use of these properties to achieve scene representation…
- also uses the principles of sequence generation and behavioral organization presented earlier